

Meson production in e^+e^- annihilation and tau-lepton decays within extended NJL model

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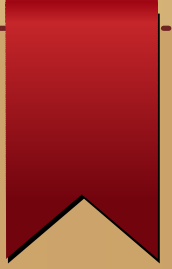
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Outline

- Motivation
- The extended NJL model
- Processes of e^+e^- annihilation into mesons
- Hadronic decays of tau leptons
- Discussion and future plans

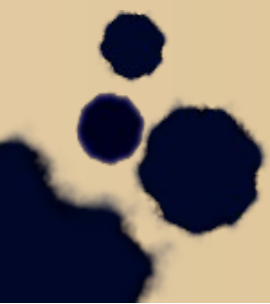
The goals and motivation



Theoretical description of **several classes of processes** with meson interactions at **energies up to 2 GeV**

- verification of the NJL models on a new class of tasks
- definition of its applicability domain
- theoretical support of experimental programs
- construction of theoretical predictions
- "pinpointing" of exotic mesons with small masses

N.B. Approaches to chiral symmetry breaking, confinement etc.



The Nambu-Jona-Lasinio model

Very simple four-fermion (current x current) interactions with **chiral symmetry**

$$\mathcal{L}_{\text{int}}^{(4)} = \frac{G_1}{2} \int d^4x \sum_{j=1}^9 \sum_{i=1}^2 [J_{S,i}^j(x) J_{S,i}^j(x) + J_{P,i}^j(x) J_{P,i}^j(x)] -$$
$$- \int d^4x \sum_{j=1}^9 \sum_{i=1}^2 \left(\frac{G_2}{2} J_{V,i}^{j,\mu}(x) J_{V,i,\mu}^j(x) + \frac{G_3}{2} J_{A,i}^{j,\mu}(x) J_{A,i,\mu}^j(x) \right)$$

The model effectively describes **spontaneous breaking of the chiral symmetry**

[1] Y. Nambu, G. Jona-Lasinio, Phys. Rev. 1961

[2] T. Eguchi, PRD 1976; K. Kikkawa, Prog. Theor. Phys. 1976

[3] D. Ebert, M.K. Volkov, Z.Phys. C 1983; M.K.Volkov, Ann. Phys. 1984

NJL model derived from QCD:

[4] B.A. Arbuzov, M.K. Volkov, I.V. Zaitsev, IJMPA 2006; ibid. 2009

NJL Model

- Currents:

$$J_{S(P),i}^j(x) = \int d^4x_1 d^4x_2 \bar{q}(x_1) F_{S(P),i}^j(x; x_1, x_2) q(x_2),$$

$$J_{V(A),i}^{j,\mu}(x) = \int d^4x_1 d^4x_2 \bar{q}(x_1) F_{V(A),i}^{j,\mu}(x; x_1, x_2) q(x_2),$$

- In the standard NJL model form factors are local like:

$$F_{A,1}^{j,\mu}(x; x_1, x_2) = \gamma_5 \gamma^\mu \tau^j \delta(x_1 - x) \delta(x_2 - x)$$

- 't Hooft 6-quark interaction is added to cure the UA(1) problem:

$$\mathcal{L}_{\text{int}}^{(6)} = -K \left\{ \det [\bar{q}(1 + \gamma_5)q] + \det [\bar{q}(1 - \gamma_5)q] \right\}$$

The extended NJL model

To include the **first radial excited states** of mesons, the NJL model is extended by introduction of simple polynomial form factors in currents, e.g. for the pseudoscalar one

$$F_{P,2}^j(\mathbf{k}^2) = i\gamma_5 \tau^j c_P^j f_j(\mathbf{k}^2)$$

$$f_j(\mathbf{k}^2) \equiv 1 + d_j \mathbf{k}^2$$

where c_P^j is a constant, d_j is the slope parameter, and \mathbf{k} is the quark 3-momentum.

Parameters c_P^j are fixed from static observables, d_j - from unchanged quark condensate

The chiral symmetry is preserved and there are no any additional parameters for interaction of the excited mesons

[1] M. K. Volkov, C. Weiss, *Phys. Rev. D* 56, 221 (1997).

[2] M. K. Volkov, A.B. Arbuzov, *Usp. Fiz. Nauk* 187 (2017).

Quark-meson Lagrangian

- Interactions:

$$L_{\text{int}} = L_{SM} + L_{NJL}$$

$$L_{NJL} = \bar{q} \left(i\gamma_5 \sum_{j=\pm} \lambda_j (a_K K^j + b_K K'^j) + \frac{1}{2} \gamma_\mu \lambda_V (a_V V_\mu + b_V V'_\mu) \right) q$$

$$a_a = \frac{1}{\sin(2\theta_a^0)} \left[g_a \sin(\theta_a + \theta_a^0) + g'_a f_a(\vec{k}^2) \sin(\theta_a - \theta_a^0) \right]$$

$$b_a = \frac{-1}{\sin(2\theta_a^0)} \left[g_a \cos(\theta_a + \theta_a^0) + g'_a f_a(\vec{k}^2) \cos(\theta_a - \theta_a^0) \right]$$

M.K. Volkov and A.B. Arbuzov, PEPAN 47, 489 (2016)

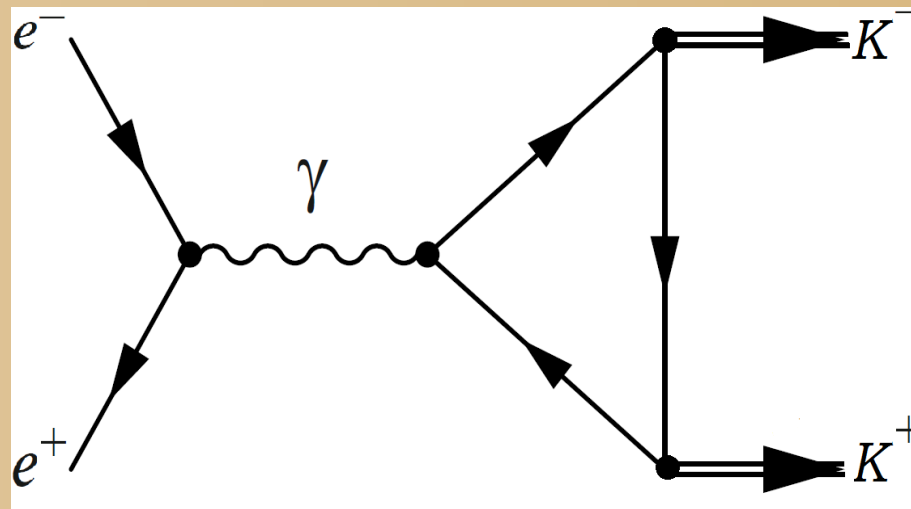
$e^+e^- \rightarrow K^+ K^-$

The amplitude includes the contact Feynman diagram with direct interaction of virtual photon with quarks and diagrams with intermediate vector mesons:

$$T = \frac{16\pi\alpha_{em}}{s} l^\mu \left[B_{(\gamma)} + B_{(\rho+\rho')} + B_{(\omega+\omega')} + e^{i\pi} B_{(\varphi+\varphi')} \right]_{\mu\nu} (p_{K^+} - p_{K^-})^\nu$$

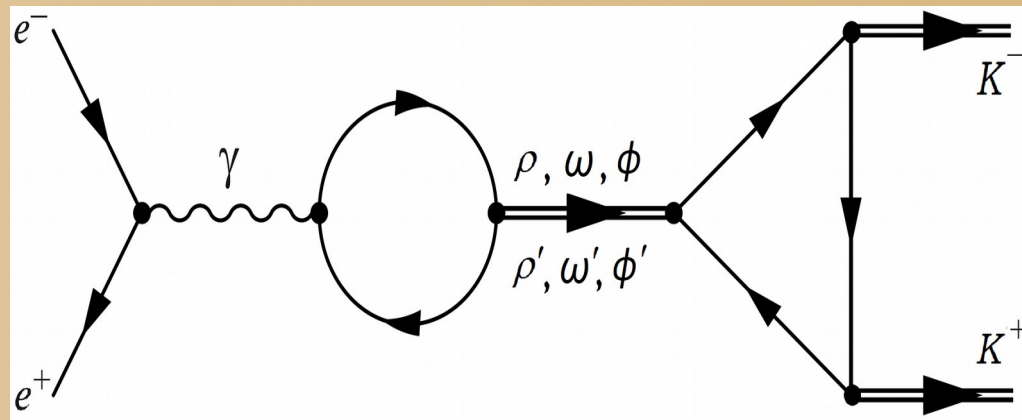
The contact diagram contribution reads

$$B_{(\gamma)}{}_{\mu\nu} = g_{\mu\nu} I_2^{a_K a_K}(m_u, m_s)$$



$e^+e^- \rightarrow K^+ K^-$

Contributions of intermediate vector mesons:



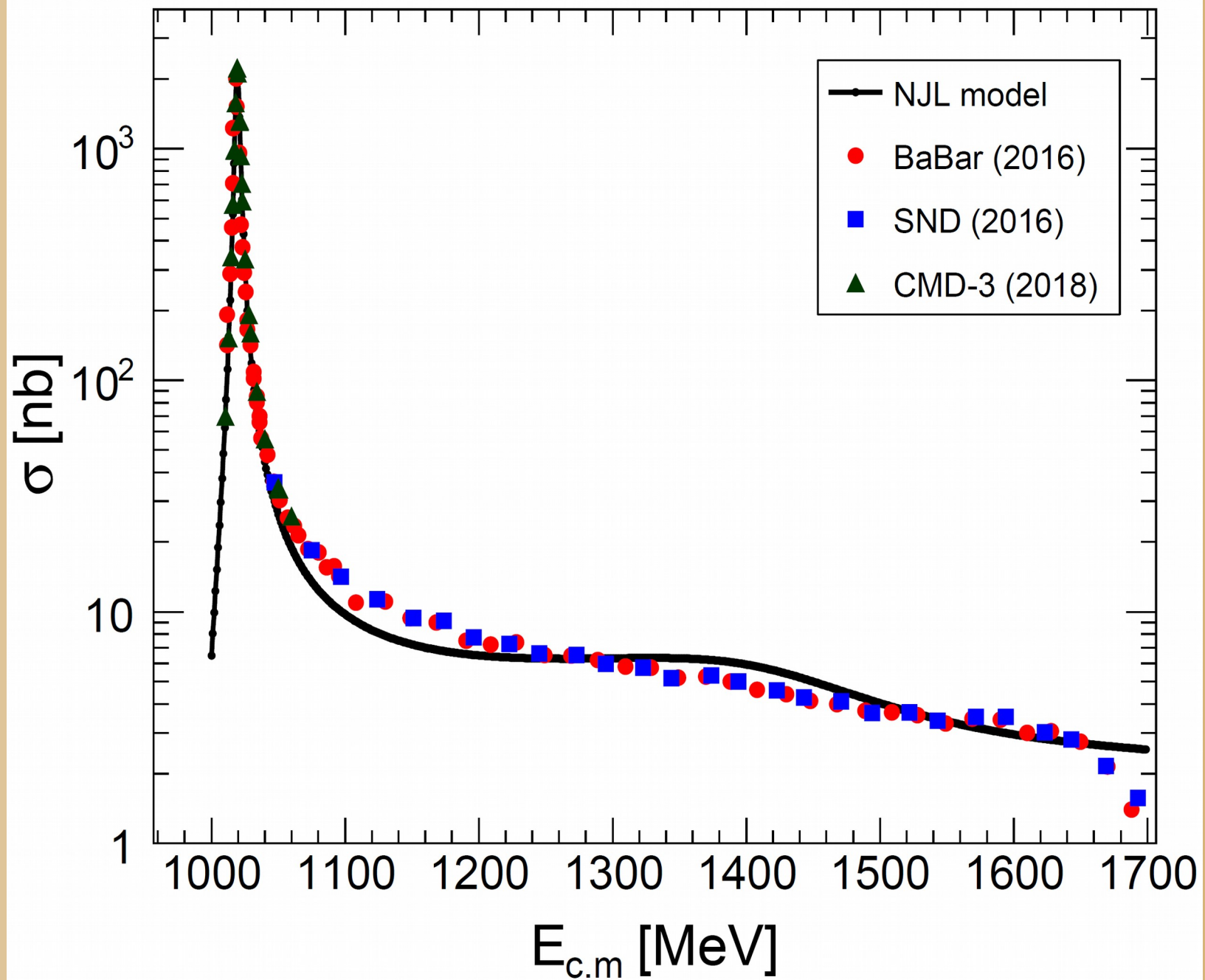
$$B_{(V+V')\mu\nu} = r_V \left[\frac{C_V}{g_V} \frac{g_{\mu\nu} s - p_\mu p_\nu}{M_V^2 - s - i\sqrt{s}\Gamma_V(s)} I_2^{a_V a_K a_K} + \frac{C_{V'}}{g_{V'}} \frac{g_{\mu\nu} s - p_\mu p_\nu}{M_{V'}^2 - s - i\sqrt{s}\Gamma_{V'}(s)} I_2^{b_V a_K a_K} \right],$$

$$r_\rho = r_{\rho'} = 1/2, r_\omega = r_{\omega'} = 1/6, r_\phi = r_{\phi'} = 1/3.$$

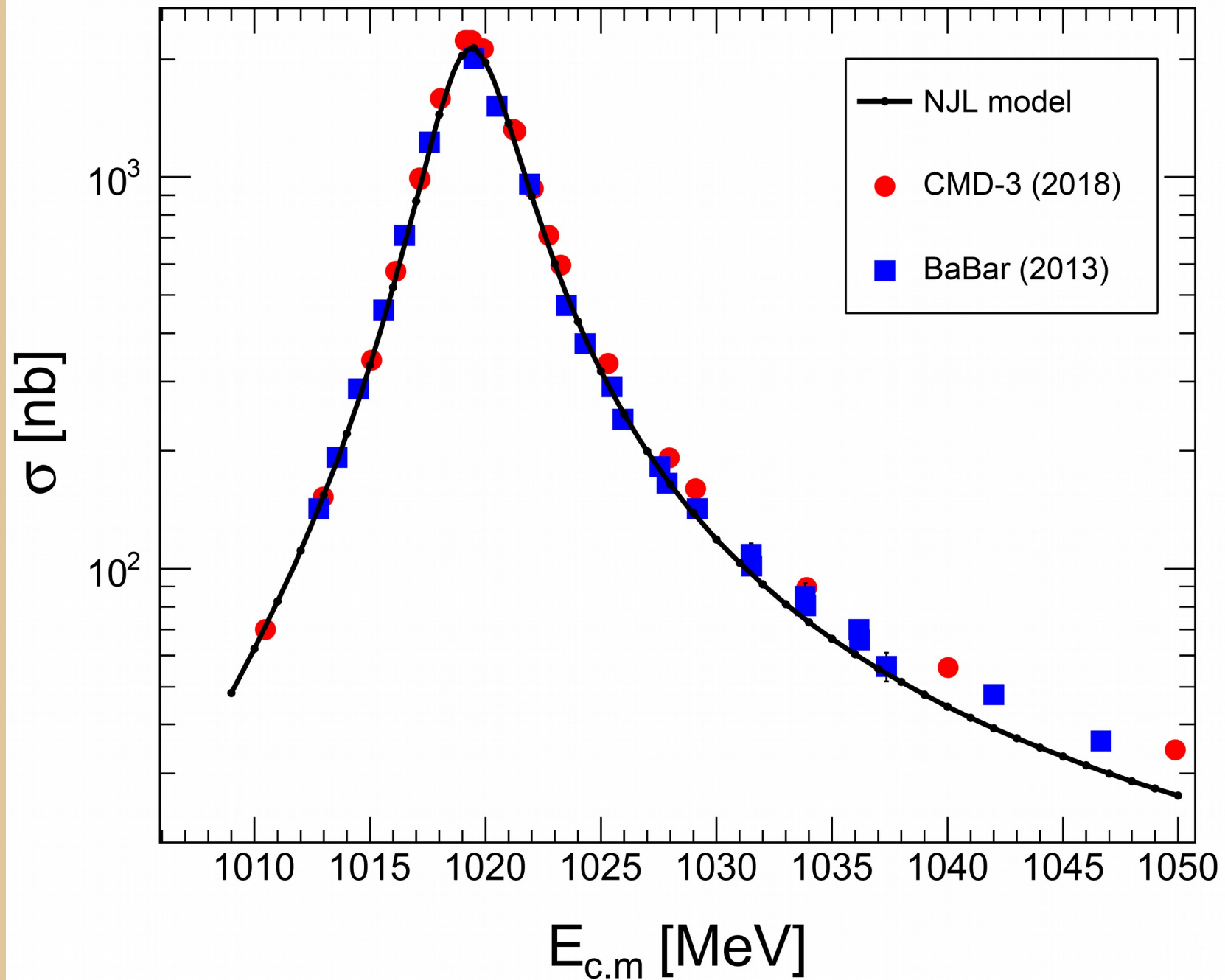
M.N. Achasov et al., Phys.Rev. D 76, 072012 (2007)

J.P. Lees et al. [BaBar Collaboration], Phys.Rev. D 88,

$e+e^- \rightarrow K^+ K^-$



$e+e^- \rightarrow K^+ K^-$



$$e+e^- \rightarrow K^+ K^-$$

Comparisons of separate contributions with results of other models:

	our result	[4]	[14]
N_ρ	0.44	0.598	0.598
N_ω	0.147	0.171	0.199
N_ϕ	0.34	0.283	0.339
$N_{\rho'}$	0.066	0.056	0.056
$N_{\omega'}$	0.022	0.016	0.019
$N_{\phi'}$	0.0005	0.005	0.006

[4] S.A. Ivashyn and A.Y. Korchin, *Eur.Phys. J. C* **49**, 697 (2007).

[14] C. Bruch, A. Khodjamirian and J.H. Kuhn, *Eur.Phys.J. C* **39**, 41 (2005).

$$e^+ e^- \rightarrow \pi^0 \gamma$$

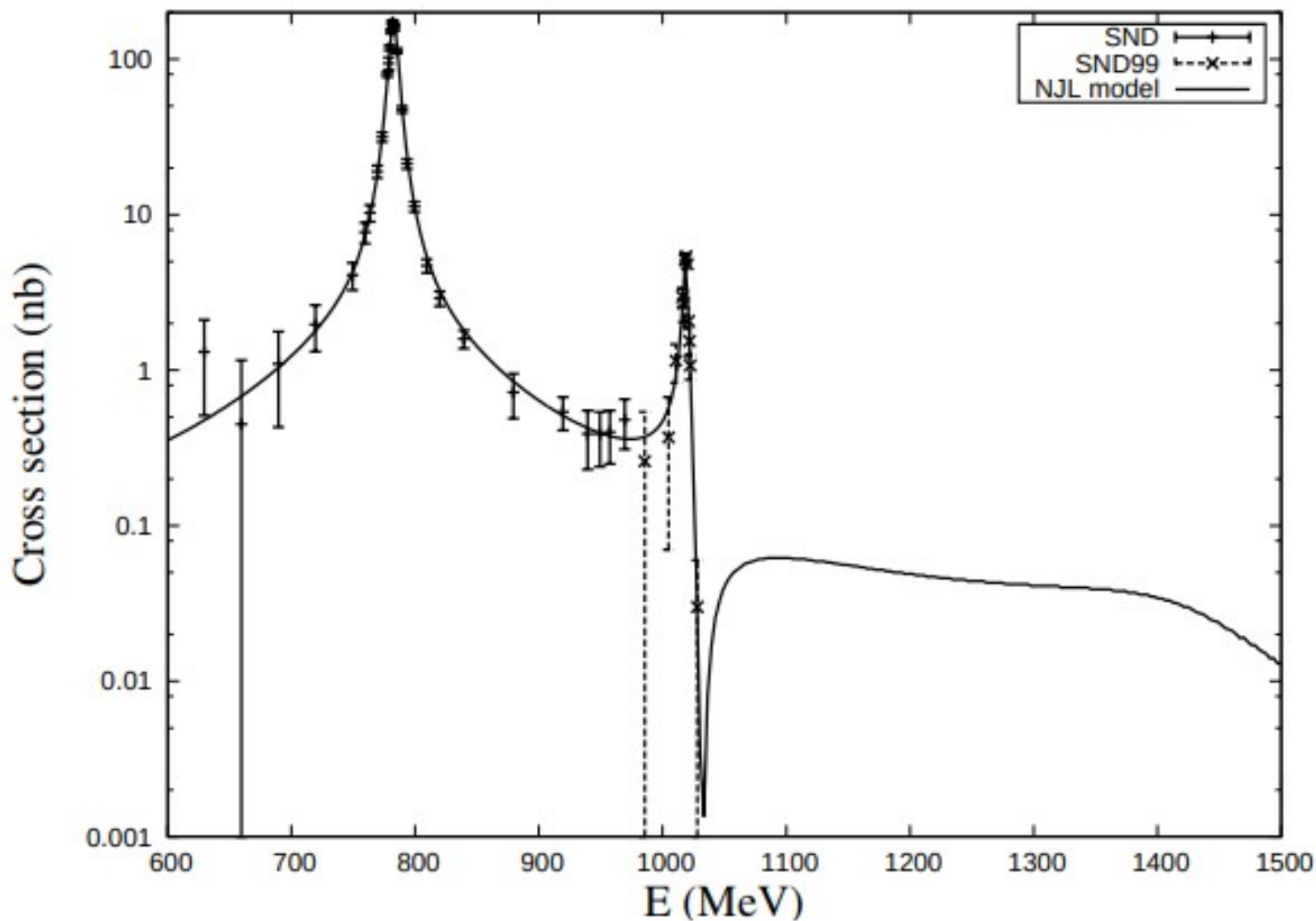


Figure 4. Comparison of experimental results for $e^+e^- \rightarrow \pi^0\gamma$ with the NJL model prediction.

A good description of the following annihilation processes was achieved:

$$e^+e^- \rightarrow [\pi, \pi(1300)]\gamma$$

A.B. Arbuzov, E.A. Kuraev, M.K. Volkov, EPJA 47 (2011) 103

$$e^+e^- \rightarrow [\eta, \eta'(958), \eta(1295), \eta(1475)]\gamma$$

A.I. Ahmadov, D.G. Kostunin, M.K. Volkov, PRC 87 (2013) 045203

$$e^+e^- \rightarrow [f_1(1285), a_1(1260)]\gamma$$

M.K. Volkov, A.A. Pivovarov and A.A. Osipov, IJMPA 32 (2017) 1750123

$$e^+e^- \rightarrow [\pi, \pi(1300)]\pi$$

M.K. Volkov and D.G. Kostunin, PRC 86 (2012) 025202

$$e^+e^- \rightarrow \omega(782)\pi^0$$

A.B. Arbuzov, E.A. Kuraev and M.K. Volkov, PRC 83 (2011) 048201

$$e^+e^- \rightarrow \rho(770)\eta$$

M.K. Volkov, K.Nurlan, A.A. Pivovarov, JETP Lett. 106 (2017) 771

$$e^+e^- \rightarrow K^\pm[K^{*\mp}(892), K^{*\mp}(1410)],$$

M.K. Volkov, A.A. Pivovarov, IJMPA 31 (2016) 1650155

$$e^+e^- \rightarrow [\eta, \eta'(958)][\phi(1020), \phi(1680)]$$

$$e^+e^- \rightarrow K^+K^-$$

M.K. Volkov, K. Nurlan, A.A. Pivovarov, PRC 98 (2018) 015206

$$e^+e^- \rightarrow [\eta, \eta'(958)]2\pi$$

M.K. Volkov, A.B. Arbuzov, D.G. Kostunin, PRC 89 (2014) 015202

Predictions were obtained for:

$$e^+e^- \rightarrow \pi(1300)\gamma,$$

$$e^+e^- \rightarrow [\eta'(958), \eta(1295), \eta(1475)]\gamma,$$

$$e^+e^- \rightarrow [f_1(1285), a_1(1260)]\gamma,$$

$$e^+e^- \rightarrow \pi(1300)\pi,$$

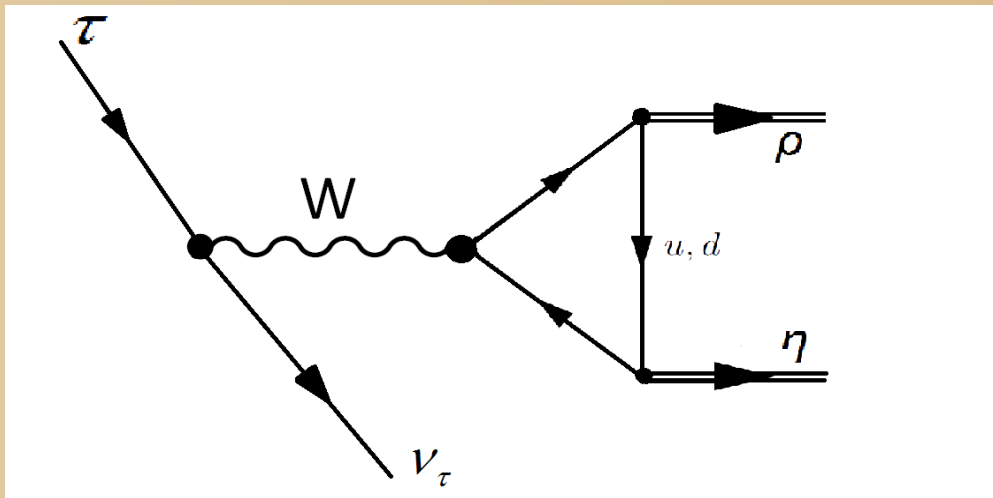
$$e^+e^- \rightarrow K^\pm K^{*\mp}(1410),$$

$$e^+e^- \rightarrow \eta'(958)\phi(1020),$$

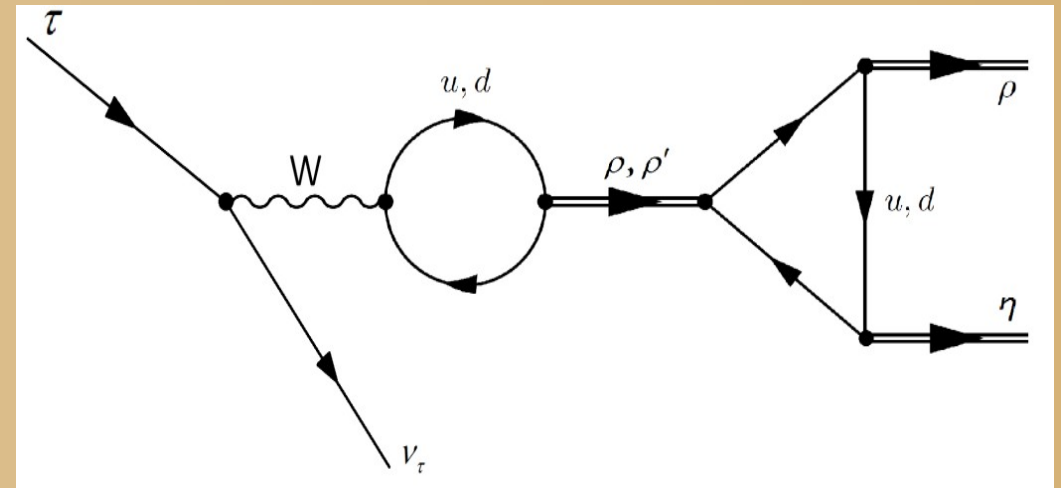
$$e^+e^- \rightarrow \eta, \phi(1680),$$

$$e^+e^- \rightarrow \eta'(958)2\pi$$

Decay $\tau \rightarrow \rho\eta\nu_\tau$



Contact diagram



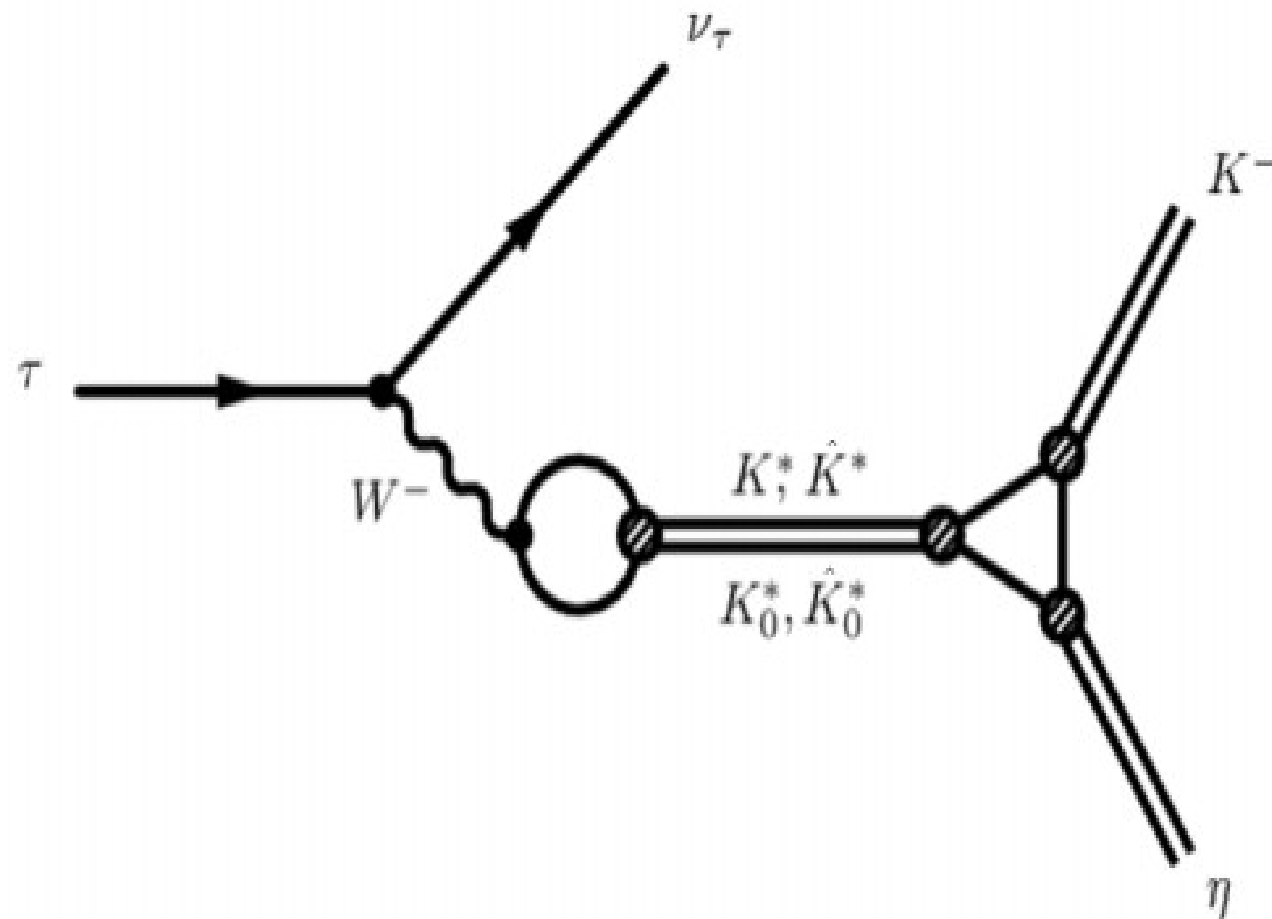
Diagrams with intermediate vector mesons

$$T = -i4m_u \frac{G_F}{\sqrt{2}} l_\mu V_{ud} \left\{ I_3^{y\rho\eta} g_{\mu\nu} + \frac{C_\rho}{g_\rho} I_3^{\rho\rho\eta} \frac{g_{\mu\nu} s - p_\mu p_\nu}{m_\rho^2 - s - i\sqrt{s}\Gamma_\rho} + \frac{C_{\rho'}}{g_\rho} I_3^{\rho'\rho\eta} \frac{g_{\mu\nu} s - p_\mu p_\nu}{m_{\rho'}^2 - s - i\sqrt{s}\Gamma_{\rho'}} \right\} \varepsilon_{\nu\lambda\delta\sigma} e_\lambda(p) p_\rho^\delta p_\eta^\sigma$$

$$Br(\tau \rightarrow \rho\eta\nu_\tau)_{NJL} = 1.44 \times 10^{-3}$$

$$Br(\tau \rightarrow 2\pi\eta\nu_\tau)_{exp} = (1.39 \pm 0.1) \times 10^{-3}$$

Decay $\tau \rightarrow \eta K^- \nu_\tau$ (I)



The decay $\tau \rightarrow \eta K^- \nu_\tau$ with the intermediate vector $K^*(892)$, $K^*(1410)$ and scalar $K_0^*(800)$, $K_0^*(1430)$

Decay $\tau \rightarrow \eta K^- \nu_\tau$ (II)

$$Br(\tau \rightarrow \eta K^- \nu_\tau) = 1.54 \cdot 10^{-4}$$

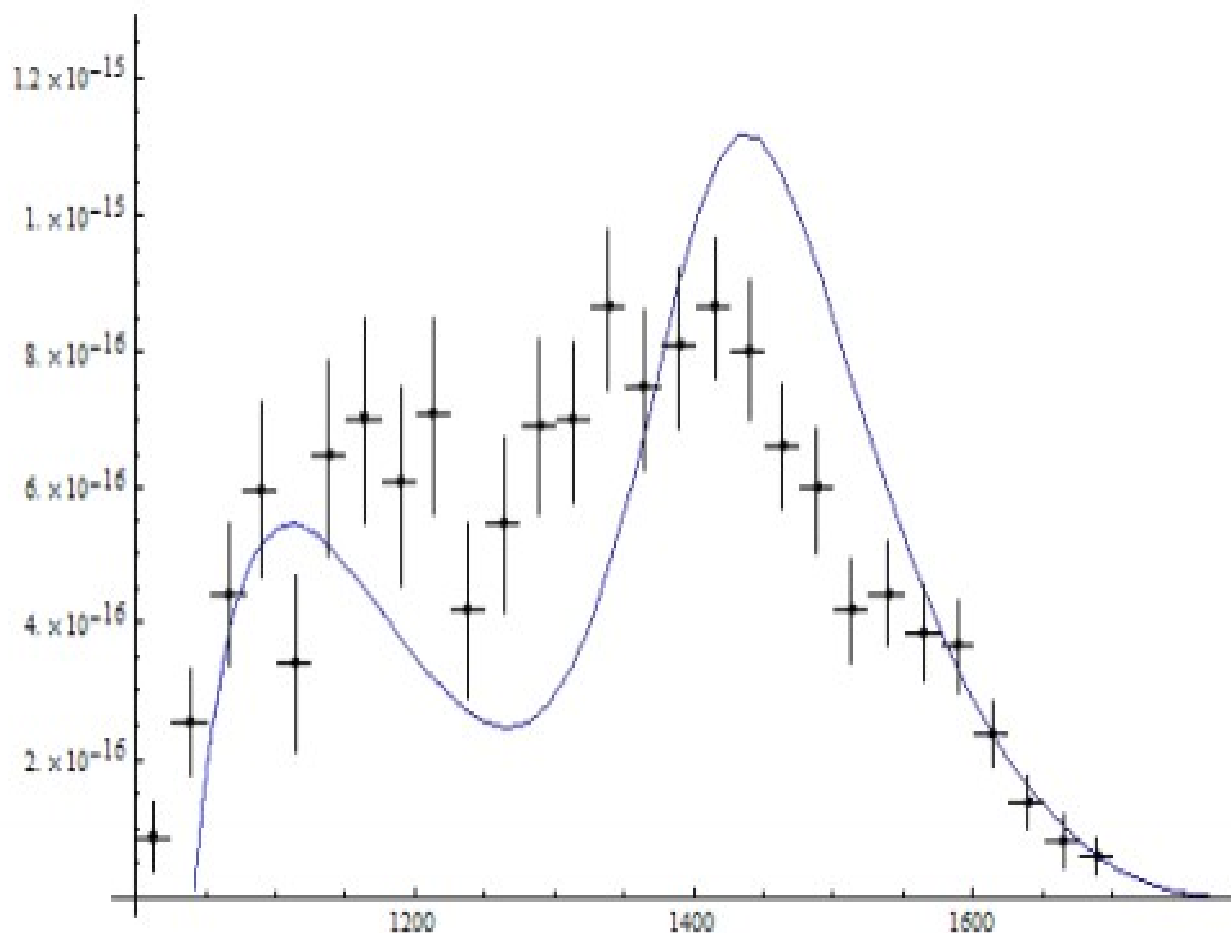
$$Br(\tau \rightarrow \eta K^- \nu_\tau)_{exp} = (1.58 \pm 0.14) \cdot 10^{-4}, [30]$$

$$Br(\tau \rightarrow \eta K^- \nu_\tau)_{exp} = (1.42 \pm 0.18) \cdot 10^{-4}, [31]$$

$$Br(\tau \rightarrow \eta K^- \nu_\tau)_{exp} = (1.52 \pm 0.08) \cdot 10^{-4}. [34]$$

$$Br(\tau \rightarrow \eta' K^- \nu_\tau) = 1.25 \cdot 10^{-6}$$

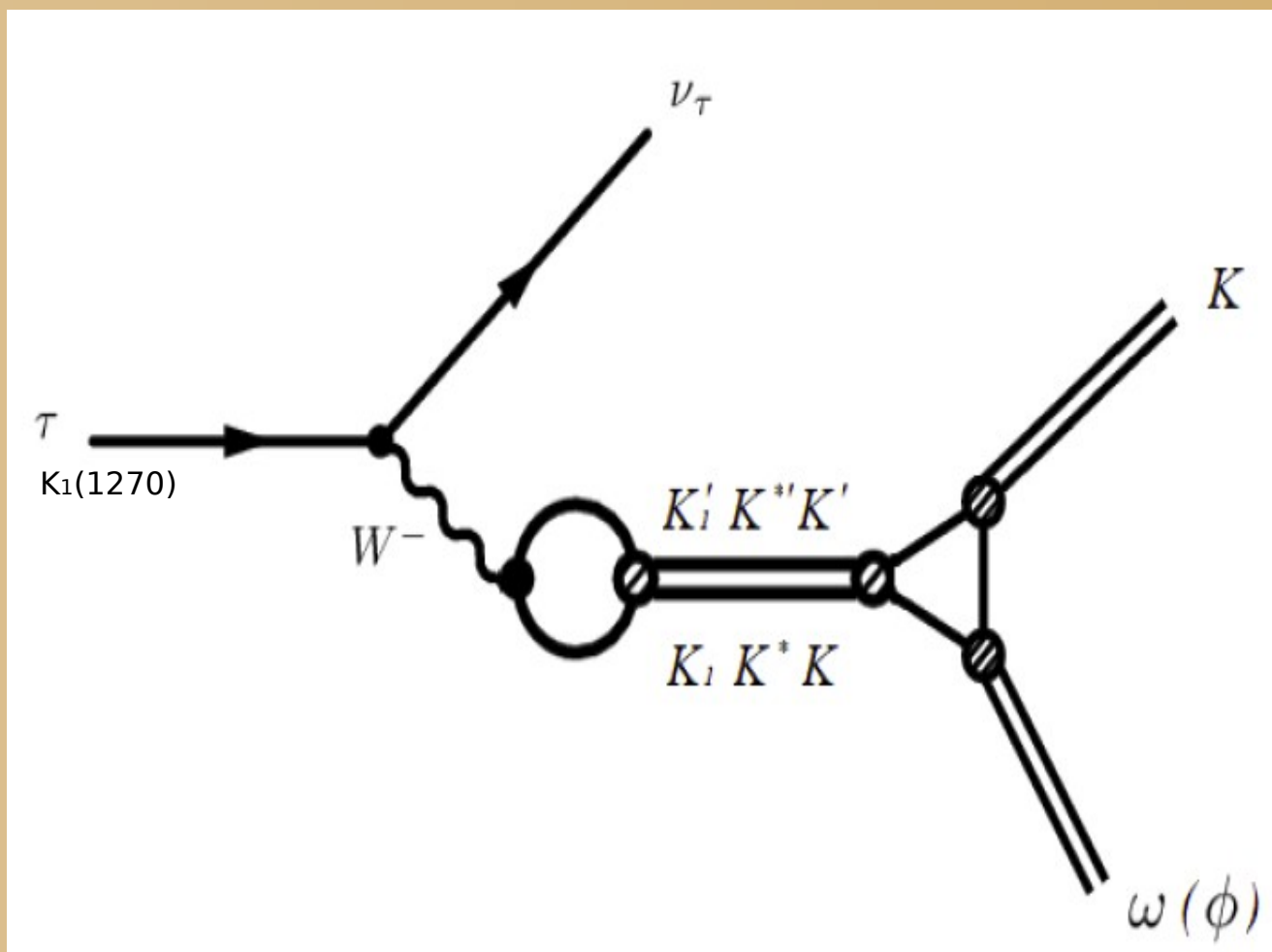
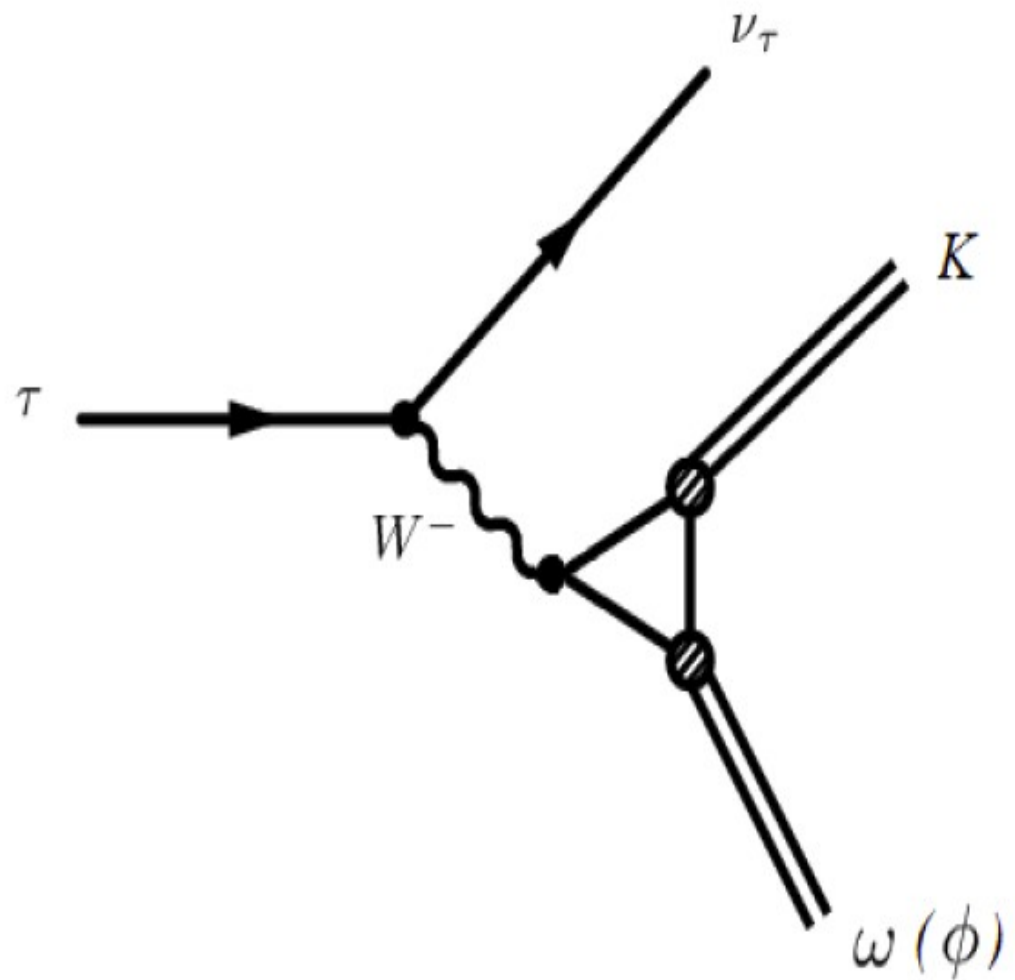
$$Br(\tau \rightarrow \eta' K^- \nu_\tau)_{exp} < 2.4 \cdot 10^{-6}$$



Process	NJL (Br)	Experiment (Br)	Publication
$\tau \rightarrow \pi\nu_\tau$	11.04%	$(10.82 \pm 0.05)\%$	M.K.Volkov, A.B.Arbutov, Phys. Usp. 60 (2017) 643
$\tau \rightarrow \pi(1300)\nu_\tau$	9.8×10^{-5}	$(10 \div 19) \times 10^{-5}$	
$\tau \rightarrow K^*(892)\nu_\tau$	1.15%	$(1.2 \pm 0.07)\%$	M.K.Volkov, K.Nurlan, PEPAN Lett. 14 (2017) 677
$\tau \rightarrow K^*(1410)\nu_\tau$	0.23%	$(0.15 + 1.4 - 1)$	
$\tau \rightarrow K_1(1270)\nu_\tau$	0.4%	$(0.47 \pm 0.11)\%$	
$\tau \rightarrow K_1(1650)\nu_\tau$	2.99×10^{-4}	-	
$\tau \rightarrow a_1(1260)\nu_\tau$	14.1%	-	
$\tau \rightarrow a_1(1640)\nu_\tau$	0.63%	-	
$\tau \rightarrow \pi^-\pi^0\nu_\tau$	24.76%	$(25.49 \pm 0.09)\%$	M.K.Volkov, D.G.Kostunin, PEPAN Lett. 10 (2013) 7
$\tau \rightarrow \pi\omega(782)\nu_\tau$	1.85%	$(1.95 \pm 0.06)\%$	M.K.Volkov, A.B.Arbutov, D.G.Kostunin, PRD 86 (2012) 057301
$\tau \rightarrow \eta\pi^-\nu_\tau$	4.72×10^{-6}	$< 9.9 \times 10^{-5}$	M.K.Volkov, D.G.Kostunin, PRD 86 (2012) 013005
$\tau \rightarrow \eta'(958)\pi^-\nu_\tau$	3.74×10^{-8}	$< 4 \times 10^{-6}$	
$\tau \rightarrow K^-\pi^0\nu_\tau$	4.13×10^{-3}	$(4.33 \pm 0.15) \times 10^{-3}$	M.K.Volkov, A.A.Pivovarov, MPLA 31 (2016) 1650043
$\tau \rightarrow \eta K^-\nu_\tau$	1.45×10^{-4}	$(1.55 \pm 0.08) \times 10^{-4}$	M.K.Volkov, A.A.Pivovarov, JETP Lett. 103 (2016) 613
$\tau \rightarrow \eta'(958)K^-\nu_\tau$	1.25×10^{-6}	$< 2.4 \times 10^{-6}$	
$\tau \rightarrow K^0K^-\nu_\tau$	1.27×10^{-3}	$(1.48 \pm 0.05) \times 10^{-3}$	M.K.Volkov, A.A.Pivovarov, MPLA 31 (2016) 1650138
$\tau \rightarrow \rho(770)\eta\nu_\tau$	1.44×10^{-3}	-	M.K.Volkov, K.Nurlan, A.A.Pivovarov, JETP Lett. 106 (2017) 771
$\tau \rightarrow \bar{K}^{*0}(892)\pi^-\nu_\tau$	1.78×10^{-3}	$(2.2 \pm 0.5) \times 10^{-3}$	M.K.Volkov, A.A.Pivovarov, JETP Lett. 108, (2018) 369
$\tau \rightarrow f_1(1285)\pi^-\nu_\tau$	3.98×10^{-4}	$(3.9 \pm 0.5) \times 10^{-4}$	M.K.Volkov, A.A.Pivovarov, A.A.Osipov, EPJA 54 (2018) 61
$\tau \rightarrow \eta 2\pi\nu_\tau$	1.46×10^{-3}	$(1.39 \pm 0.07) \times 10^{-3}$	M.K.Volkov, A.B.Arbutov, D.G.Kostunin, PRC 89 (2014) 015202
$\tau \rightarrow \eta'(958)2\pi\nu_\tau$	9×10^{-7}	$< 1.2 \times 10^{-5}$	

Preliminary:

$$\tau \rightarrow [\omega(782), \phi(1020)] K^- \nu_\tau$$



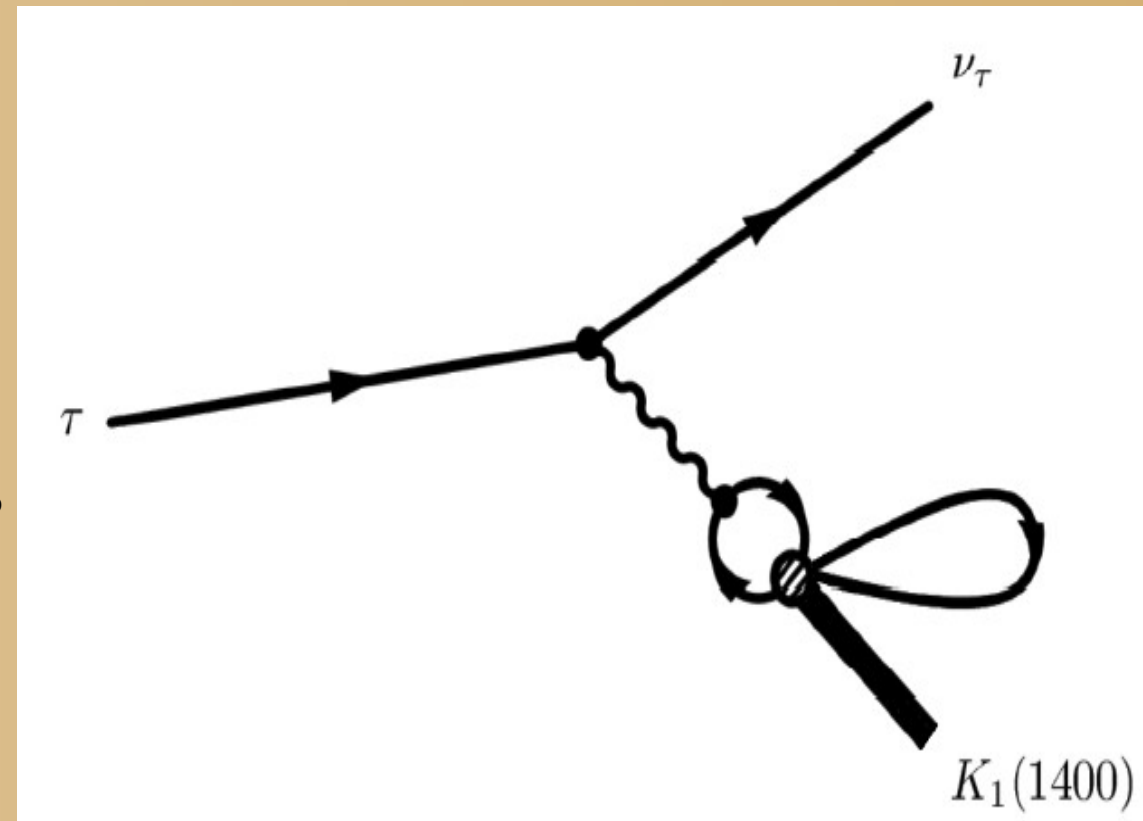
Preliminary: $\tau \rightarrow [\omega(782), \phi(1020)]K^- \nu_\tau$

- Intermediate mesons: K, K(1460), $K_1(1270)$, $K_1(1560)$, $K^*(892)$, $K^*(1410)$ but **$K_1(1400)$** is also possible. It is a **tetraquark** candidate
- NJL + tetraquark gives:

$$Br(\tau \rightarrow K\omega(782)\nu_\tau)_{tot} = 3.2 \times 10^{-4}$$

$$Br(\tau \rightarrow K\phi(1020)\nu_\tau)_{tot} = 2.3 \times 10^{-5}$$

- PDG: $4.1(0.9) 10^{-4}$ and $4.4(1.6) 10^{-5}$



Mixing of eta mesons

- Four-by-four mixing scheme is adapted: the ground and the first radial excited states of eta and eta prime are mixed all together:
- $\eta(550)$, $\eta'(958)$,
- $\eta(1295)$, $\eta(1475)$

$R_{i,j}$	η	$\hat{\eta}$	η'	$\hat{\eta}'$
φ_1^8	0,71	0,62	-0,32	0,56
φ_2^8	0,11	-0,87	-0,48	-0,54
φ_1^9	0,62	0,19	0,56	-0,67
φ_2^9	0,06	-0,66	0,30	0,82

- But $\eta(1405)$ is missed, it is a pseudoscalar **glueball** candidate

The Vector Meson Dominance

- The vector meson dominance (VMD) (Sakurai '1960) automatically appears in the standard NJL model after summing up the contributions with intermediate photon and ρ meson in the ground state
- In tau lepton decays the VMD appears in transitions of W boson into axial-vector mesons
- In the extended NJL model the **VMD** works only for the ground states of intermediate mesons, but **fails for the first radial excited states**

Discussion and future plans

- A series of **30+** papers on the subject was published by M.K.Volkov & co. since 2011
- Whole classes of processes are systematically considered within **the same model without changing parameter values**
- Theoretical **predictions** for future experiments were constructed
- The extended NJL model successfully works for e^+e^- annihilation and tau lepton decays at **energies up to ~ 2 GeV** ^{a_0}
- A certain number of **problems** was revealed, they can be treated as indications of **light exotic mesons**: $\eta(1405)$, $a_0(980)$, $a_1(1410)$, $f_0(1500)$ etc.
- The nearest plans concern processes with **axial-vector currents**
- General lessons on **effective QFT models**: hints on **symmetry breaking** and **energy scales**
- Applications for **hot dense matter**...