The decay $J/\psi \to \gamma X(J^P) \to \gamma\phi\phi$: Dynamical analysis of $X(J^P) \to \phi\phi$ resonance contributions.

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Outline

1. Introduction
2. Amplitudes and partial widths.
3. Results of fitting
4. Discussion
5. Conclusion

Abstract

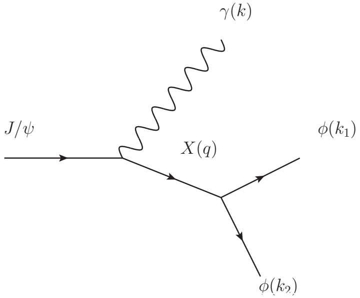
The dynamics of the $J^{PC} = 0^{-+}$, 0^{++}, and 2^{++} resonance contributions to the decay $J/\psi \to \gamma X(J^{PC}) \to \gamma\phi\phi$ is analysed using the data obtained by BESIII collaboration. The effective coupling constants parameterising invariant amplitudes of the transitions $J/\psi \to \gamma X(J^{PC})$ and $X(J^{PC}) \to \phi\phi$ and masses of $X(J^{PC})$ resonances are found from the fits. They are used for evaluation of the branching fractions $B_{X(J^{PC})\to\phi\phi}$, relative branching fractions $B_{J/\psi\to\gamma X(J^{PC})\to\gamma\phi\phi}$, and for obtaining the photon angular distributions.

Introduction

- The interest in the decay $J/\psi \to \gamma\phi\phi$ is related with the possible existence of the exotic glueball state decaying into the $\phi\phi$ pair (Lindenbaum and Longacre; Etkin et al., 1985).
- The spin-parity quantum numbers of the resonance states $X(J^P)$ decaying into $\phi\phi$ are reported to be $J^P = 0^+, 0^-$, and 2^+.
- The data on the reaction $e^+e^- \to J/\psi \to \gamma\phi\phi$ obtained by BESIII Collaboration, PRD**93**, 112011 (2016) were analysed using the simple Breit-Wigner formula for the individual resonance contribution,

$$M_X \propto \frac{1}{m_X^2 - s - im_X\Gamma_X}.$$

- Dominant contribution to the amplitude of decay $J/\psi \to \gamma\phi\phi$:



- But the dynamics of the decay chain $J/\psi \to \gamma X(J^{PC})$, $X(J^{PC}) \to \phi\phi$ is relatively simple only in case of $J^{PC} = 0^{-+}$ resonance admitting the single p wave contribution in both vertices.
- In general, one should include the different spin-orbital momentum structures for different spin-parities of the $X(J^{PC})$ resonances in the $\phi\phi$ system, especially in case of the tensor contribution $J^{PC} = 2^{++}$ where a number of independent spin structures enter the amplitudes $J/\psi \to \gamma X(2^{++})$ and $X(2^{++}) \to \phi\phi$.
- This talk is devoted to analysis of BESIII data starting from effective invariant amplitudes parametrized by some unknown constants determined from fits and to use them for evaluation of relevant branching fractions and angular distributions of final photons.

Notations

- Four-momenta assignment:
  $J/\psi(Q) \to \gamma(k)X(q) \to \gamma(k)\phi(k_1)\phi(k_2)$
- Polarization vectors assignments: ϵ_μ, $\epsilon_{1\mu}$, $\epsilon_{2\mu}$ $(\boldsymbol{\xi}, \boldsymbol{\xi}_1, \boldsymbol{\xi}_2)$ are, respectively, the polarization 4-vectors of J/ψ and ϕ's (their 3D counterparts in their respective rest frame); $e_\mu = (0, \boldsymbol{e})$ is polarization 4-vector of the photon.
- The energy-momentum 4-vector of $\phi\phi$ state in J/ψ rest frame is $q = (q_0, \boldsymbol{q})$,

$$q_0 = \frac{m^2_{J/\psi} + m^2_{12}}{2m_{J/\psi}},$$

$$\boldsymbol{q} = -\boldsymbol{k} = -\boldsymbol{n}\frac{m^2_{J/\psi} - m^2_{12}}{2m_{J/\psi}}.$$

Notations cont'd

- $\boldsymbol{n}$ is the unit vector in the direction of the photon, m_{12} is invariant mass of $\phi\phi$ pair.
- Energy-momentum of one of the ϕ mesons, $k_{1\mu} = (k^*_{10}, \boldsymbol{k}^*_1)$, in c.m.s of $\phi\phi$ pair:

$$
\begin{aligned}
k^*_{10} &= \frac{1}{2} m_{12}, \\
\boldsymbol{k}^*_1 &= \frac{\boldsymbol{n}_1}{2} \sqrt{m^2_{12} - 4m^2_\phi},
\end{aligned}
$$

$\boldsymbol{n}_1$ is the unit vector in the direction of the motion of ϕ meson.

Scheme of calculation

- General expressions:

$$M_{J_X=0}(J/\psi \to \gamma X \to \gamma\phi\phi) = \frac{[M(J/\psi \to \gamma X)][M(X \to \phi\phi)]}{D_X},$$

$$M_{J_X=2}(J/\psi \to \gamma X \to \gamma\phi\phi) = \sum_{\lambda_X}[M_{\mu\nu}(J/\psi \to \gamma X)T^{(\lambda_X)}_{\mu\nu}] \times [M_{\alpha\beta}(X \to \phi\phi)T^{(\lambda_X)}_{\alpha\beta}] \cdot D_X^{-1},$$

- Each vertex is calculated in the respective rest frame using Lorentz-transformed polarization tensor $T_{\mu\nu} \equiv T^{(\lambda_X)}_{\mu\nu}$:

$$\begin{aligned} T_{00} &= \frac{t_{ij} q_i q_j}{m_{12}^2}, \\ T_{0i} &= \frac{q_j}{m_{12}} \left[t_{ij} + \frac{t_{jk} q_i q_k}{m_{12}(q_0 + m_{12})} \right], \\ T_{ij} &= t_{ij} + \frac{(t_{ik} q_j + t_{jk} q_i) q_k}{m_{12}(q_0 + m_{12})} + \frac{t_{kl} q_i q_j q_k q_l}{m_{12}^2 (q_0 + m_{12})^2}, \end{aligned}$$

  $t_{ij} \equiv t^{(\lambda_X)}_{ij}$ is polarization tensor in the rest frame.
- Sum over polarizations of the intermediate tensor resonance:

$$\begin{aligned} \sum_{\lambda_X} t^{(\lambda_X)}_{ij} t^{(\lambda_X)}_{kl} &= \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - \frac{1}{3}\delta_{ij}\delta_{kl} \\ &\equiv \Pi_{ij,kl}. \end{aligned}$$

Pseudoscalar case

- Radiative transition amplitude and partial width:

$$M_{J/\psi\to\gamma X(0^-)} = g_{J/\psi\gamma X(0^-)}\epsilon_{\mu\nu\lambda\sigma}Q_\mu\epsilon_\nu k_\lambda e_\sigma = g_{J/\psi\gamma X(0^-)}m_{J/\psi}|\boldsymbol{k}|(\boldsymbol{n}\cdot[\boldsymbol{\xi}\times\boldsymbol{e}]),$$

$$\Gamma_{J/\psi\to\gamma X(0^-)}(m_{12}) = \frac{g^2_{J/\psi\gamma X(0^-)}}{12\pi}|\boldsymbol{k}|^3.$$

- $X(0^-)\to\phi\phi$ transition amplitude and partial width:

$$M_{X(0^-)\to\phi\phi} = g_{X(0^-)\to\phi\phi}\epsilon_{\mu\nu\lambda\sigma}k_{1\mu}\epsilon_{1\nu}k_{2\lambda}\epsilon_{2\sigma} = g_{X(0^-)\phi\phi}m_{12}|\boldsymbol{k}_1^*|(\boldsymbol{n}_1\cdot[\boldsymbol{\xi}_1\times\boldsymbol{\xi}_2]),$$

$$\Gamma_{X(0^-)\to\phi\phi}(m_{12}) = \frac{g^2_{X(0^-)\phi\phi}}{8\pi}|\boldsymbol{k}_1^*|^3.$$

- Amplitude of interest in case of a number of resonances is

$$M_{J/\psi\to\gamma X(0^-)\to\gamma\phi\phi} = A^{(0^-)} m_{J/\psi} m_{12} |\boldsymbol{k}||\boldsymbol{k}_1^*| \times (\boldsymbol{\xi}[\boldsymbol{n}\times\boldsymbol{e}])(\boldsymbol{n}_1[\boldsymbol{\xi}_1\times\boldsymbol{\xi}_2])$$

- The dynamics of process is included through the factor $A^{(0^-)}$ to be specified below.

Scalar case

- Radiative transition amplitude and partial width:

$$M_{J/\psi\to\gamma X(0^+)} = -g_1(\epsilon e) = g_1(\boldsymbol{\xi}\boldsymbol{e}),$$
$$\Gamma_{J/\psi\to\gamma X(0^+)} = \frac{g_1^2|\boldsymbol{k}|}{12\pi m^2_{J/\psi}}.$$

D-wave drops because $\boldsymbol{en} = 0$.

- $X(0^+) \to \phi\phi$ transition amplitude:

$$M_{X(0^+)\to\phi\phi} = -f_1(\epsilon_1\epsilon_2) - f_2(\epsilon_1 k_2)(\epsilon_2 k_1) = f_{00}^{(0^+)}(\boldsymbol{\xi}_1\boldsymbol{\xi}_2) + f_{22}^{(0^+)}(\boldsymbol{\xi}_1\boldsymbol{n}_1)\times(\boldsymbol{\xi}_2\boldsymbol{n}_1);$$
$$f_{00}^{(0^+)} = f_1,\ f_{22}^{(0^+)} = (2f_1 + f_2 m_{12}^2)\frac{\boldsymbol{k}_1^{*2}}{m_\phi^2},$$

- $\phi\phi$ partial width:

$$\Gamma_{X(0^+)\to\phi\phi} = \frac{|\boldsymbol{k}_1^*|}{16\pi m_{12}^2}\left(2\left|f_{00}^{(0^+)}\right|^2 + \left|f_{00}^{(0^+)} + f_{22}^{(0^+)}\right|^2\right).$$

  Dynamical content of the $J/\psi \to \gamma X(0^+) \to \gamma\phi\phi$ component of $\phi\phi$ spectrum will be specified below.
- Here $f_{SL}^{(J^P)}$ corresponds to the assignment of spin S and orbital angular momentum L of $\phi\phi$ state with given spin-parity.
- Can be generalized to the case of several resonance contributions.

Tensor case

- Radiative transition amplitude:

$$M_{J/\psi\to\gamma X(2^+)} = [c_1(\epsilon e)Q_\mu Q_\nu + c_2(\epsilon k)e_\mu k_\nu + c_3\epsilon_\mu e_\nu]\, T_{\mu\nu} \equiv [g_{02}(\boldsymbol{\xi}\cdot\boldsymbol{e})n_i n_j + g_{12}(\boldsymbol{\xi}\cdot\boldsymbol{n})e_i n_j + g_{20}\xi_i e_j]\, t_{ij},$$

where

$$g_{02} = -c_1\frac{m^2_{J/\psi}\boldsymbol{k}^2}{m^2_{12}},$$
$$g_{12} = -\frac{\boldsymbol{k}^2}{m_{12}}\left(c_2 q_0 + \frac{c_3}{q_0+m_{12}}\right),$$
$$g_{20} = -c_3.$$

- Radiative width in case of fixed J/ψ polarization:

$$\Gamma^{(\lambda_{J/\psi})}_{J/\psi\to\gamma X(2^+)} = \frac{|\boldsymbol{k}|}{8\pi m^2_{J/\psi}}\int\left\{\left(|g_{02}|^2+\frac{3}{2}|g_{20}|^2-\frac{1}{3}|g_{02}+g_{20}|^2\right)\right.$$
$$\left.\times[\boldsymbol{\xi}^{(\lambda_{J/\psi})}\times\boldsymbol{n}]^2+|g_{12}+g_{20}|^2(\boldsymbol{\xi}^{(\lambda_{J/\psi})}\boldsymbol{n})^2\right\}\frac{d\Omega_{\boldsymbol{n}}}{4\pi},$$

- Tracing fixed J/ψ polarization is necessary for obtaining angular distribution of photons in the reaction $e^+e^-\to J/\psi\to\gamma\phi\phi$.

- $X(2^+) \to \phi\phi$ transition amplitude:

$$M_{X(2^+)\to\phi\phi} = \{g_1\epsilon_{1\mu}\epsilon_{2\nu} + k_{1\mu}k_{2\nu}[g_2(\epsilon_1\epsilon_2) + g_3(\epsilon_1 k_2)(\epsilon_2 k_1)] + g_4[\epsilon_{1\mu}k_{2\nu}(\epsilon_2 k_1) + \epsilon_{2\mu}k_{1\nu}(\epsilon_1 k_2)]\} T_{\mu\nu} \equiv [f_{20}\xi_{1i}\xi_{2j} + f_{02}(\boldsymbol{\xi}_1\cdot\boldsymbol{\xi}_2)n_{1i}n_{1j} + f_{22}[(\boldsymbol{\xi}_1\cdot\boldsymbol{n}_1)\xi_{2i} + (\boldsymbol{\xi}_2\cdot\boldsymbol{n}_1)\xi_{1i}]n_{1j} + f_{24}(\boldsymbol{\xi}_1\cdot\boldsymbol{n}_1)(\boldsymbol{\xi}_2\cdot\boldsymbol{n}_1)n_{1i}n_{1j}]\, t_{ij},$$

$$f_{20} = g_1,$$
$$f_{02} = g_2\boldsymbol{k}_1^{*2},$$
$$f_{22} = \frac{\boldsymbol{k}_1^{*2}}{m_\phi}\left(\frac{g_1}{k_{10}^* + m_\phi} + g_4 m_{12}\right),$$
$$f_{24} = \frac{\boldsymbol{k}_1^{*4}}{m_\phi^2}\left[\frac{g_1}{(k_{10}^* + m_\phi)^2} + 2g_2 + g_3 m_{12}^2 + 2g_4\frac{m_{12}}{k_{10}^* + m_\phi}\right].$$

- $X(2^+) \to \phi\phi$ decay width:

$$\Gamma_{X(2^+)\to\phi\phi} = \frac{|\boldsymbol{k}_1^*|}{240\pi m_{12}^2}\left(10|f_{20}+f_{22}|^2+3|f_{20}|^2+2|f_{20}+f_{24}|^2+ 2|f_{02}+f_{24}|^2+4|f_{02}+f_{22}|^2+4|f_{22}+f_{24}|^2- 4|f_{22}|^2-6|f_{24}|^2\right),$$

0^- resonance

- All coupling constants in effective vertices are assumed real.
- $\phi\phi$ spectrum in reaction $e^+e^- \to J/\psi \to \gamma\phi\phi$:

$$\frac{dN^{(0^-)}}{dm_{12}} = \frac{\mathcal{N}}{(2\pi)^3 \times 6}\left|A^{(0^-)}\right|^2 m_{12}^2 |\mathbf{k}|^3 |\mathbf{k}_1^*|^3.$$

- Three pseudoscalar resonances are included: $X_1 = \eta(2225)$, $X_2 = \eta(2100)$, $X_3 = X(2500)$.
- Since pseudoscalar case is kinematically simple (the only p-wave structure in both vertices), two dynamical modes are considered.

- General expression includes the mixing via the common $\phi\phi$ mode (model A):

$$A^{(0^-)} = \begin{pmatrix} g_{J/\psi\gamma X_1} & g_{J/\psi\gamma X_2} & g_{J/\psi\gamma X_3} \end{pmatrix} \times \begin{pmatrix} D_1 & -\Pi_{12} & -\Pi_{13} \\ -\Pi_{12} & D_2 & -\Pi_{23} \\ -\Pi_{13} & -\Pi_{23} & D_3 \end{pmatrix}^{-1} \begin{pmatrix} g_{X_1\phi\phi} \\ g_{X_2\phi\phi} \\ g_{X_3\phi\phi} \end{pmatrix},$$

$$D_{X_i(J^P)}(m_{12}^2) = m^2_{X_i(J^P)} - m_{12}^2 - im_{12}\Gamma_{X_i(J^P)\to\phi\phi}(m_{12}) - im_{X_i(J^P)}\Gamma'_{X_i},$$

$$\Pi_{ij} \equiv \Pi_{ij}(m_{12}^2) = \mathrm{Re}\Pi_{ij} + im_{12}g_{X_i\phi\phi}g_{X_j\phi\phi} \times \frac{|\boldsymbol{k}_1^*|^3}{8\pi}.$$

- Γ'_{X_i} takes into account other possible modes, $\mathrm{Re}\Pi_{ij}$ are assumed to be constants.

- The model where mixing is neglected, $\Pi_{ij} \equiv 0$ (model B).
- The fitted are m_{X_i}, Γ'_{X_i}, $g_{J/\psi\gamma X_i}\sqrt{\mathcal{N}}$, $g_{X_i\phi\phi}$, $i = 1, 2, 3$ in both models, and $a_{12} \equiv \mathrm{Re}\Pi_{12}$, $a_{13} \equiv \mathrm{Re}\Pi_{13}$, $a_{23} \equiv \mathrm{Re}\Pi_{23}$ in model A. Using these parameters evaluated are
- strong interaction branching fractions

$$B_{X_i \to \phi\phi} = \frac{2}{\pi} \int_{2m_\phi}^{2.7\mathrm{GeV}} \frac{m_{12}^2 \Gamma_{X_i \to \phi\phi}(m_{12})}{|D_{X_i}|^2} dm_{12}$$

- relative production characteristics $\mathcal{N}\Gamma_{J/\psi} B_{J/\psi \to \gamma X_i \to \gamma\phi\phi}$, $i = 1, 2, 3$, and

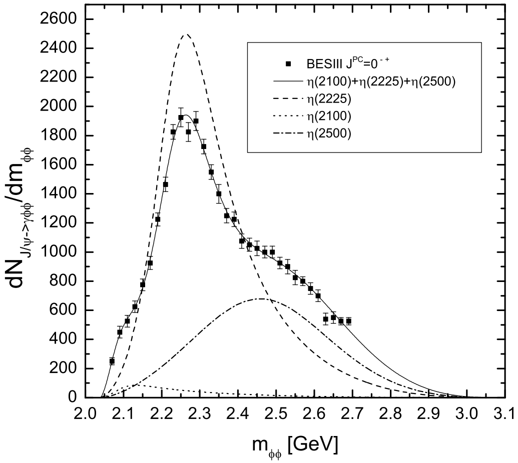
$$N^{(0^-)} \equiv \mathcal{N}\Gamma_{J/\psi} B_{J/\psi \to \gamma(X_1 + X_2 + X_3) \to \gamma\phi\phi} = \int_{2m_\phi}^{m_{J/\psi}} \frac{dN^{(0^-)}}{dm_{12}} dm_{12}$$

- In general,

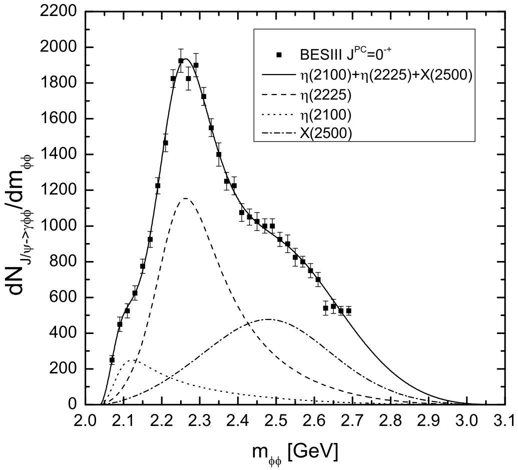
$$B_{J/\psi\to\gamma X_i\to\gamma\phi\phi} = \frac{1}{\Gamma_{J/\psi}}\int_{2m_\phi}^{m_{J/\psi}} \Gamma_{J/\psi\to\gamma X_i}(m_{12}) \times \frac{2m_{12}^2\Gamma_{X_i\to\gamma\phi\phi}(m_{12})}{\pi|D_{X_i}(m_{12}^2)|^2} dm_{12}.$$

- It reduces to $B_{J/\psi\to\gamma X_i\to\gamma\phi\phi} = B_{J/\psi\to\gamma X_i} \times B_{X_i\to\phi\phi}$ in the limit of narrow width of X resonance.

$J^P = 0^-$ component (model A)



$J^P = 0^-$ component (model B)



parameter	model A	model B
$m_{X_1(0^-)}$ [GeV]	2.2312 ± 0.0015	2.252 ± 0.002
$\Gamma'_{X_1(0^-)}$ [GeV]	0.227 ± 0.002	0.189 ± 0.002
$B_{X_1(0^-)\to\phi\phi}$	$(1.55 \pm 0.02) \times 10^{-2}$	0.218 ± 0.003
$m_{X_2(0^-)}$ [GeV]	2.0757 ± 0.0025	2.077 ± 0.002
$\Gamma'_{X_2(0^-)}$ [GeV]	0.136 ± 0.005	0.118 ± 0.005
$B_{X_2(0^-)\to\phi\phi}$	$(2.94 \pm 0.43) \times 10^{-4}$	$(1.03 \pm 0.08) \times 10^{-3}$
$m_{X_3(0^-)}$ [GeV]	2.6590 ± 0.0028	2.705 ± 0.003
$\Gamma'_{X_3(0^-)}$ [GeV]	0.51 ± 0.01	0.34 ± 0.01
$B_{X_3(0^-)\to\phi\phi}$	$(9.72 \pm 0.26) \times 10^{-2}$	0.126 ± 0.003
a_{12} [GeV^2]	0.128 ± 0.003	-
a_{13} [GeV^2]	-0.087 ± 0.004	-
a_{23} [GeV^2]	-0.005 ± 0.004	-
$N^{(0^-)}$	710 ± 13	708 ± 16
$\chi^2/n_{\rm d.o.f.}$	$24.4/18 \approx 1.4$	$25.8/21 \approx 1.2$

- Interference:

$$I = \mathcal{N}\Gamma_{J/\psi}\left[B_{J/\psi\to\gamma(X_1(0^-)+X_2(0^-)+X_3(0^-))\to\gamma\phi\phi} - \sum_{i=1,2,3} B_{J/\psi\to\gamma X_i(0^-)\to\gamma\phi\phi}\right] = -262 \pm 33 \text{ (model A)}; \; 151 \pm 19 \text{ (model B)}$$

0^+ resonance

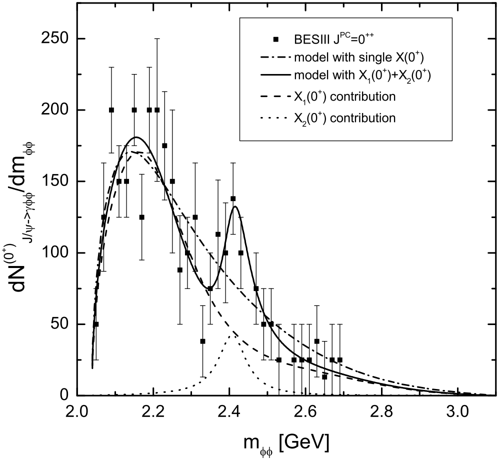
- Single 0^+ resonance results in poor fit, $\chi^2/n_{\rm d.o.f.} = 37.8/28 = 1.4$.
- Spectrum generalized to include two resonances:

$$\frac{dN^{(0^+)}}{dm_{12}} = \frac{\mathcal{N}}{(2\pi)^3 \times 12 m^2_{J/\psi}} |\boldsymbol{k}||\boldsymbol{k}_1^*| \left(2|A_0|^2 + |A_0 + A_2|^2\right),$$

$$A_0 = \frac{g_{11} f_{001}}{D_{X_1(0^+)}} + \frac{g_{12} f_{002}}{D_{X_2(0^+)}},$$

$$A_2 = \frac{g_{11} f_{221}}{D_{X_1(0^+)}} + \frac{g_{12} f_{222}}{D_{X_2(0^+)}}.$$

- Fit is improved: $\chi^2/n_{\rm d.o.f.} = 19.7/23 = 0.9$.


BESIII J^PC=0^++
model with single X(0+)
model with X1(0+)+X2(0+)
X1(0+) contribution
X2(0+) contribution
250
200
150
100
50
0
2.0
2.2
2.4
2.6
2.8
3.0
dN(0+) J/ψ->γφφ /dm φφ
m φφ [GeV]

- Results of fitting and evaluation:

$$
\begin{aligned}
m_{X_1(0^+)} &= 2.190 \pm 0.009 \text{ GeV}, \\
\Gamma'_{X_1(0^+)} &= 0.00 \pm 0.01 \text{ GeV}, \\
B_{X_1(0^+)\to\phi\phi} &= 0.70 \pm 0.04, \\
m_{X_2(0^+)} &= 2.409 \pm 0.010 \text{ GeV}, \\
\Gamma'_{X_2(0^+)} &= 0.003 \pm 0.021 \text{ GeV}, \\
B_{X_2(0^+)\to\phi\phi} &= 0.86 \pm 0.19, \\
N^{(0^+)} &\equiv \mathcal{N}\Gamma_{J/\psi} B_{J/\psi\to\gamma(X_1(0^+)+X_2(0^+))\to\gamma\phi\phi} = \\
& 63 \pm 5, \\
\mathcal{N}\Gamma_{J/\psi} B_{J/\psi\to\gamma X_1(0^+)\to\gamma\phi\phi} &= 52 \pm 4, \\
\mathcal{N}\Gamma_{J/\psi} B_{J/\psi\to\gamma X_2(0^+)\to\gamma\phi\phi} &= 5 \pm 2;
\end{aligned}
$$

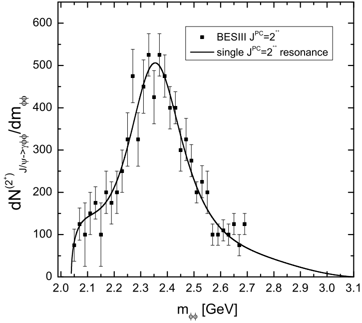
interference: $I = 6 \pm 7$.

2^+ resonance

- 9 free parameters characterize single contribution. Restricted statistics prevents from inclusion of more than one resonance.
- $\phi\phi$ mass spectrum in reaction $e^+e^- \to J/\psi \to \gamma\phi\phi$:

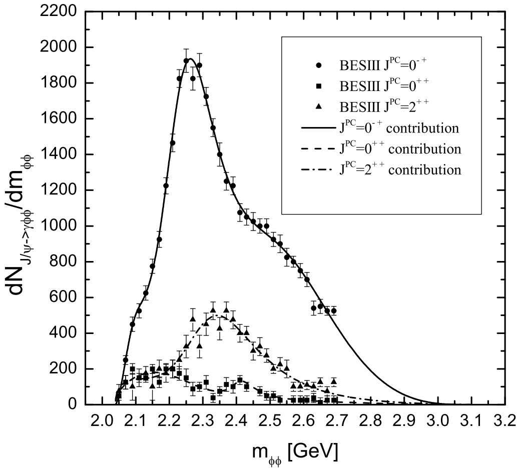
$$\frac{dN^{(2^+)}}{dm_{12}} = \mathcal{N}\frac{2m_{12}^2\left\langle\Gamma_{J/\psi\to\gamma X(2^+)}\right\rangle_{\lambda_{J/\psi}=\pm1}}{\pi|D_{X(2^+)}|^2}\times \Gamma_{X(2^+)\to\phi\phi},$$

$$\left\langle\Gamma_{J/\psi\to\gamma X(2^+)}\right\rangle_{\lambda_{J/\psi}=\pm1} = \frac{|\boldsymbol{k}|}{72\pi m_{J/\psi}^2}\left[4g_{02}^2+7g_{20}^2-4g_{02}g_{20}+3(g_{12}+g_{20})^2\right].$$



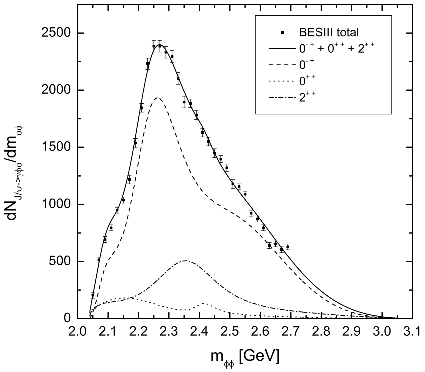
$$m_{X(2^+)} = 2.621 \pm 0.012 \text{ GeV}, \; \Gamma'_{X(2^+)} = 0.005 \pm 0.018 \text{ GeV},$$
$$B_{X(2^+)\to\phi\phi} = 0.21 \pm 0.01, \; \chi^2/n_{d.o.f.} = 19.7/24 \approx 0.8.$$

Summary of fitting



Consistency check

- Take sum of fitted J^P contributions and compare with experiment:



Photon angular distribution

- Angular distribution of photons in reaction $e^+e^- \to J/\psi \to \gamma\phi\phi$:

$$\frac{dN}{d\cos\theta_\gamma} = \frac{3}{8}(1+\cos^2\theta_\gamma)\left[N^{(0^-)} + N^{(0^+)}\right] + N_1^{(2^+)} + N_2^{(2^+)}\cos^2\theta_\gamma,$$

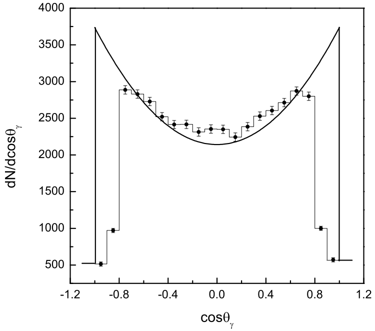
$$N_{1,2}^{(2^+)} = \frac{\mathcal{N}}{32\pi m_{J/\psi}^2}\int_{2m_\phi}^{m_{J/\psi}} dm_{12}\frac{m_{12}^2\Gamma_{X(2^+)\to\phi\phi}(m_{12})}{\pi\left|D_{X(2^+)}\right|^2}|\boldsymbol{k}| \times \left[|g_{02}|^2 + \frac{3}{2}|g_{20}|^2 - \frac{1}{3}|g_{02}+g_{20}|^2 \pm |g_{12}+g_{20}|^2\right].$$

- Calculation with the fitted parameters gives $N^{(0^-)} = 708$, $N^{(0^+)} = 63$ and $N_1^{(2^+)} = 89$, $N_2^{(2^+)} = -8$.

- Multiplying unnormalized distribution by

$$N^{-1} = \left[N^{(0^-)} + N^{(0^+)} + 2N_1^{(2^+)} + \frac{2}{3}N_2^{(2^+)}\right]^{-1}$$

followed by multiplication by area under experimental histogram results in



- Resonance parameters cited by BESIII and PDG were obtained in the fixed width approximation.
- Correct comparison of the present work with BESIII data and PDG entries requires evaluation of the effective resonance peak positions and widths.
- A rough estimate can be obtained upon neglecting the resonance peak distortion due to the effects of the phase space volume by peak locations and evaluating widths at half of height of the resonance peaks.

In case of 0^- one finds

- masses $m_{X_1(0^-)} \equiv m_{\eta(2250)} \approx 2260$ MeV, $m_{X_2(0^-)} \equiv m_{\eta(2100)} \approx 2120$ MeV, and $m_{X_3(0^-)} \equiv m_{\eta(2500)} \approx 2480$ MeV
- effective widths $\Gamma_{X_1(0^-)} \equiv \Gamma_{\eta(2250)} \approx 220$ MeV, $\Gamma_{X_2(0^-)} \equiv \Gamma_{\eta(2100)} \approx 210$ MeV, and $\Gamma_{X_3(0^-)} \equiv \Gamma_{\eta(2500)} \approx 400$ MeV.
- Within one or two magnitudes of the experimental uncertainty they agree with the values given by BESIII.

- In case of 0^+ resonance contribution, $X_1(0^+)$ has effective peak characteristics which agree within the experimental accuracy with those of $f_0(2100)$ observed by BESIII. $X_2(0^+)$ included here to achieve the better description of the data is new. The data with improved statistics could resolve the issue.
- Effective characteristics of the tensor resonance obtained here agree with those of $f_2(2340)$ cited by PDG.

- Dynamical analysis of the resonance contributions to $J/\psi \to \gamma X \to \gamma\phi\phi$ decay amplitude is performed based on the effective amplitudes of the transitions $J/\psi \to \gamma X$ and $X \to \phi\phi$.
- Resonances with quantum numbers $J^{PC} = 0^{-+}, 0^{++}, 2^{++}$ are taken into account to describe $\phi\phi$ mass spectrum in the reaction $e^+e^- \to J/\psi \to \gamma X(J^{PC}) \to \gamma\phi\phi$ studied by BESIII collaboration.
- Two models, with and without mixing of three $X(0^{-+})$ resonances, are considered when fitting the pseudoscalar component of the spectrum. It is shown that both above models give satisfactory description of the data, hence one cannot distinguish between them with the present accuracy of the data.

- The scalar component of the $\phi\phi$ spectrum is better described in the model with two scalar resonances.
- Surprisingly, the tensor component requires only one resonance, because the non-trivial behaviour at the left shoulder of the resonance peak is due to $\phi\phi$ mass dependencies in $X(2^{++}) \to \phi\phi$ vertex.
- Masses and effective coupling constants parameterising invariant amplitudes are extracted from the fits and used for evaluation of branching fractions. Consistency of fits is supported by evaluating the incoherent sum of 0^{-+}, 0^{++}, 2^{++} contributions to mass spectrum and by calculating the photon angular distribution.

- Some details (values of coupling constants, expressions of amplitudes in terms of helicity ones, verification of incoherence of resonance contributions with different quantum numbers, etc.) are given in the paper A.A.K., Phys. Rev. D**99**, 014019 (2019).

Thank You!