Off-shell fermion polarization and t-quark production

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Introduction

It is well known how to calculate polarization of a final electron in framework of QFT, see e.g. textbook *Quantum Electrodynamics* of Berestetskii, Pitaevskii and Lifshitz.

If we are interested in polarization of intermediate fermion, first of all we need to give an accurate definition for this value. However, the concept of polarization in intermediate state is used for a long time in particle physics. One can recall the account of polarization in the method of equivalent photons (V. M. Budnev, I. F. Ginzburg, G. V. Meledin and V. G. Serbo, Phys. Rep. 15, 181 (1975)).

Another example — experimental and theoretical activity concerning of polarization of t-quark produced in hadron collisions, see e.g. review W. Bernreuther and P. Uwer, Nucl. Part. Phys. Proc. 261-262, 414 (2015). Note that in the case of t-quark the naive definition for polarization is used in analogy with on-mass-shell particle.

In method of equivalent electrons (V. N. Baier, E. A. Kuraev, V. S. Fadin and V. A. Khoze, Phys. Rep. 78, 293 (1981)) polarization was not taken into account.

Outline

- Polarization of final fermion
  - Standard method of calculation
  - Problem of looking for axis of complete polarization
  - Equivalence of two problems
- Spectral representation of fermion propagator
  - Bare propagator
  - Dressed propagator and generalized spin projectors
- Polarization in intermediate state
- Parametrization of t-quark propagator

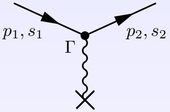
Details:

A.E.Kaloshin, V.P.Lomov. On the polarization of fermion in an intermediate state. Int.J.Mod.Phys. A32 (2017) 1750096

A.E.Kaloshin, V.P.Lomov. Mixing of fermions and spectral representation of propagator. Int.J.Mod.Phys. A31 (2016) 1650031

Polarization of final electron – standard approach

Let electron from initial state with momentum $p_{1\mu}$ and polarization vector $s_{1\mu}$ is turned into final state with $p_{2\mu}$ and $s_{2\mu}$ (polarization selected by a detector)



Matrix element

$$\mathscr{M} = \bar{u}_2(p_2, s_2)\Gamma u_1(p_1, s_1).$$

Square of matrix element

$$|\mathscr{M}|^2 = \mathrm{Sp}\left(u_2\bar{u}_2\Gamma u_1\bar{u}_1\tilde{\Gamma}\right) = A + B_\mu s_2^\mu = A\left(1 + \frac{B_\mu}{A}s_2^\mu\right), \quad \tilde{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0. \tag{1}$$

Here we wrote terms dependent and independent on s_2. Since $(s_2p_2) = 0$, only transverse part of vector B_μ remains

$$B_\mu^\perp = \left(g_{\mu\nu} - \frac{p_{2\mu}p_{2\nu}}{m^2}\right) \cdot B^\nu.$$

Polarization of final electron – standard approach

The matrix element square (1) is in fact projection of the scattered electron density matrix (its spin part is defined by the vector $s_\mu^{(f)}$) onto the detector density matrix ρ'. Thus comparison of (1) with

$$\mathrm{Sp}(\rho'\rho) = \mathrm{Sp}\left(\frac{m+\hat{p}_2}{2m}\cdot\frac{1+\gamma^5\hat{s}_2}{2}\cdot\frac{1+\gamma^5\hat{s}^{(f)}}{2}\right) = \frac{1}{2}\Big(1-\big(s_2 s^{(f)}\big)\Big) \quad (2)$$

gives the final electron polarization $s_\mu^{(f)}$ as such:

$$s_\mu^{(f)} = -\frac{B_\mu^\perp}{A}. \quad (3)$$

Let us introduce short notations for final state projectors

$$\begin{aligned}
\Lambda_2^\pm &= \Lambda_m^\pm(n_2) = \frac{1}{2}\big(1\pm\hat{n}_2\big), \quad \hat{n}_2 = \frac{\hat{p}_2}{m}, \quad n_2^2 = 1, \\
\Sigma_2 &= \Sigma_0(s_2) = \frac{1}{2}\big(1+\gamma^5\hat{s}_2\big), \quad s_2^2 = -1, \quad (s_2 n_2) = 0
\end{aligned} \quad (4)$$

and similarly for initial state.

Polarization of final electron – standard approach

The matrix element square in these notations is proportial to

$$|\mathscr{M}|^2 \sim \mathrm{Sp}\left(\Lambda_2^+ \Sigma_2 \Gamma \Lambda_1^+ \Sigma_1 \tilde{\Gamma}\right) = \mathrm{Sp}\left(\Sigma_2 X\right) = \mathrm{Sp}\left(\frac{1}{2}(1+\gamma^5 \hat{s}_2) X\right). \quad (5)$$

Here we have introduced matrix, which is used below

$$X = \Lambda_2^+ \Gamma \Lambda_1^+ \Sigma_1 \tilde{\Gamma} \Lambda_2^+. \quad (6)$$

The coefficients A and B_μ in (1) are calculated like

$$A = \frac{1}{2}\,\mathrm{Sp}\left(X\right), \qquad B_\mu = \frac{1}{2}\,\mathrm{Sp}\left(\gamma^5 \gamma_\mu X\right), \quad (7)$$

and the orthogonality property $B_\mu p_2^\mu = 0$ is seen from it.

Polarization of final electron – standard approach

Let us find the decomposition of the matrix X in γ-matrix basis. Simple calculations show that all coefficients are easily expressed by means of p_2, A, B_μ

$$X = \frac{1}{2}(1+\hat{n}_2)\big(A - \gamma^5 \hat{B}\big) = \frac{A}{2}(1+\hat{n}_2)\big(1 - \gamma^5 \hat{B}/A\big). \tag{8}$$

Recall that $s_\mu^{(f)} = -B_\mu/A$ is final electron polarization.

The search for complete polarization axis of bispinor

After scattering, described by the amplitude (4) we have a new state

$$u(p_1, s_1) \to \Lambda_2^+ \Gamma u(p_1, s_1) = \Lambda_2^+ \Gamma \Lambda_1^+ \Sigma_1 u(p_1, s_1). \qquad (9)$$

Let us consider the problem of search for complete polarization axis z_μ of bispinor of scattered electron

$$\gamma^5 \hat{z} \cdot \Lambda_2^+ \Gamma u_1 = \Lambda_2^+ \Gamma u_1, \quad (z n_2) = 0. \qquad (10)$$

We know in advance that this problem has a solution. Let us rewrite the equation in equivalent form

$$\frac{1 + \gamma^5 \hat{z}}{2} \cdot \Lambda_2^+ \Gamma u_1 = \Lambda_2^+ \Gamma u_1. \qquad (11)$$

Equivalence of two problems

Let us show that the complete polarization axis z coincides with $s^{(f)}$. Start from the problem

$$\frac{1+\gamma^5\hat{z}}{2}\cdot\Lambda_2^+\Gamma u_1=\Lambda_2^+\Gamma u_1. \tag{12}$$

If we take Hermitian adjoint of this equation

$$\bar{u}_1\tilde{\Gamma}\Lambda_2^+\cdot\frac{1+\gamma^5\hat{z}}{2}=\bar{u}_1\tilde{\Gamma}\Lambda_2^+. \tag{13}$$

Multiplying both equations by each other and substituting the density matrix of initial electron $u_1\bar{u}_1$ one gets the matrix relation

$$\frac{1+\gamma^5\hat{z}}{2}\cdot\left(\Lambda_2^+\Gamma\Lambda_1^+\Sigma_1\tilde{\Gamma}\Lambda_2^+\right)\cdot\frac{1+\gamma^5\hat{z}}{2}=\left(\Lambda_2^+\Gamma\Lambda_1^+\Sigma_1\tilde{\Gamma}\Lambda_2^+\right)\equiv X. \tag{14}$$

This relation can be transformed into equation connecting the complete polarization axis z_μ and final electron polarization $s_\mu^{(f)}$.

Equivalence of two problems

We see that the matrix X satisfies the equations

$$(1-\gamma^5\hat{z})\cdot X = X\cdot(1-\gamma^5\hat{z}) = 0. \tag{15}$$

If to use for X the expression (8) found above we obtain two equations

$$(1+\hat{n}_2)(1-\gamma^5\hat{z})(1+\gamma^5\hat{s}^{(f)}) = (1+\hat{n}_2)(1+\gamma^5\hat{s}^{(f)})(1-\gamma^5\hat{z}) = 0. \tag{16}$$

Let us multiply the arisen spin matrices

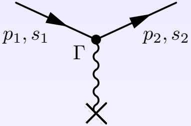
$$(1-\gamma^5\hat{z})(1+\gamma^5\hat{s}^{(f)}) = 1+(zs^{(f)})+\gamma^5(\hat{s}^{(f)}-\hat{z})+\sigma^{\mu\nu}z_\mu s^{(f)}_\nu = 0,$$
$$(1+\gamma^5\hat{s}^{(f)})(1-\gamma^5\hat{z}) = 1+(zs^{(f)})+\gamma^5(\hat{s}^{(f)}-\hat{z})-\sigma^{\mu\nu}z_\mu s^{(f)}_\nu = 0.$$

It immediately follows that these two vectors coincide $s^{(f)} = z$.

These two problems are equivalent, but in the problem of looking for the axis of complete polarization the amplitude is used instead of its square. This makes possible to apply the same method for calculation of fermion polarization both for final and intermediate states.

Electron scattering in an external field

As a simple example let us consider scattering of electron in an external field.



The vertex factor Γ contains Fourier transform of the external field and corresponding γ-matrix.
The problem of looking for axis (35) can be rewritten as

$$\frac{1+\gamma^5 \hat{z}}{2} \cdot \Lambda_2^+ \Gamma \Lambda_1^+ \Sigma_1 \chi = \Lambda_2^+ \Gamma \Lambda_1^+ \Sigma_1 \chi, \tag{17}$$

where χ is an arbitrary bispinor. The found equivalence $s^{(f)} = z$ tells that vector z does not depend on bispinor χ.

So one cat rewrite the problem as matrix one

$$\frac{1+\gamma^5\hat{z}}{2}\cdot\Lambda_2^+\Gamma\Lambda_1^+\Sigma_1 = \Lambda_2^+\Gamma\Lambda_1^+\Sigma_1. \tag{18}$$

and it gives convenient method to look for vector z with use of γ-matrix basis. We checked that for external fields of different kinds (S, P, V, A) the solution z of the problem (18) coincides $s^{(f)}$ (for definite polarization of initial fermion, $s_1^2 = -1$).

For vector vertex $\Gamma = \gamma^\mu A_\mu(q)$ we obtain

$$\begin{aligned}
z_\mu &= s_{1\mu} - a_1 p_{1\mu} - a_2 p_{2\mu} - a_3 A_\mu, \\
a_1 &= -a_2 = \frac{(p_2 s_1)(AA) - 2(p_2 A)(s_1 A)}{D}, \\
a_3 &= 2\frac{(p_1 p_2)(s_1 A) - (p_1 A)(p_2 s_1) - (s_1 A)m^2}{D}, \\
D &= (p_1 p_2)(AA) - 2(p_1 A)(p_2 A) - (AA)m^2.
\end{aligned} \tag{19}$$

Spectral representation of propagator

We want to apply the problem of looking for complete polarization axis (35) to the case of fermion in an intermediate state.
To construct SR one needs to solve the eigenvalue problem for inverse propagator.

$$S\Pi = \lambda\Pi. \tag{20}$$

Having found eigenvalues λ_i and eigenprojectors Π_i

$$\Pi_i\Pi_k = \delta_{ik}\Pi_i, \quad i,k = 1,2, \tag{21}$$

we can construct the spectral representation of inverse propagator

$$S(p) = \lambda_1\Pi_1 + \lambda_2\Pi_2. \tag{22}$$

If the system of projectors is complete, then this expression can be easily reversed and propagator looks like this:

$$G(p) = \frac{1}{\lambda_1}\Pi_1 + \frac{1}{\lambda_2}\Pi_2, \tag{23}$$

i.e. propagator poles are zeroes of eigenvalues λ_i.

Bare propagator case

The eigenprojectors Π_i for a bare propagator are the known off-shell projector operators $\Lambda^{\pm}_W$

$$\Lambda^{\pm}_W = \frac{1}{2}\Big(1 \pm \frac{\hat{p}}{W}\Big), \quad p^2 = W^2, \tag{24}$$

where W is center-of-mass energy. As a result the bare propagator

$$G_0(p) = \frac{1}{\hat{p} - m_0} = \frac{1}{W - m_0}\Lambda^{+}_W + \frac{1}{-W - m_0}\Lambda^{-}_W \tag{25}$$

looks as a sum of poles with positive and negative energies. It is necessary to stress that we have covariant separation of poles $1/(W \pm m_0)$. Eigenvalues are

$$\lambda_1 = W - m_0, \quad \lambda_2 = -W - m_0$$

Dressed propagator case

With account of interaction

$$S(p) = \hat{p} - m_0 - \Sigma(p). \tag{26}$$

SR looks differently depending on interaction.

- If theory conserves parity, then $\Sigma(p)$ contains unit matrix and $\hat{p}$

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2) = \Sigma^+(W)\Lambda^+_W + \Sigma^-(W)\Lambda^-_W, \tag{27}$$

  where $\Sigma^\pm(W) = A(W^2) \pm W B(W^2)$. In this case

$$G(p) = \frac{1}{W - m_0 - \Sigma^+(W)}\Lambda^+_W + \frac{1}{-W - m_0 - \Sigma^-(W)}\Lambda^-_W. \tag{28}$$

- In theory with γ^5 the self-energy also has γ^5 terms

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2) + \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2), \tag{29}$$

  and eigenprojectors Π_i do not coincide with $\Lambda^\pm_W$.

Dressed propagator in theory with parity violation

In case of parity violation the eigenprojectors:

$$\Pi_{1,2}(p) = \frac{1}{2}\big(1 \pm \hat{n}\tau\big), \quad \hat{n} = \frac{\hat{p}}{W},$$
$$\tau = \frac{1}{R}\Big(1 - B - \gamma^5 D - \hat{n}\gamma^5 \frac{C}{W}\Big),$$
$$R = \sqrt{(1-B)^2 - D^2 + C^2/W^2}, \tag{30}$$

and eigenvalues $\lambda_i(W)$ are

$$\lambda_{1,2}(W) = -m_0 - A(W^2) \pm W R(W^2). \tag{31}$$

Dressed propagator and spin projectors

An essential aspect – the existence of spin projectors commuting with propagator. Note, that the standard spin projectors

$$\Sigma_0(s) = \frac{1 + \gamma^5 \hat{s}}{2}, \quad s^2 = -1, \quad (sp) = 0, \tag{32}$$

cease to commute with propagator in the presence of γ^5 in a vertex. Nevertheless, there exist the generalized spin projectors (Kaloshin, Lomov (2015)) having all desired properties.

"Under observation" of the energy eigenprojector $\Pi_i(p)$ the generalized spin projectors take simple form

$$\Pi_i(p)\Sigma(s) = \Pi_i(p)\,\frac{1}{2}\big(1 + \gamma^5 \hat{s}\hat{n}\big), \quad n^\mu = p^\mu/W. \tag{33}$$

So, in theory with parity violation

$$\Sigma_0(s) = \frac{1}{2}(1 + \gamma^5 \hat{s}) \quad \Rightarrow \quad \Sigma(s) = \frac{1}{2}(1 + \gamma^5 \hat{s}\hat{n}) \tag{34}$$

Axis of complete polarization for virtual fermion

The problem of looking for complete polarization axis of bispinor

$$\frac{1+\gamma^5\hat{z}}{2}\cdot\Lambda_2^+\Gamma u_1=\Lambda_2^+\Gamma u_1. \tag{35}$$

can be easily extended for fermion in intermediate state.
Look again at free propagator

$$G_0(p)=\frac{1}{\hat{p}-m}=\frac{1}{(W-m)}\Lambda_W^+ +\frac{1}{-(W+m)}\Lambda_W^-. \tag{36}$$

One needs only to change projector Λ_2^+ in (35) to one of off-shell projectors $\Lambda_W^\pm(p_2)=(1\pm\hat{p}_2/W)/2$, $p_2^2=W^2$. The problem is turned into:

$$\frac{1}{2}(1+\gamma^5\hat{z}^\pm)\cdot\Lambda_W^\pm(p_2)\Gamma u_1=\Lambda_W^\pm(p_2)\Gamma u_1,\quad (z^\pm p_2)=0, \tag{37}$$

and such problem also has solution: there always exists a vector $z_\mu^\pm$, such as $(z^\pm p_2)=0$, $(z^\pm)^2=-1$.

Axis of complete polarization for virtual fermion

The aforesaid is also true for dressed propagator in theory with γ^5.

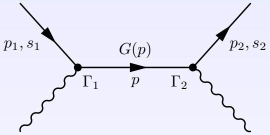
Let us write down the problem of looking for complete polarization axis for the dressed energy and spin projectors:

$$\Sigma(z^{\pm}) \cdot \Pi^{\pm}(p_2)\Gamma\Lambda_1^{+}\Sigma_1 u_1 = \Pi^{\pm}(p_2)\Gamma\Lambda_1^{+}\Sigma_1 u_1, \quad (z^{\pm}p_2) = 0. \tag{38}$$

Properties of the problem are the same, but here the “dressed” projectors take part.

Polarization of fermion in an intermediate state

The spectral representation of propagator allows to give an accurate definition of fermion polarization in an intermediate state.
Consider some process with intermediate fermion



The corresponding amplitude

$$\mathscr{M} = \bar{u}_2(p_2, s_2)\Gamma_1 G(p)\Gamma_2 u_1(p_1, s_1). \tag{39}$$

For the case of bare propagator or theory without γ^5 the fermion propagator in the intermediate state has form

$$G(p) = \frac{1}{\lambda_1}\Lambda_W^+ + \frac{1}{\lambda_2}\Lambda_W^-, \qquad \Lambda^\pm = \frac{1}{2}\left(1 + \frac{\hat{p}}{W}\right) \tag{40}$$

where $\Lambda_W^\pm$ are off-shell energy projectors.

Polarization of fermion in an intermediate state

If to recall the problem of looking for complete polarization axis involving $\Lambda_W^{\pm}$ (37), the propagator in the amplitude (39) can be rewritten as following

$$G = \frac{1}{\lambda_1}\Sigma_0(z^+)\Lambda_W^+ + \frac{1}{\lambda_2}\Sigma_0(z^-)\Lambda_W^-. \tag{41}$$

It gives a correct definition for polarization of fermion in an intermediate state.

Polarization of fermion in an intermediate state

What if there is γ^5 in vertex? In this case dressed fermion propagator in intermediate state may be represented as

$$G(p) = \frac{1}{\lambda_1}\Pi_1(p) + \frac{1}{\lambda_2}\Pi_2(p), \tag{42}$$

where $\Pi_{1,2}(p)$ are the energy projectors (30). Using the problem (38) one can see that the dressed propagator inside the diagram acquires spin projectors

$$G \to \tilde{G} = \frac{1}{\lambda_1} \cdot \frac{(1+\gamma^5 \hat{z}_1 \hat{n})}{2}\Pi_1(p) + \frac{1}{\lambda_2} \cdot \frac{(1+\gamma^5 \hat{z}_2 \hat{n})}{2}\Pi_2(p), \quad (z^\pm)^2 = -1. \tag{43}$$

It should be pointed out that the dressed energy projectors $\Pi_i(p)$ presented here contain self-energy contributions and should be renormalized.

Renormalization (stable fermion)

Simplest method – to use On-Mass-Shell (OMS) scheme.
Dressed inverse propagator (CP is conserved)

$$S = \hat{p} - m - \Sigma^r(p), \qquad \Sigma^r(p) = \hat{p} B^r(p^2) + \hat{p}\gamma^5 D^r(p^2)$$

Recall the off-shell eigenprojectors ($\hat{n} = \hat{p}/W$)

$$\Pi^r_{1,2}(p) = \frac{1}{2}\big(1 \pm \hat{n}\tau\big), \quad \tau = (1 - B^r - \gamma^5 D^r)/R, \\ R = \sqrt{(1 - B^r)^2 - (D^r)^2}, \tag{44}$$

and eigenvalues $\lambda^r_i(W)$

$$\lambda^r_{1,2}(W) = -m - A^r \pm W R(W^2). \tag{45}$$

Renormalization (stable fermion)

Renormalized loops (Imaginary part=0)

$$\begin{aligned} A^r &= m\varkappa, \\ B^r &= \tilde{B}(W^2) - \varkappa \\ D^r &= \tilde{D}(W^2). \end{aligned}$$

Notations: $\tilde{D}(W^2) \equiv D(W^2) - D(m^2)$, $\varkappa = 2m^2 B'(m^2)$
It leads to simple properties of eigenvalues and eigenprojectors

$$\begin{aligned} \lambda_1^r(m) &= 0, \\ (\lambda_1^r)'(m) &= 1, \\ \Pi_1(W=m) &= \frac{1}{2}(1+\frac{\hat{p}}{m}) \end{aligned}$$

and similarly for λ_2, Π_2 at $W=-m$.

Renormalization (unstable fermion)

If loop contributions have imaginary part at $W = m$, one can use GOMS scheme.
Approximate form of eigenvalue in vicinity of $W = m$

$$\lambda_1 = W - m + i\frac{\Gamma(W)}{2} + O(g^2(W-m)) + O(g^4)$$

and eigenprogectors

$$\Pi_{1,2} = \frac{1}{2}(1 + \hat{n}\tau) \tag{46}$$

$$\tau(W^2) = 1 + i\frac{\Gamma(W)}{W}\gamma^5 + O(g^2(W^2 - m^2)) + O(g^4) \tag{47}$$

In matrix density generalized spin projectors may be written in two equivalent forms

$$\Sigma(s) = \frac{1}{2}\left(1 + \gamma^5 \hat{s}\tau\right) \quad \text{or} \quad \frac{1}{2}\left(1 + \gamma^5 \hat{s}\hat{n}\right)$$

About t-quatk resonance curve

We will write down an approximate expression

$$G(p) = \frac{1}{W - m + i\Gamma(W)/2} \cdot \Pi_1(p) \cdot \Sigma(z_1) + (\text{negative energy pole})$$

Modified energy and spin projectors

$$\Pi_1(p) = \frac{1}{2}\left[1 + \hat{n}(1 + i\frac{\Gamma(W)}{W}\gamma^5)\right]$$

$$\Sigma(z_1) = \frac{1}{2}\left[1 + \gamma^5 \hat{z_1}(1 + i\frac{\Gamma(W)}{W}\gamma^5)\right]$$

Notations:

$$W = \sqrt{p^2}, \quad n^\mu = p^\mu / W$$
$$z_1^\mu p_\mu = 0, \quad z_1^\mu z_{1\ \mu} = -1$$

Summary

We suggested to reformulate calculation of polarization of final electron as a problem of looking for the complete polarization axis of a produced state. As a result, one can use the same approach for fermion in intermediate state, if to write propagator in form of spectral representation.

These two poins together:
(i) problem of looking for the complete polarization axis and
(ii) spectral representation of fermion propagator
allow to give a correct definition for polarization of intermediate fermion.

Most interesting is the case of fermion resonance in theory with P-parity violation. Corresponding the energy and spin projection operators are modified in theory with γ^5 – we found their form.

The obtained projectors are used to give the most accurate parametrization of t-quark resonance curve including for its off-shell polarization.

Thank you for attention!

My thanks to Organizers !