Axial-vector meson LbL contribution to g-2 of muon in nonlocal quark model

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Plan

1. Motivation
2. Nonlocal quark model in pseudoscalar-scalar sector
3. Axial-vector contribution
4. Conclusions

Motivation

Cosmology tell us that 95% of matter is not described in text-books yet. Dark Matter surrounds us! Where it is ?

Two search strategies

1. High energy physics to excite heavy degrees of freedom. No any evidence till now.
2. Low energy physics to produce Rare processes in view of huge statistics.

There are some rough edges of SM.

Anomalous magnetic moment of the muon $(g-2)_\mu$ is most famous and stable example

Motivation

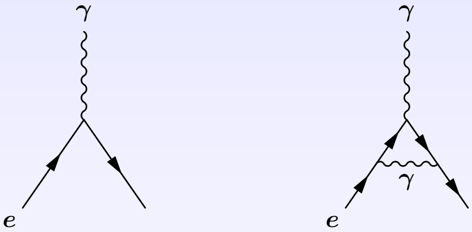
Dirac Equation Predicts for free point-like spin $\frac{1}{2}$ charged particle:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{p^2}{2m} - \frac{e}{2m}\left(\vec{L} + 2\vec{S}\right)\cdot\vec{B}\right]\psi$$

$g = 2$, $a = (g-2)/2 = 0$ (no anomaly at tree level)

a becomes nonzero due to interactions resulting in fermion substructure

Motivation. One loop QED radiative correction



$$\Gamma_\mu = e\gamma_\mu + a\frac{ie}{2m}\sigma_{\mu\nu}q_\nu$$

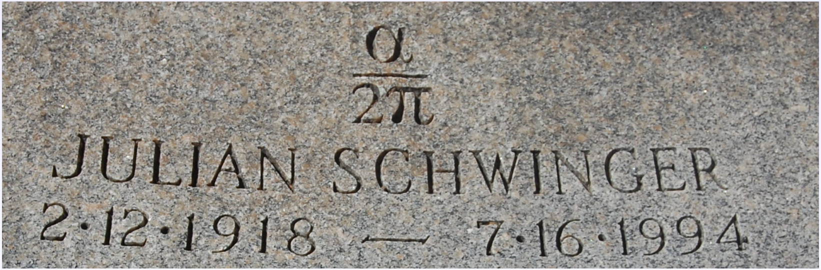
$$a_e = \frac{\alpha}{2\pi} = 0.001162,^{1}$$

$$a_e^{exp} = 0.001145 \pm 0.00004,^{2}$$

[1] J. S. Schwinger, Phys. Rev. **73** (1948) 416.

[2] H. M. Foley and P. Kusch, Phys. Rev. **72**, 1256 (1947).

Motivation. One loop QED radiative correction



Anomalous magnetic momentum of electron.

1. To measurable level a_e arises entirely from virtual electrons and photons

$$a_e^{\text{Harvard}} = 1\ 159\ 652\ 180.73\ (0.28) \times 10^{-12} \quad [0.24\ \text{ppb}].^{3}$$

2. In standard model

$$a_e = \left\{a_e^{\text{QED}} + a_e^{\text{weak}} + a_e^{\text{hadr}}\right\}^{\text{SM}}, \quad a_e^{\text{QED}} = \sum_{n=1}^{\infty}\left(\frac{\alpha}{\pi}\right)^n a_e^{(2n)},$$

3. This result leads to the determination of the fine structure constant α with the extraordinary precision [4]

$$\alpha^{-1} = 137.0359991570(29)(27)(18)(331)$$

where uncertainties are from the eighth-order, tenth-order, <u>and hadronic and EW terms</u>, and the measurement of a_e.

[3]D. Hanneke, S. Fogwell and G. Gabrielse, PRL **100** , 120801 (2008).

[4]T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, PRD **91**, 033006 (2015).

Motivation. Anomalous magnetic momentum of muon.

1. Anomalous magnetic momentum of muon $a_\mu = (g-2)_\mu$ is measured in experiment E821 (BNL) with high precision[5]

$$a_\mu^{\rm exp} = 11\,659\,209.1(6.3) \times 10^{-10}$$

2. The nonzero lepton AMMs are induced by radiative corrections. In the SM are induced by QED, weak and strong (hadronic) interactions.

$$a_\mu = \left\{ a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hadr} \right\}^{\rm SM} + ???$$

3. Tenth-order QED contribution[6] to a_μ

$$a_\mu^{\rm QED} = 11\ 658\ 471.8951\ (0.0080) \times 10^{-10}$$

4. Weak contribution[7]

$$a_\mu^{\rm weak} = 15.36\ \ (0.1) \times 10^{-10}$$

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[5] G.W.Bennett,et al.PRD73,072003(2006); P.J.Mohr,et al.RMP84,1527(2012).

[6] T.Aoyama, M.Hayakawa, T.Kinoshita, M.Nio, PRL 109, 111807 (2012).

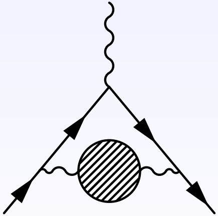
[7] A.Czarnecki, W.J.Marciano, A.Vainshtein, PRD67, 073006 (2003); C.Gnendiger, D.Stockinger and H.Stockinger-Kim, PRD88, 053005 (2013)

Motivation. Anomalous magnetic momentum. HVP

5. Strong contribution separated into three terms

$$a_\mu^{\text{hadr}} = a_\mu^{\text{HVP,LO}} + (a_\mu^{\text{HVP,NLO}} + a_\mu^{\text{HVP,NNLO}} + ..) + a_\mu^{\text{HLbL}}$$

- Contribution of hadron vacuum polarization can be extracted from experimental data for process $e^+e^- \to$ in hadrons (or hadronic τ-lepton decays)



$$a_\mu^{\text{HVP,LO}} = \begin{cases} 693.1 \ (3.4) \times 10^{-10}, & ^{8} \\ 693.26 \ (2.46) \times 10^{-10}. & ^{9} \end{cases}$$

[8] M.Davier, A.Hoecker, B.Malaescu, Z.Zhang, EPJC 77 (2017) 827

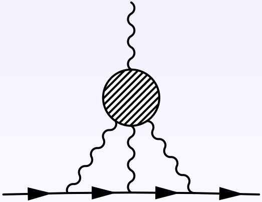
[9] A. Keshavarzi, D. Nomura, T. Teubner PRD 97 (2018) 114025

Motivation. Anomalous magnetic momentum. HLbL

6. Higher orders hadronic contribution to HVP [10, 11]

$$a_\mu^{\mathrm{HVP,NLO}} = -9.82\,(0.04) \times 10^{-10},$$
$$a_\mu^{\mathrm{HVP,NNLO}} = 1.24(0.01) \times 10^{-10}.$$

7. The "Glasgow consensus" value for the hadronic light-by-light contribution [12]



$$a_\mu^{\mathrm{HLbL}}(\mathrm{Glasgow}) = 10.5\ \ (2.6) \times 10^{-10}$$

[10] K.Hagiwara,R.Liao,A.D.Martin,D.Nomura,T.Teubner, JPG 38, 085003 (2011)

[11] A. Kurz, T. Liu, P. Marquard and M. Steinhauser, PLB 734, 144 (2014).

[12] J.Prades, E.de Rafael, A.Vainshtein, in *Advanced series on directions in high energy physics, Vol. 20* [arXiv:0901.0306 [hep-ph]].

Motivation. Anomalous magnetic momentum.

8. Combining all SM contributions one obtains

$$a_\mu^{\mathrm{SM},8} = 116\ 591\ 82.3(4.3) \times 10^{-10}$$
$$a_\mu^{\mathrm{SM},9} = 116\ 591\ 82.04(3.56) \times 10^{-10}$$
$$a_\mu^{\mathrm{SM},^{13}} = 116\ 591\ 78.3(3.5) \times 10^{-10}$$

9. The resulting difference between the experimental result and the full SM prediction are

$$a_\mu^{\mathrm{BNL}} - a_\mu^{\mathrm{SM},8} = 26.8\ \ (7.6) \times 10^{-10}(3.5\sigma),$$
$$a_\mu^{\mathrm{BNL}} - a_\mu^{\mathrm{SM},9} = 27.06\ \ (7.26) \times 10^{-10}(3.7\sigma).$$
$$a_\mu^{\mathrm{BNL}} - a_\mu^{\mathrm{SM},13} = 30.6\ \ (7.2) \times 10^{-10}(4.3\sigma).$$

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[13] F. Jegerlehner, Springer Tracts Mod. Phys. **274** (2017) 1

Motivation. Anomalous magnetic momentum. HLbL

The SM theoretical error is dominated by the hadronic contributions. Theoretical predictions of HVP and HLbL contributions to a_μ should be of the same level or better than the precision of planed experiments.
LbL scattering amplitude is a complicated object. It is a sum of different diagrams, the quark loop, the meson exchanges, the meson loops and the iterations of these processes. However, there is hierarchy connected to existence of two small parameters: the inverse number of colors $1/N_c$ and the ratio of the characteristic internal momentum to the chiral symmetry parameter $m_\mu/(4\pi f_\pi) \sim 0.1$.

Lagrangian of nonlocal model

The Lagrangian of the nonlocal model has the form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{4q} + \mathcal{L}_{tH}$$
$$\mathcal{L}_{free} = \bar{q}(x)(i\hat{\partial} - m_c)q(x)$$

m_c – current quark mass matrix with diagonal elements $m_c^u = m_c^d$, m_c^s

$$\mathcal{L}_{4q} = \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)]$$
$$\mathcal{L}_{tH} = -\frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_P^a(x)J_P^b(x)J_P^c(x)]$$

Nonlocal quark currents are

$$J_M^a(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x-x_1)\,\Gamma_M^a q(x+x_2),$$

where $M = S, P$ and $\Gamma_S^a = \lambda^a$, $\Gamma_P = i\gamma^5\lambda^a$, and $f(x)$ is a form factor reflecting the nonlocal properties of the QCD vacuum.

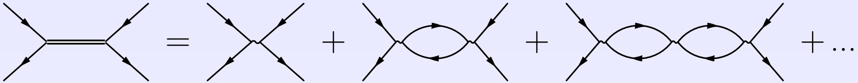
Lagrangian

The model can be bosonized using the stationary phase approximation which leads to the system of gap equations for the dynamical quark masses $m_{d,i} \quad (i = u, d, s)$

$$
\begin{aligned}
m_{d,u} + G S_u + \frac{H}{2} S_u S_s &= 0, \\
m_{d,s} + G S_s + \frac{H}{2} S_u^2 &= 0, \\
S_i = -8 N_c \int \frac{d_E^4 k}{(2\pi)^4} \frac{f^2(k^2) m_i(k^2)}{D_i(k^2)} &,
\end{aligned}
$$

where $m_i(k^2) = m_{c,i} + m_{d,i} f^2(k^2)$ is the dynamical quark mass, $D_i(k^2) = k^2 + m_i^2(k^2)$, $f(k^2)$ is the nonlocal form factor in the momentum representation.

T matrix



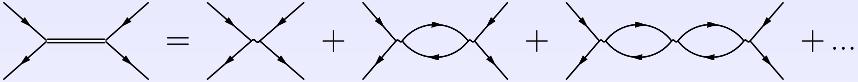
The vertex functions and the meson masses can be found from the Bethe-Salpeter equation. For the separable interaction the quark-antiquark scattering matrix in pseudoscalar channel becomes

$$\mathbf{T} = \hat{\mathbf{T}}(p^2)\delta^4\left(p_1 + p_2 - (p_3 + p_4)\right)\prod_{i=1}^{4} f(p_i^2),$$

$$\hat{\mathbf{T}}(p^2) = i\gamma_5\lambda_k\left(\frac{1}{-\mathbf{G}^{-1} + \mathbf{\Pi}(p^2)}\right)_{kl} i\gamma_5\lambda_l,$$

where p_i are the momenta of external quark lines, $\mathbf{G}$ and $\mathbf{\Pi}(p^2)$ are the corresponding matrices of the four-quark coupling constants and the polarization operators of pseudoscalar mesons $(p = p_1 + p_2 = p_3 + p_4)$.

T matrix



The meson masses can be found from the zeros of determinant $\det(\mathbf{G}^{-1} - \mathbf{\Pi}(-M^2)) = 0$. The $\hat{\mathbf{T}}$-matrix for the system of mesons in each neutral channel can be expressed as

$$\hat{\mathbf{T}}_{ch}(P^2) = \sum_{M_{ch}} \frac{\overline{V}_{M_{ch}}(P^2) \otimes V_{M_{ch}}(P^2)}{-(P^2 + \mathrm{M}^2_{M_{ch}})},$$

where M_M are the meson masses, $V_M(P^2)$ are the vertex functions $\left(\overline{V}_M(p^2) = \gamma^0 V_M^\dagger(P^2)\gamma^0\right)$. The sum is over full set of light mesons: $(M_{PS} = \pi^0, \eta, \eta')$ in the pseudoscalar channel and $(M_S = a_0(980), f_0(980), \sigma)$ in the scalar one.

External fields

The gauge-invariant interactions with external photon field can be introduced with Schwinger phase factor

$$q(y) \to Q(x,y) = \mathcal{P}\exp\left\{ i \int\limits_x^y dz^\mu V_\mu^a(z) T^a \right\} q(y),$$

apart from kinetic term the additional terms in nonlocal interations are generated

$$J_I(x) = \int d^4x_1 d^4x_2\, f(x_1) f(x_2)\, \bar{Q}(x - x_1, x)\, \Gamma_I\, Q(x, x + x_2)$$

External fields

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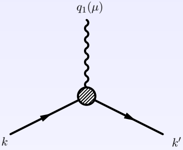
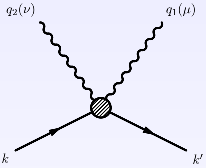
The following equations are used for obtaining of nonlocal vertices

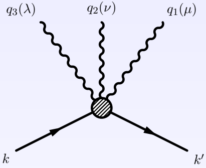
$$\frac{\partial}{\partial y^\mu} \int\limits_x^y dz^\nu\ F_\nu(z) = F_\mu(y), \qquad \delta^{(4)}(x-y) \int\limits_x^y dz^\nu\ F_\nu(z) = 0.$$

Nonlocal vertices

As a result the nonlocal vertices with arbitrary number of photon fields are generated







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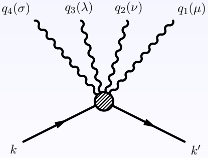
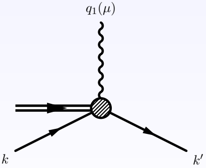
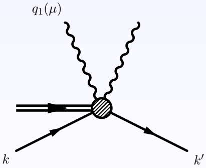






N_c counting rules.

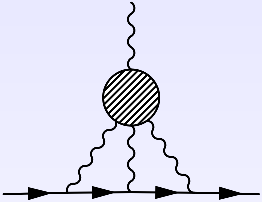
In order to have correspondence with QCD the quark mass should scale as N_c^0 for large number of colors

$$m_d = GN_c \cdot 8 \int \frac{d_E^4 k}{(2\pi)^4} f^2(k) \frac{m(k)}{k^2 + m^2(k)}$$

This means that four-quark coupling constant should scales as $G \sim 1/N_c$. As a result meson propagator leads to $1/N_c$ suppression of diagrams

$$\mathrm{D}_p^{\mathrm{M}} = \frac{1}{-G^{-1} + \Pi_p^{\mathrm{M}}} \to \frac{1}{N_c},$$

Light-by-light hadronic contribution to the muon AMM



Muon AMM can be extracted by using the projection

$$a_\mu^{\mathrm{HLbL}} = \frac{1}{48 m_\mu} \mathrm{Tr}\left((\hat{p} + m_\mu)[\gamma^\rho, \gamma^\sigma](\hat{p} + m_\mu) \Pi_{\rho\sigma}(p, p) \right),$$

$$\Pi_{\rho\sigma}(p', p) = -ie^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1 + q_2 - k)^2} \times$$

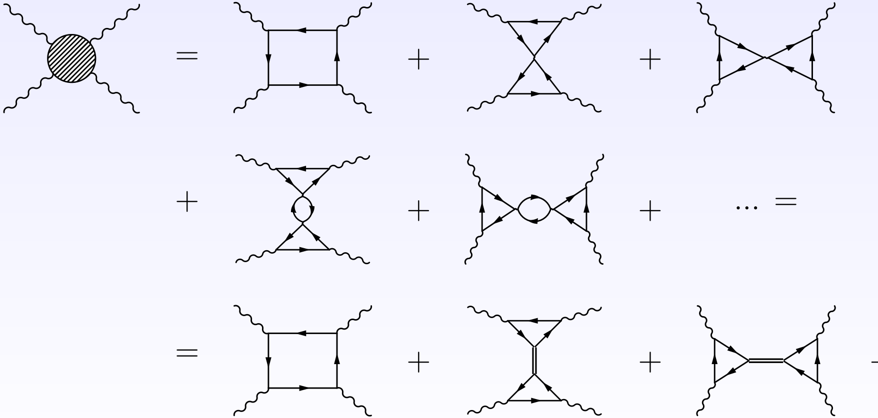
$$\times \gamma^\mu \frac{\hat{p}' - \hat{q}_1 + m_\mu}{(p' - q_1)^2 - m_\mu^2} \gamma^\nu \frac{\hat{p} - \hat{q}_1 - \hat{q}_2 + m_\mu}{(p - q_1 - q_2)^2 - m_\mu^2} \gamma^\lambda \times$$

$$\times \frac{\partial}{\partial k^\rho} \Pi_{\mu\nu\lambda\sigma}(q_1, q_2, k - q_1 - q_2),$$

m_μ is the muon mass, $k_\mu = (p' - p)_\mu$, static limit $k_\mu \to 0$.

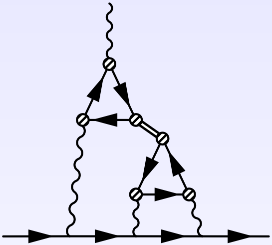
Four-rank polarization tensor

To the leading $1/N_c$ order four-rank polarization tensor $\Pi_{\mu\nu\lambda\sigma}$ can be represented in the form [14]



[14] The nonlocal multi-photon vertices are not shown for simplicity.

HLbL resonance contribution to the muon AMM



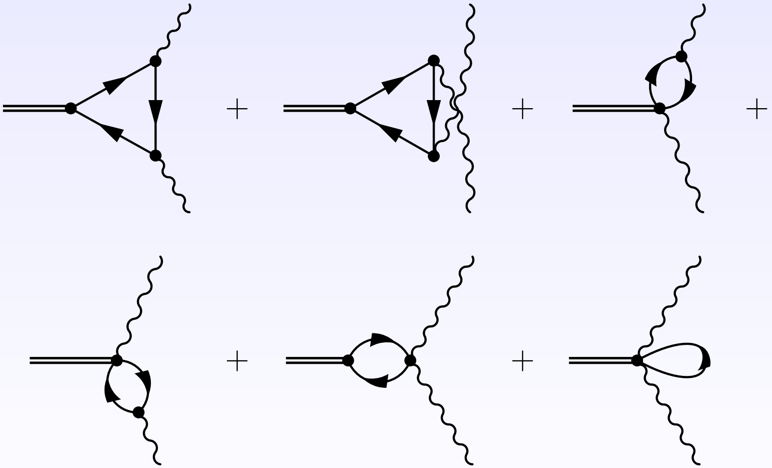
$$\frac{\partial}{\partial k^\rho}\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2) =$$

$$i\frac{\Delta^{\mu\nu}(q_1+q_2,q_1,q_2)}{(q_1+q_2)^2-M^2}\frac{\partial}{\partial k^\rho}\Delta^{\lambda\sigma}(q_1+q_2,-q_1-q_2,k)$$

$$+\,i\frac{\Delta^{\nu\lambda}(-q_1,q_2,-q_1-q_2)}{q_1^2-M^2}\frac{\partial}{\partial k^\rho}\Delta^{\mu\sigma}(-q_1,q_1,k)$$

$$+\,i\frac{\Delta^{\mu\lambda}(-q_2,q_1,-q_1-q_2)}{q_2^2-M^2}\frac{\partial}{\partial k^\rho}\Delta^{\nu\sigma}(-q_2,q_2,k)+O(k)$$

Meson–photon–photon transition amplitude



Two-photon–pseudoscalar(scalar) meson. I

Triangular diagram with external pseudoscalar (scalar) meson and two photon legs with arbitrary virtualities can be written as

$$A\left(\gamma^*_{(q_1,\epsilon_1)}\gamma^*_{(q_2,\epsilon_2)} \to P^*_{(p)}\right) = -ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma \mathrm{F}_{P^*\gamma^*\gamma^*}\left(p^2; q_1^2, q_2^2\right),$$

$$A\left(\gamma^*_{(q_1,\mu)}\gamma^*_{(q_2,\nu)} \to S^*_{(p)}\right) = e^2\Delta^{\mu\nu}_{S^*\gamma^*\gamma^*}(q_3, q_1, q_2) =$$

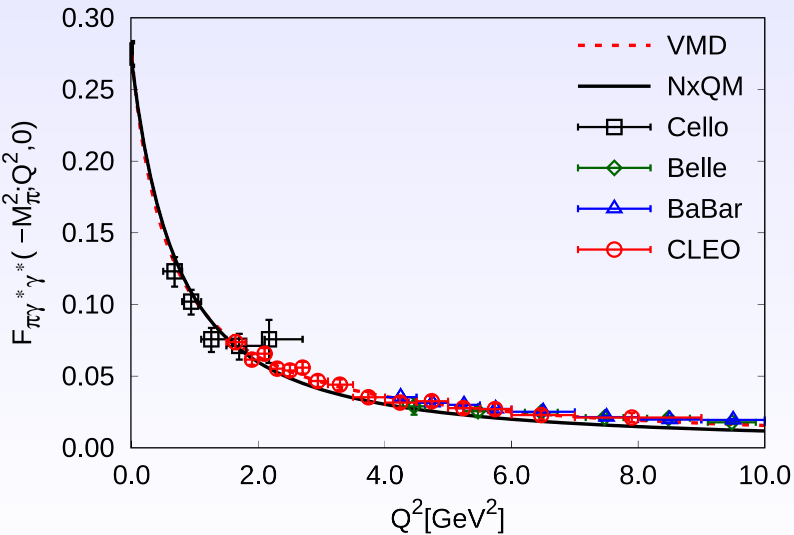
$$= e^2\left[A_{S^*\gamma^*\gamma^*}\left(p^2; q_1^2, q_2^2\right)T_A^{\mu\nu}(q_1,q_2)\right.$$

$$\left. + B_{S^*\gamma^*\gamma^*}(p^2; q_1^2, q_2^2)T_B^{\mu\nu}(q_1,q_2)\right],$$

$$T_A^{\mu\nu}(q_1,q_2) = \left(g^{\mu\nu}(q_1\cdot q_2) - q_1^\nu q_2^\mu\right)$$

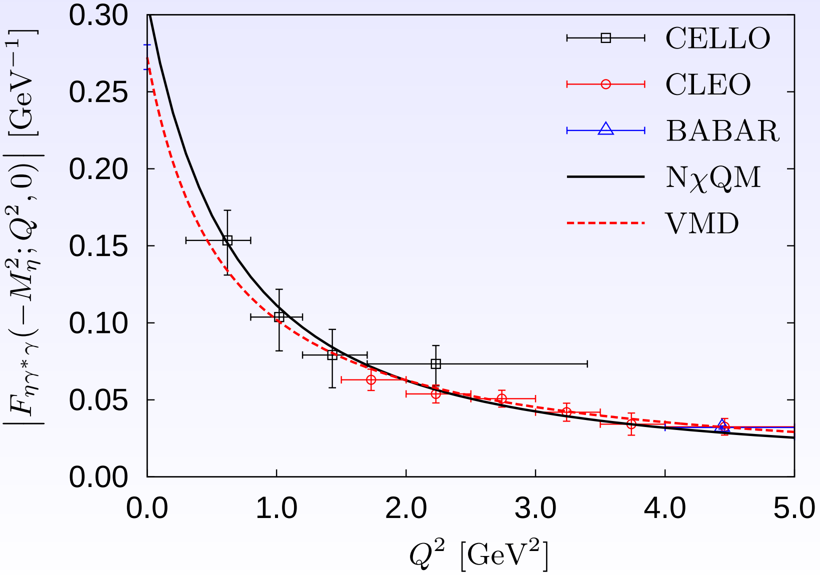
$$T_B^{\mu\nu}(q_1,q_2) = \left(q_1^2 q_2^\mu - (q_1\cdot q_2)q_1^\mu\right)\left(q_2^2 q_1^\nu - (q_1\cdot q_2)q_2^\nu\right),$$

Pion FF



VMD
NxQM
Cello
Belle
BaBar
CLEO
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
2.0
4.0
6.0
8.0
10.0
$F_{\pi\gamma^*\gamma^*}(-M_\pi^2, Q^2, 0)$
$Q^2[\mathrm{GeV}^2]$

η FF



0.30
0.25
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0.15
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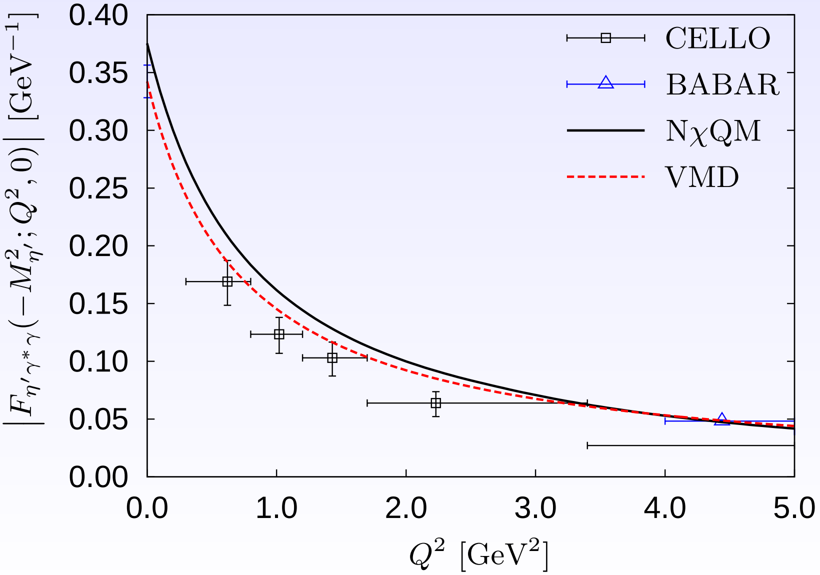
0.0
1.0
2.0
3.0
4.0
5.0
$\left|F_{\eta\gamma^*\gamma}(-M_\eta^2; Q^2, 0)\right|$ [GeV^{-1}]
Q^2 [GeV^2]
CELLO
CLEO
BABAR
NχQM
VMD

η' FF



0.40
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1.0
2.0
3.0
4.0
5.0
CELLO
BABAR
NχQM
VMD
$|F_{\eta'\gamma^*\gamma}(-M_{\eta'}^2;Q^2,0)|$ [GeV^{-1}]
Q^2 [GeV^2]

HLbL contribution to the muon AMM

Model	a_μ^{HLbL}	Reference
LMD+V	8.0(1.2)	(Knecht [1])
ENJL	8.3(3.2)	(Bijnens [2])
Mesons+ π,quark loops+ resc.	8.7(1.3)	(Danilkin [12])
VMD, HLS	8.96(1.54)	(Hayakawa [3])
Mesons+ π, K,quark loops	10.34(2.88)	(Jegerlehner [11])
Glasgow consensus	10.5(2.6)	(Prades [4])
LENJL	10.77(1.68)	(Bartos [5])
oLMDV	11.6(4.0)	(Nyffeler [6])
(LMD+V)$'$	13.6(2.5)	(Melnikov [7])
Q-box	14.05	(Pivovarov [8])
CχQM	15.0(0.3)	(Greynat [9])
Our work	16.8(1.25)	
DS	18.8(0.4)	(Goecke [10])

Table: Model estimates of the HLbL contribution to a_μ obtained in different works. All numbers are given in 10^{-10}.

Lagrangian in presence of vector–axial-vector interaction

The Lagrangian of the nonlocal model has the form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{4q} + \mathcal{L}_{tH} + \mathcal{L}_{V,AV},$$

$$\mathcal{L}_{V,AV} = \frac{G_2}{2}[J_V^a(x)J_V^a(x) + J_{AV}^a(x)J_{AV}^a(x)]$$

Nonlocal quark currents are

$$J_M^a(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x - x_1)\,\Gamma_M^a q(x + x_2),$$

where $M = V, AV$ and $\Gamma_V = \gamma^\mu \lambda^a$, $\Gamma_{AV} = \gamma^5 \gamma^\mu \lambda^a$.

Meson-photon-photon interaction vertices (AV I)

$$T_\alpha^{\mu\nu} = \varepsilon_{\rho\sigma\tau\alpha}\Big\{ A_1 q_1^\tau g^{\mu\rho} g^{\sigma\nu} + A_2 q_2^\tau g^{\mu\rho} g^{\sigma\nu} + A_3 q_1^\nu q_1^\rho q_2^\sigma g^{\tau\mu} + \\ + A_4 q_2^\nu q_1^\rho q_2^\sigma g^{\tau\mu} + A_5 q_1^\mu q_1^\rho q_2^\sigma g^{\tau\nu} + A_6 q_2^\mu q_1^\rho q_2^\sigma g^{\tau\nu} \Big\}$$

where $A_i \equiv \mathcal{A}_i(p^2, q_1^2, q_2^2)$ and due to gauge invariance

$$-A_2 + q_1^2 A_5 + (q_1 \cdot q_2) A_6 = 0,$$
$$-A_1 + (q_1 \cdot q_2) A_3 + q_2^2 A_4 = 0$$

and by Bose symmetry

$$\mathcal{A}_1(p^2, q_1^2, q_2^2) = -\mathcal{A}_2(p^2, q_2^2, q_1^2)$$
$$\mathcal{A}_3(p^2, q_1^2, q_2^2) = -\mathcal{A}_6(p^2, q_2^2, q_1^2)$$
$$\mathcal{A}_4(p^2, q_1^2, q_2^2) = -\mathcal{A}_5(p^2, q_2^2, q_1^2)$$

Meson-photon-photon interaction vertices (AV II)

$$T_{\alpha}^{\mu\nu} = \varepsilon_{\rho\sigma\tau\alpha}\Big\{R^{\mu\rho}(q_1,q_2)R^{\nu\sigma}(q_1,q_2)\,(q_1-q_2)^{\tau}\,\frac{(q_1\cdot q_2)}{m_A^2}\,F^{(0)}_{AV\gamma^*\gamma^*}(p^2,q_1^2,q_2^2)$$

$$+\;R^{\nu\rho}(q_1,q_2)Q_1^{\mu}q_1^{\sigma}\,q_2^{\tau}\,\frac{1}{m_A^2}\,F^{(1)}_{AV\gamma^*\gamma^*}(p^2,q_1^2,q_2^2)$$

$$+\;R^{\mu\rho}(q_1,q_2)Q_2^{\nu}q_2^{\sigma}\,q_1^{\tau}\,\frac{1}{m_A^2}\,F^{(1)}_{AV\gamma^*\gamma^*}(p^2,q_2^2,q_1^2)\Big\},$$

$$R^{\mu\nu}(q_1,q_2) = -g^{\mu\nu} + \frac{1}{X}\left\{(q_1\cdot q_2)\,(q_1^{\mu}\,q_2^{\nu} + q_2^{\mu}\,q_1^{\nu}) - q_1^2\,q_2^{\mu}\,q_2^{\nu} - q_2^2\,q_1^{\mu}\,q_1^{\nu}\right\},$$

$$Q_1^{\mu} = \left(q_1^{\mu} - \frac{q_1^2}{\nu}q_2^{\mu}\right), \quad Q_2^{\nu} = \left(q_2^{\nu} - \frac{q_2^2}{\nu}q_1^{\nu}\right),$$

where $X = (q_1\cdot q_2)^2 - q_1^2 q_2^2$ and $R^{\mu\nu}(q_1,q_2)$ is fully transverse tensor, Q_1^{μ} and Q_2^{ν} are transverse with respect to q_1 and q_2

$$q_{1(2),\nu}R^{\mu\nu}(q_1,q_2) = 0,\; q_{1(2),\mu}R^{\mu\nu}(q_1,q_2) = 0,$$

$$q_{1\mu}Q_1^{\mu} = 0, \quad q_{2\nu}Q_2^{\nu} = 0.$$

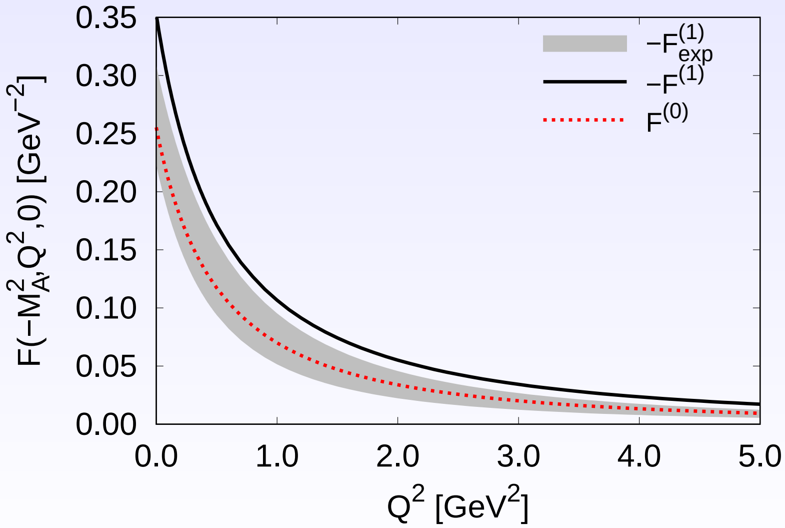
Two-photon-decay width of AV-meson

Axial-vector meson can not decay into two real photons, according to the Landau–Yang theorem. The coupling of 1^{++} mesons to two photons is still possible in the case when one or both photons are virtual. Two-photon "decay" width defined as

$$\tilde{\Gamma}_{\gamma^*\gamma^*}(AV) = \lim_{Q^2\to 0} \frac{M_A^2}{Q^2}\Gamma^{\mathrm{TS}}_{\gamma\gamma^*} = \frac{\pi\alpha^2 M_A^5}{12}[F^{(1)}_{AV\gamma^*\gamma^*}(M_A^2, 0, 0)]^2$$

At present we have only few experimental data on form factor of transition of 1^{++} meson to two photons $F_{AV\gamma^*\gamma^*}(t^2, k^2, k^2)$. The L3 Collaboration studied the reaction $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^- f_1(1285) \to e^+e^-\eta\pi^+\pi^-$ and $f_1(1285)$ transition form factor for the case when one of the photons is real and another one is virtual.

f_1 FF



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
1.0
2.0
3.0
4.0
5.0
Q2 [GeV2]
F(−M2A,Q2,0) [GeV−2]
−F(1)exp
−F(1)
F(0)

g-2 axial-vector exchange – numerical results

model	AV contribution in 10^{-10}	
ENJL	0.25 ± 0.1	[2]
HLS	0.2 ± 0.1	[3]
MV	2.2 ± 0.5	[7]
Empirical estimations	0.755 ± 0.271	[11]
Our work	0.34	

In nonlocal model the separate result for contribution of $a_1(1260)$, $f_1(1285)$ is $0.67 \cdot 10^{-11}$. However, due to decrease of pion contribution because of $\rho - \gamma$ and $\pi - a_1$ mixing one can estimate axial-vector a_1, f_1 contribution as $0.34 \cdot 10^{-11}$.

Conclusions. I

- Our result decrease difference between experimental result and theoretical estimations
- Within the nonlocal quark model the main contribution to the light-by-light process comes from contact and pion contributions.
- The pseudoscalar meson contributions to muon AMM are systematically lower then the results obtained in the other works.
- This is due to full kinematic dependence – off-shell effects.
- Contact term is large since the quarks in the loop are dynamical, i.e. at zero virtuality they have constituent mass but for large momenta their mass become current. Contact term diverges for zero quark mass. This is somewhat similar to the result of DSE– BSE calculations.

Conclusions. II

- The value of axial-vector contribution in nonlocal quark model have the same order as estimations in the ENJL, HLS models and empirical estimations.
- No trace of rather big axial-vector contribution as in Melnikov–Vainshtein work.
- $\rho - \gamma$ and $\pi - a_1$ mixing for pseudoscalar contribution partially decrease axial-vector contribution.
- Result seems stable for the possible extension to vector–axial-vector sector.

Conclusions. Questions

- The sign of scalar meson contribution is positive while in some estimations it is negative. In our calculation the scalar meson mainly influence error bar but not the main value.
- How relate our model calculations to the dispersive approach to the hadronic light-by-light contribution to the muon g-2 (Prof. Gilberto Colangelo talk)?
- Next-to-leading $1/N_c$ expansion terms can contribute. Charged pion loop gave negative contribution but there are a lot of $1/N_c$ corrections in quark model (e.g. correction to quark self-energy, dressing of mesons by two-mesons intermediate state, etc.).

HFS

- The contribution of axial-vector mesons in the nonlocal model to hyperfine splitting of levels of muonic hydrogen within the error bar of empirical estimation – talk by A.E.Dorokhov.

THANKS !

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