Light mesons from JPAC+COMPASS analyses

Misha Mikhasenko

Joint Physics Analysis Center,
COMPASS @ CERN,
CERN, Switzerland

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Contents

Introduction

Joint Physics Analysis Center

[collage by Vincent Mathieu]

Emilie Passemar Indiana U.
Andrew Jackura Indiana U.
Nathan Sherrill Indiana U.
Tim Londergan Indiana U.

Joint Physics Analysis Center

Astrid Hiller Blin Mainz U.
Misha Mikhasenko Bonn U.
VM JLab
Jannes Nys Ghent U.
Adam Szczepaniak Indiana U.

Cesar Fernández-Ramírez UNAM

Alessandro Pilloni JLab ⟶ ECT*

Łukasz Bibrzycki Cracow P. U.

Arkaitz Rodas Bilbao Madrid U.

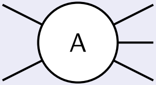
Viktor Mokeev JLab

Miguel Albaladejo JLab

JPAC effort

Collaboration of theoreticians and experimentalists

Amplitude analysis



- Writting amplitudes using general QFT constraints
- Analysis of experimental data
- Analytic continuation, pole search

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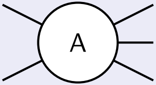

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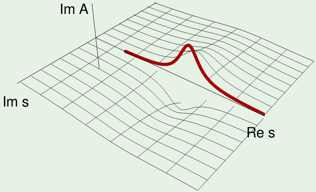
General properties of the scattering amplitude



Analyticity + Unitary + Crossing symmetry

- Scattering amplitude is an analytic function in $s = E^2$ complex plane,
- The Real axis $\to$ physical world,
- Resonances = poles of the unphysical sheet.

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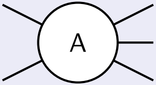

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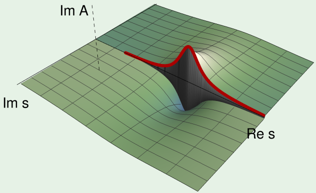

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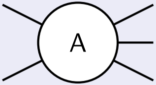

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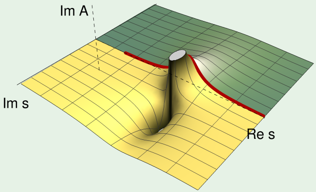

Analyticity + Unitary + Crossing symmetry

- Scattering amplitude is an analytic function in $s = E^2$ complex plane,
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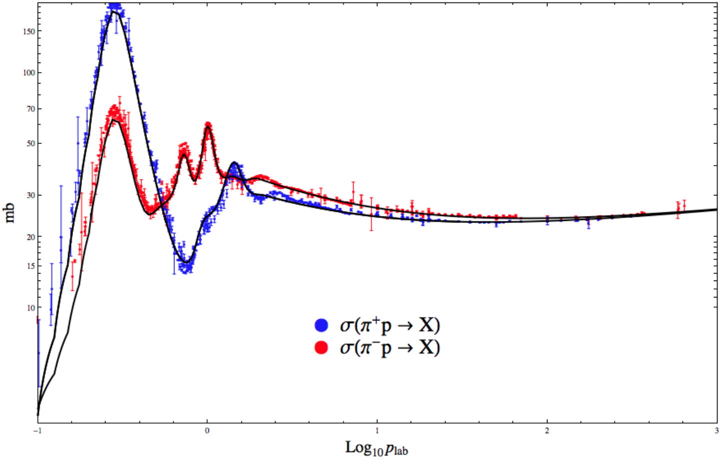
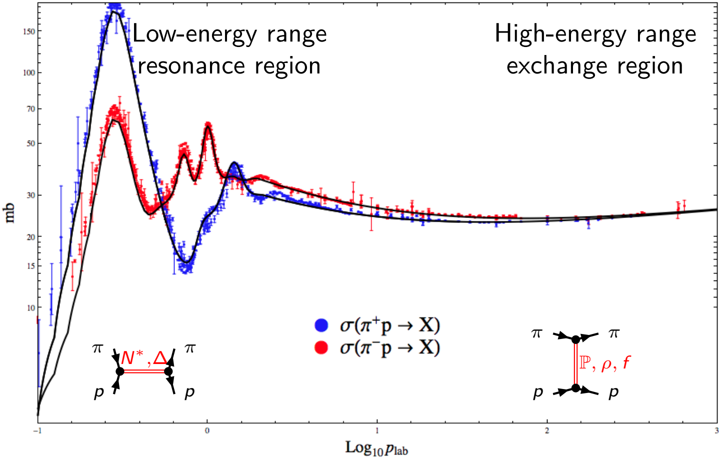
Two regimes of scattering

Hadronic duality [V.Mathieu,et al.,PRD92 (2015), 074004]



Two regimes of scattering

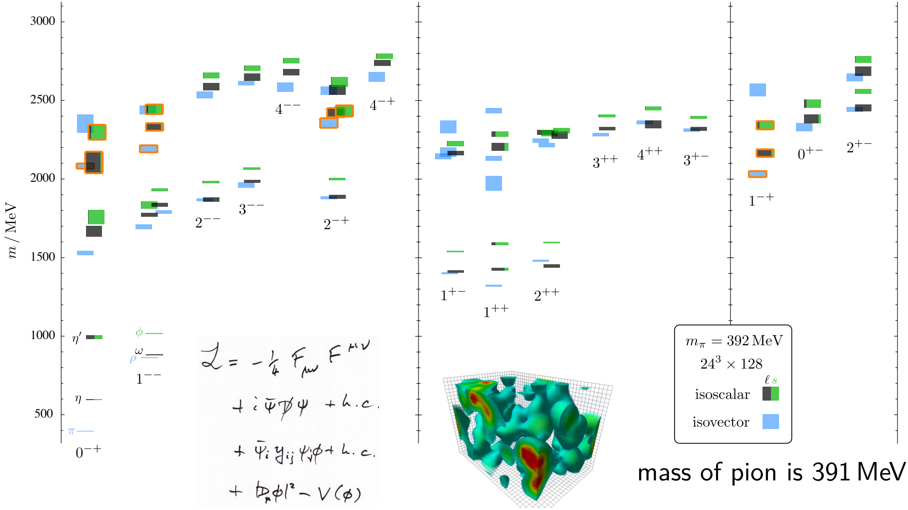
Hadronic duality [V.Mathieu,et al.,PRD92 (2015), 074004]



Hadronic excitations

Results of lattice QCD

[Dudek et al., PRD 88, 094505 (2013)]



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi} \not{D} \Psi + h.c. + \bar{\Psi}_i y_{ij} \Psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

mass of pion is 391 MeV

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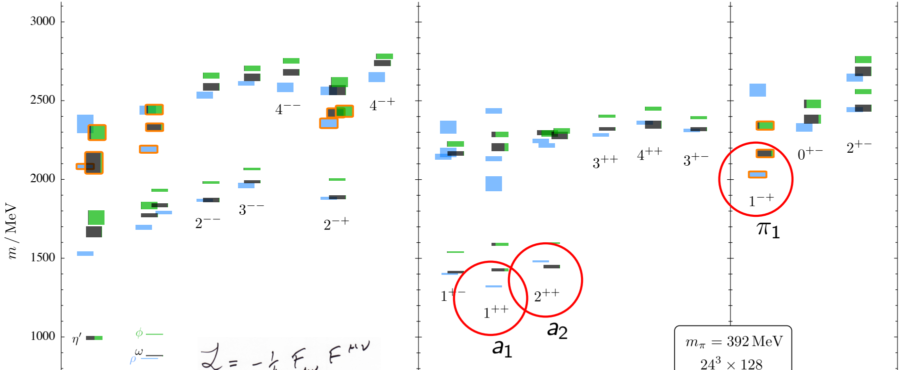

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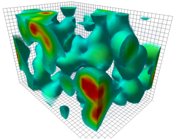
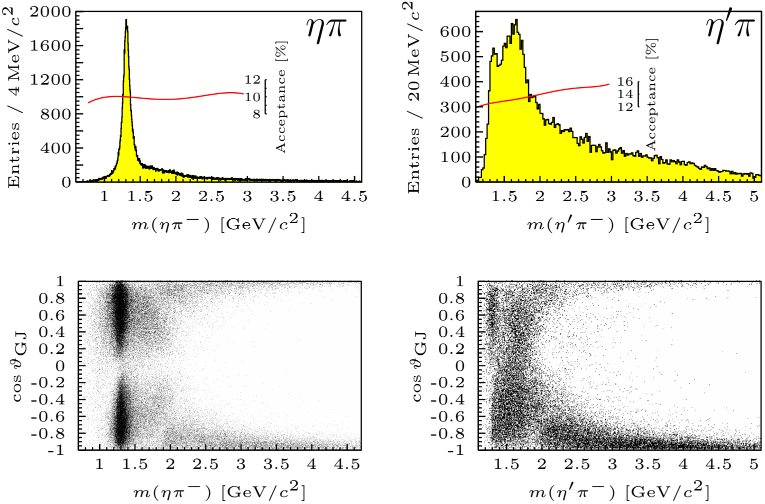


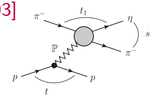
mass of pion is 391 MeV

$\eta^{(\prime)}\pi$ analyses

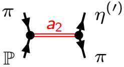
$\eta\pi$ vs $\eta'\pi$ at COMPASS

[(COMPASS) PLB 740 (2015) 303]

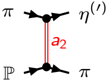




Resonance production



η-forward production



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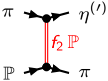
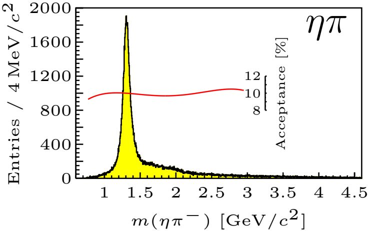
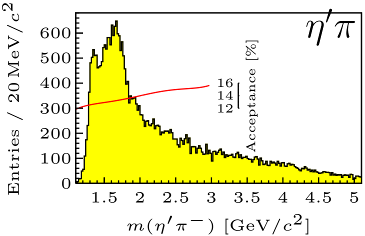


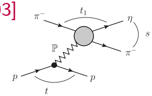
$\eta^{(\prime)}(0^-)\pi(0^-)$, $J^{PC} = L^{P+}$ $\Rightarrow$ $\cos\theta_{\rm GJ}$ asymmetry $\Rightarrow$ exotic waves!

$\eta\pi$ vs $\eta'\pi$ at COMPASS

[(COMPASS) PLB 740 (2015) 303]







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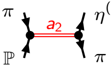
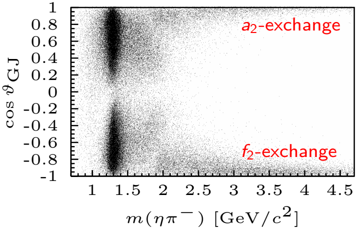
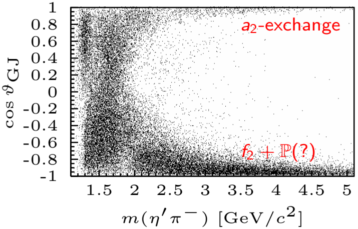
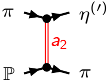






η-forward production



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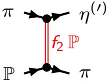

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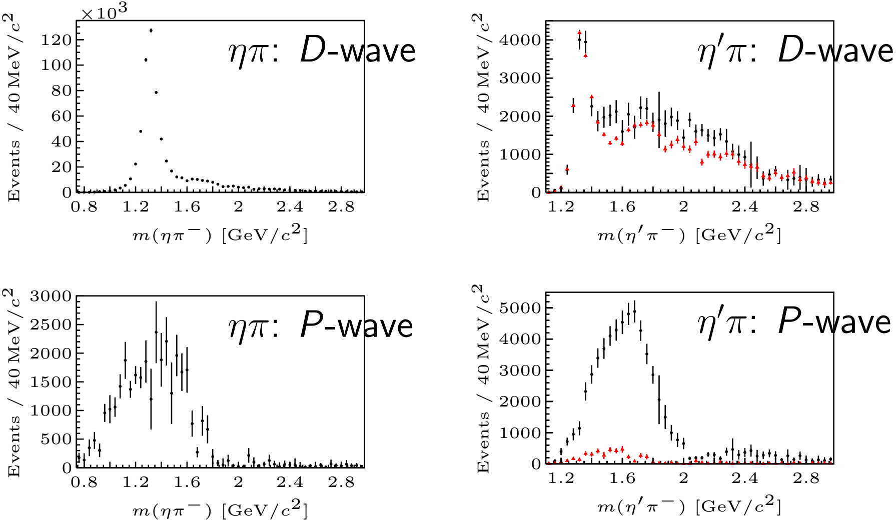
$\eta^{(\prime)}\pi$ partial wave analysis

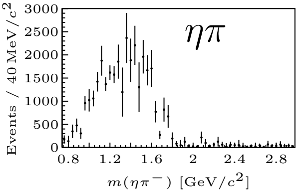
[(COMPASS) PLB 740 (2015) 303]

Two-main contribution: *P*- and *D*-waves



PDG status: exotic π_1 states

Two candidates



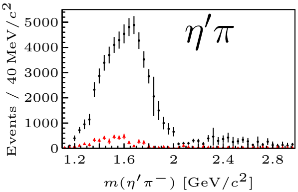
$\pi_1(1400)$ $I^G(J^{PC}) = 1^-(1^{-+})$

See also the mini-review under non-$q\bar{q}$ candidates in PDG 2006 , Journal of Physics G33 1 (2006).

$\pi_1(1400)$ MASS	1354 ± 25 MeV (S = 1.8)
$\pi_1(1400)$ WIDTH	330 ± 35 MeV

Decay Modes

Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P (MeV/c)
Γ_1	$\eta\pi^0$	seen		557
Γ_2	$\eta\pi^-$	seen		556
Γ_3	$\eta'\pi$			318



$\pi_1(1600)$ $I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1600)$ MASS	1662^{+8}_{-9} MeV
$\pi_1(1600)$ WIDTH	241 ± 40 MeV (S = 1.4)

Decay Modes

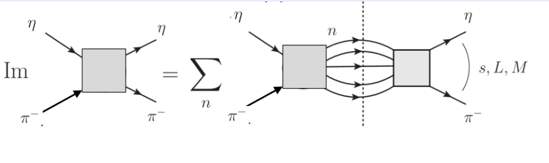
Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P (MeV/c)
Γ_1	$\pi\pi\pi$	seen		803
Γ_2	$\rho^0\pi^-$	seen		641
Γ_3	$f_2(1270)\pi^-$	not seen		318
Γ_4	$b_1(1235)\pi$	seen		357
Γ_5	$\eta'(958)\pi^-$	seen		543
Γ_6	$f_1(1285)\pi$	seen		314

Amplitude for $\eta\pi$ production [A.Jackura,MM,A.Pilloni,et al. (JPAC-COMPASS), PLB779, 464-472]

N-over-*D* method

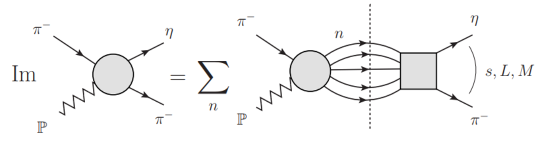
Scattering amplitude: $\eta\pi \to \eta\pi$, *D*-wave

$$T = \frac{N(s)}{D(s)}$$



Production amplitude: $\pi\mathbb{P} \to \eta\pi$, *D*-wave

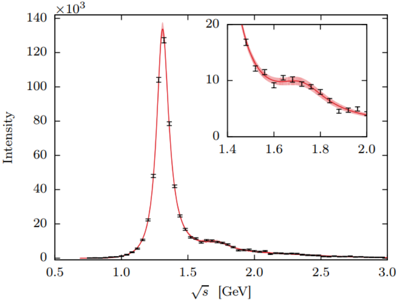
$$a = \frac{n(s)}{D(s)}$$



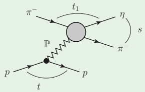
- $D(s)$ is universal, has only the right-hand cut.
- $N(s)$ and $n(s)$ have the left-hand cut only (exchanges)

Tensor mesons ($J^{PC} = 2^{++}$)

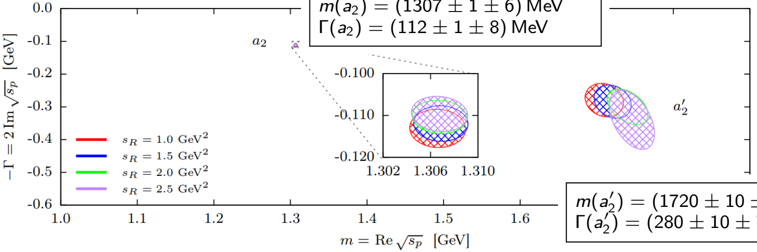
Advanced $\eta\pi$ analysis [A.Jackura,MM,A.Pilloni,et al. (JPAC-COMPASS), PLB779, 464-472]



Single channel: $\eta\pi$ D-wave



- Elastic unitarity
- Two CDD-poles



$m(a_2) = (1307 \pm 1 \pm 6)$ MeV
$\Gamma(a_2) = (112 \pm 1 \pm 8)$ MeV

$m(a_2') = (1720 \pm 10 \pm 60)$ MeV
$\Gamma(a_2') = (280 \pm 10 \pm 70)$ MeV

Coupled-channel amplitude [A.Rodas,A.Pilloni,MM,et al. (JPAC), PRL122 (2019)]

Scattering amplitude: $\eta^{(\prime)}\pi \to \eta^{(\prime)}\pi$, P/D-waves

$$T = \frac{N(s)}{D(s)}$$

- 2 K-matrix pole for D-wave
- 1 K-matrix pole for P-wave

$$\rho N^J_{ki}(s') = \delta_{ki} \frac{(p_{\eta^{(\prime)}\pi}\sqrt{s}/2)^{2J+1}}{(s' + s_L)^{2J+1+\alpha}},$$

$$D^J_{ki}(s) = \left[K^J(s)^{-1}\right]_{ki} - \frac{s}{\pi}\int_{s_k}^{\infty} ds' \frac{\rho N^J_{ki}(s')}{s'(s' - s - i\epsilon)}.$$

Production amplitude: $\pi\mathbb{P} \to \eta^{(\prime)}\pi$, P/D-waves

$$a = \frac{n(s)}{D(s)}$$

- left poles to model unknown production function $n(s)$

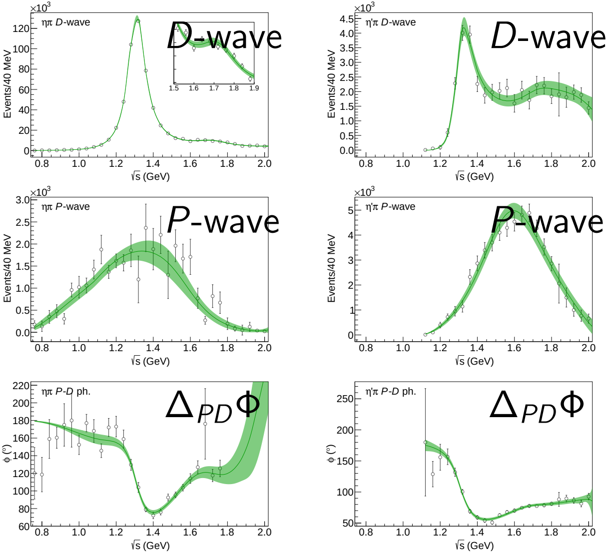
$$a^J_i(s) = q^{J-1} p^J_i \sum_k n^J_k(s) \left[D^J(s)^{-1}\right]_{ki}$$

- $D(s)$ has only the right-hand cut.
- $N(s)$ and $n(s)$ have the left-hand cut only (exchanges)

Fit to the data

[A.Rodas,A.Pilloni,MM,et al. (JPAC), PRL122 (2019)]

$\chi^2/\mathrm{ndf} = 162/122$, the band - 2σ bootstrap error



D-wave difference

- Kinematics ($m_{\eta'} > m_\eta$)

$\Rightarrow$ Same amplitude.

P-wave difference

- production mechanism
- + kinematics.

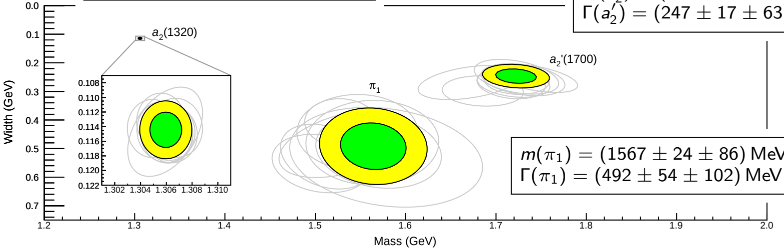
Results: pole positions

[A.Rodas,A.Pilloni,MM,et al. (JPAC), PRL122 (2019)]

$m(a_2) = (1306 \pm 0.8 \pm 1.3)$ MeV
$\Gamma(a_2) = (114.4 \pm 1.6 \pm 0.0)$ MeV

$m(a_2') = (1722 \pm 15 \pm 67)$ MeV
$\Gamma(a_2') = (247 \pm 17 \pm 63)$ MeV

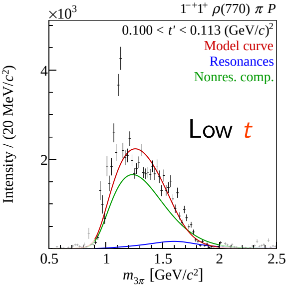
$m(\pi_1) = (1567 \pm 24 \pm 86)$ MeV
$\Gamma(\pi_1) = (492 \pm 54 \pm 102)$ MeV

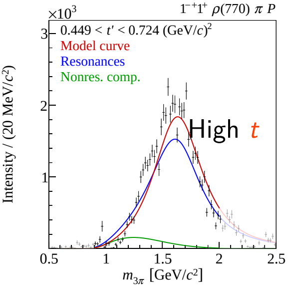


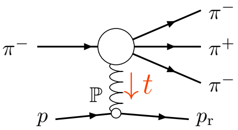
- Change parametrization of the denominator $\rho N^J_{ki}(s') = \delta_{ki}\,\frac{(p_{\eta^{(\prime)}\pi}\sqrt{s}/2)^{2J+1}}{(s'+s_L)^{2J+1+\alpha}}$,
  - $s_R = 1\,\text{GeV} \to 0.8, 1.8\,\text{GeV}$.
  - $\alpha = 2 \to 1\text{GeV}$.
  - Different function, $\rho N^J_{ki}(s') = \delta_{ki}\, Q_J(z_{s'})\, s'^{-\alpha} \lambda^{-1/2}(s', m^2_{\eta^{(\prime)}}, m^2_\pi)$
- Change of parameters in the numerator $n(s)$
  - Effective transferred momentum $t_{\text{eff}} = -0.1\,\text{GeV}^2 \to -0.5\,\text{GeV}^2$.
  - Order of the polynomial 3rd-order $\to$ 4th-order.

Same π_1 as in 3π?

[See a talk of B.Ketzer, afternoon]

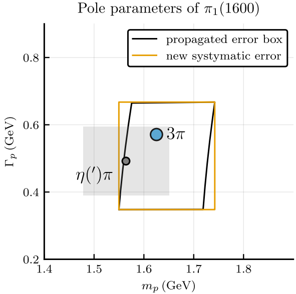






The COMPASS fit:

- Signal by BW amplitude
- Flexible background



Consistent results on $\pi_1(1600)$ pole:

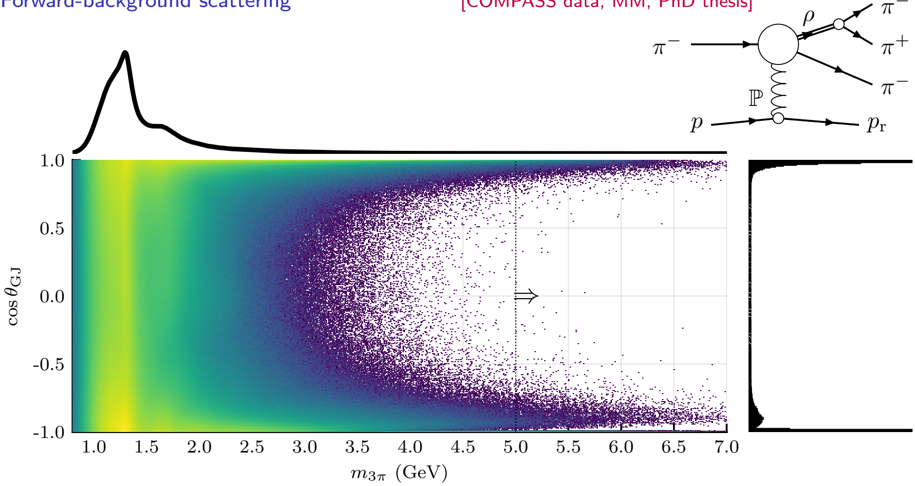
- $\rho\pi$ Breit-Wigner parameters
  $\Rightarrow$ pole position
- $\eta^{(\prime)}\pi$ systematic margins

Three-pions physics

Diffractive production of 3π off proton target

Forward-background scattering

[COMPASS data, MM, PhD thesis]

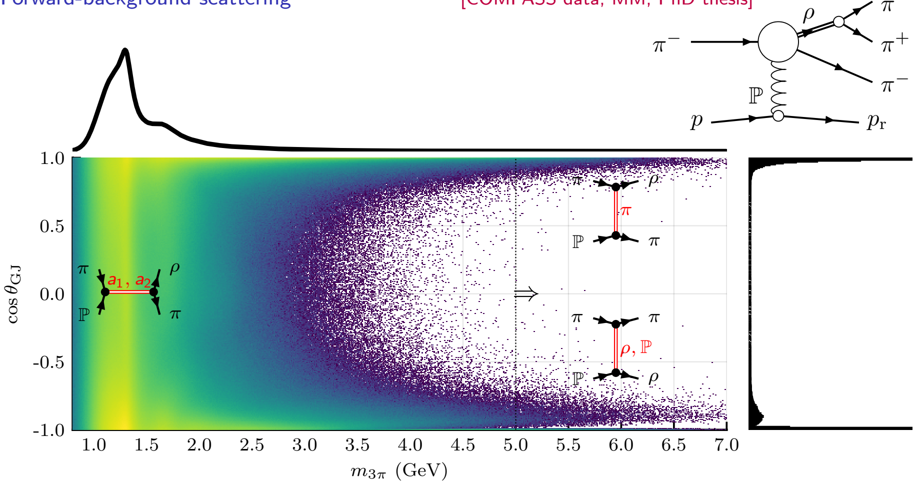


The high-energy exchange processes penetrate to the low energy and make resonance characterization difficult

Diffractive production of 3π off proton target

Forward-background scattering

[COMPASS data, MM, PhD thesis]

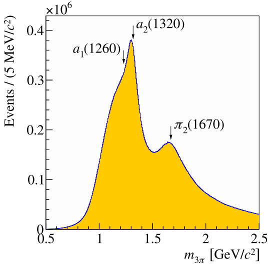


The high-energy exchange processes penetrate to the low energy and make resonance characterization difficult

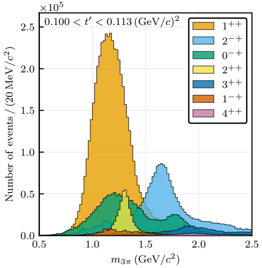
Three-pion resonances

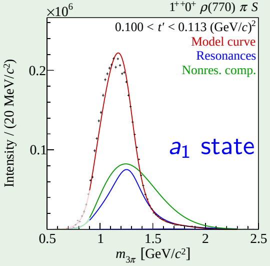
[see talk by D.Ryabchikov, afternoon]

COMPASS PWA [PRD95 (2017) 032004]



Partial Wave Analysis →





COMPASS Fit:

- Signal by BW amplitude $\sim 50\%$
- Background $\sim 50\%$

$m_{a_1}^{(\mathrm{BW})} = (1299^{+12}_{-28})$ MeV,

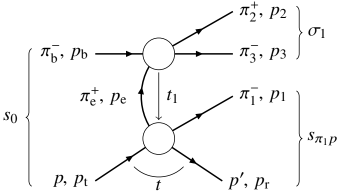
$\Gamma_{a_1}^{(\mathrm{BW})} = (380 \pm 80)$ MeV.

Large uncertainty due to unknonwn background.

Model for the forward scattering

[MM, A.Jackura (JPAC) in preparation]

Deck effect



$$= (T_{\pi_1 p})_{\lambda\lambda'} \frac{1}{m_\pi^2 - t_1} T_{\pi_2\pi_3} \qquad (1)$$

- Two diagrams (π^- symmetrization)

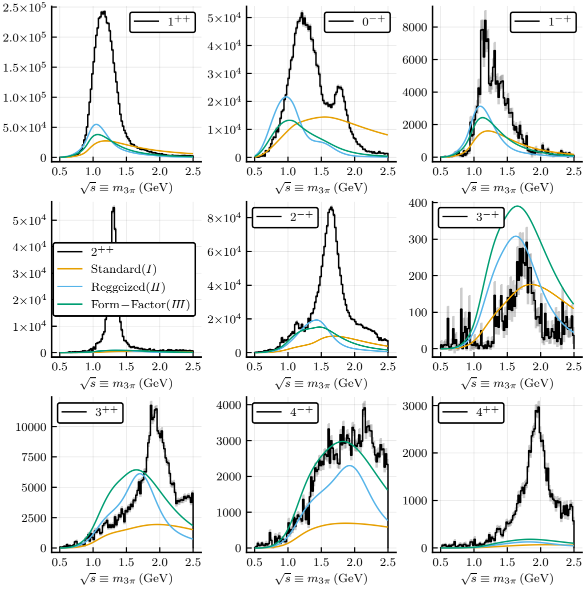
$$\mathcal{B}_{\lambda\lambda'} = \mathcal{B}^{(1)}_{\lambda\lambda'} + \mathcal{B}^{(3)}_{\lambda\lambda'}$$

- High energy $p\pi$ scattering
- $\pi\pi$ scattering dominated by resonances in lower partial waves
  - Relative strength of S, P, D-waves is controlled by unitarity

$$\mathcal{B}^{(1)} = s_{\pi p} F(t) \frac{\mathrm{FF}(t_1)}{m_\pi^2 - t_1} \left[\frac{2}{3}\, t^{(\sigma_1, f_0)}(\sigma_1) + 3\, t^{(\rho)}(\sigma_1, t_1) P_1(\cos\theta_{\pi\pi}) + \frac{10}{3}\, t^{(f_2)}(\sigma_1, t_1) P_2(\cos\theta_{\pi\pi}) \right].$$

Comparison with the COMPASS data

[MM PhD thesis]



Pion propagator:

- Standard Deck

$$\frac{1}{m_\pi^2 - t_1},$$

- Regge Deck

$$\frac{e^{-i\pi\alpha(t_1)/2}}{m_\pi^2 - t_1}\left(\frac{s' - u'}{2s_{\mathrm{sc}}}\right)^{\alpha(t_1)}$$

- Form-factored Deck

$$\frac{e^{bt_1}}{m_\pi^2 - t_1},$$

$a_1(1260)$ state – isospin parter of ρ

$a_1(1260)$ WIDTH

INSPIRE search

VALUE (MeV)	EVTS		DOCUMENT ID		TECN	COMMENT
250 to 600	**OUR ESTIMATE**					
389 ± 29	**OUR AVERAGE** Error includes scale factor of 1.3.					
430 ±24 ±31			DARGENT	2017	RVUE	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
367 ±9 $^{+28}_{-25}$	420k		ALEKSEEV	2010	COMP	190 $\pi^- \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •						
410 ±31 ±30		1	AUBERT	2007AU	BABR	10.6 $e^+ e^- \to \rho^0\rho^\pm\pi^\mp\gamma$
520 - 680	6360	2	LINK	2007A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ±20		3	GOMEZ-DUMM	2004	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ±41	90k		SALVINI	2004	OBLX	$\bar{p}\, p \to 2\pi^+ 2\pi^-$
460 ±85	205	4	DRUTSKOY	2002	BELL	$B^{(*)} K^- K^{*0}$
814 ±36 ±13	37k	5	ASNER	2000	CLE2	10.6 $e^+ e^- \to \tau^+\tau^-$, $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$

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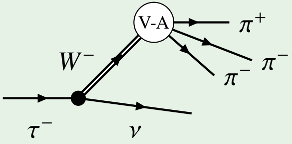
$\tau^- \to \pi^-\pi^+\pi^-\nu$

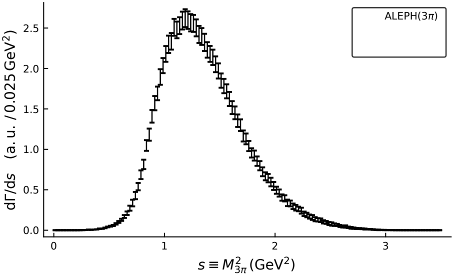


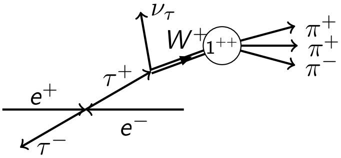
- V-A: Vector (1^{--}) or Axial (1^{++})
- Isospin 1 due to the charge
- Negative G-parity $\Rightarrow$ positive C-parity

$\Rightarrow J^{PC} = 1^{++}$

Fit to ALEPH data

[data from ALEPH, Phys.Rept.421 (2005)]





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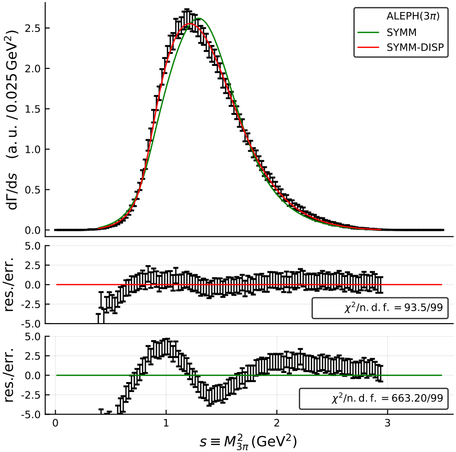


Dispersive model vs Non-dispersive model

- Difference: LH singularities
- The dispersive model fits significantly better

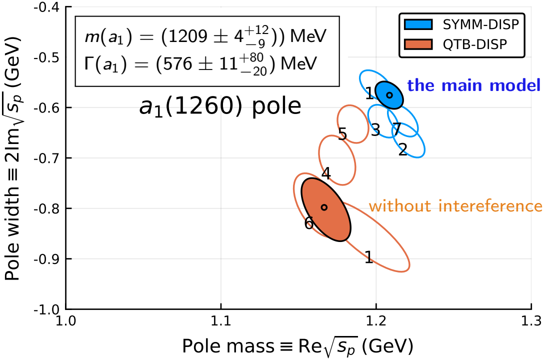
Fit function

$$\chi^2(c, m, g) = (\vec{D} - \vec{M}(c, m, g))^T C_{\text{stat}}^{-1} (\vec{D} - \vec{M}(c, m, g)),$$

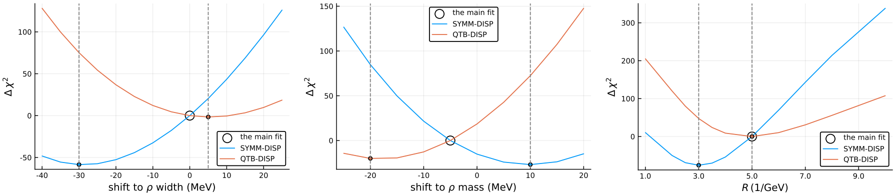
- Stat. cov. matrix is used in the fit
- Syst. cov. matrix – in the bootstrap

First measurement of the $a_1(1260)$ pole position

The result and systematic studies [MM (JPAC), PRD98 (2018), 096021]



#	Fit studies
1	$s < 2\ \mathrm{GeV}^2$
2	$R' = 3\ \mathrm{GeV}^{-1}$
3	$m'_\rho = m_\rho + 10$ MeV
4	$m'_\rho = m_\rho - 10$ MeV
5	$m'_\rho = m_\rho - 20$ MeV
6	$\Gamma'_\rho = \Gamma_\rho + 5$ MeV
7	$\Gamma'_\rho = \Gamma_\rho - 30$ MeV



Summary

Meson spectroscopy

- Using hadronic scattering as QCD excitation laboratory.
- Mapping gluonic degrees of freedom to structures of excited states is an essential test of QCD.
- Non-perturmative methods are required

Resent impact of JPAC to light meson spectroscopy

Extensive analyses and extraction of resonance poles:

- Tensor states: $a_2(1320)$ and $a_2(1700)$
- Establishing single exotic $\pi_1(1600)$
- Ground axial state $a_1(1260)$

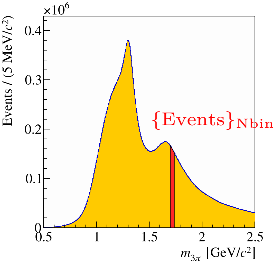
JPAC effort

- $>$ 50 research papers in PRD, PLB, PRL, EJPC ($>$ 10 in 2018)
- $>$ 100 invited talks and seminars
- Collaboration with GlueX, CLAS12, COMPASS, MAMI, BaBar, LHCb,...
- Summer Schools on Reaction Theory (2015, 2017)

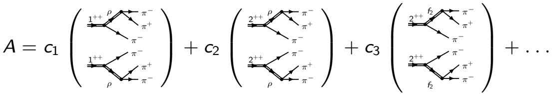
Thank you

Three-particles PW technique

COMPASS@PWA

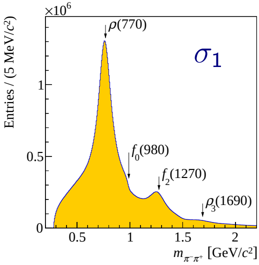


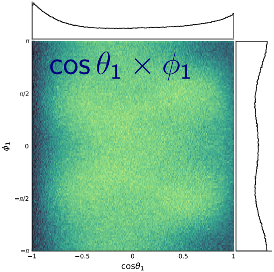
Model: a sum of 88* partial waves

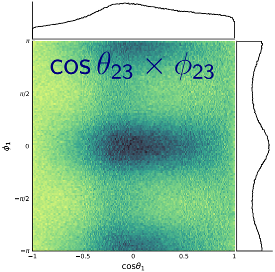


Likelihood Fit: a product of probability per event

$$L = \prod_{e=1}^{\mathrm{Nevents}} \frac{A(\tau_e)}{\int \mathrm{d}\tau_e A(\tau_e)}, \quad \tau = (\sigma_1, \cos\theta_1, \phi_1, \cos\theta_{23}, \phi_{23})$$







Tour to the complex plane

[MM (JPAC), PRD98 (2018), 096021]

Analytical continuation

$$|t_I^{-1}(s)| = \left| \frac{m^2 - s}{g^2} - i \left(\frac{\tilde{\rho}(s)}{2} + \rho(s) \right) \right|.$$

- Analytical continuation of $\rho(s)$: integral over the Dalitz plot for the complex energy

$$\rho(s) = \sum_\lambda \int \mathrm{d}\Phi_3 \left| f_\rho(\sigma_1) d_{\lambda 0}(\theta_{23}) - f_\rho(\sigma_3) d_{\lambda 0}(\hat{\theta}_3 + \theta_{12}) \right|^2$$

- Analytic contuation of ρ-meson decay amplitude f_ρ
  - Breit-Wigner amplitude with the dynamic width
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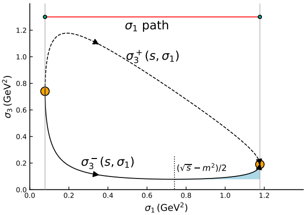

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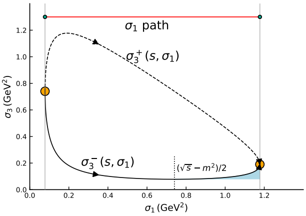

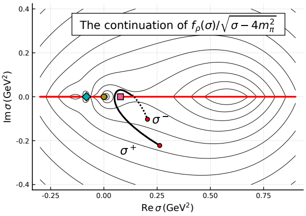

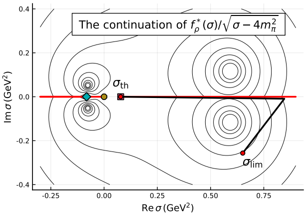

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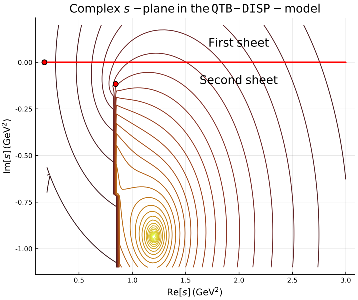

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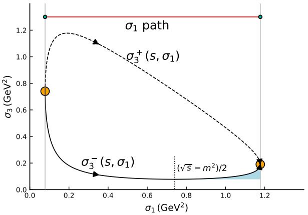

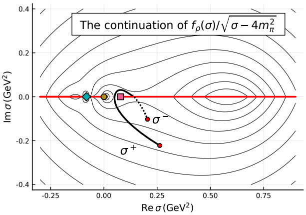

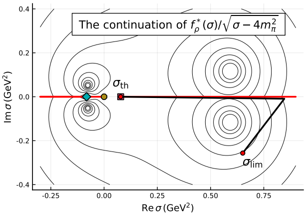

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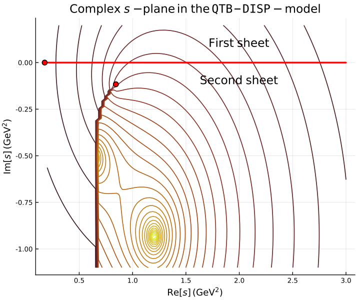

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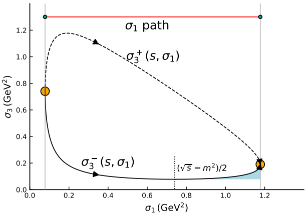

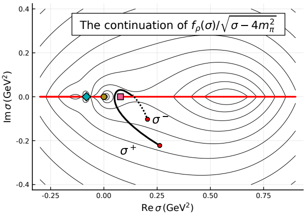

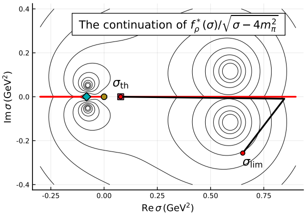

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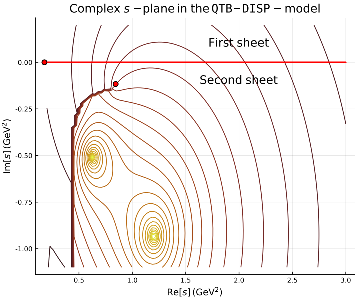

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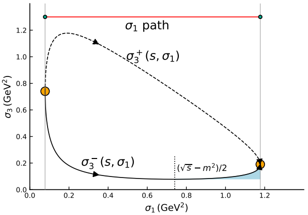

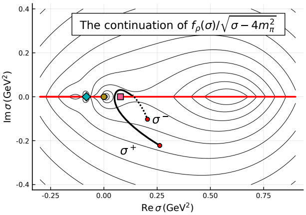

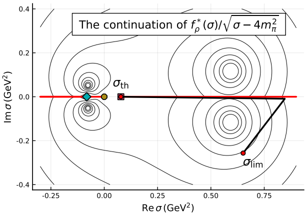

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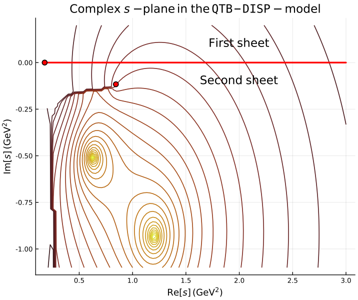

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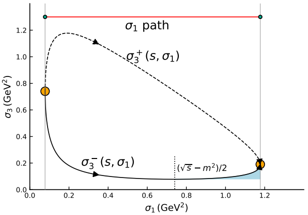
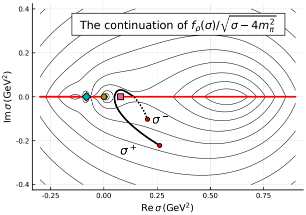
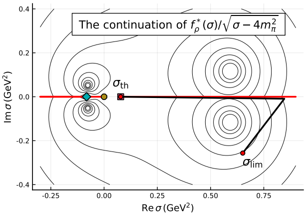




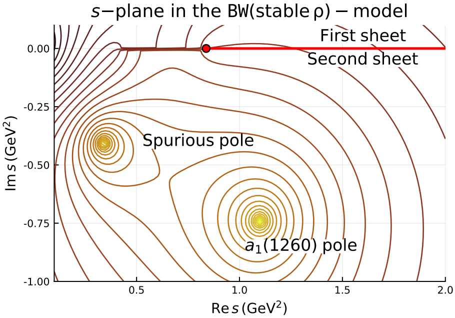


The spurious pole in the Breit-Wigner model

Energy dependent width, stable particles

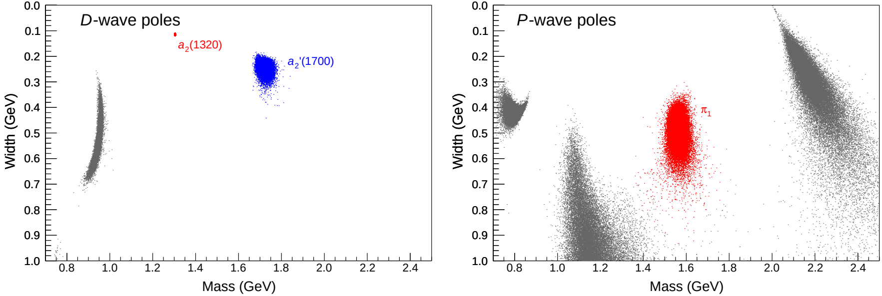
$$t(s) = \frac{1}{m^2 - s - im\Gamma(s)}, \quad \Gamma(s) = \Gamma_0 \frac{p(s)}{p(m^2)} \frac{m}{\sqrt{s}}, \quad p(s) = \frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{2\sqrt{s}}.$$

Example: $m_1 = 140\,\text{MeV}$, $m_2 = 770\,\text{MeV}$, $m = 1.26\,\text{GeV}$, $\Gamma_0 = 0.5\,\text{GeV}$



Live demo

Bootstrap: stability of the poles



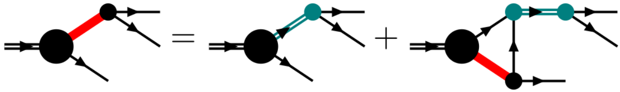
- Statistical bands are obtained by 50k bootstrap samples

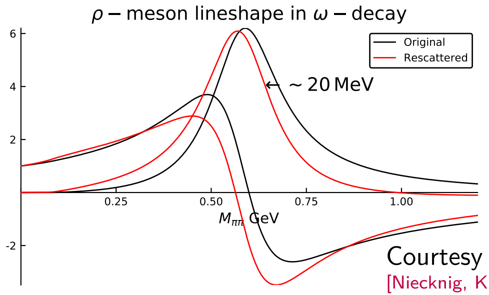
Subchannel dynamics

Khuri-Treiman equations

Consistency equations for the isobar lineshape

- Governed by two-body unitarity
- Model: only RHC for the isobar amplitude
- Uses Analyticity / Cauchy theorem / Omnès trick





KT analysis of $\omega \to 3\pi$

- $\rho \to \rho$ rescattering
- Solved by iteration

Courtesy of Tobias Isken

[Niecknig, Kubis, JHEP 1510 (2015) 142]