



# Amplitude analysis (AA) of $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ and $e^+e^- \rightarrow 2\pi^+2\pi^-$ reactions with the CMD-3 detector

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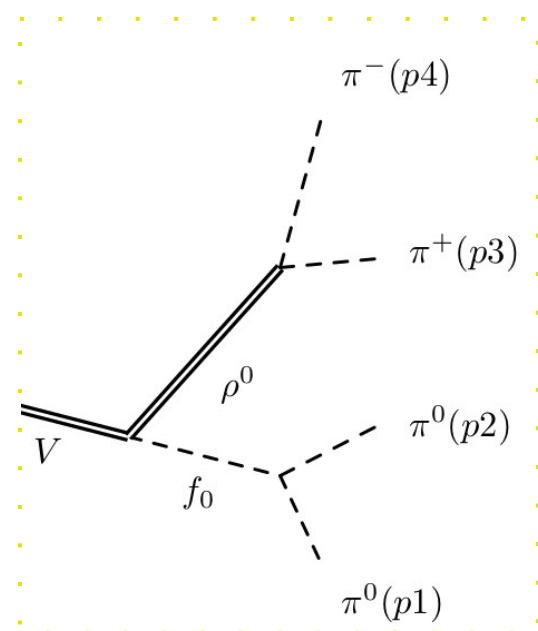
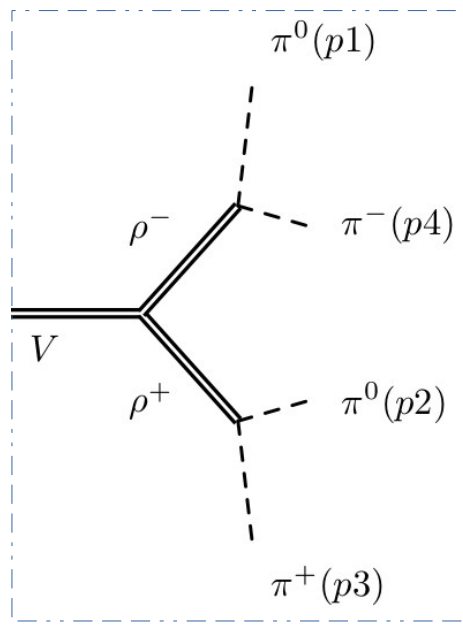
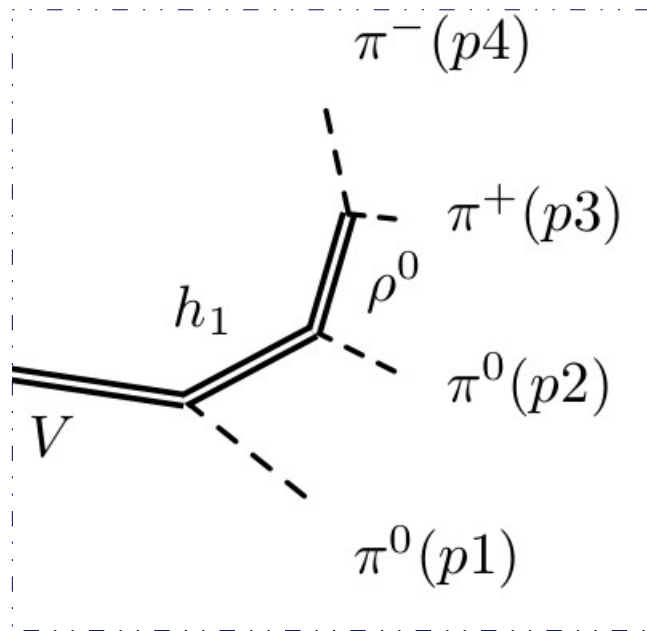
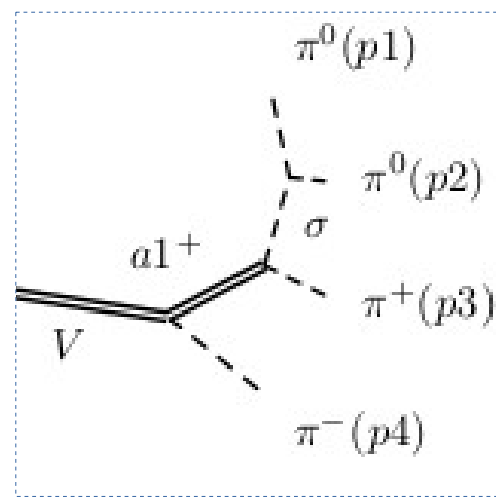
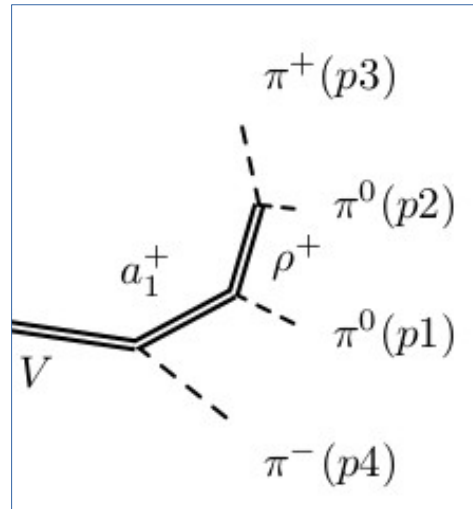
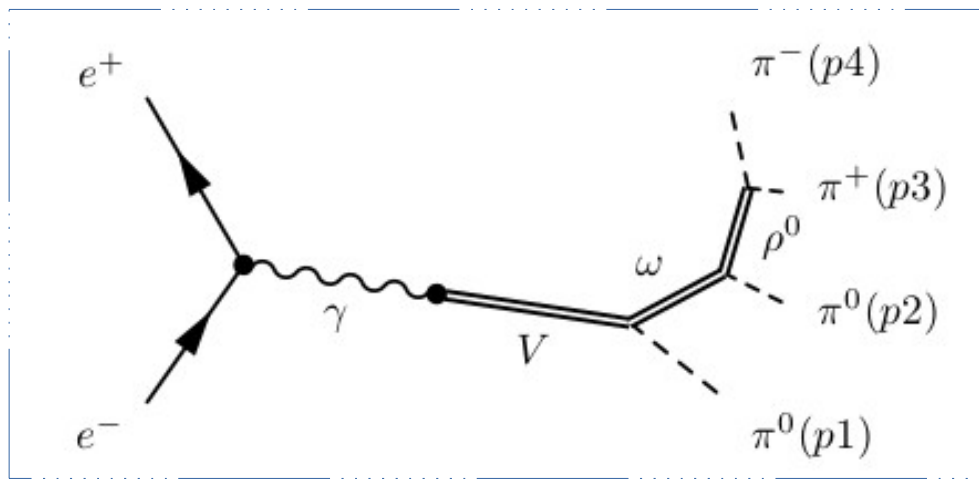


1. For the measurement of the **cross section of the  $e^+e^- \rightarrow 4\pi$**  reaction we are required to find out which intermediate states are involved in this process in order to obtain a correct value of detection efficiency.

2. The AA allows to describe experimental data by phenomenological **model**.

3. Actually, the phenomenological approach is far from the **fundamental understanding** of the process of the hadronization of the produced pair of quarks and of four pions production.





## The amplitude analysis at CMD-2 ( $5.8 \text{ pb}^{-1}$ )

- The data in the  $ee \rightarrow \pi^+\pi^-2\pi^0$  (22128 events) with  $\sqrt{s} = [1.05-1.38] \text{ GeV}$  is used

- The dominance of the  $\omega\pi$  and  $a_1\pi$  is proved

- The data in  $ee \rightarrow \pi^+\pi^-2\pi^0$  and  $ee \rightarrow 2\pi^+2\pi^-$  (28552) is used for the estimation:  
 $B(a_1 \rightarrow \sigma\pi)/B(a_1 \rightarrow \rho\pi) \sim 0.3$

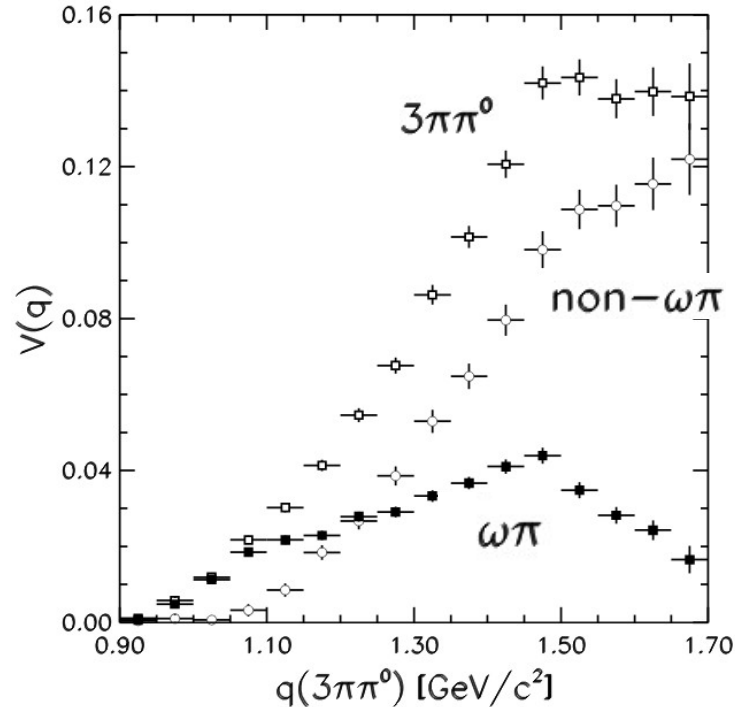
- The measured cross section are systematically shifted from other measurement.

The results of a search for the admixture of other possible states

Model	$L_{\min}$	$r_X$ [%]	U.L. [%]
$\omega\pi^0 + a_1\pi$	1264	—	—
$\omega\pi^0 + a_1\pi + \rho\sigma$	1256	$2.1^{+1.2}_{-0.9}$	4.3
$\omega\pi^0 + a_1\pi + h_1\pi$	1263	$0.1^{+0.2}_{-0.1}$	0.4
$\omega\pi^0 + a_1\pi + a_2\pi$	1263	$0.2^{+0.4}_{-0.2}$	0.8
$\omega\pi^0 + a_1\pi + \pi'\pi$	1250	$9.5^{+3.2}_{-2.8}$	15.
$\omega\pi^0 + a_1\pi + \rho^+\rho^-$	1246	$4.7^{+2.0}_{-1.6}$	7.7

*R.R. Akhmetshin et al., Physics Letters B **466**, 392–402 (1999)*

# Amplitude analysis of $\tau \rightarrow 3\pi\pi^0\nu_\tau$ at CLEO (1999)



Spectral functions

$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau 2\pi^- \pi^+ \pi^0)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = \frac{3 \cos^2 \theta_c}{2\pi\alpha^2 m_\tau^8} \int_0^{m_\tau^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2) \cdot \left[ \frac{1}{2} \sigma_{e^+e^- \rightarrow 2\pi^- 2\pi^+}(Q^2) + \sigma_{e^+e^- \rightarrow \pi^+\pi^- 2\pi^0}(Q^2) \right]$$

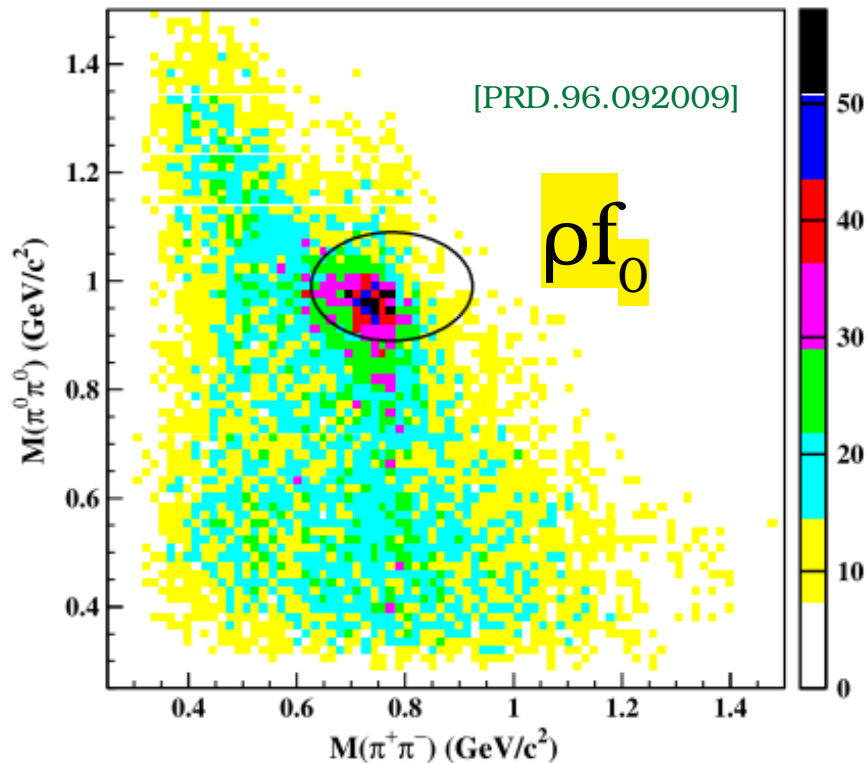
Model	Integrated amplitudes	Goodness-of-fit
Model 2	$R_{\omega\pi} = 0.38 \pm 0.02 \pm 0.02$ $R_{a_1\pi} = 0.43 \pm 0.02 \pm 0.02$	< 5%
Model 3	$R_{\omega\pi} = 0.38 \pm 0.02 \pm 0.01$ $R_{a_1\pi} = 0.49 \pm 0.02 \pm 0.02$ $R_{\sigma\rho} = 0.01 \pm 0.02 \pm 0.01$ $R_{f_0\rho} = 0.01 \pm 0.01 \pm 0.01$	20%

Fit results for various models

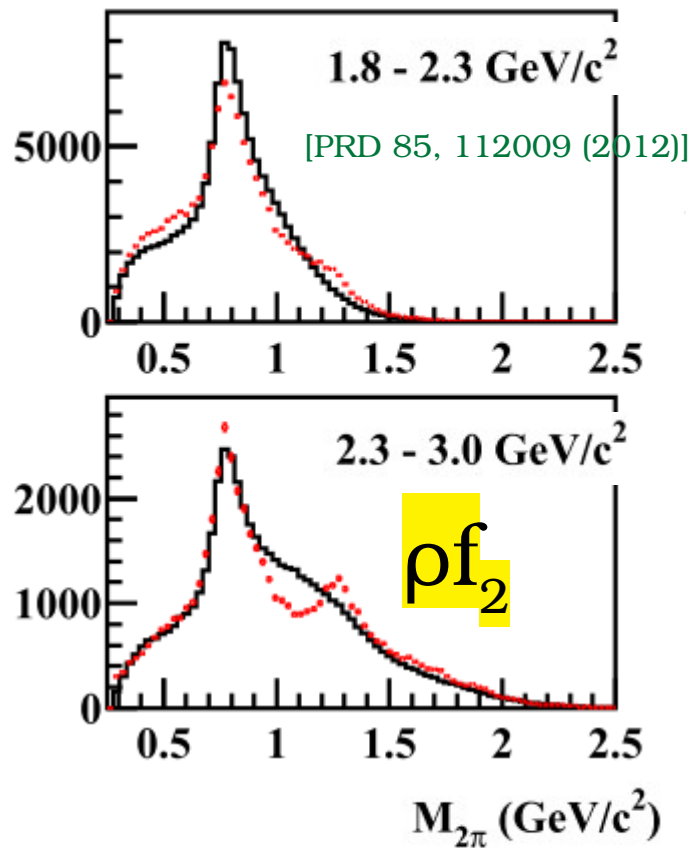
- Model with  $\omega\pi$  and  $a_1\pi$  and  $\rho\sigma$ ,  $\rho f_0$  provides the best description of the data.

*Physical Review D - Particles, Fields, Gravitation and Cosmology*, **61**, 1-16 (2000).

There are evidences of  $\rho f_0$  and  $\rho f_2$  at higher energies with BaBar detector



The evidence of  $\rho f_0$  in the process  $e+e^- \rightarrow 2\pi^0\pi^+\pi^-$  with BaBar



Two pion invariant mass spectra in the process  $e+e^- \rightarrow 2\pi^+\pi^-$

# General strategy

**Signal selection**  
**( $ee \rightarrow \pi^+\pi^-2\pi^0$ )**

**Signal selection**  
**( $ee \rightarrow 2\pi^+2\pi^-$ )**

**Building of the amplitudes formalism**

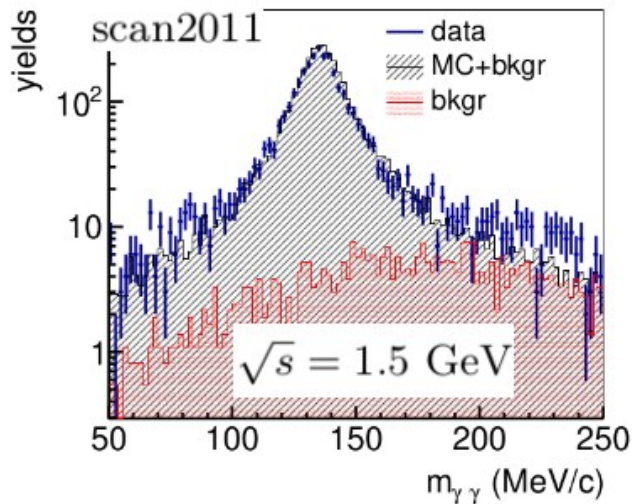
**The definition and minimization  
of likelihood function (L)**

**Model vs Experiment comparison**

# Signal selection

**( $ee \rightarrow \pi^+\pi^-2\pi^0$ ) 64 kevents**

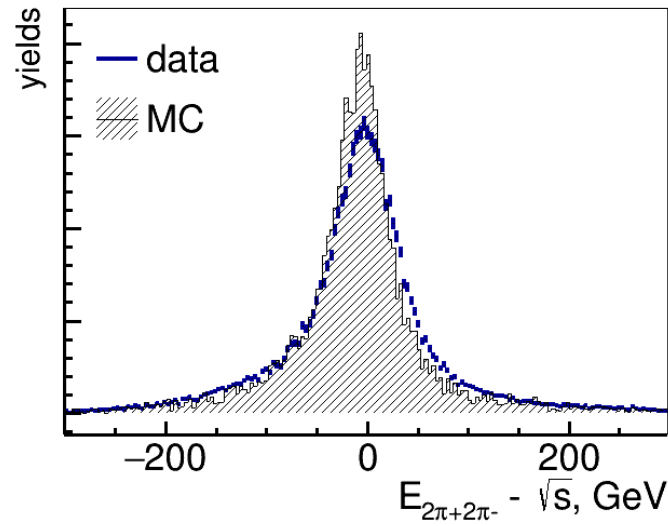
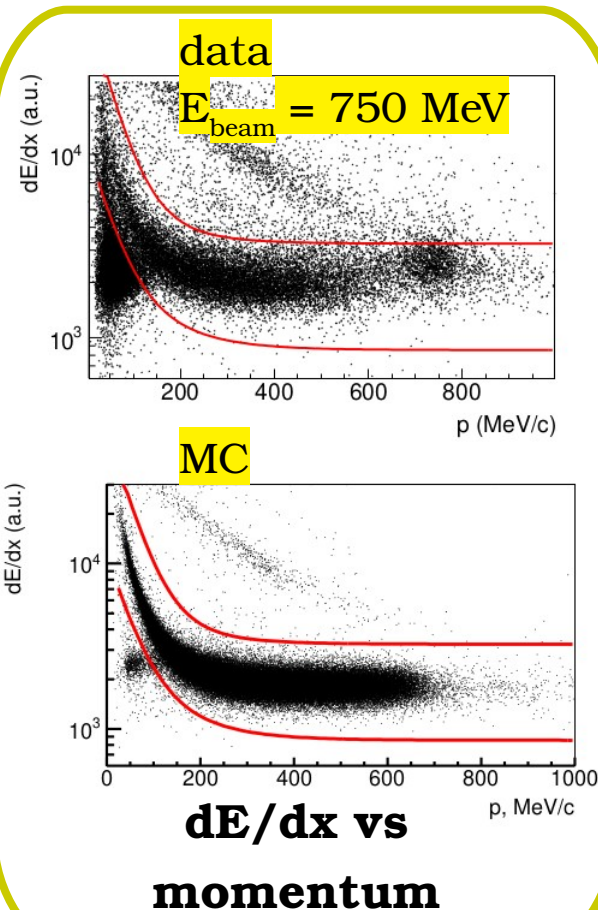
- $0.7 < \theta_{\pi,\gamma} < \pi - 0.7$  rad
- Total  $(E-\sqrt{s})$  and  $P < 150$  MeV/(c)
- Two candidates for  $\pi^0$
- 5C kinematic fit
- The **invariant mass** spectrum of 3rd and 4th **photons** is used for the estimation of the contribution of background:



# Signal selection

**( $ee \rightarrow 2\pi^+2\pi^-$ ) 72 kevents**

- $0.7 < \theta_{\pi} < \pi - 0.7$  rad
- Total  $(E-\sqrt{s})$  and  $P < 150$  MeV/(c)
- 4C kinematic fit
- The spectrum of **total energy of four tracks**  $(E-\sqrt{s})$  is used for the estimation of the contribution of background:





The production of  $4\pi$  system can proceed via a list of intermediate states:

- $\omega[1^{--}] \pi^0[0^{-+}]$  ( Only  $2\pi^\pm 2\pi^0$ )
- $a_1(1200)[1^+] \pi[0^-]$
- $\rho[1^{--}] f_0(980)[0^{++}]$
- $\rho[1^{--}] \sigma(500)[0^{++}]$
- $\rho[1^{--}] f_2(1270)[2^{++}]$
- $\rho^+[1^-] \rho^-[1^-]$  ( Only  $2\pi^\pm 2\pi^0$ )
- $a_2(1320)[2^{++}] \pi[0^-]$
- $h_1(1170)[1^{+-}] \pi^0[0^{-+}]$  ( Only  $2\pi^\pm 2\pi^0$ )

To get magnitudes of amplitudes we perform unbinned likelihood amplitude analysis. The relative number of events  $I$  at a particular point  $\Omega$  in phase space can be represented as

$$I(\Omega) = \sum_{\alpha} |V_{\alpha} A_{\alpha}(\Omega)|^2 \cdot d\Phi(\Omega),$$

where the sum runs over all intermediate states,  $V_{\alpha}$  - the complex production amplitude (the free parameter) and  $A_{\alpha}(\Omega)$  - the amplitude at a particular point in phase space.

The likelihood for model under test is

$$L = -\log \prod_{i=\text{signal}} \frac{\pi^+ \pi^- 2\pi^0 I(\Omega_i)}{\int \varepsilon I(\Omega) d\Omega} - \log \prod_{i=\text{signal}} \frac{2\pi^+ 2\pi^- I(\Omega_i)}{\int \varepsilon I(\Omega) d\Omega}$$

The limited acceptance and efficiency of the detector is taken into account by summing only over simulated events that pass the reconstruction and analysis cuts.

$$\int \varepsilon I(\Omega) d\Omega = \frac{1}{N_{MC}^{gen}} \sum_{rec}^{phase\ space\ MC} |V_\alpha A_\alpha(\Omega)|^2$$

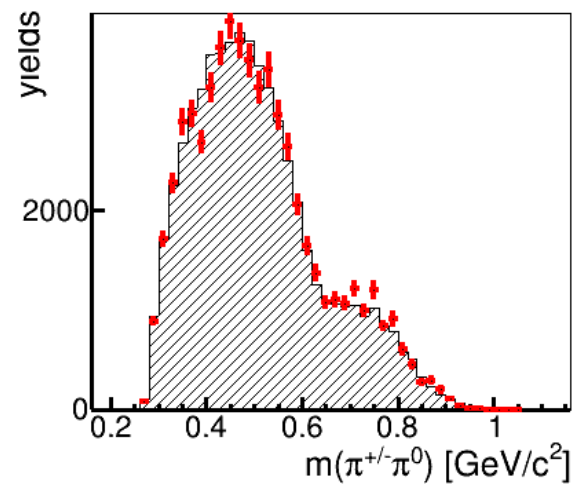
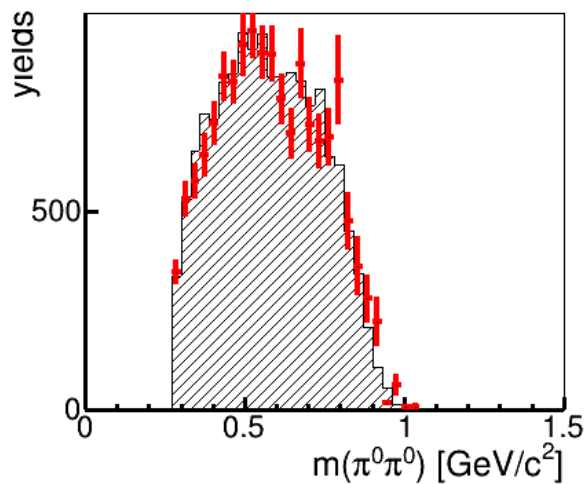
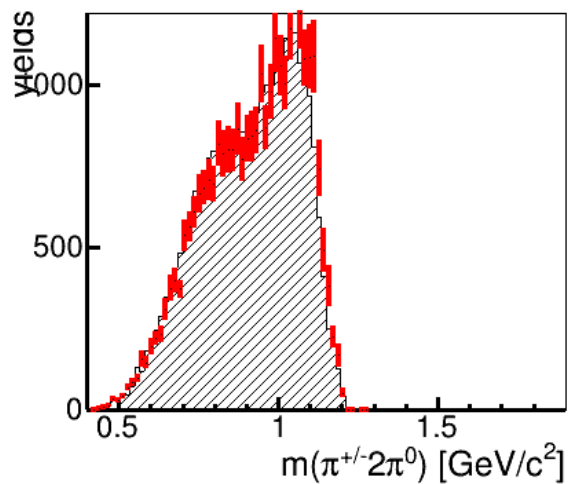
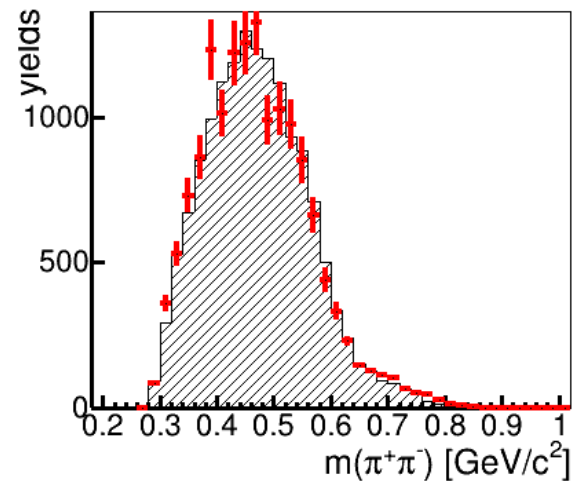
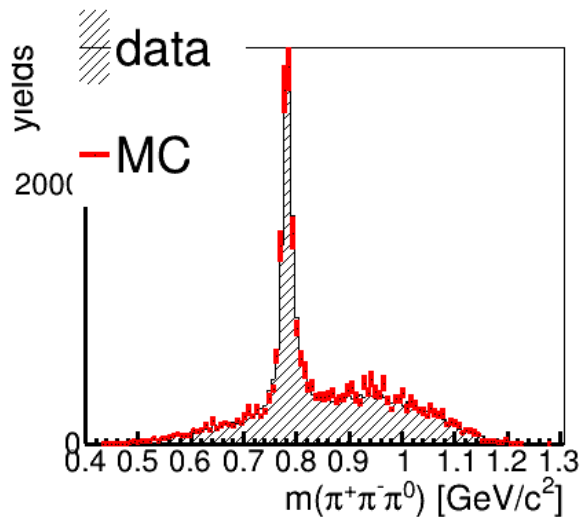
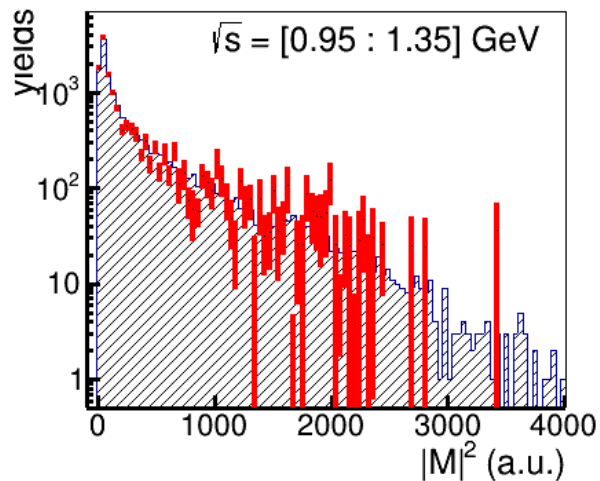
- An amplitude is normalized to 1:  $\int |A_\alpha(\Omega)|^2 d\Omega = 1$ ;
- The  $\omega\pi^0$  amplitude is clearly seen at all energies, so  $A_{\omega\pi^0}$  fixes at 1;

- The amplitudes is symmetric (anti-symmetric) with respect to the interchange of the momenta of neutral (charged) mesons according to Bose symmetry and C-parity conservation.

$$\begin{aligned}
L(\omega\rho\pi) &= g_{\omega\rho\pi} \cdot \epsilon_{\mu\nu\rho\sigma} \cdot \delta^{ab} \cdot \omega_\mu \cdot d_\nu \pi^{*a} \cdot (d_\rho \rho_\sigma^{*b} - d_\sigma \rho_\rho^{*b}), \\
L(a_1\rho\pi) &= g_{a_1\rho\pi} \cdot \epsilon^{abc} \cdot a_{1\mu}^a \cdot d_\nu \pi^{*b} \cdot (d_\mu \rho_\nu^{*c} - d_\nu \rho_\mu^{*c}), \\
L(a_1\sigma\pi) &= g_{a_1\sigma\pi} \cdot \delta^{ab} \cdot (d_\mu a_{1\nu}^a - d_\nu a_{1\mu}^a) \cdot d_\mu \phi^*(\sigma) \cdot d_\nu \phi^{*b}(\pi), \\
L(\rho'\rho f_0) &= g_{\rho'\rho f_0} \cdot \delta^{ab} \cdot (d_\mu \rho_\nu'^a - d_\nu \rho_\mu'^a) (d_\mu \rho_\nu^{*b} - d_\nu \rho_\mu^{*b}) \cdot \phi_{f_0}^*, \\
L(\rho'\rho^+\rho^-) &= g_{\rho'\rho^+\rho^-} \cdot \epsilon^{abc} (d_\mu \rho_\nu'^a - d_\nu \rho_\mu'^a) \cdot (d_\alpha \rho_\nu^{*b} - d_\nu \rho_\alpha^{*b}) \cdot (d_\mu \rho_\alpha^{*c} - d_\alpha \rho_\mu^{*c}), \\
L(\rho'h_1\pi^0) &= g_{\rho'h_1\pi^0} \cdot \delta^{ab} (d_\mu \rho_\nu'^a - d_\nu \rho_\mu'^a) \cdot (d_\mu h_{1\nu}^{*b} - d_\nu h_{1\mu}^{*b}) \phi_\pi^*,
\end{aligned}$$

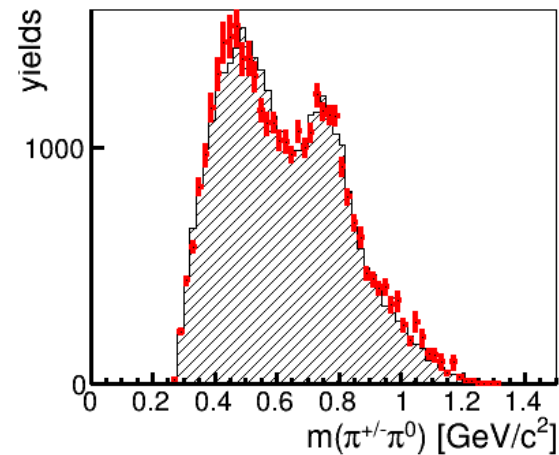
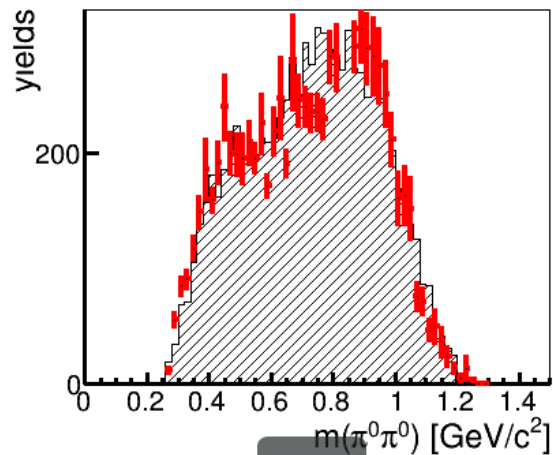
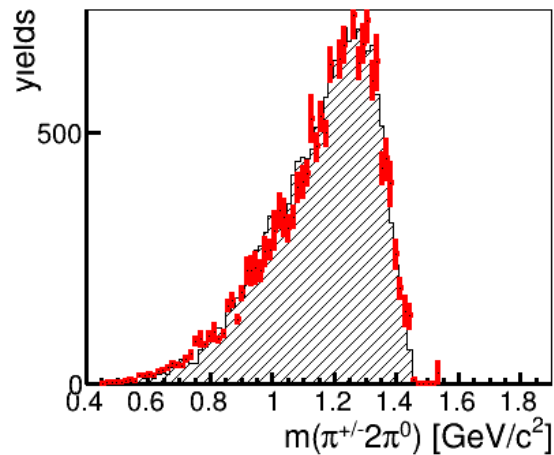
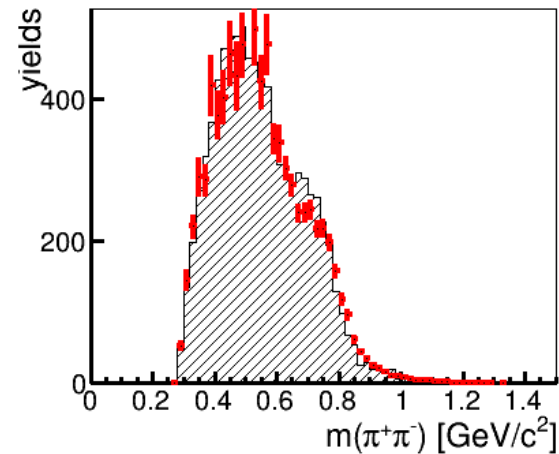
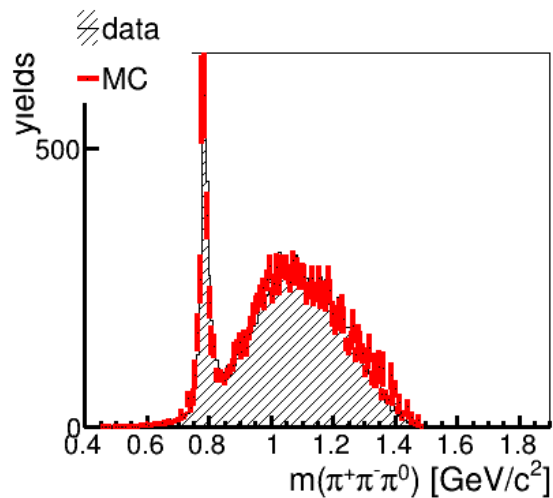
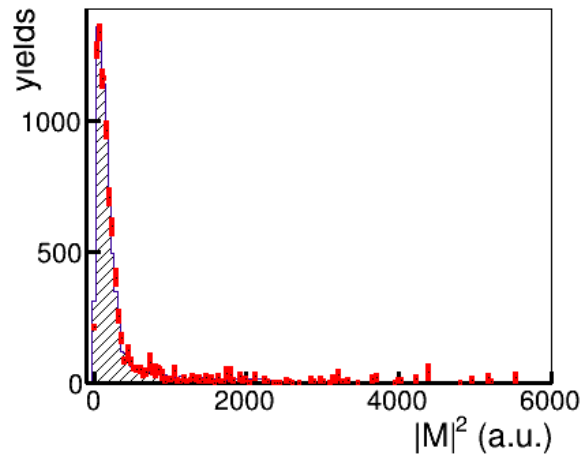
- Masses and central values of widths of resonances are fixed according to PDG.
- Spectral function obtained from the Dyson equation with simplest  $\pi$ - $\sigma$  coupling is used as the propagator of  $\sigma(500)$  [*Phys. Rev. C* **76** 065204 (2007)]
- Flatté distribution is used for the propagator of  $\rho(770)$  (1980).

# $(ee \rightarrow \pi^+\pi^-\pi^0)$

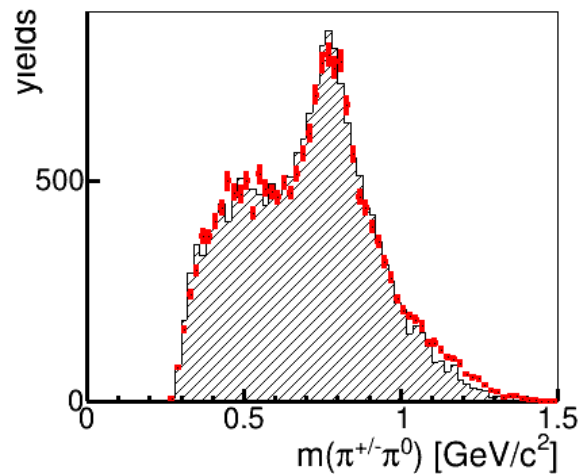
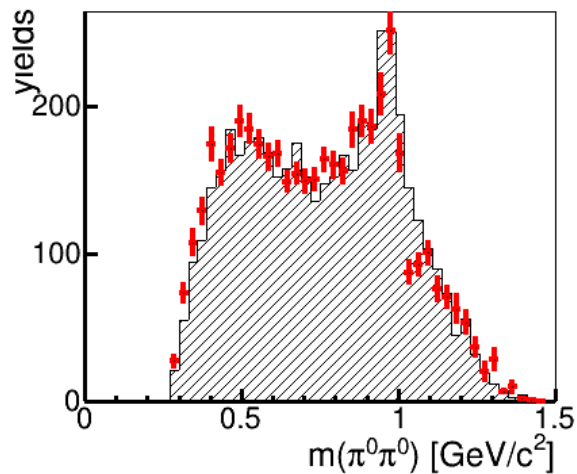
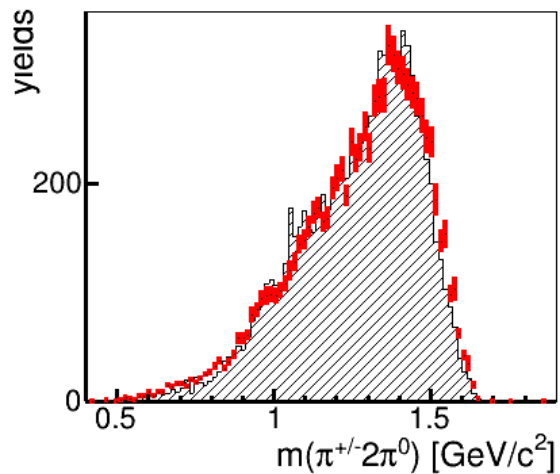
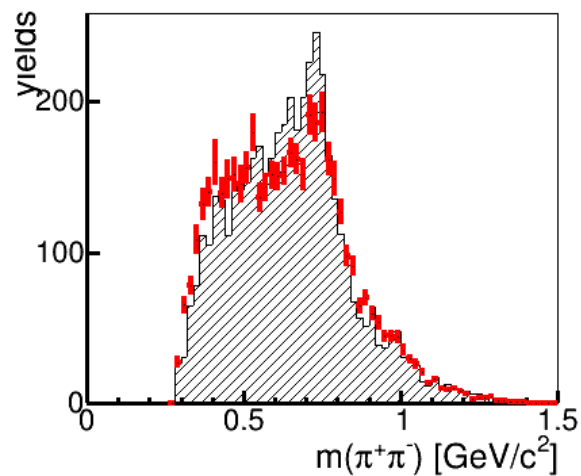
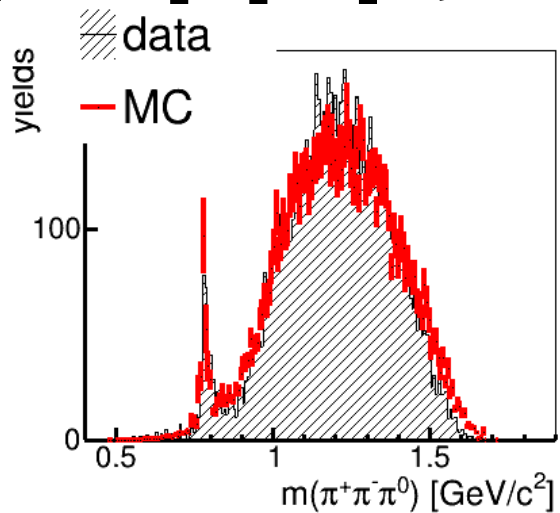
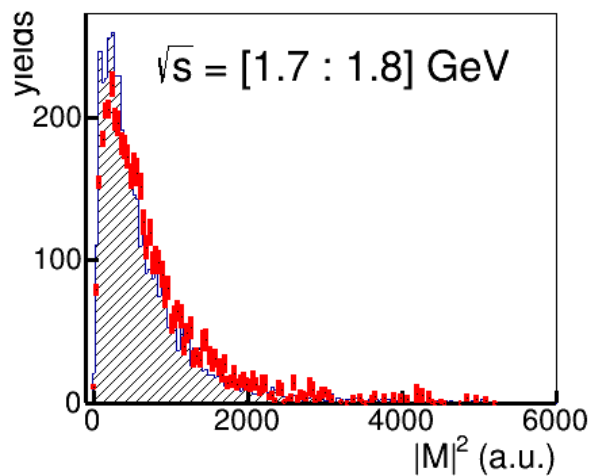


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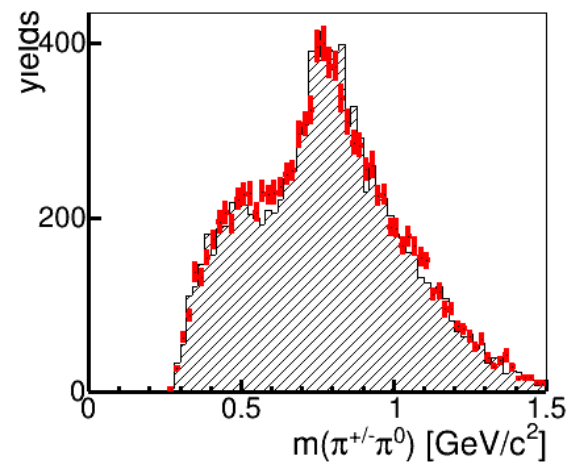
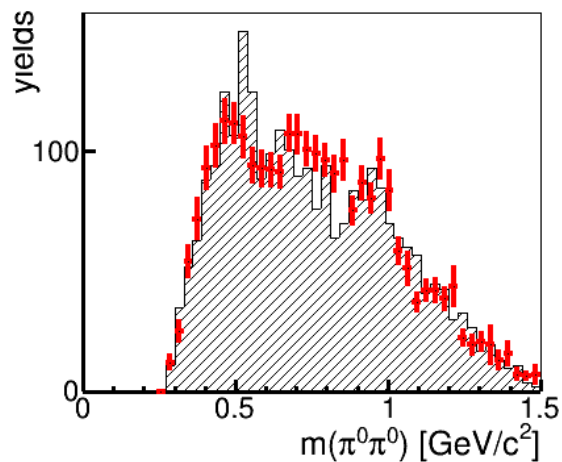
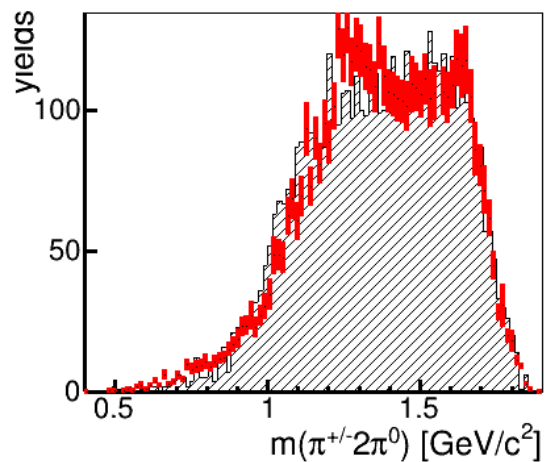
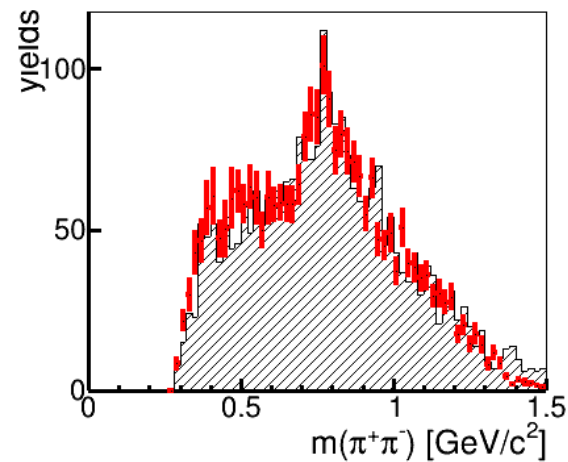
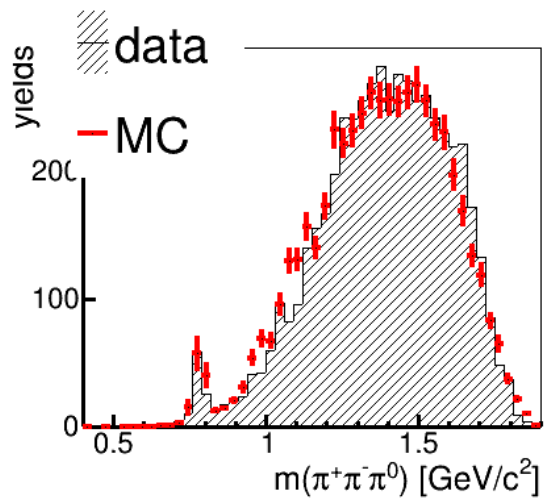
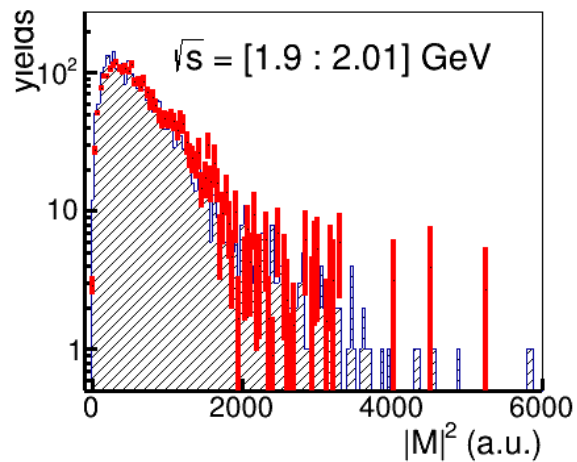
$\sqrt{s} = [1.5 : 1.6] \text{ GeV}$



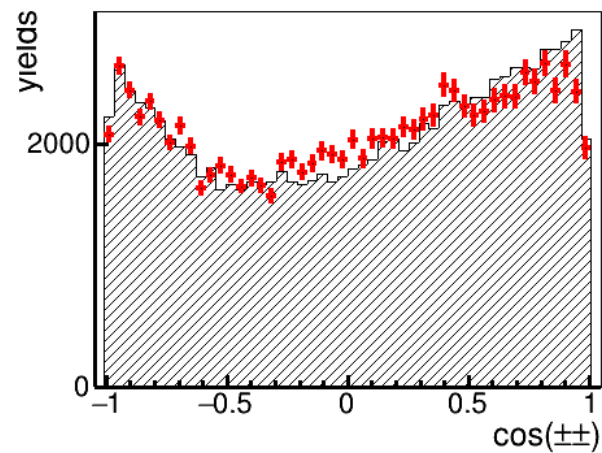
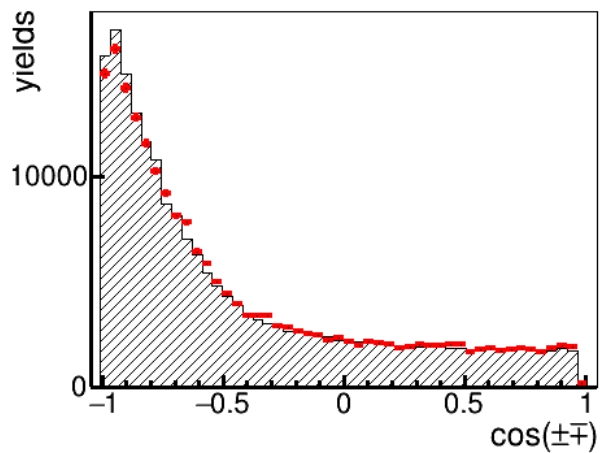
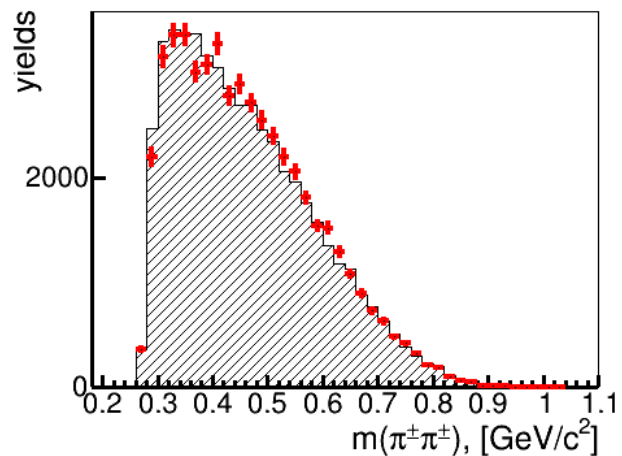
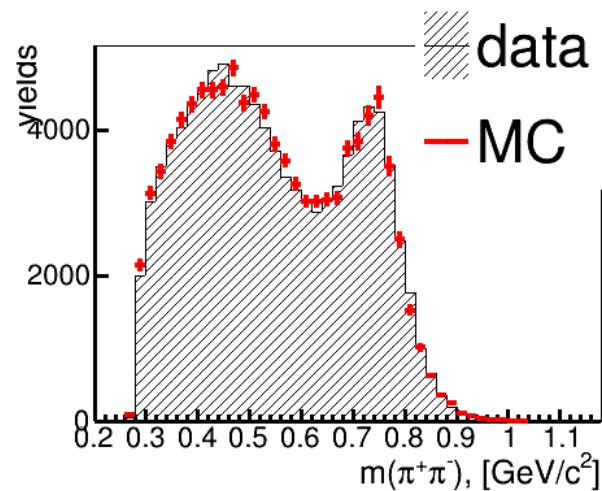
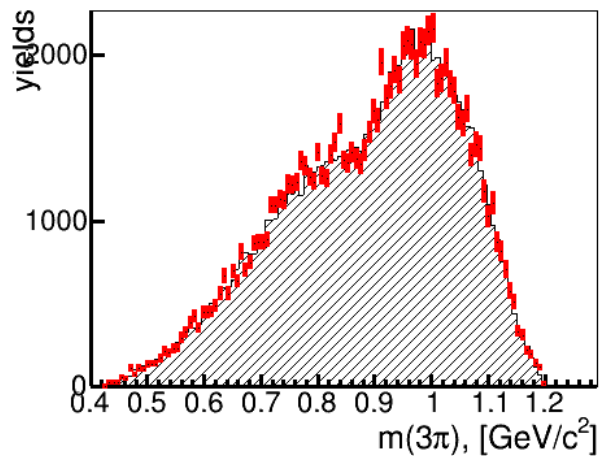
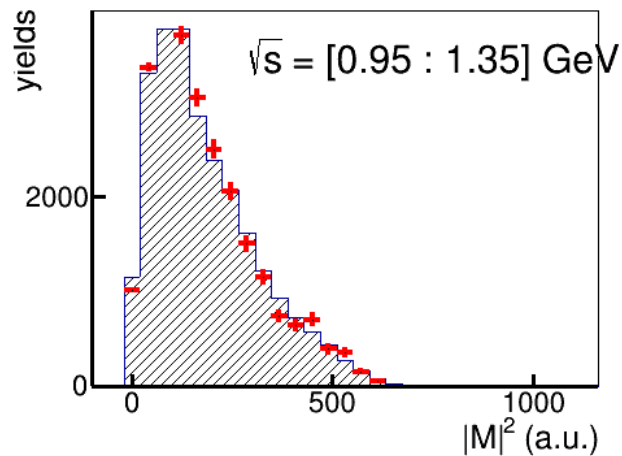
# $(ee \rightarrow \pi^+\pi^-\pi^0)$



# $(ee \rightarrow \pi^+\pi^-\pi^0)$

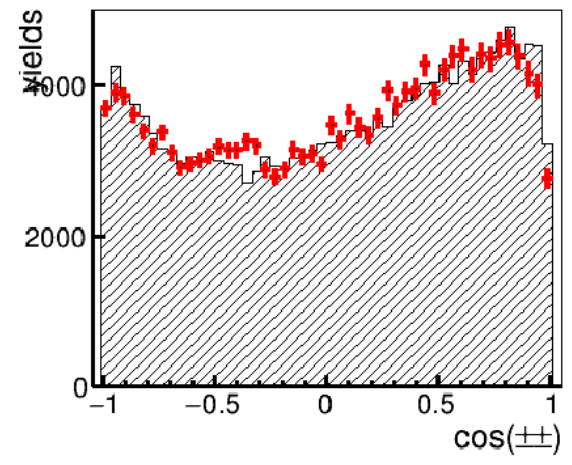
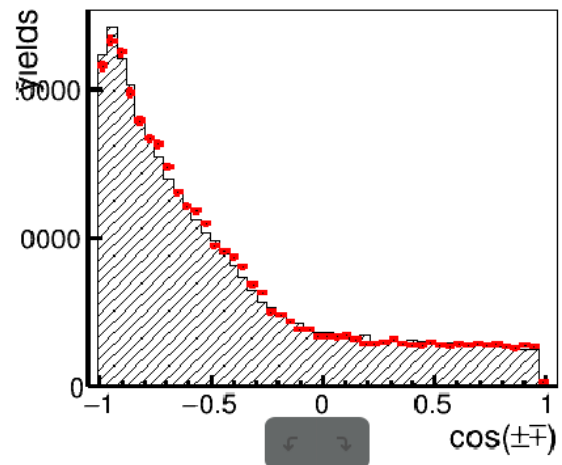
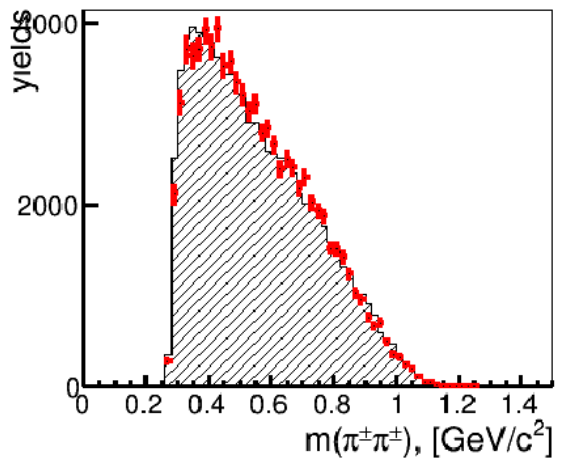
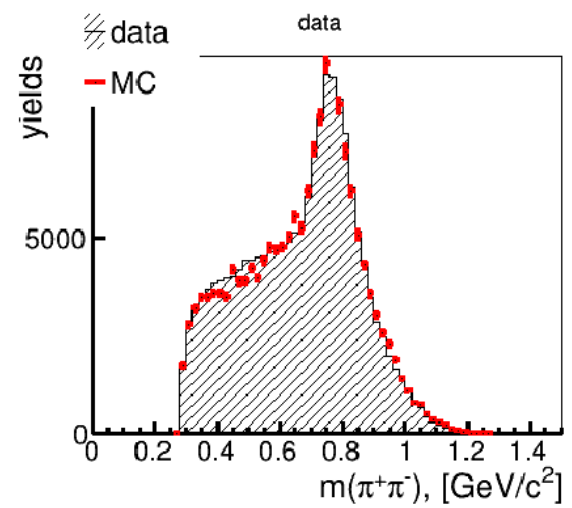
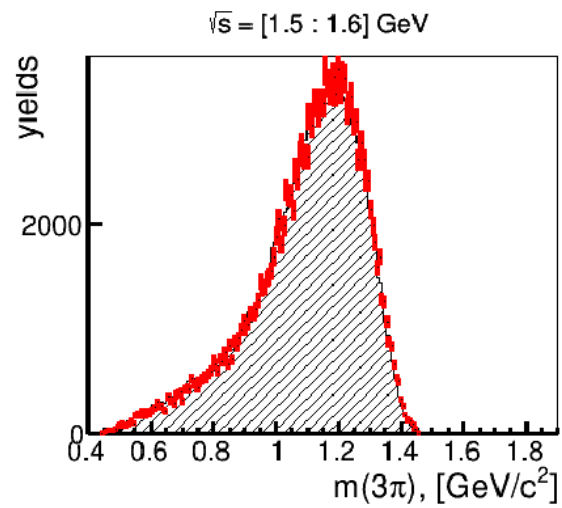
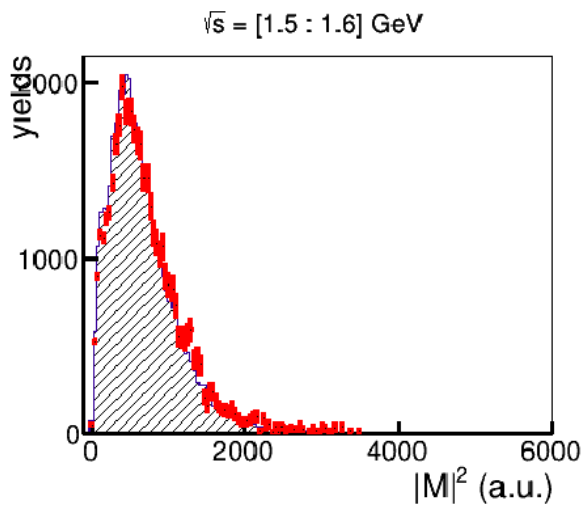


# $(e e \rightarrow 2\pi^+ 2\pi^-)$

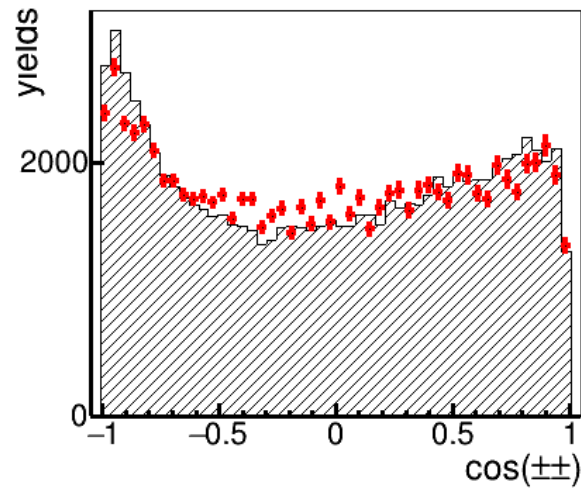
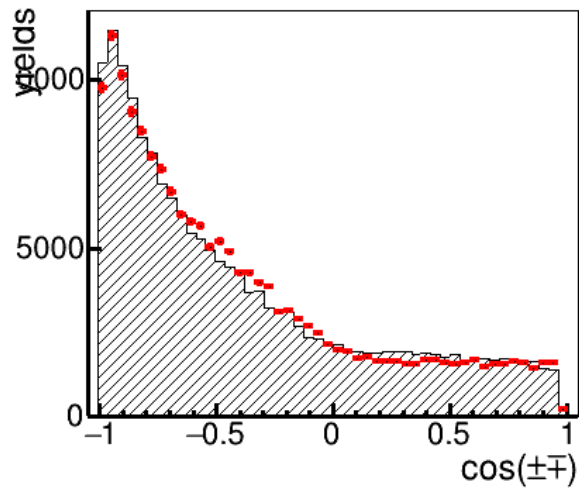
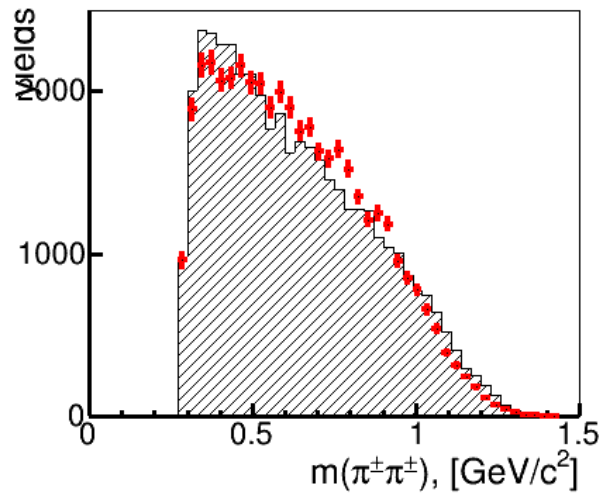
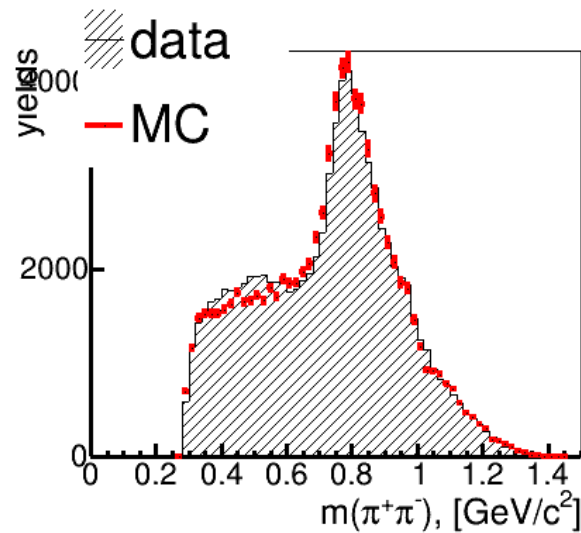
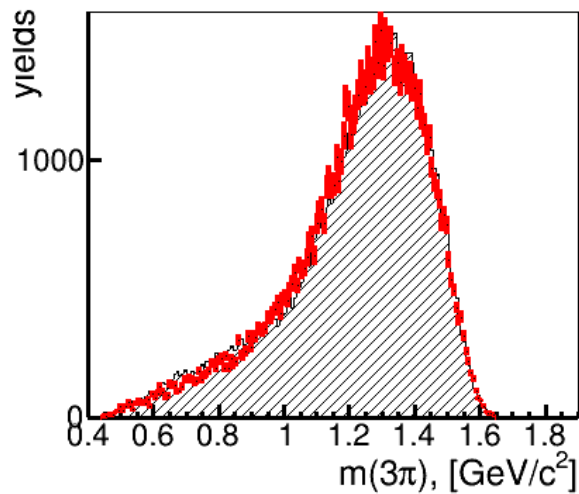
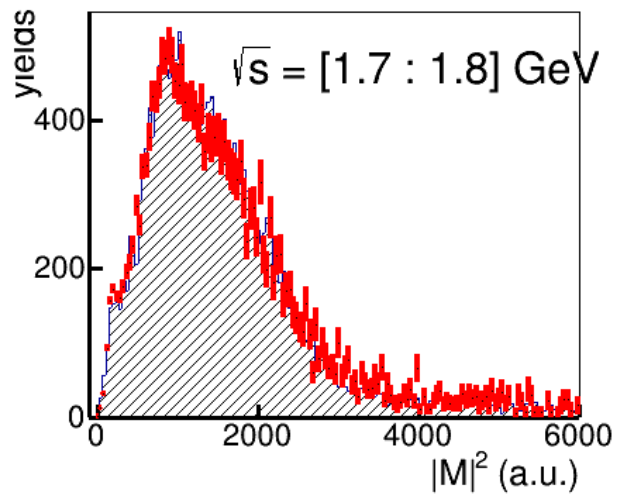




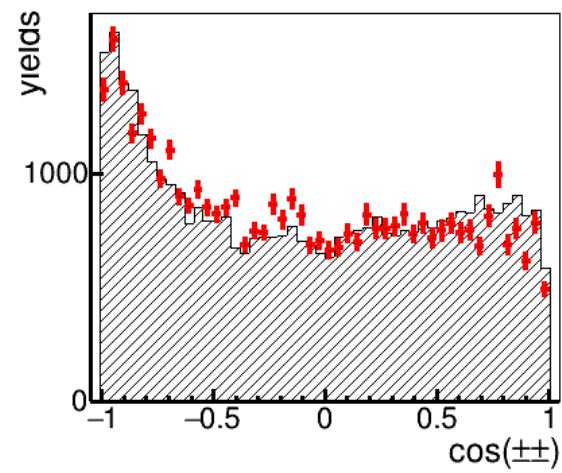
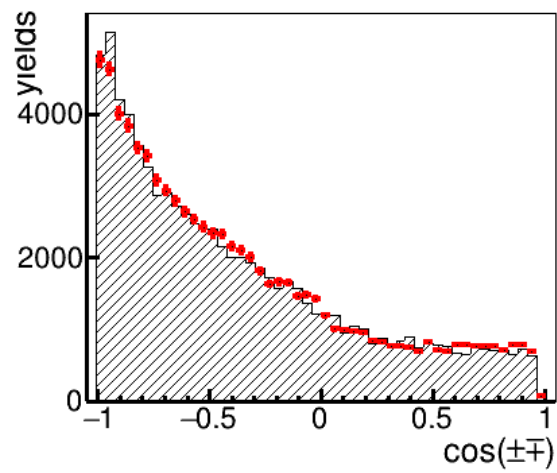
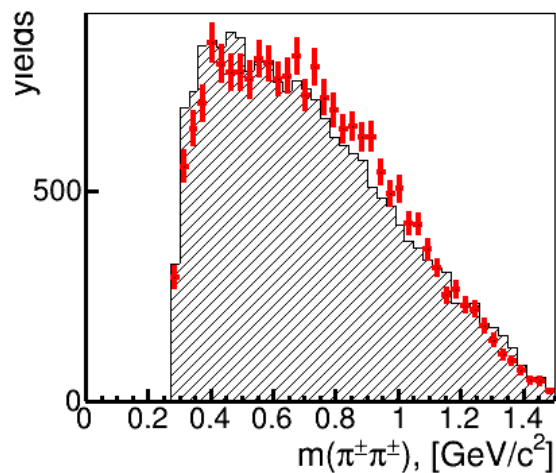
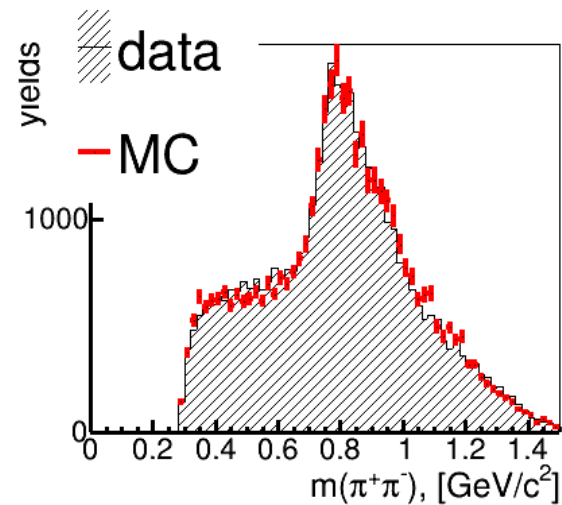
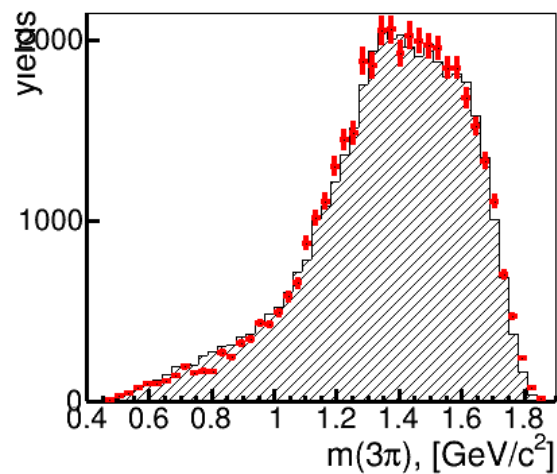
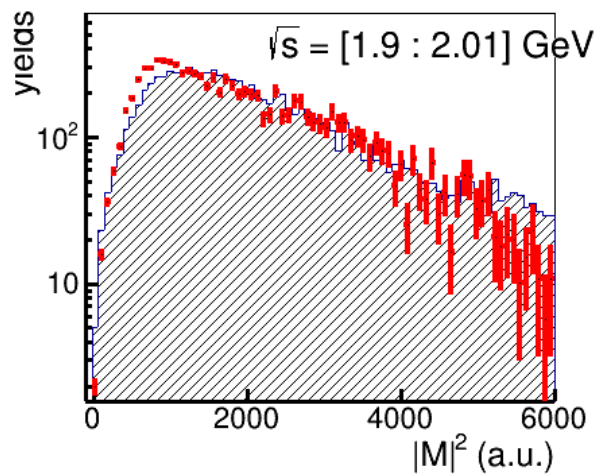
# $(ee \rightarrow 2\pi^+2\pi^-)$

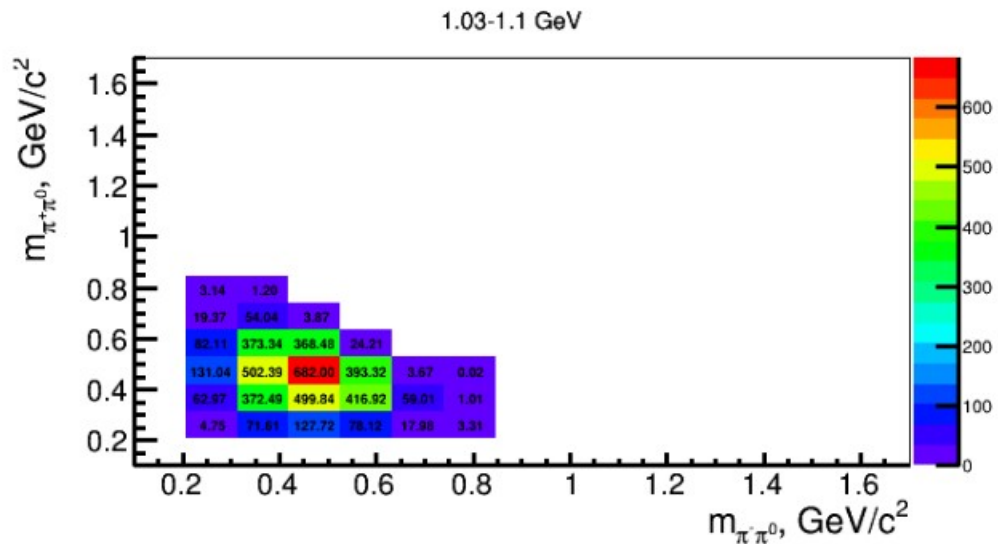
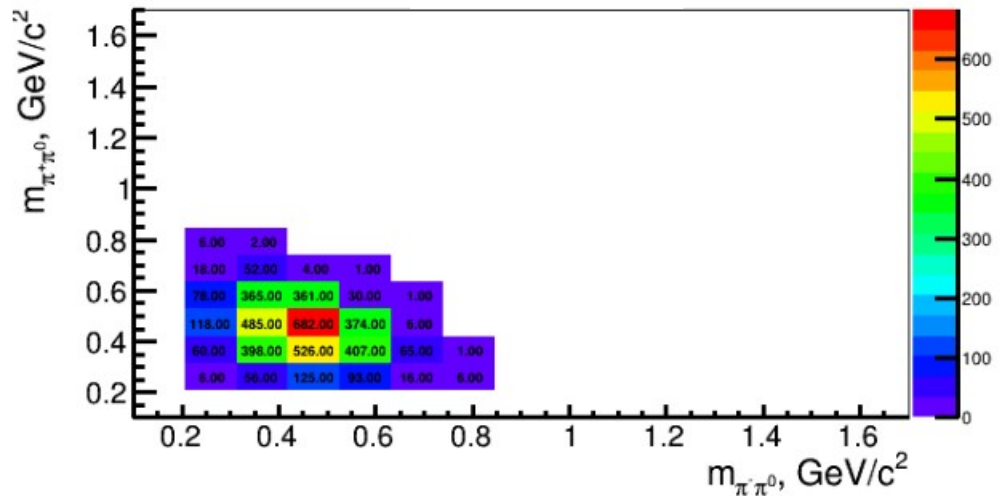
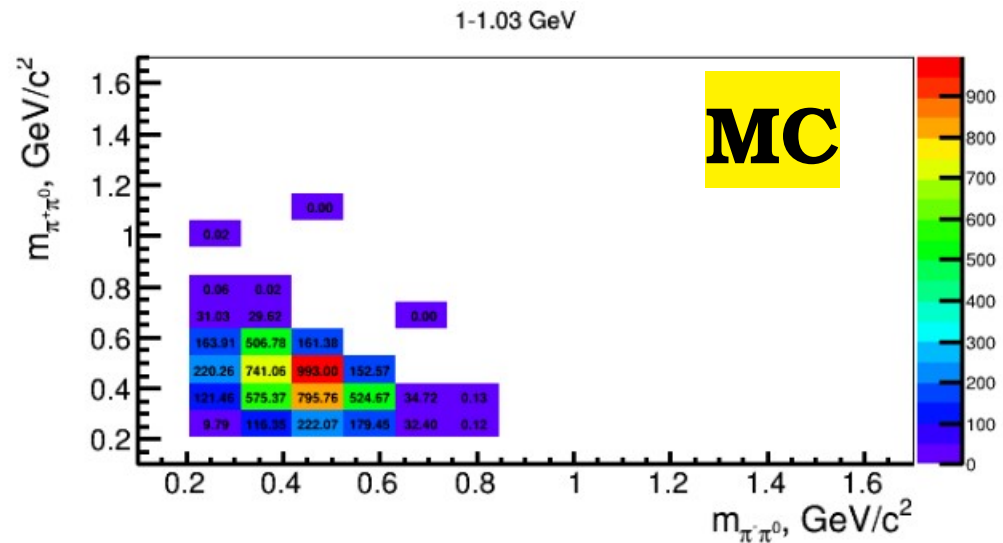
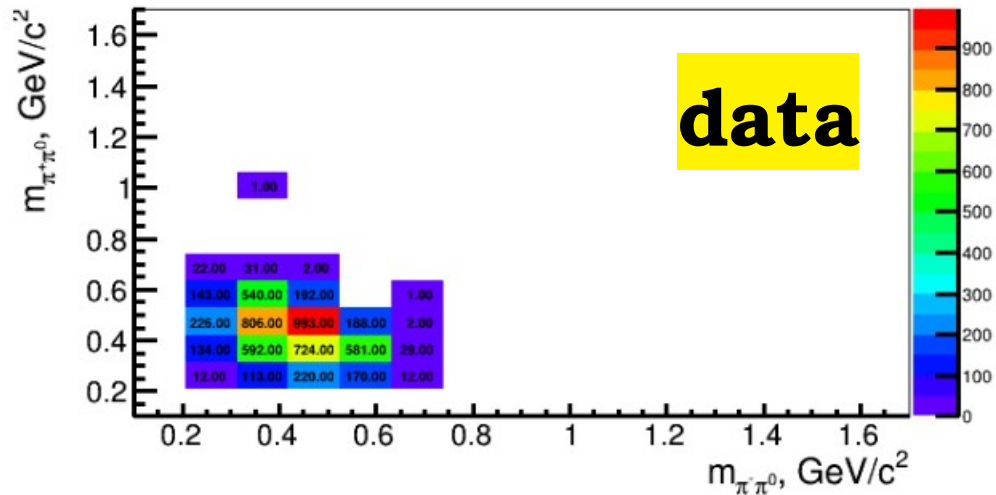


# $(ee \rightarrow 2\pi^+2\pi^-)$



# $(ee \rightarrow 2\pi^+2\pi^-)$

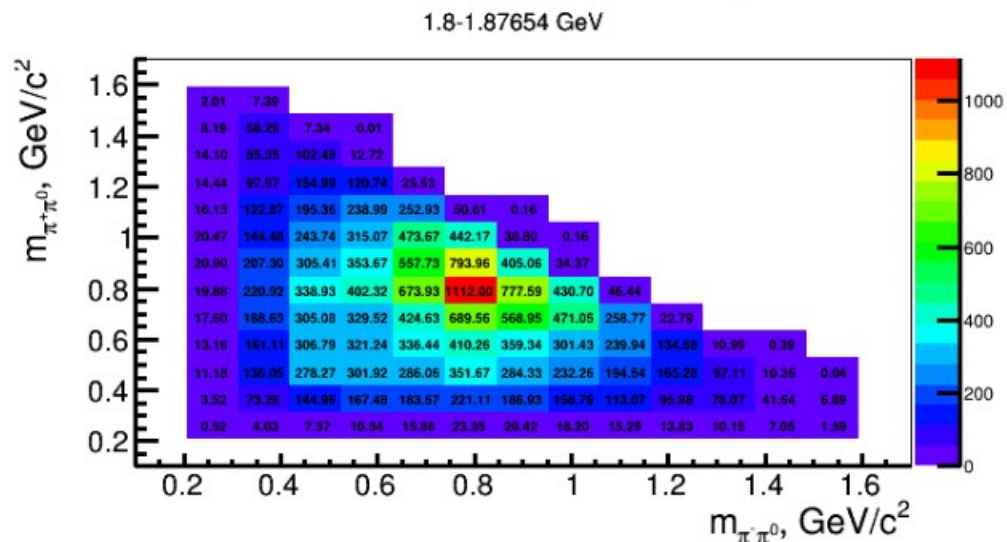
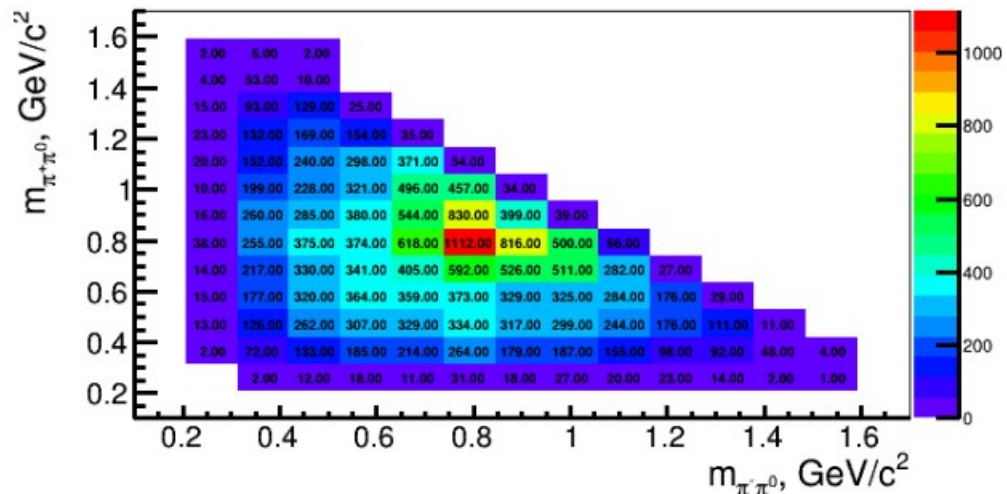
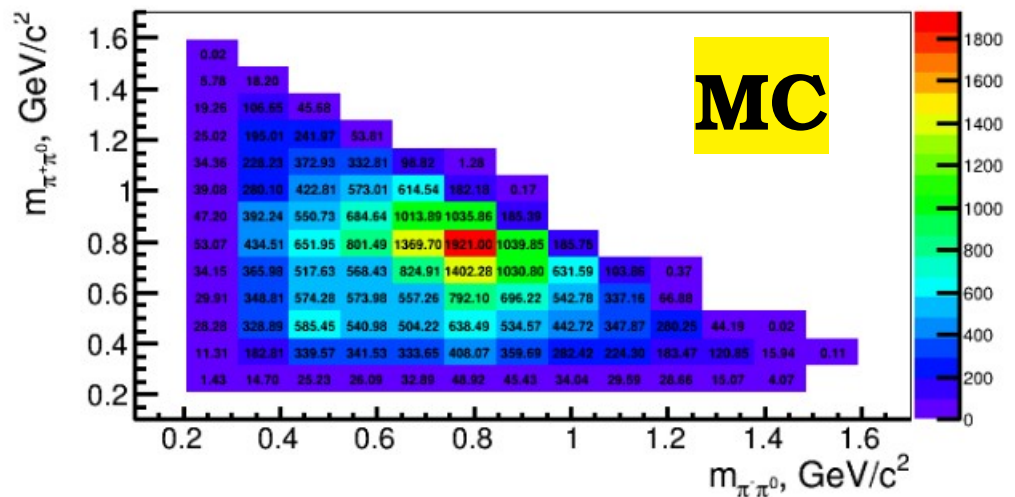
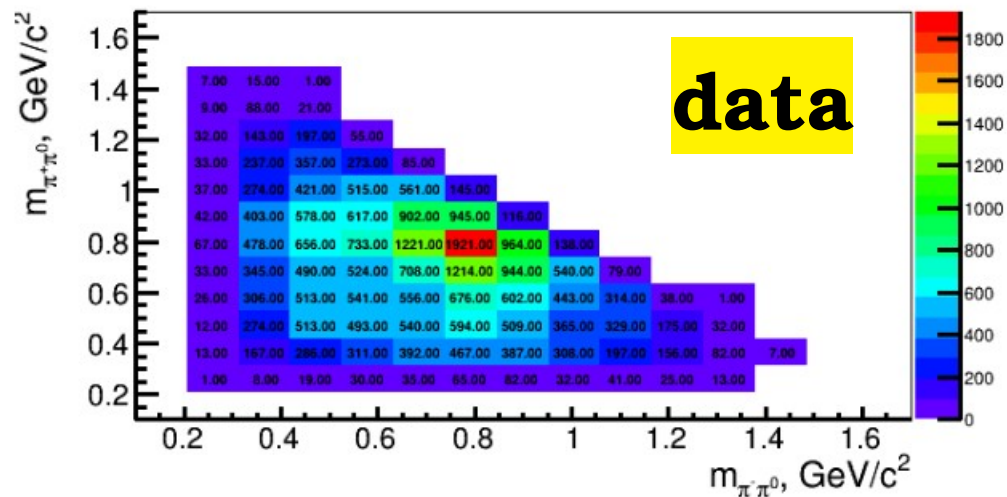






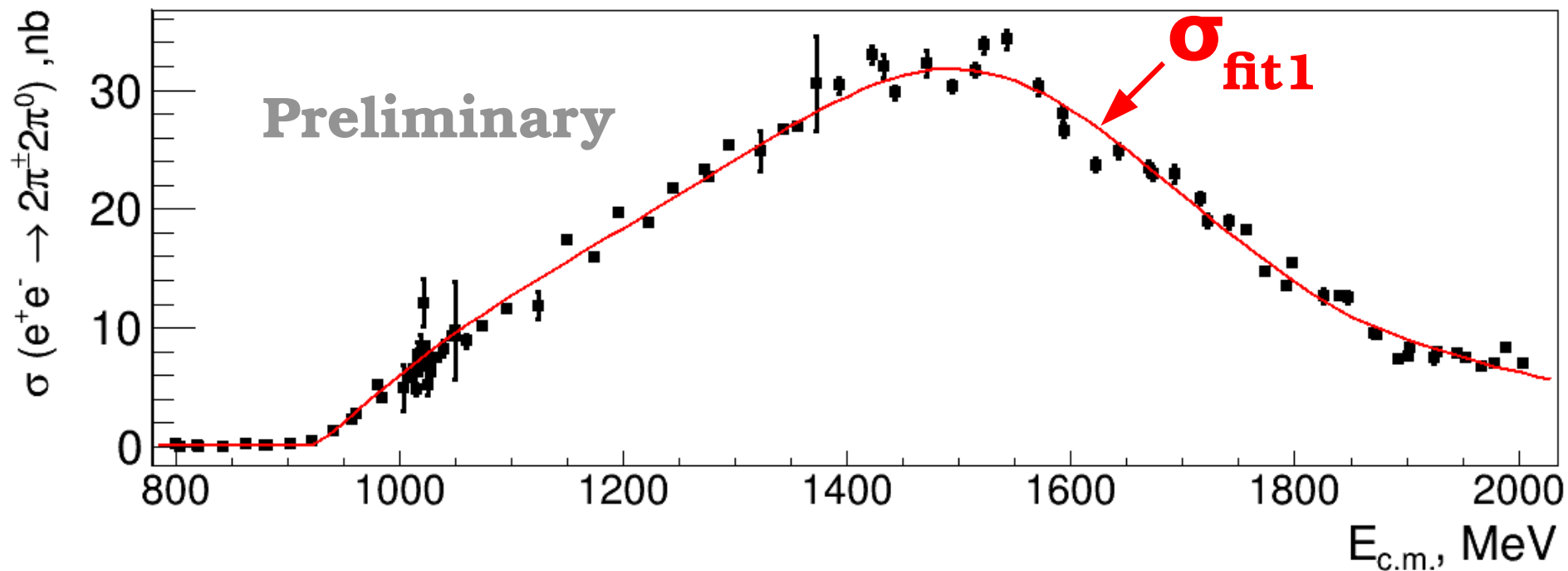






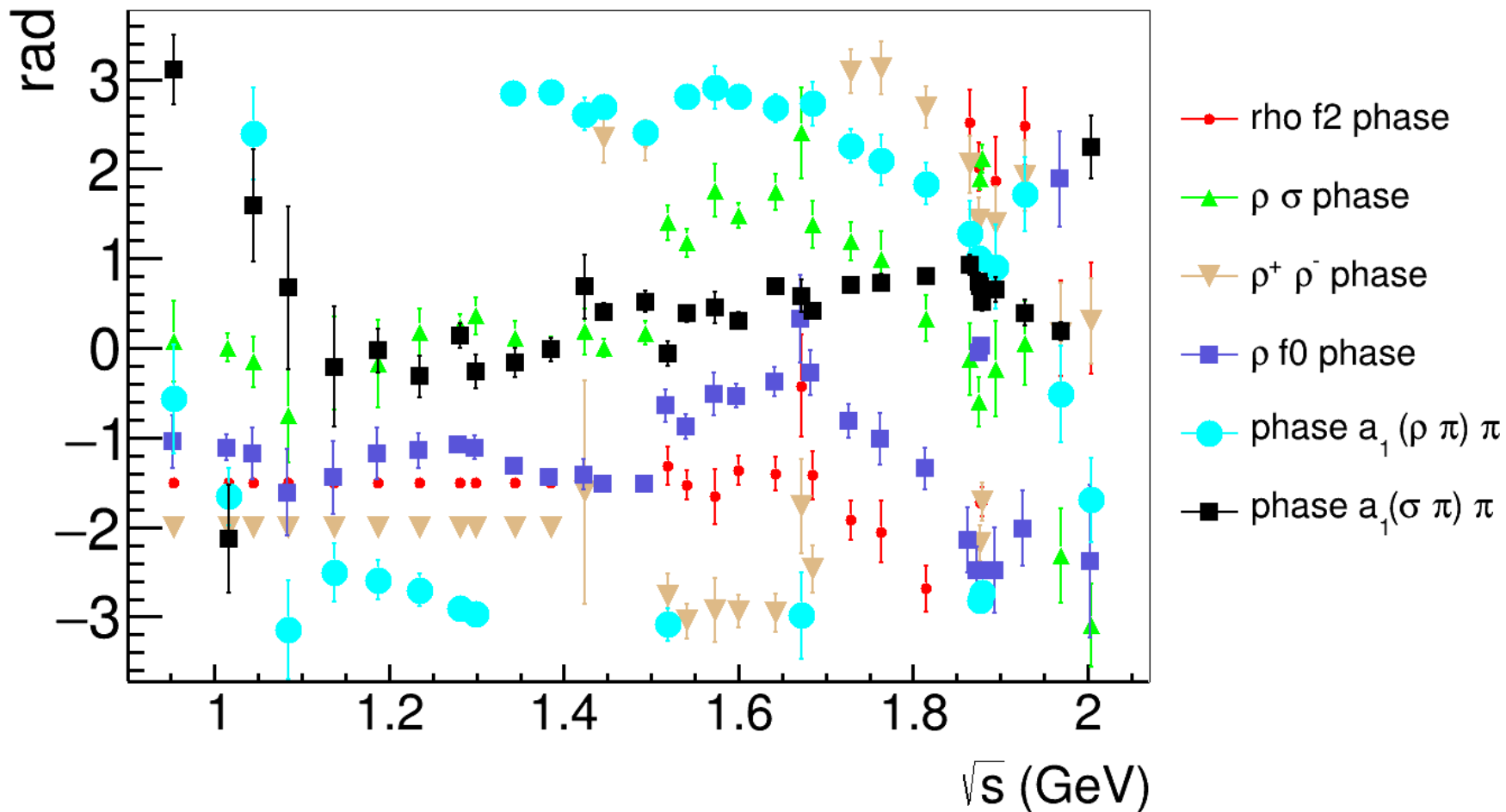






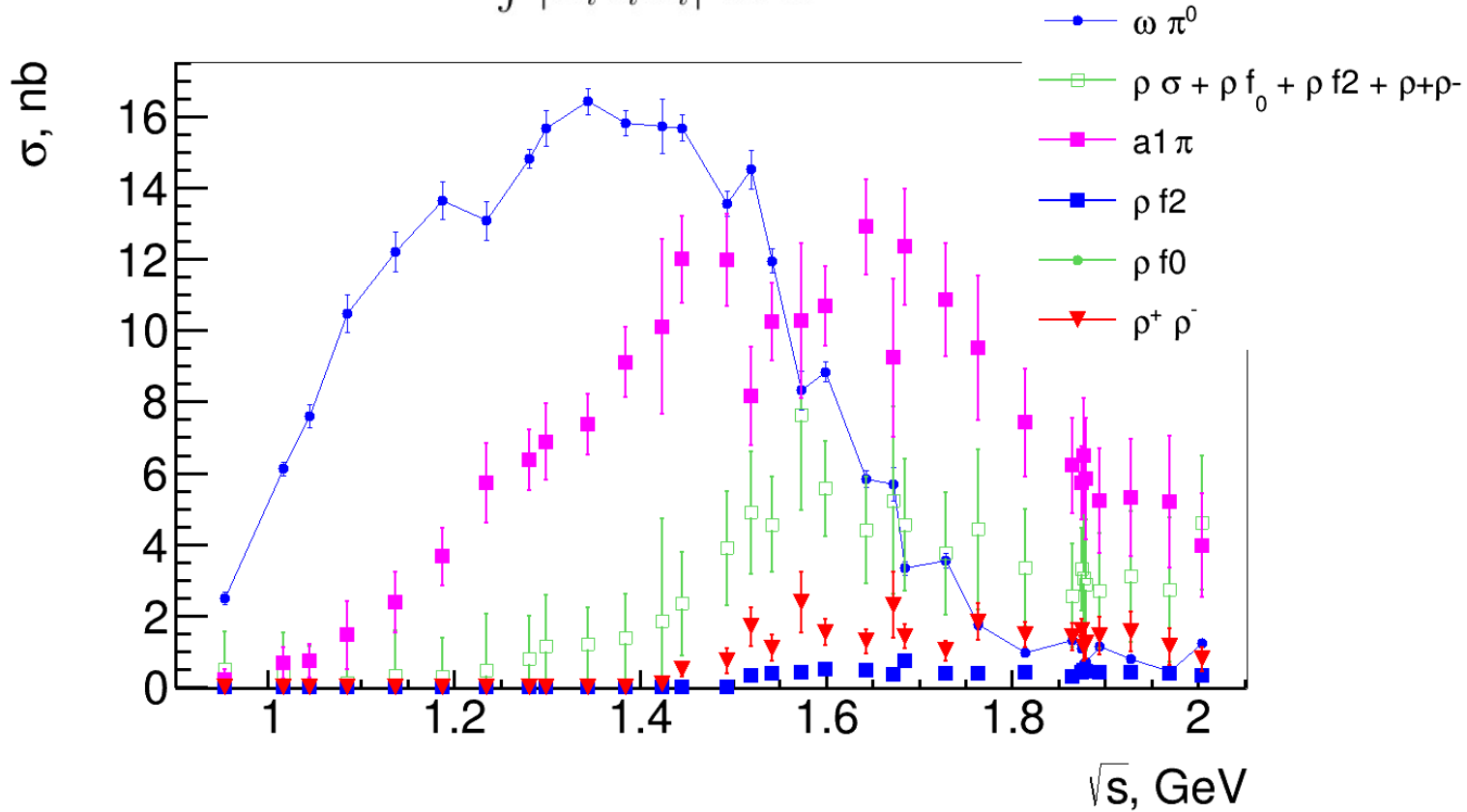
The cross-section of  $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$  obtained by using the model described above

# results

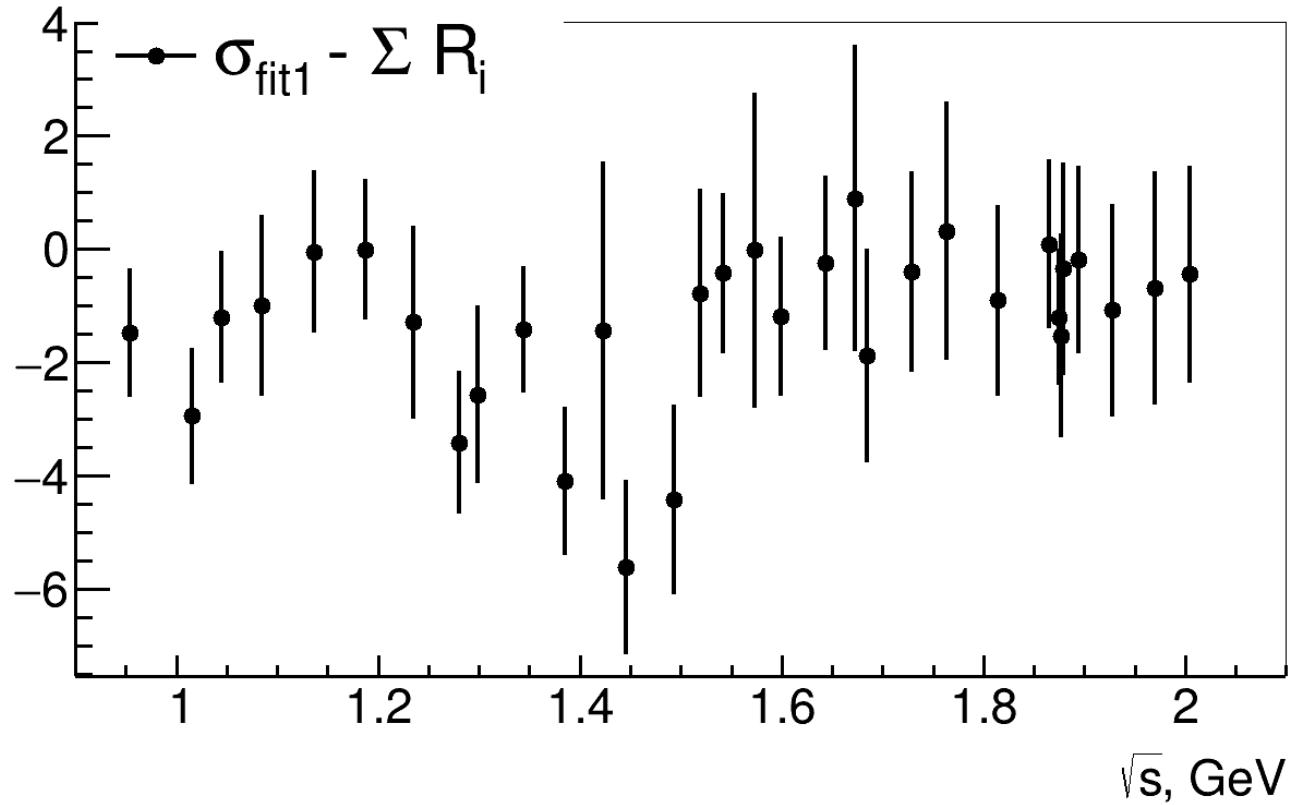


- Neither parameterization of the amplitude does not describe the energy dependence of the phase

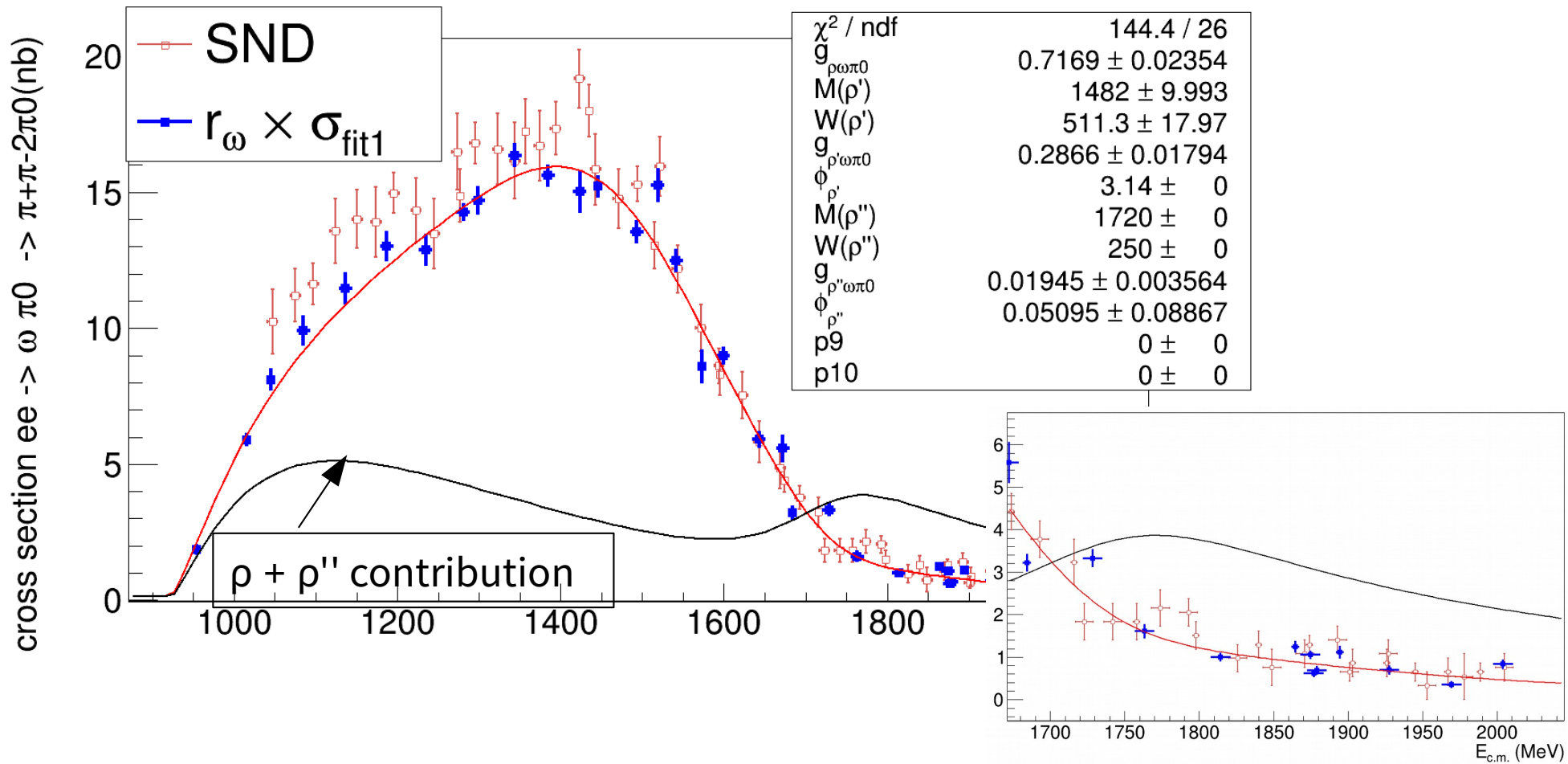
$$R_i = \frac{\int |A_i|^2 dPh}{\int |\sum_i V_i A_i|^2 dPh} \cdot \sigma_{2\pi^0\pi^+\pi^-}$$



The cross-section of different contribution vs  $E_{\text{c.m.}}$



The cross-section of  $e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^+\pi^-2\pi^0$  with subtracted sum of  $R_i$



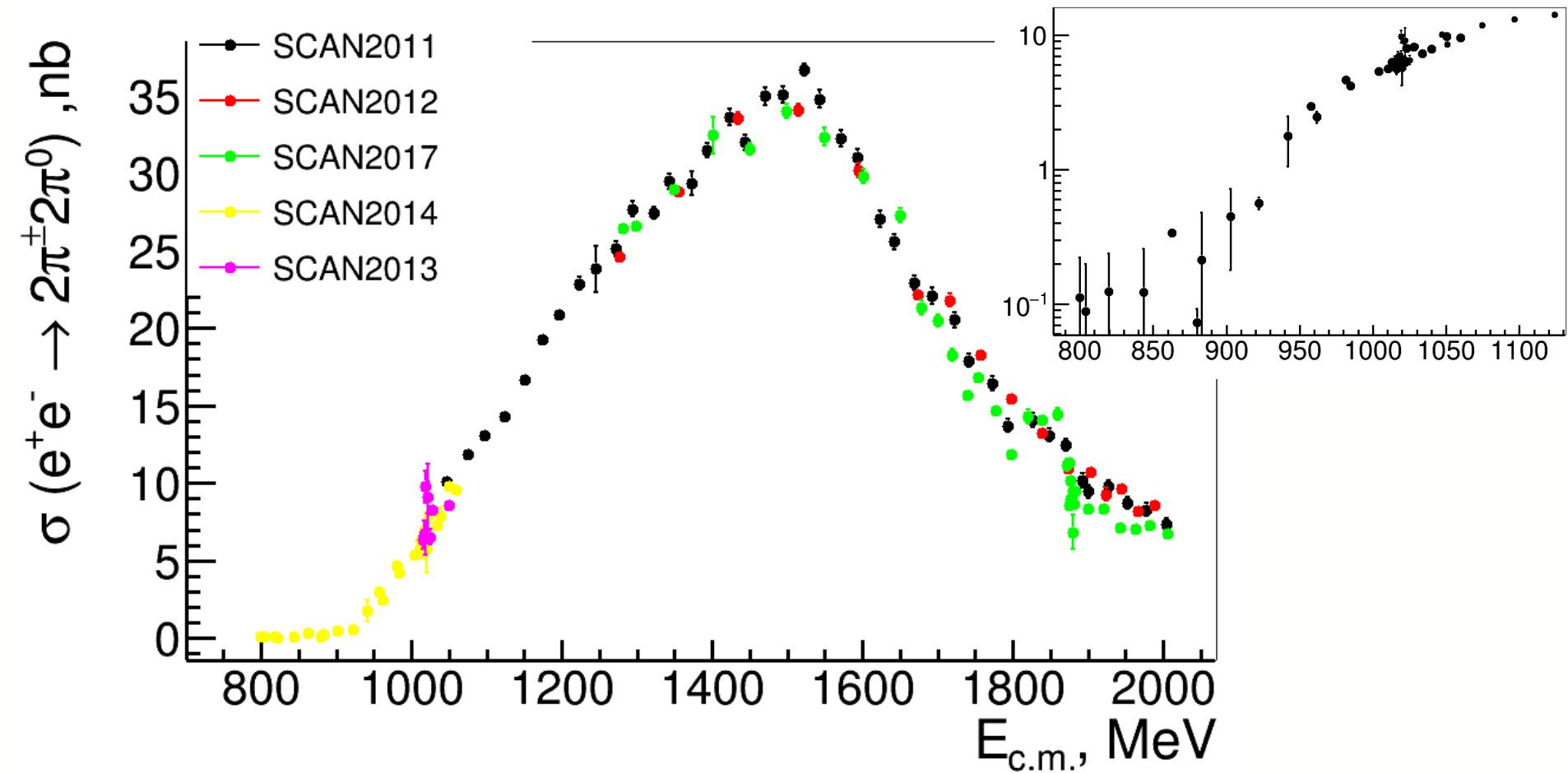
The cross-section of  $e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^+\pi^-2\pi^0$  vs  $E_{\text{c.m.}}$

# Summary

- The internal dynamics of the reaction  $e^+e^- \rightarrow 4\pi$  is studied preliminary in the energy range  $\sqrt{s} = [0.95-2.01]$  GeV
- The dominance of the channels  $\omega\pi$  and  $a_1\pi$  is confirmed.  
The  $\omega\pi$  channel is dominated in the process  $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$  up to
- $\sqrt{s} \sim 1.5$  GeV
- The estimation of the contributions of  $\rho\sigma$ ,  $\rho f_0$ ,  $\rho^-\rho^+$ ,  $\rho f_2$  is shown.
- The simultaneous analysis of  $e^+e^- \rightarrow 2\pi^+2\pi^-$ ,  $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ ,  $\tau^- \rightarrow \pi^-3\pi^0\nu$ ,  $\tau^- \rightarrow 2\pi^-\pi^+\pi^0\nu$  is proposed

back-up



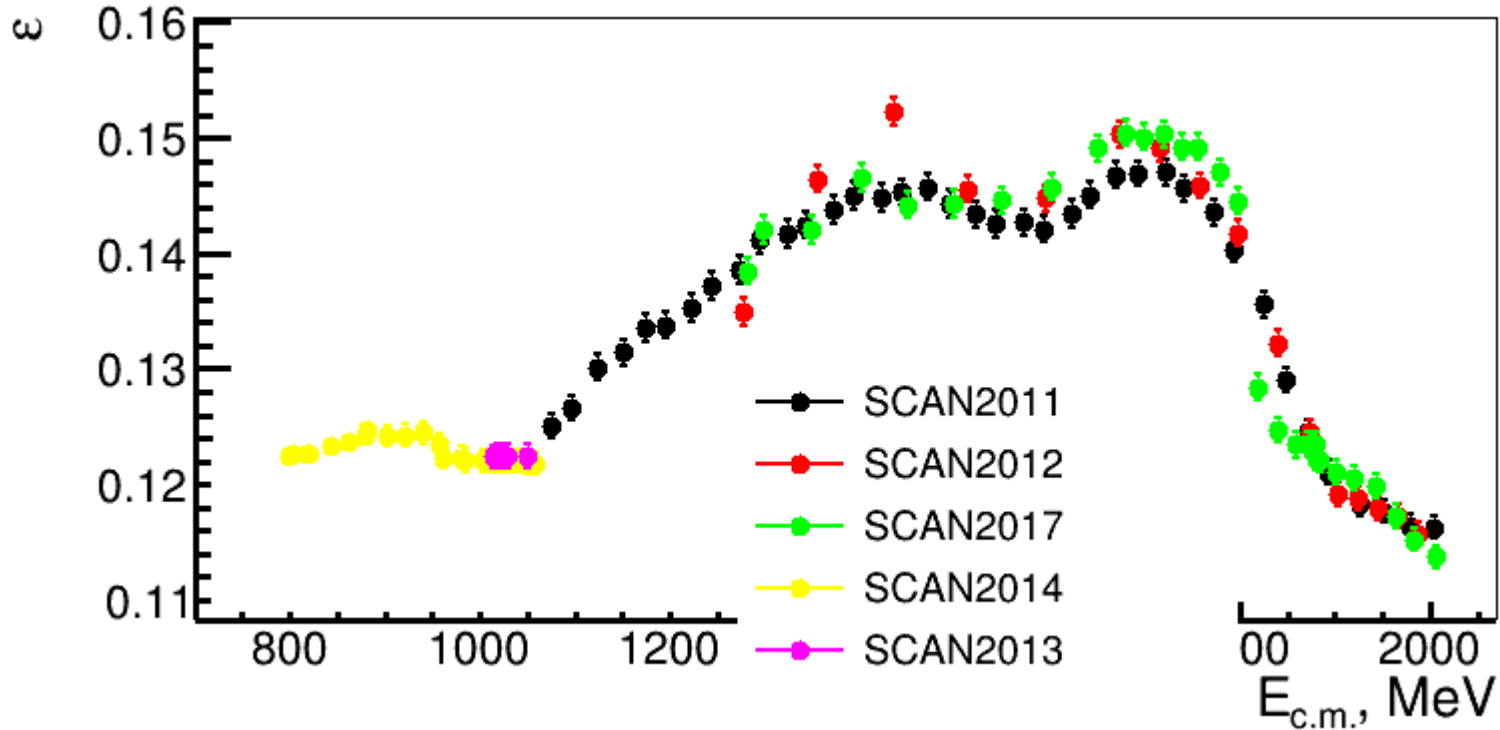


The cross-section of  $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$  vs  $E_{c.m.}$  for different scans

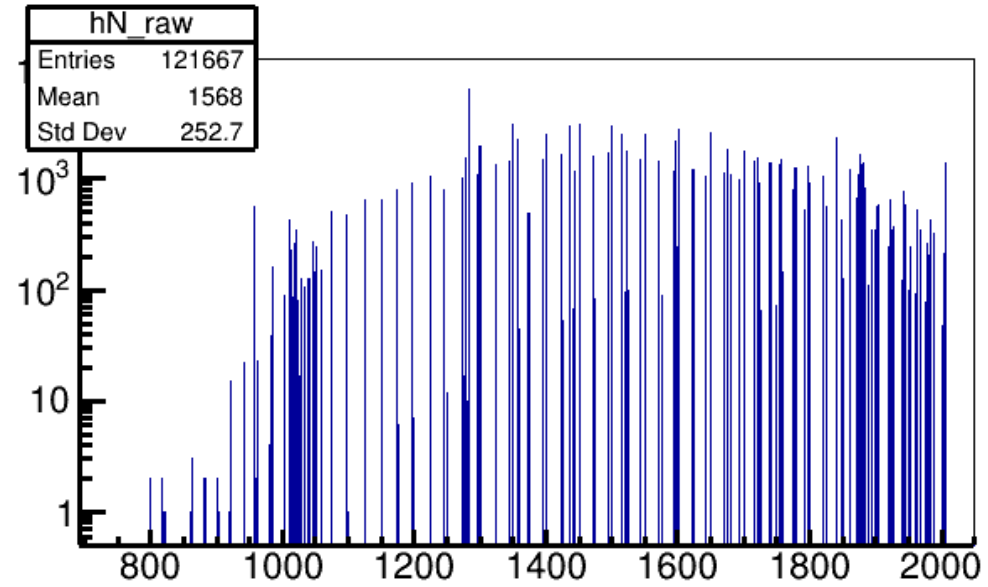
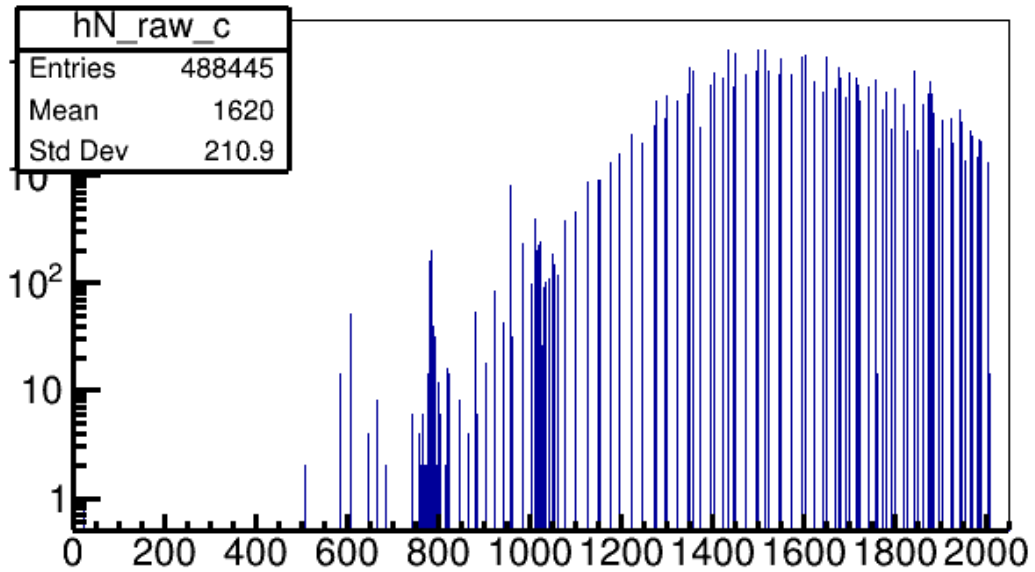
Soon we will publish the cross section of the cross section

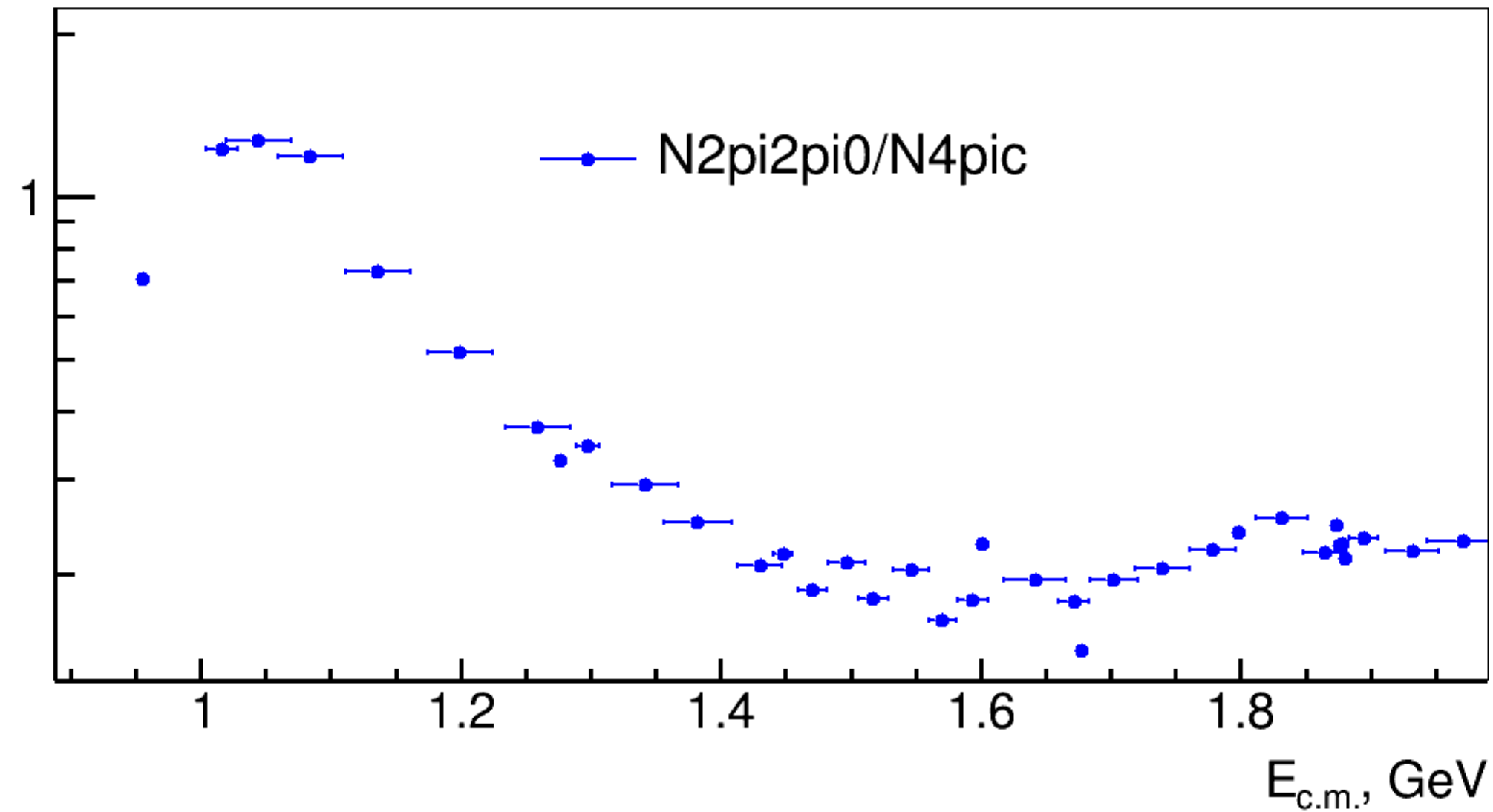
$$\varepsilon_{4\pi} = \frac{\sum_{det}^{ph.sp.} (|M(p_{\pi^0}, p_{\pi^0}, p_{\pi^+}, p_{\pi^-})|^2)}{\sum_{gen}^{ph.sp.} (|M(p_{\pi^0}, p_{\pi^0}, p_{\pi^+}, p_{\pi^-})|^2)}$$

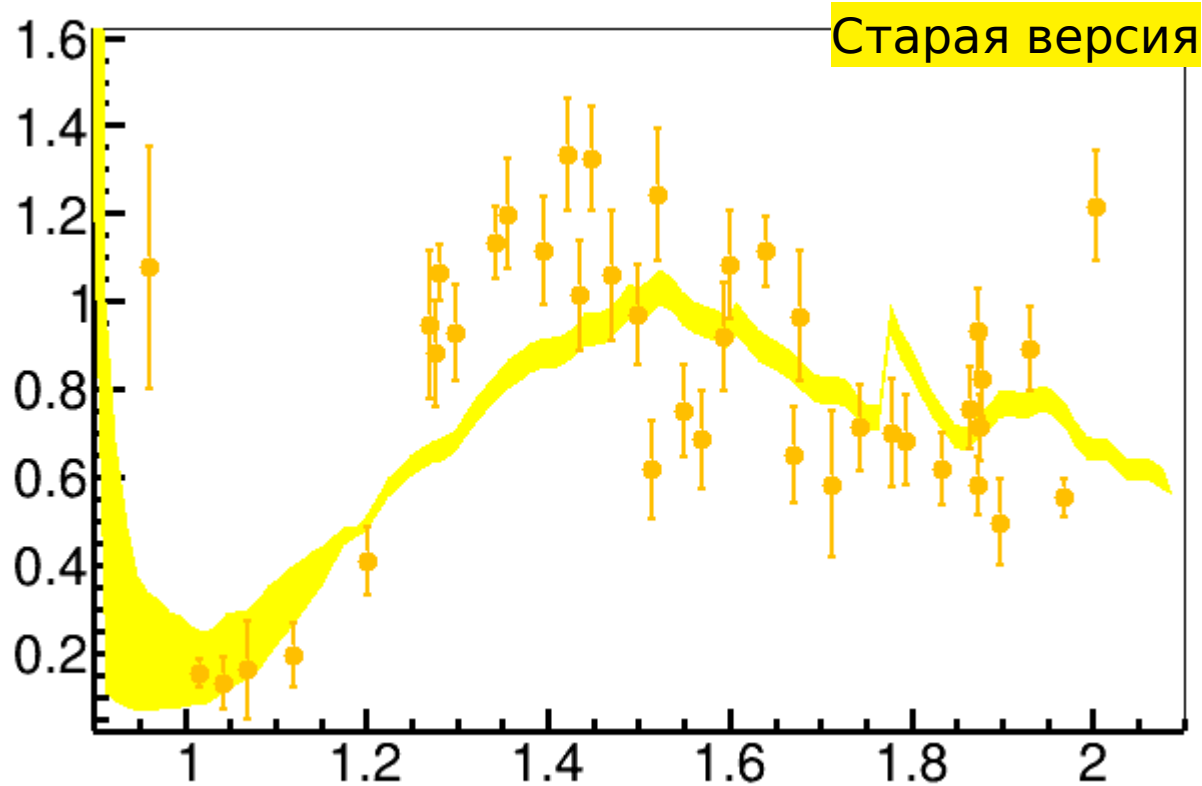
The max energy of simulated ISR is 0.2 GeV.



Detection efficiency vs. c.m. energy.



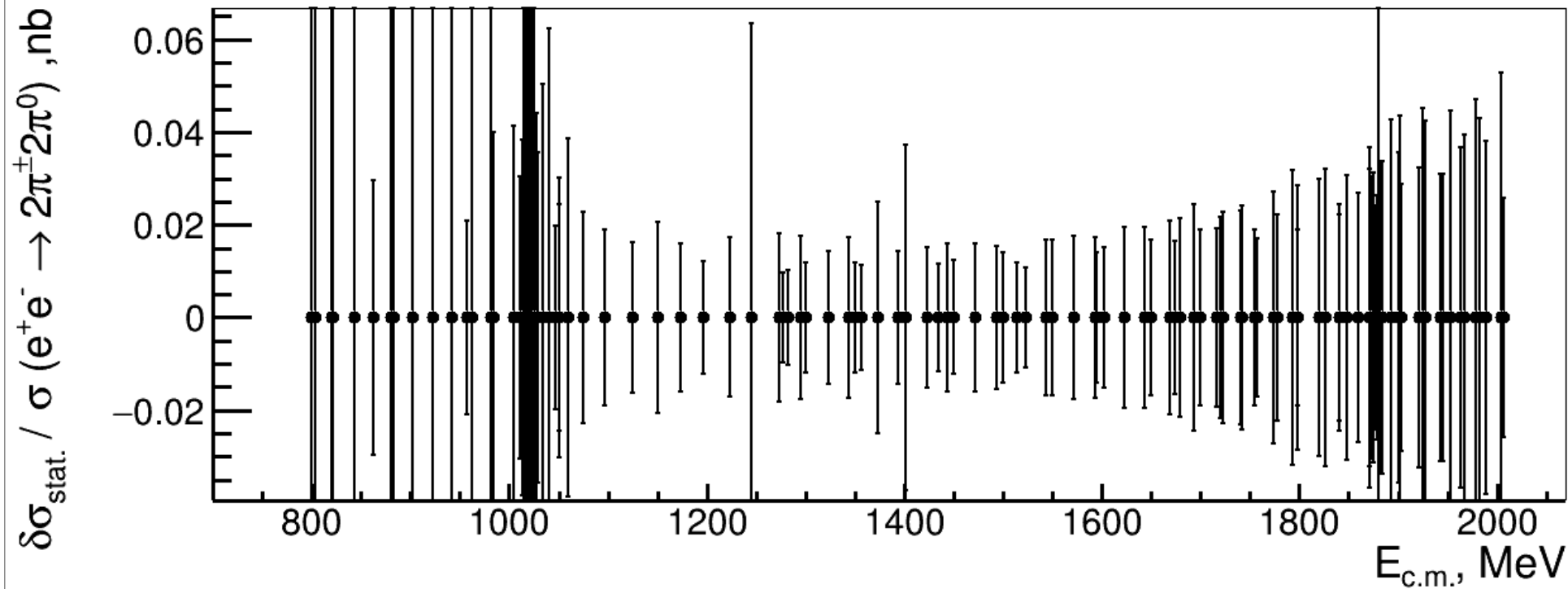




Отношение сечений  $e^+e^- \rightarrow 2\pi^+2\pi^-$  к  $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ .

Закрашенная область — эксперимент (из др. работ).

Точки с ошибками — результат амплитудного анализа (из данной работы).



Statistical precision is about 2-3% in average