R measurement at KEDR

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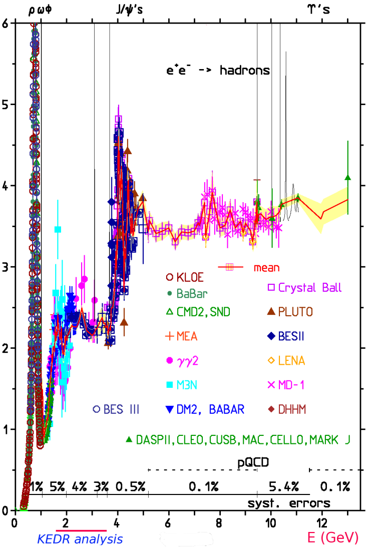
KEDR collaboration

27 February 2019

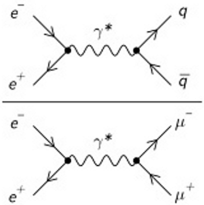
International Workshop on e^+e^- collisions from ϕ to ψ

"Priore loco" (In first)



F. Jegerlehner arXiv:1511.0447

$$R = \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^-\mu^+)} \approx$$



In first approximation:

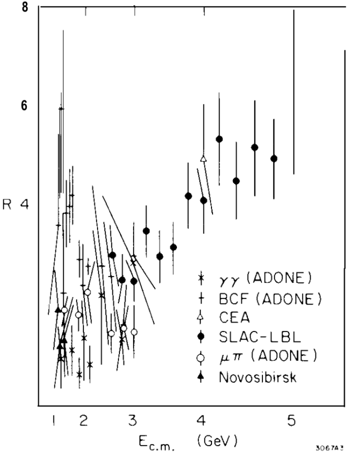
$$R(s) \simeq 3 \sum e_q^2$$

$R(s)$ is used to determine:

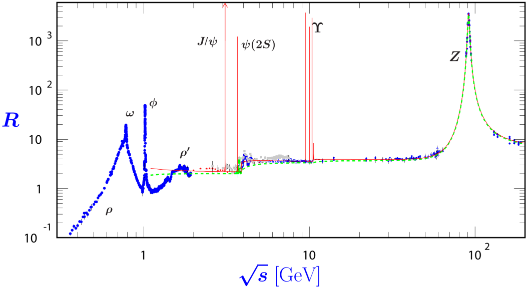
- $\alpha_s(s)$
- $(g_\mu - 2)/2$
- $\alpha(M_Z^2)$

$R(s)$ measurement. Experimental data.

"Confer!" (Compare!)



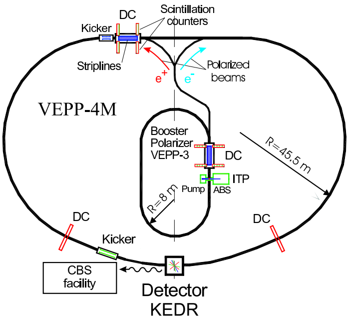
"The ratio R as of July 1974"
Presented by Richter at the
London conference in July 1974.



PDG data at the present time.

VEPP-4M and KEDR

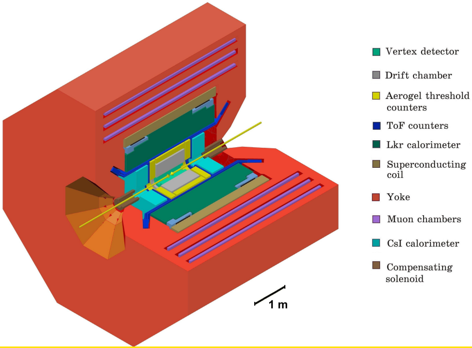
"Actum atque tractatum" (done and discussed)



Beam energy	$1 \div 5$ GeV
Number of bunches	2×2
Luminosity 1.8 GeV	1.5×10^{30} cm^{-2} s^{-1}

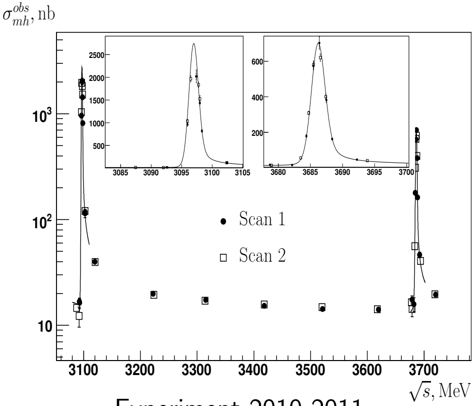
Energy measurement:

- Resonant depolarization method:
  Instant measurement accuracy ~ 1 keV
  Energy interpolation accuracy $10 \div 30$ keV
- Compton backscattering method ~ 100 keV

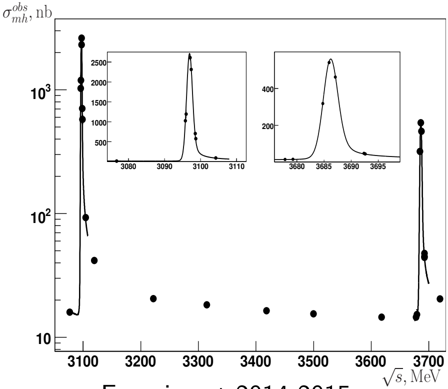


"Consideratio naturae"(contemplation of nature)

The observed multihadron cross section as a function of the c.m. energy



Experiment 2010-2011



Experiment 2014-2015

- The c.m. energy range between 3.076 and 3.72 GeV studied
- An integrated luminosity of 2.7 pb^{-1} collected at 9 energies 3.077, 3.120,3.223, 3.315, 3.418, 3.500, 3.521, 3.618, 3.719 GeV
- $\sim (2-6)\times 10^3$ m.h. events per point, $\sim 38\times 10^3$ in total

"Modus operandi" (procedure;method of operating)

The way that we are measuring R:

$$R = \frac{\sigma_{obs}(s) - \sum \varepsilon_{\psi}^{tail}(s)\sigma_{\psi}^{tail}(s) - \sum \varepsilon_{bg}^{i}(s)\sigma_{bg}^{i}(s)}{\varepsilon(s)(1+\delta(s))\sigma_{\mu\mu}^{0}}$$

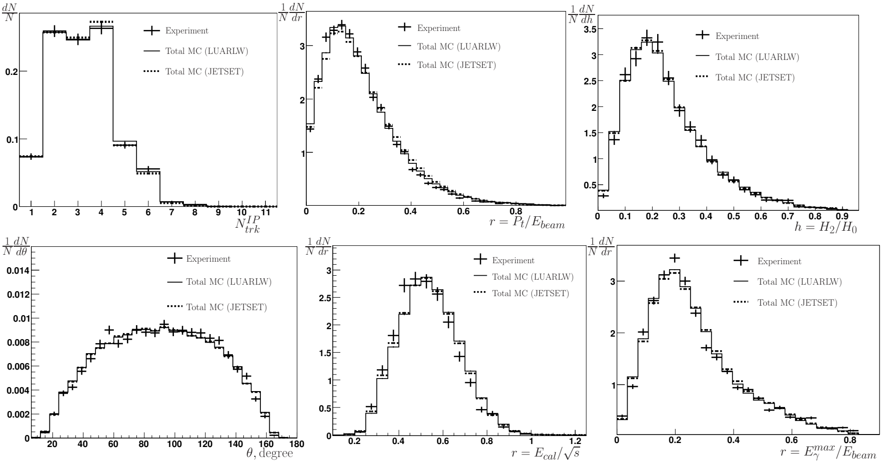
with $\sigma_{obs}(s) = \frac{N_{mh} - N_{res.bg.}}{\int \mathcal{L}dt}$ where N_{mh} represent all events pass hadronic selection criteria, $N_{res.bg.}$ – residual machine background
$\sum \varepsilon_{\psi}^{tail}(s)\sigma_{\psi}^{tail}(s)$ is contribution from J/ψ and $\psi(2S)$ resonances
$\sum \varepsilon_{bg}^{i}(s)\sigma_{bg}^{i}(s)$ is contribution from physical processes: $e^+e^- \to l^+l^-$, $\gamma\gamma$-processes.
$\varepsilon(s)$ – multihadron efficiency.

$$1+\delta(s) = \int dx \frac{1}{1-x} \frac{\mathcal{F}(s,x)}{\left|1-\tilde{\Pi}(s(1-x))\right|^2} \frac{\tilde{R}(s(1-x))\varepsilon(s(1-x))}{R(s)\varepsilon(s)}$$

$\mathcal{F}(s,x)$ – radiative correction kernel (**E.A.Kuraev, V.S.Fadin Sov.J.Nucl.Phys.41(466-472)1985**) Here $\tilde{\Pi}$ and $\tilde{R}$ does not includes J/ψ and $\psi(2S)$ resonances. **To determine the contributions of the J/ψ and $\psi(2S)$ without external data, the additional data samples of about 0.4 pb^{-1}(2010-2011) and 0.34 pb^{-1}(2014-2015) were collected in the vicinity of peak regions.**

Simulation: JETSET and LUARLW

“Punctum saliens” (The most important thing)



Properties of hadronic events produced in the uds continuum at 3.119 GeV (2014-2015).

Here N is the number of events, N_{trk}^{IP} is the number of tracks originated from IP, P_t is a transverse momentum of the track, H_2 and H_0 are Fox-Wolfram moments, θ is a polar angle of the track, E_{cal} is energy deposited in the calorimeter, E_γ^{max} is energy of the most energetic photon.

Systematic uncertainties

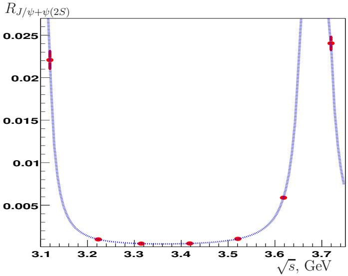
"Satius est supervacua discere quam nihil"
(Better to learn more than necessary than nothing at all)

Seneca

Source	Syst. uncertainty, %		
	Scan 1 and 2 (2010-2011)	Scan 2014-2015	Correlated
Luminosity	1.1	0.9	0.4
Rad. corr.	$0.4 \div 0.6$	$0.5 \div 0.8$	$0.2 \div 0.4$
uds simulation	$1.3 \div 2.0$	1.1	0.9
Track reconstruction	0.5	0.4	–
J/ψ	$0.1 \div 2.7$	$0.1 \div 1.8$	–
$\psi(2S)$ (at 3.72 GeV)	1.4	1.1	–
l^+l^-	$0.1 \div 0.2$	$0.3 \div 0.4$	$0.1 \div 0.2$
e^+e^-X	$0.1 \div 0.2$	0.1	0.1
Trigger	0.2	0.2	0.2
Nuclear interaction	0.2	0.2	0.2
Machine background	$0.5 \div 1.1$	$0.4 \div 0.8$	–
Cuts	0.6	0.6	–
Total	$2.1 \div 3.6$ (correlated $1.8 \div 2.5$)	$1.9 \div 2.7$	1.1

R for $\sqrt{s}$= 3.12 – 3.72 GeV

"Restitutio in integrum" (Restoring the thing to its original state)



Using J/ψ and $\psi(2S)$ parameters, we obtain $R_{uds}(s) + R_{J/\psi+\psi(2S)} \Longrightarrow R(s)$

Data 2010-2011		Data 2014-2015		Combination	
$\sqrt{s}$, MeV	$R_{\mathbf{uds}}(s)$	$\sqrt{s}$, MeV	$R_{\mathbf{uds}}(s)$	$\sqrt{s}$, MeV	$R_{\mathbf{uds}}(s)\{R(s)\}$
-	-	3076.7 ± 0.2	2.188 ± 0.056 ± 0.042	3076.7 ± 0.2	2.188 ± 0.056 ± 0.042
3119.9 ± 0.2	2.215 ± 0.089 ± 0.066	3119.2 ± 0.2	2.211 ± 0.046 ± 0.060	3119.6 ± 0.4	2.212{2.235} ± 0.042 ± 0.049
3223.0 ± 0.6	2.172 ± 0.057 ± 0.045	3221.8 ± 0.2	2.214 ± 0.055 ± 0.042	3222.5 ± 0.8	2.194{2.195} ± 0.040 ± 0.035
3314.7 ± 0.7	2.200 ± 0.056 ± 0.043	3314.7 ± 0.4	2.233 ± 0.044 ± 0.042	3314.7 ± 0.6	2.219{2.219} ± 0.035 ± 0.035
3418.2 ± 0.2	2.168 ± 0.050 ± 0.042	3418.3 ± 0.4	2.197 ± 0.047 ± 0.040	3418.3 ± 0.3	2.185{2.185} ± 0.032 ± 0.035
-	-	3499.6 ± 0.4	2.224 ± 0.054 ± 0.040	3499.6 ± 0.4	2.224{2.224} ± 0.054 ± 0.040
3520.8 ± 0.4	2.200 ± 0.050 ± 0.044	-	-	3520.8 ± 0.4	2.200{2.201} ± 0.050 ± 0.044
3618.2 ± 1.0	2.201 ± 0.059 ± 0.044	3618.1 ± 0.4	2.220 ± 0.049 ± 0.042	3618.2 ± 0.7	2.212{2.218} ± 0.038 ± 0.035
3719.4 ± 0.7	2.187 ± 0.068 ± 0.060	3719.6 ± 0.2	2.213 ± 0.047 ± 0.049	3719.5 ± 0.5	2.204{2.228} ± 0.039 ± 0.042

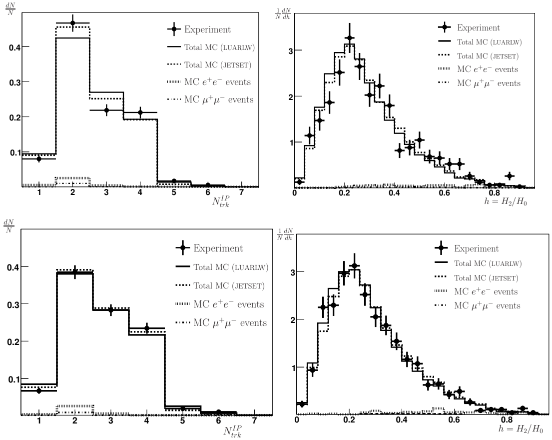
V.V.Anashin et al., Phys.Lett. B 753, 533-541 (2016).[arXiv:1510.02667]
V.V.Anashin et al., Phys.Lett. B 788, 42-51 (2019).[arXiv:1805.06235]

R for $\sqrt{s}$= 1.84 – 3.05 GeV

"Fac simile" (to make alike;reproduction exacte)

- An integrated luminosity 0.66 pb^{-1} collected at 13 equidistant points with a step $\sim$ 0.1 GeV: 1.841, 1.937 ... 3.048 GeV
- $\sim 10^3$ hadronic events per point, 14.8×10^3 events in total
- Simulation of the *uds* continuum based on the LUARLW generator, tuned JETSET alternatively used at 6 points for a cross-check.

Experimental distribution and two variants of MC simulation based on LUARLW and tuned JETSET are plotted ($\sqrt{s} = 1.94$ GeV and $\sqrt{s} = 2.14$ GeV).



R for $\sqrt{s} = 1.84 - 3.05$ GeV

"Ut supra" (as (described) above)

Measured value of $R = \frac{\sigma_{obs}(s) - \sum \varepsilon^i_{bg}(s)\sigma^i_{bg}(s)}{\varepsilon(s)(1+\delta(s))\sigma^0_{\mu\mu}}$

$\sqrt{s}$, MeV	$R(s)$
1841.0	$2.226 \pm 0.139 \pm 0.158$
1937.0	$2.141 \pm 0.081 \pm 0.073$
2037.3	$2.238 \pm 0.068 \pm 0.072$
2135.7	$2.275 \pm 0.072 \pm 0.055$
2239.2	$2.208 \pm 0.069 \pm 0.053$
2339.5	$2.194 \pm 0.064 \pm 0.048$
2444.1	$2.175 \pm 0.067 \pm 0.048$
2542.6	$2.222 \pm 0.070 \pm 0.047$
2644.8	$2.220 \pm 0.069 \pm 0.049$
2744.6	$2.269 \pm 0.065 \pm 0.050$
2849.7	$2.223 \pm 0.065 \pm 0.047$
2948.9	$2.234 \pm 0.064 \pm 0.051$
3048.1	$2.278 \pm 0.075 \pm 0.048$

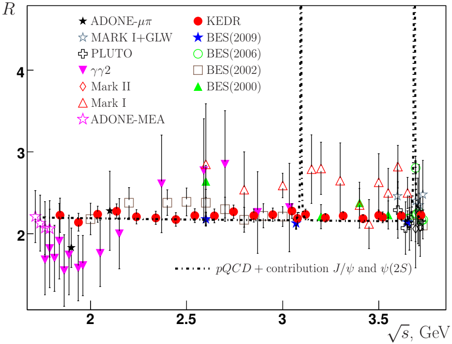
The main systematic uncertainties in the R:

Source	Error,%
Luminosity	1.2
Rad. corr.	$0.5 \div 2.0$
uds simulation	$1.2 \div 6.6$
l^+l^-	$0.3 \div 0.6$
e^+e^-X	0.2
Trigger	0.3
Nuclear interaction	0.4
Machine background	$0.4 \div 0.9$
Cuts	0.7
Total	$2.1 \div 7.1$

V.V. Anashin et al., Phys.Lett. B 770C, 174 (2017), [arXiv:1610.02827]

Comparison with others experiments

"De omnibus dubitandum"(All is to be doubted)
René Descartes



The quantity R versus the c.m. energy and the sum of the prediction of perturbative QCD and a contribution of narrow resonances.

In the c.m.energy range 3.08-3.72 GeV the weighted average $\overline{R}_{\mathrm{uds}} = 2.204 \pm 0.014 \pm 0.026$ is approximately one sigma higher than that theoretically expected, $R_{\mathrm{uds}}^{\mathrm{pQCD}} = 2.16 \pm 0.01$ calculated according to the pQCD In the lower c.m.energy range 1.84-3.05 GeV the weighted average is $2.225 \pm 0.020 \pm 0.047$ (the pQCD prediction of 2.18 ± 0.02).

An application of the $R(s)$

"Natura appetit perfectum, ita est lex"

(Nature desires perfection, so also does the law)

Correlated uncertainties of R_{uds} in %

Source	Uncertainty in %	
	Data 2010	Data 2010 / 2011,2014
Luminosity		
Cross section calc.	0.5	0.4
Calorimeter response	0.7	-
Calorimeter alignment	0.2	0.2
Rad. correction		
Π approx.	0.3	0.1
$\delta R_{uds}(s)$	0.2	0.2
$\delta\epsilon(s)$	0.3	0.2
Continuum simulation	1.2	$0.4 \div 0.8$
Track reconstr.	0.5	0.4
e^+e^-X contribution	0.2	0.1
l^+l^- contribution	0.3	0.2
Trigger efficiency	0.3	0.2
Nuclear interaction	0.4	0.2
Sum in quadrature	1.8	$0.8 \div 1.1$

$$R_{uds}(s) \simeq 2\times\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s^2}{\pi^2}\times\left(\frac{365}{24}-9\zeta_3-\frac{11}{4}\right)\right)$$

where ζ is the Euler-Riemann zeta function,

$$\alpha_s(s) = \frac{1}{b_0 t}\left(1 - \frac{b_1 l}{b_0^2 t} + \frac{b_1(l^2 - l - 1) + b_0 b_2}{b_0^4 t^2} + \frac{b_1^3(-2l^3 + 5l^2 + 4l - 1) - 6b_0 b_2 b_1 l + b_0^2 b^3}{2b_0^6 t^3}\right)$$

with $t = \ln\frac{s}{\Lambda^2}$, $l = \ln t$ parametrized in terms of the QCD scale parameter Λ and coefficients b_0, b_1, b_3 (can be found in PDG). To determine Λ, we minimise the χ^2 function

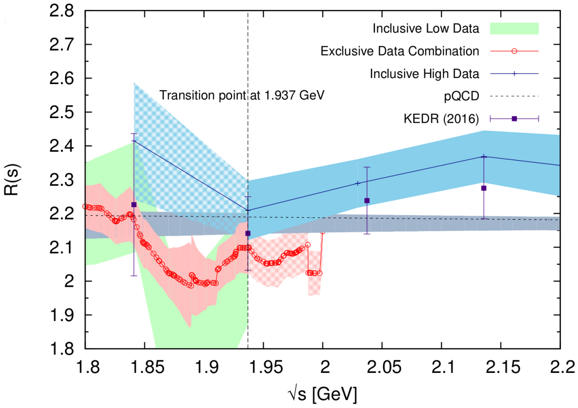
$$\chi^2 = \sum_i\sum_j\left(R^{meas}_{uds}(s_i)-R^{calc}_{uds}(s_i)\right)C^{-1}_{ij}\left(R^{meas}_{uds}(s_j)-R^{calc}_{uds}(s_j)\right),$$

The obtained value of $\Lambda = 0.361^{+0.155}_{-0.174}$ GeV corresponds to $\alpha_s(m_\tau) = 0.332^{+0.100}_{-0.092}$. If the next order of pQCD is included in the expansion of R_{uds}, the fitting results are as follows: $\Lambda = 0.437^{+0.210}_{-0.215}$ GeV and $\alpha_s(m_\tau) = 0.378^{+0.173}_{-0.120}$.

$\alpha_s(m_\tau)$ determined from our $R(s)$ results is consistent with obtained in semileptonic τ decays ($\alpha_s(m_\tau) = 0.331 \pm 0.013$)

Comparison with exclusive data

"Ex uno disces omnes"(From one thing you can discern all)



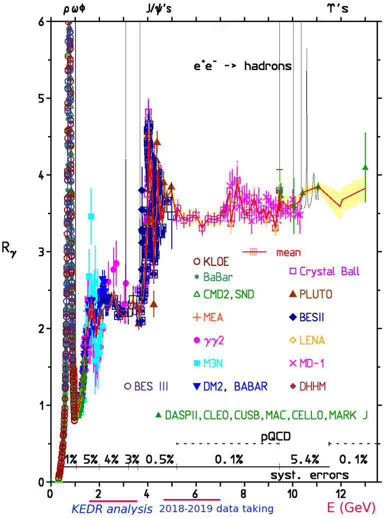
A. Keshavarzi, D. Nomura and T. Teubner.
The muon $g-2$ and $\alpha(M_Z^2)$: a new data-based analysis.
Phys. Rev. D **97**, 114025 (2018).[arXiv:1802.02995].

Outlook

"Omne futurum incertum"(Every future thing is uncertain)

R measurement in the energy range 4.56-6.96 GeV.

- First scan finished in 2018. An integrated luminosity $\sim 4\ pb^{-1}$ collected at 8 equidistant points with a step ~ 0.3 GeV from 4.71 to 6.81 GeV
- In 2019 we plan to start the second scan (10 equidistant points in the energy range $4.56 \div 6.96$ GeV).



F. Jegerlehner arXiv:1511.0447

Summary

"Jucundi acti labores"(past labors are pleasant)
Cicero

- KEDR measured the R values at 22 center-of-mass energies between 1.84 and 3.72 GeV.
  In the energy range between 1.84 and 3.05 GeV the achieved accuracy is about or better than 3.9% at most of the energy points with a systematic uncertainty less than 2.4%.
  For the energies above J/ψ resonance the total error is about or better than 2.6%
  and a systematic uncertainty of about 1.9%.

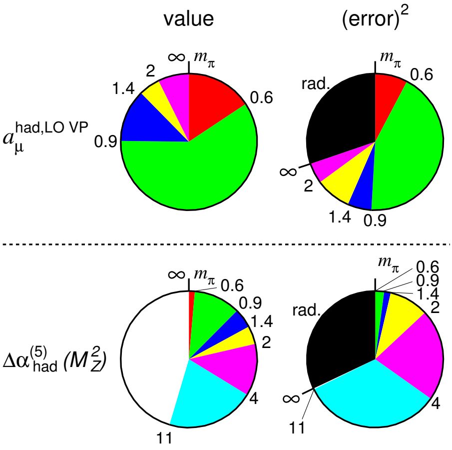
- We plan to take data at the energy range from 4.56 to 6.96 GeV

"Gratia gratiam parit"(thanks begets thanks)

Thank you for your time and attention

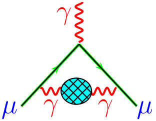
BACKUP SLIDES

R contribution in a_μ and $\alpha(M_Z^2)$



K.Hagiwara et al. arxiv:1105.3149

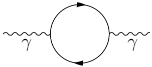
$$a_\mu^{exp} = (g_\mu - 2)/2$$



$$a_\mu^{LO\ \ VP} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{K(s)R(s)}{s}ds$$

Low energy contributions dominate

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}$$

$$\Delta\alpha = \sum_f$$  $$= \Delta\alpha_{\text{lep}}(s)+\Delta\alpha_{\text{had}}(s)$$

$$\Delta\alpha^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\,\text{Re}\int_{m_\pi^2}^{\infty}\frac{R(s)ds}{s(s-M_Z^2-i\epsilon)}$$

In the soft photon approximation analytical expression for the annihilation cross section nearby a narrow resonance.
Ya.I. Azimov *et al.* JETP Lett. 21 (1975) 172. With up-today modifications one has

$$\sigma^{e^+e^-\to\text{hadr}}(s) = \sigma^{e^+e^-\to\text{hadr}}_{\text{continuum}} + \frac{12\pi}{s}\,(1+\delta_{sf})\left[\frac{\Gamma_{ee}\tilde{\Gamma}_h}{\Gamma M}\,\mathrm{Im}\,f(s) - \frac{2\alpha\sqrt{R\,\Gamma_{ee}\tilde{\Gamma}_h}}{3\sqrt{s}}\,\lambda\,\mathrm{Re}\,\frac{f^*(s)}{1-\Pi_0}\right],$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{ee\to ee} = \left(\frac{d\sigma}{d\Omega}\right)^{ee\to ee}_{\text{QED}} + \frac{1}{s}\,(1+\delta_{sf})\left\{\frac{9}{4}\frac{\Gamma_{ee}^2}{\Gamma M}(1+\cos^2\theta)\,\mathrm{Im}\,f - \frac{3\alpha}{2}\frac{\Gamma_{ee}}{M}\left[(1+\cos^2\theta)\,\mathrm{Re}\,\frac{f^*}{1-\Pi_0(s)} - \frac{(1+\cos\theta)^2}{(1-\cos\theta)}\,\mathrm{Re}\,\frac{f^*}{1-\Pi_0(t)}\right]\right\},$$

Recently it was verified in the work X. Y. Zhou, Y. D. Wang and L. G. Xia, Chin. Phys. C **41** (2017) no.8,083001

$$\delta = \frac{3}{4}\beta + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + \beta^2\left(\frac{37}{96} - \frac{\pi^2}{12} - \frac{L}{72}\right),\quad L = \ln\left(s/m_e^2\right),\ \beta = \frac{2\alpha}{\pi}\,(L-1),$$

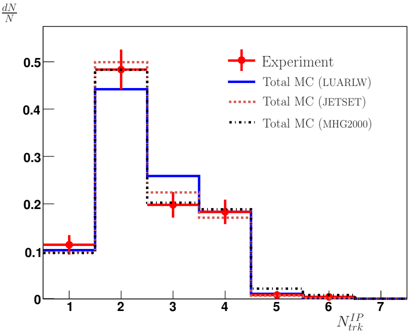
$$f(s) = \frac{\pi\beta}{\sin\pi\beta}\left(\frac{s}{M^2 - s - iM\Gamma}\right)^{1-\beta}$$

Γ_{ee}, Γ, M – 'dressed' parameters including corrections to the vacuum polarization, $\Gamma_{ee} = \Gamma^{(0)}_{ee}/|1-\Pi_0|^2$, λ–parameter controls the resonance–continuum interference, $\tilde{\Gamma}_h \neq \Gamma_h$

Numerical convolution with the collision energy distribution is used to fit resonance.

Detection efficiency uncertainty in the energy range $\sqrt{s} = 1.84 \div 3.05$ GeV

- Used two essentially different MC generators (LUARLW and tuned JETSET)
- We validated our estimate of the systematic uncertainty related to simulation of the *uds* continuum using an unfolding method (Chinise Physics C Vol. 37, No. 6 (2013) 063001).
- The estimate at the most problematic energy point 1.84 GeV was additionally verified using the exclusive generator MHG2000.



Detection efficiency uncertainties obtained by different methods

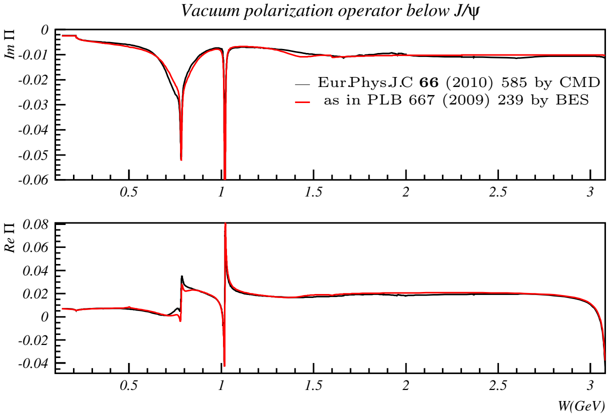
Energy, MeV	$\delta\epsilon/\epsilon$		
	LUARLW JETSET	Unfolding method	LUARLW MHG2000
1841.0	6.6%	3.6%	3.8%
1937.0 ÷ 2135.7	2.5%	1.9%	–
2135.7 ÷ 3048.1	1.2%	0.5%	–

Unfolding method

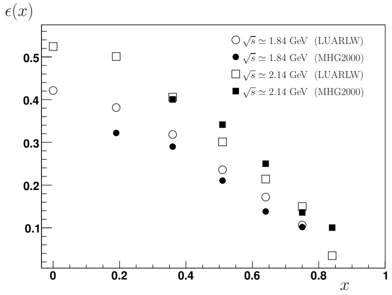
- An efficiency matrix ϵ_{ij} describes the efficiency of an event generated with j charged tracks to be reconstructed with i charged tracks.
- The distribution of the number of observed charged track events in data, N_i^{obs} , is known. The true multiplicity distribution in data can be estimated from the observed multiplicity distribution in data and the efficiency matrix by minimizing the χ^2.
- $$\chi^2 = \sum_{i=1}^{i=8} \frac{N_i^{obs} - \sum_{i=1}^{i=8} \epsilon_{ij} \times N_j}{N_i^{obs}}$$

  where the N_j $(j = 0, 2, 4, 6, 8)$ describe the true multiplicity distribution in data and are taken as floating parameters in the fit.
- The total «true» number of events in data can be obtained by summing all fitted N_j.

Π(s) calculation



Radiation correction calculation in the energy range 1.84 – 3.05 GeV



Detection efficiency vs variable x at 1.84 and 2.14 GeV.

$$1+\delta(s) = \int \frac{dx}{1-x} \frac{\mathcal{F}(s,x)}{|1-\Pi((1-x)s)|^2} \frac{R((1-x)s)\varepsilon((1-x)s)}{R(s)\varepsilon(s)}$$

$$R(s) = -\frac{3}{\alpha} \operatorname{Im} \Pi_{\text{hadr}}(s)$$

Vacuum polarization according to CMD-2 data compilation: Eur. Phys. J. C66 (2010) 585

Radiative correction factor $1+\delta$

$\sqrt{s}$, MeV	$1+\delta$	$\sqrt{s}$, MeV	$1+\delta$
1841.0	1.0423 ± 0.0208	2542.6	1.0739 ± 0.0054
1937.0	1.0429 ± 0.0156	2644.8	1.0796 ± 0.0054
2037.3	1.0515 ± 0.0126	2744.6	1.0809 ± 0.0054
2135.7	1.0634 ± 0.0106	2849.7	1.0823 ± 0.0054
2239.2	1.0645 ± 0.0096	2948.9	1.0774 ± 0.0054
2339.5	1.0664 ± 0.0075	3048.1	1.0584 ± 0.0053
2444.1	1.0684 ± 0.0064		

Selection criteria

Selection criteria for hadronic events which were used by AND.

Variable	Allowed range	
	3.12-3.72 GeV (2010-2011)	1.84 - 3.05 GeV
$N_{\text{track}}^{\text{IP}}$	≥ 1	≥ 1
E_{obs}	> 1.6 GeV	> 1.4 GeV (> 1.3 GeV if $E_{beam} < 1.05$ GeV)
$E_{\gamma}^{\text{max}}/E_{\text{beam}}$	< 0.8	< 0.8
$E_{\text{obs}} - E_{\gamma}^{\text{max}}$		> 1.2 GeV (> 1.1 GeV if $E_{beam} < 1.05$ GeV)
E_{cal}	> 0.75 GeV	> 0.55 GeV
H_2/H_0	< 0.85	< 0.9
$\lvert P_z^{\text{miss}}/E_{\text{obs}}\rvert$	< 0.6	< 0.7
$E_{\text{LKr}}/E_{\text{cal}}^{\text{tot}}$	> 0.15	> 0.15
$\lvert Z_{\text{vertex}}\rvert$	< 20.0 cm	< 15.0 cm
	$N_{\text{particles}} \geq 4$ or $\tilde{N}_{\text{track}}^{\text{IP}} \geq 2$	$N_{\text{particles}} \geq 3$ or $\tilde{N}_{\text{track}}^{\text{IP}} \geq 2$

The correlation matrix for systematic uncertainties of the R value obtained in the KEDR experiments

Point Correlation Matrix

1	1	0.139	0.143	0.193	0.192	0.212	0.212	0.216	0.207	0.211	0.216	0.201	0.222	0.096	0.046	0.096	0.105	0.110	0.098	0.089	0.114	0.071
2		1	0.309	0.418	0.408	0.445	0.437	0.466	0.446	0.457	0.467	0.434	0.480	0.200	0.097	0.201	0.225	0.229	0.212	0.189	0.244	0.151
3			1	0.423	0.425	0.470	0.470	0.480	0.460	0.463	0.480	0.442	0.486	0.212	0.101	0.212	0.232	0.243	0.218	0.198	0.253	0.158
4				1	0.575	0.635	0.635	0.649	0.622	0.610	0.649	0.598	0.637	0.287	0.137	0.286	0.314	0.329	0.295	0.268	0.342	0.213
5					1	0.621	0.621	0.642	0.615	0.629	0.643	0.598	0.661	0.280	0.134	0.280	0.310	0.322	0.293	0.262	0.336	0.208
6						1	0.677	0.709	0.679	0.695	0.710	0.661	0.730	0.306	0.148	0.305	0.342	0.351	0.323	0.287	0.371	0.229
7							1	0.709	0.679	0.695	0.710	0.661	0.730	0.304	0.148	0.305	0.342	0.348	0.323	0.287	0.371	0.229
8								1	0.695	0.710	0.725	0.675	0.745	0.320	0.153	0.320	0.351	0.368	0.330	0.299	0.382	0.238
9									1	0.681	0.695	0.647	0.715	0.307	0.146	0.306	0.336	0.352	0.316	0.287	0.366	0.228
10										1	0.710	0.654	0.701	0.314	0.150	0.313	0.344	0.360	0.323	0.293	0.374	0.233
11											1	0.675	0.745	0.321	0.153	0.320	0.351	0.368	0.330	0.300	0.382	0.238
12												1	0.687	0.298	0.142	0.298	0.327	0.342	0.307	0.279	0.356	0.222
13													1	0.330	0.157	0.329	0.361	0.378	0.339	0.308	0.393	0.245
14														1	0.288	0.396	0.405	0.394	0.356	0.317	0.403	0.333
15															1	0.345	0.347	0.345	0.305	0.275	0.345	0.288
16																1	0.486	0.475	0.427	0.380	0.483	0.400
17																	1	0.486	0.427	0.387	0.486	0.405
18																		1	0.427	0.380	0.483	0.400
19																			1	0.340	0.427	0.356
20																				1	0.384	0.318
21																					1	0.403
22																						1