

New information on the strong isospin symmetry breaking in the reactions of the $a_0^0(980)$ and $f_0(980)$ resonance production

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ABSTRACT

We discuss the isotopic symmetry breaking as a tool of studying the production mechanisms and nature of light scalar mesons.

OUTLINE

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OUTLINE

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Here we are based, in particular, on the reviews:

N.N. Achasov and G.N. Shestakov, Nucl. Part. Phys. Proc. 287–288, 89-94 (2017) and Uspekhi Fiz. Nauk 189, No. 1, 3-32 (2019).

Introduction

The forty years ago we discovered theoretically a threshold phenomenon known as the mixing of $a_0^0(980)$ and $f_0(980)$ resonances that breaks the isotopic invariance considerably, since the effect is

$$\sim \sqrt{2(M_{K^0} - M_{K^+})/M_{K^0}} \approx 0.13$$

in the module of the amplitude, but not $\sim (M_{K^0} - M_{K^+})/M_{K^0} \approx 1/126$, i.e., by the order of magnitude greater than it could be expected from the naive considerations. This effect appears as a narrow resonant structure with the width of about $2(M_{K^0} - M_{K^+}) \approx 8 \text{ MeV}$ between the $K^+ K^-$ and $K^0 \bar{K}^0$ thresholds due to $a_0^0(980) \rightarrow K \bar{K} \rightarrow f_0(980)$ transition and vice versa.

N.N. Achasov , S.A. Devyanin and G.N. Shestakov, Phys. Lett. B 88, 367 (1979).

Since that time many new proposals were appeared, concerning both the searching $a_0^0(980) - f_0(980)$ mixing and estimating the effects related with this phenomenon. A short list of references on this subject is presented below.

Introduction

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[33] M. Ablikim et al., Phys. Rev. Lett. 108, 182001 (2012). [34] M. Ablikim et al., Phys. Rev. D 92, 012007 (2015). [35] N. N. Achasov, A. A. Kozhevnikov, and G. N. Shestakov, Phys. Rev. D 92, 036003 (2015). [36] N. N. Achasov, A. A. Kozhevnikov, and G. N. Shestakov, Phys. Rev. D 93, 114027 (2016). [37] M. Bayar, V. R. Debastiani, Phys. Lett. B 775, 94 (2017). [38] N. N. Achasov, G. N. Shestakov, Phys. Rev. D 96, 036013 (2017), ibid 96, 016027 (2017), ibid 96, 091501(R) (2017), ibid D 97 054033 (2018). [39] M. Ablikim et al., Phys. Rev. Lett. 121, 022001 (2018). [40] X. D. Cheng, H. B. Li, R. M. Wang, and M. Z. Yang, Phys. Rev. D 99, 014024 (2019).

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Nowadays this phenomenon is discovered experimentally and studied with the help of detectors **VES in Protvino and BESIII in Beijing** in the following reactions:

Introduction

V. Dorofeev et al., Eur. Phys. J. A 38, 149 (2008), ibid 47, 68 (2011),

$$\pi^- N \rightarrow \pi^- f_1(1285) N \rightarrow \pi^- f_0(980) \pi^0 N \rightarrow \pi^- \pi^+ \pi^- \pi^0 N,$$

M. Ablikim et al., Phys. Rev. D 83, 032003 (2011); M. Ablikim et al.,
Phys. Rev. Lett. 121, 022001 (2018),

$$J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0(980) \rightarrow \phi \eta \pi^0,$$

$$\chi_{c1}(1P) \rightarrow a_0(980) \pi^0 \rightarrow f_0(980) \pi^0 \rightarrow \pi^+ \pi^- \pi^0,$$

M. Ablikim et al., Phys. Rev. Lett. 108, 182001 (2012),

$$J/\psi \rightarrow \gamma \eta(1405) \rightarrow \gamma f_0(980) \pi^0 \rightarrow \gamma 3\pi,$$

M. Ablikim et al., Phys. Rev. D 92, 012007 (2015),

$$J/\psi \rightarrow \phi f_1(1285) \rightarrow \phi f_0(980) \pi^0 \rightarrow \phi 3\pi.$$

Introduction

After these experiments, it has become clear, [N.N. Achasov, A.A. Kozhevnikov, and G.N. Shestakov, Phys. Rev. D 92, 036003 \(2015\), D 93, 114027 \(2016\); N.N. Achasov and G.N. Shestakov, Nucl. Part. Phys. Proc. 287–288, 89 \(2017\)](#), that the similar isospin breaking effects can appear not only due to the $a_0^0(980) - f_0(980)$ mixing, but also for any mechanism of the production of the $K\bar{K}$ pairs with a definite isospin in the S wave,^a

$$X_{I=0} \rightarrow (K^+ K^- + K^0 \bar{K}^0) \rightarrow a_0^0(980) \rightarrow \eta \pi^0,$$

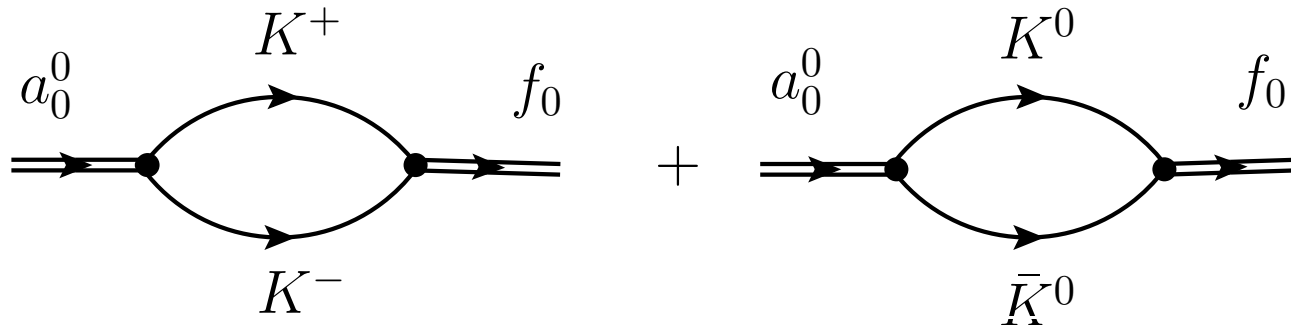
$$X_{I=1} \rightarrow (K^+ K^- + K^0 \bar{K}^0) \rightarrow f_0(980) \rightarrow \pi^+ \pi^-.$$

Thus [a new tool](#) to study the production mechanisms and the nature of light scalars is emerged.

^aEach such mechanism reproduces both the narrow resonant peak and sharp jump of the phase in the reaction amplitude between the $K^+ K^-$ and $K^0 \bar{K}^0$ thresholds.

What is the $a_0^0(980) - f_0(980)$ mixing?

The main contribution to the $a_0^0(980)$ - $f_0(980)$ mixing amplitude caused by the diagrams



has the form

$$\Pi_{a_0^0 f_0}(m) \approx \frac{g_{a_0 K^+ K^-} g_{f_0 K^+ K^-}}{16\pi} \left[i \left(\rho_{K^+ K^-}(m) - \rho_{K^0 \bar{K}^0}(m) \right) \right],$$

where the invariant virtual mass of scalar resonances $m \geq 2m_{K^0}$ and $\rho_{K\bar{K}}(m) = \sqrt{1 - 4m_K^2/m^2}$; if $0 < m < 2m_K$, then $\rho_{K\bar{K}}(m) \rightarrow i|\rho_{K\bar{K}}(m)|$.

$a_0^0(980) - f_0(980)$ mixing

In the region between the $K^+ K^-$ and $K^0 \bar{K}^0$ thresholds, which is the 8 MeV wide, the $a_0^0(980) - f_0(980)$ transition amplitude is

$$|\Pi_{a_0 f_0}(m)| \approx \frac{|g_{a_0 K^+ K^-} - g_{f_0 K^+ K^-}|}{16\pi} \sqrt{\frac{2(m_{K^0} - m_{K^+})}{m_{K^0}}}$$

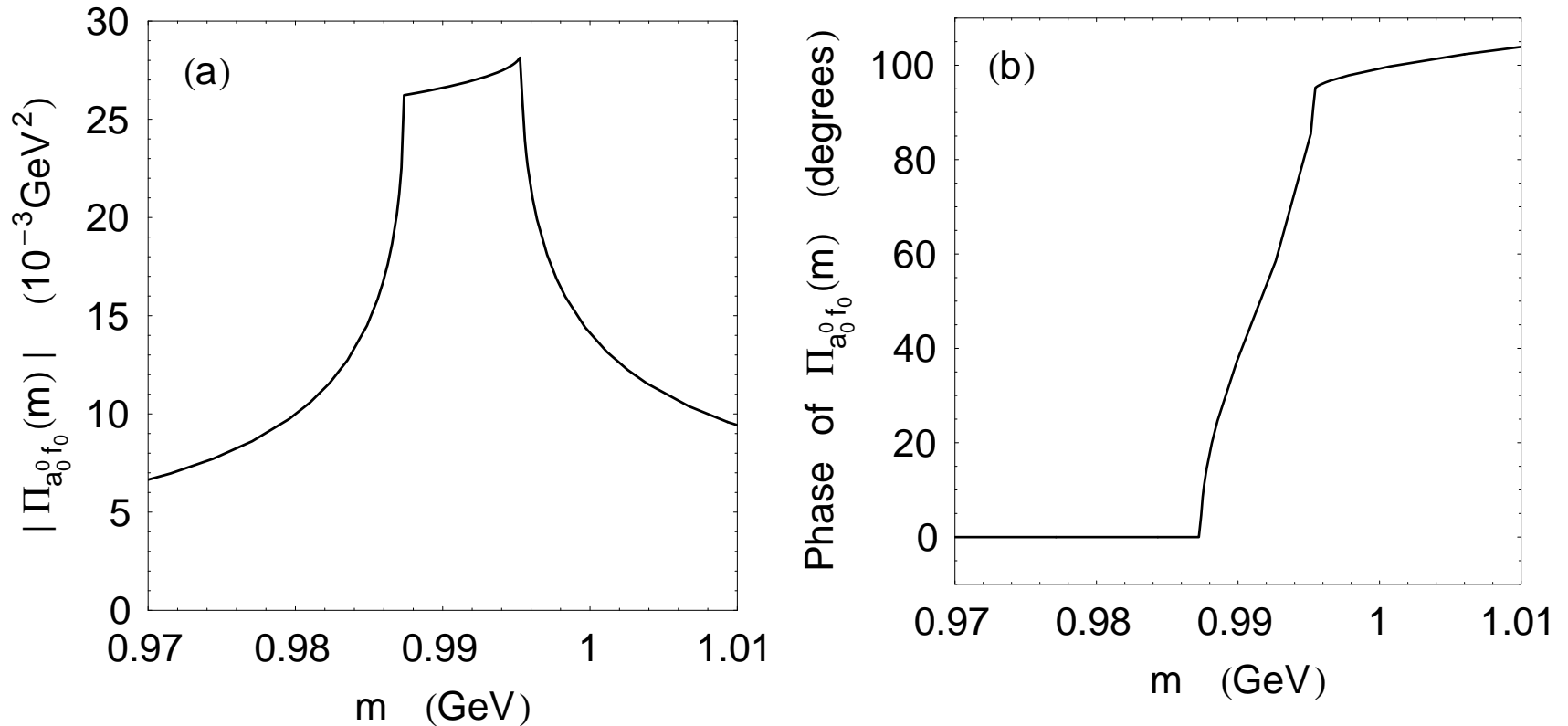
$$\approx 0.127 |g_{a_0 K^+ K^-} - g_{f_0 K^+ K^-}| / 16\pi \simeq 0.03 \text{ GeV}^2$$

$$\approx m_K \sqrt{m_{K^0}^2 - m_{K^+}^2} \approx m_K^{3/2} \sqrt{m_d - m_u}.$$

Note that the $\rho^0 - \omega$ and $\pi^0 - \eta$ mixing amplitudes are an order of magnitude smaller:

$$|\Pi_{\rho^0 \omega}| \approx |\Pi_{\pi^0 \eta}| \approx 0.003 \text{ GeV}^2 \approx (m_d - m_u) \times 1 \text{ GeV}.$$

$a_0^0(980) - f_0(980)$ mixing



(a) An example of the modulus and (b) the phase of the $a_0^0(980) - f_0(980)$ mixing amplitude in the region of the $K^+ K^-$ and $K^0 \bar{K}^0$ thresholds.

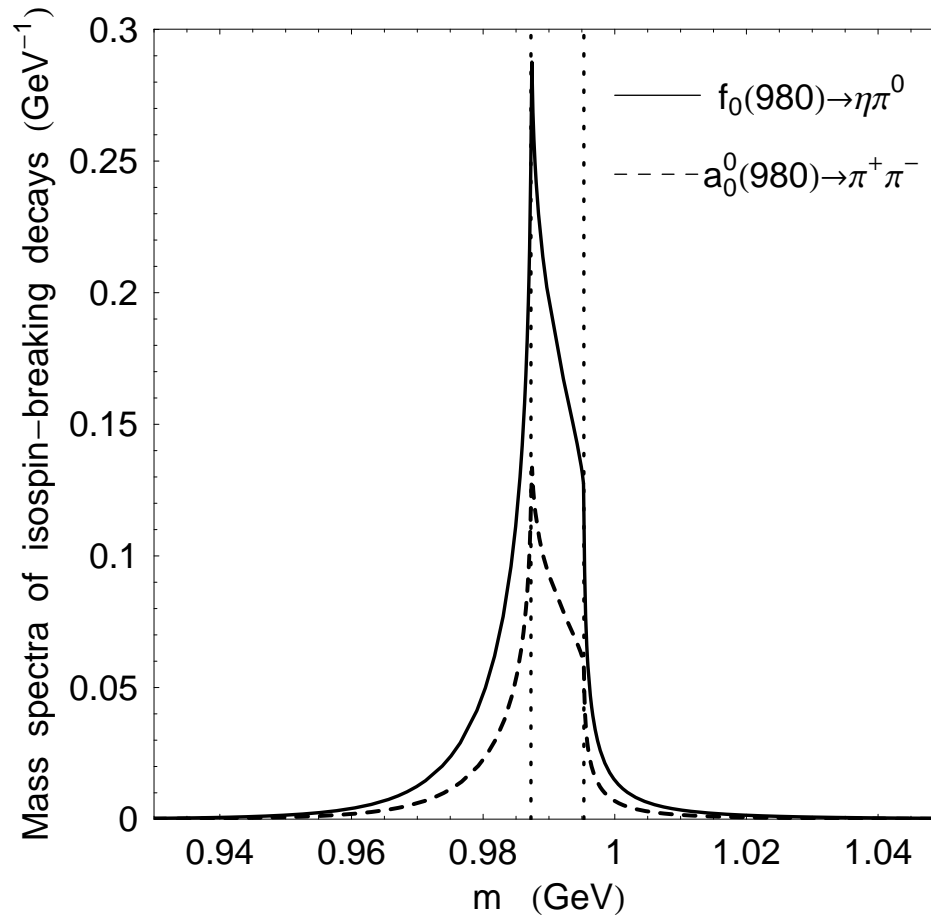
$a_0^0(980) - f_0(980)$ mixing

$$BR(f_0(980) \rightarrow K\bar{K} \rightarrow a_0^0(980) \rightarrow \eta\pi^0) = \int \Gamma_{a_0^0 \rightarrow \eta\pi^0}(m) \\ \times \frac{2m^2}{\pi} \left| \frac{\Pi_{a_0^0 f_0}(m)}{D_{a_0^0}(m)D_{f_0}(m) - \Pi_{a_0^0 f_0}^2(m)} \right|^2 dm \approx 0.3\%,$$

$$BR(a_0^0(980) \rightarrow K\bar{K} \rightarrow f_0(980) \rightarrow \pi^+\pi^-) = \int \Gamma_{f_0 \rightarrow \pi\pi}(m) \\ \times \frac{2m^2}{\pi} \left| \frac{\Pi_{a_0^0 f_0}(m)}{D_{a_0^0}(m)D_{f_0}(m) - \Pi_{a_0^0 f_0}^2(m)} \right|^2 dm \approx 0.14\%,$$

where $D_{a_0^0}(m)$ and $D_{f_0}(m)$ are the propagators of the $a_0^0(980)$ and $f_0(980)$ resonances, respectively. The following figure shows the mass spectra corresponding to the integrands in the above equations.

$a_0^0(980) - f_0(980)$ mixing



Mass spectra in the isospin-violating decays $f_0(980) \rightarrow \eta\pi^0$ and $a_0^0(980) \rightarrow \pi^+\pi^-$, caused by the $a_0^0(980) - f_0(980)$ mixing.

$a_0^0(980) - f_0(980)$ mixing

Comment.

Thus, one can see that the $a_0^0(980) - f_0(980)$ mixing cuts a narrow resonant structure from the $f_0(980)$ and $a_0^0(980)$ resonance distributions, having in the $\pi^+\pi^-$ and $\eta\pi^0$ channels, respectively, the normal widths of about 50-100 MeV.

Here one has used the values of the coupling constants of the $f_0(980)$ with the $\pi\pi$ and $K\bar{K}$ channels and the $a_0^0(980)$ with the $K\bar{K}$ and $\eta\pi$ channels obtained from the BESIII data (2011) for the central values of the $f_0(980) \rightarrow a_0^0(980)$ and $a_0^0(980) \rightarrow f_0(980)$ transition intensities measured in the reactions

$$J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0(980) \rightarrow \phi \eta \pi^0 \text{ and}$$

$$\chi_{c1}(1P) \rightarrow a_0(980) \pi^0 \rightarrow f_0(980) \pi^0 \rightarrow \pi^+ \pi^- \pi^0.$$

$a_0^0(980)$ - $f_0(980)$ mixing in polarization phenomena

The phase jump of the $a_0^0(980)$ - $f_0(980)$ mixing amplitude suggests the idea to study this mixing in polarization phenomena; [N. N. Achasov and G. N. Shestakov, Phys. Rev. Lett. 92, 182001 \(2004\), Phys. Rev. D 70, 074015 \(2004\)](#). If a process amplitude with a spin configuration is dominated by the $a_0^0(980)$ - $f_0(980)$ mixing then the spin asymmetry of the cross section jumps near the $K \bar{K}$ thresholds. An example is the reaction on a polarized proton target

$$\pi^- p_{\uparrow} \rightarrow (a_0^0(980) + f_0(980)) n \rightarrow a_0^0(980) n \rightarrow \eta \pi^0 n.$$

$$\frac{d^3\sigma}{dt dm d\psi} = \frac{1}{2\pi} \left[|M_{++}|^2 + |M_{+-}|^2 + 2\Im(M_{++}M_{+-}^*) P \cos \psi \right].$$

The dimensionless normalized spin asymmetry is

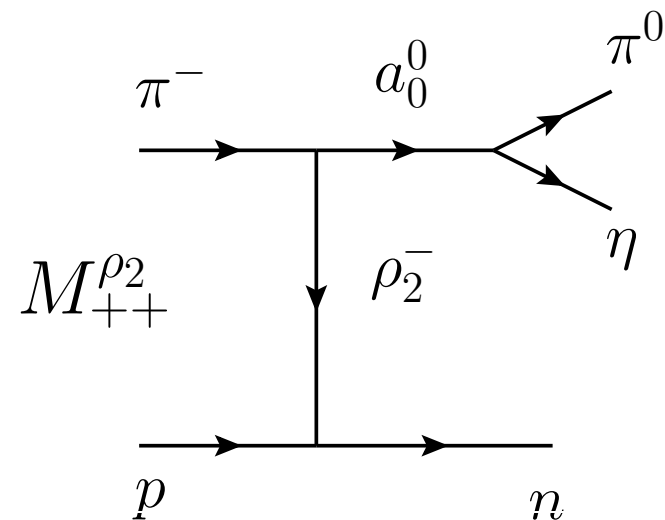
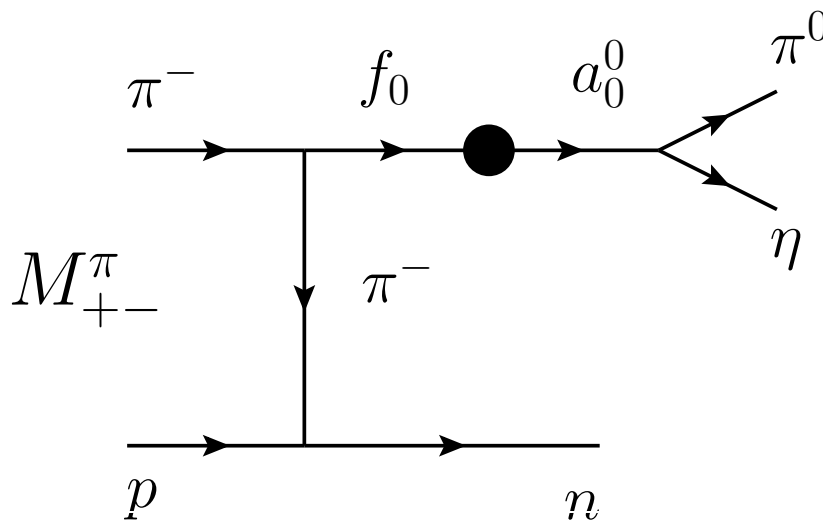
$$A(t, m) = 2 \Im(M_{++}M_{+-}^*) / [|M_{++}|^2 + |M_{+-}|^2],$$

$$-1 \leq A(t, m) \leq 1.$$

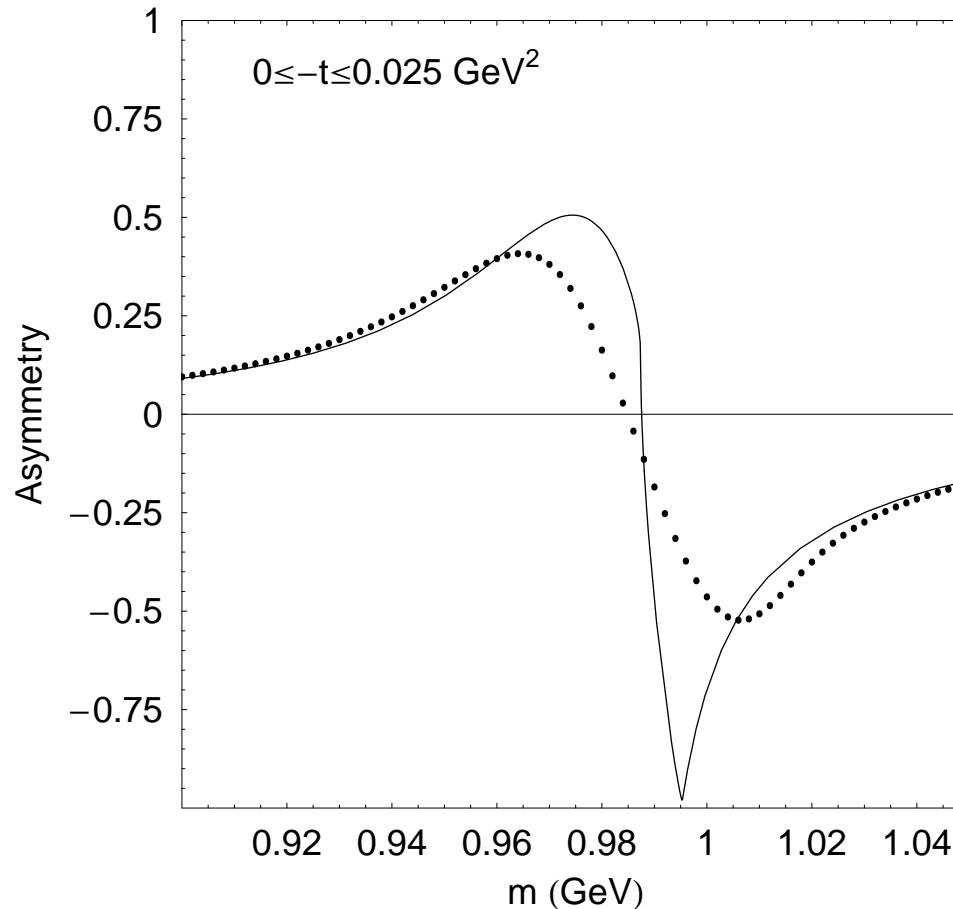
$a_0^0(980)$ - $f_0(980)$ mixing in polarization phenomena

Comment.

Here M_{+-} and M_{++} are the s-channel helicity amplitude with and without nucleon helicity flip **interfering in the polarized experiment**, ψ is the angle between the normal to the reaction plane formed by the momenta of the π^- and $\eta\pi^0$ system, and the transverse (to the π^- beam axis) polarization of the the proton target, and P is a degree of this polarization.



Spin asymmetry



The figure illustrates the strong asymmetry jump which is the direct manifestation of the $a_0^0(980)$ - $f_0(980)$ mixing amplitude M_{+-}^π interfering with the isospin allowed amplitude $M_{++}^{\rho_2}$ in the ρ_2 and π Regge exchange model.

These polarization effects are still in waiting for their investigators.

Observation of $a_0^0(980) - f_0(980)$ mixing

Recently, the BESIII Collaboration (2018) has reported a new observation of $a_0^0(980) - f_0(980)$ mixing in the decays of

$$J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0(980) \rightarrow \phi \eta \pi^0 \quad \text{and}$$

$$\chi_{c1}(1P) \rightarrow a_0(980) \pi^0 \rightarrow f_0(980) \pi^0 \rightarrow \pi^+ \pi^- \pi^0,$$

(after studying of it in 2011) using data samples of 1.31×10^9 J/ψ events and 4.48×10^8 $\psi(3686)$ events accumulated with the BESIII detector. The signals of $f_0(980) \rightarrow a_0^0(980)$ and $a_0^0(980) \rightarrow f_0(980)$ mixing have been observed at levels of statistical significance of 7.4σ and 5.5σ , respectively.

The corresponding branching fractions and mixing intensities have been measured.

Note that one of the most important feature of the $a_0^0(980) - f_0(980)$ mixing has been observed in this experiment. Namely, the width of the $f_0(980)$ signal in the $\eta \pi^0$ decay channel appears significantly narrower than the world average value of the $f_0(980) \rightarrow \pi \pi$ decay width.

The data on $a_0^0(980) - f_0(980)$ mixing intensities

$$\begin{aligned}\xi_{fa} &= \frac{BR(J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0^0(980) \rightarrow \phi \eta \pi^0)}{BR(J/\psi \rightarrow \phi f_0(980) \rightarrow \phi \pi \pi)} \\ &= (0.99 \pm 0.16(stat.) \pm 0.30(sys.) \pm 0.09(para.))\%, \\ &= (0.41 \pm 0.13(stat.) \pm 0.17(sys.) \pm 0.13(para.))\%;\end{aligned}$$

$$\begin{aligned}\xi_{af} &= \frac{BR(\chi_{c1} \rightarrow a_0^0(980) \pi^0 \rightarrow f_0(980) \pi^0 \rightarrow \pi^+ \pi^- \pi^0)}{BR(\chi_{c1} \rightarrow a_0^0(980) \pi^0 \rightarrow \eta \pi^0 \pi^0)} \\ &= (0.40 \pm 0.07(stat.) \pm 0.14(sys.) \pm 0.07(para.))\%.\end{aligned}$$

As for the coupling constants $g_{f_0 K^+ K^-}$ and $g_{a_0^0 K^+ K^-}$, their values estimated using these data are in agreement with many previous experimental results and also with the $q^2 \bar{q}^2$ model for the $f_0(980)$ and $a_0^0(980)$ mesons.

Decay $f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$

A very interesting situation takes place in the case of the $f_1(1285)$ resonance. According to the VES result, the isospin breaking decay of the $f_1(1285)$ into $f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ is so strong,

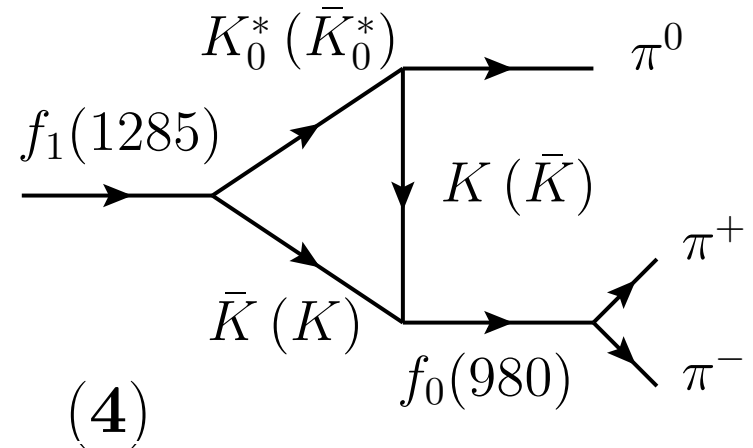
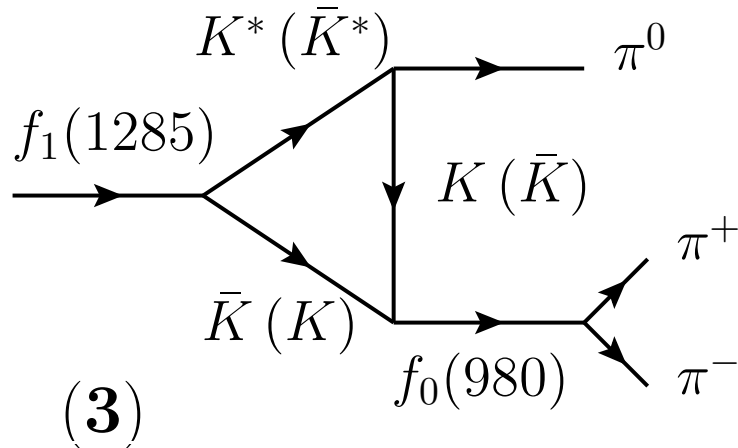
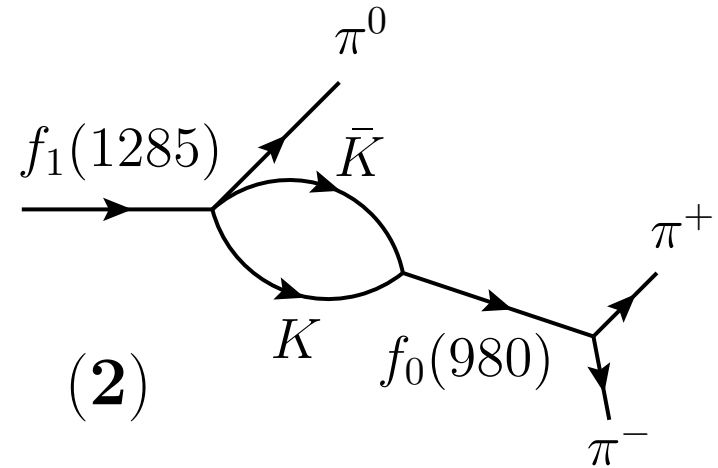
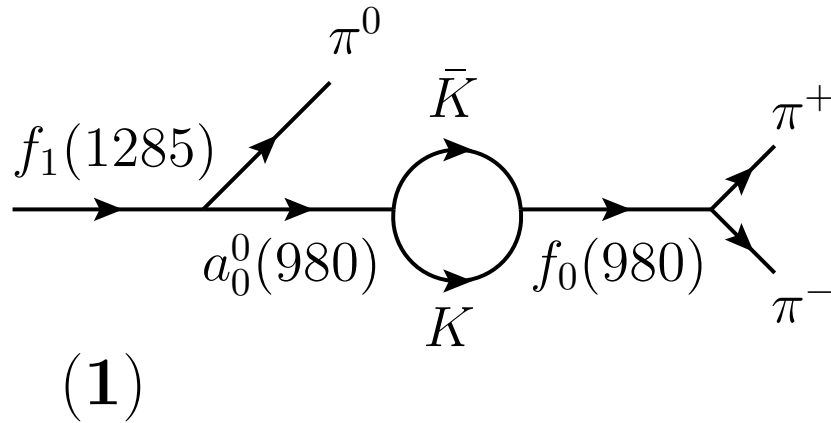
$$\frac{BR(f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{BR(f_1(1285) \rightarrow a_0^0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)} = (2.5 \pm 0.9)\%,$$

$$BR(f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0) = (0.30 \pm 0.09)\%,$$

that its description due to the transition mechanism $f_1(1285) \rightarrow a_0^0(980)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ requires the "terrible" $a_0^0(980)$ - $f_0(980)$ mixing and, as a result, the inconvenient values of the coupling constants of the scalar mesons with the pseudo-scalar ones in the many cases.

In fact, the strong isospin breaking effect discovered in the decay $f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ denotes a more general $K\bar{K}$ loop mechanism of the isospin breaking in this decay.

Decay $f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$



We have analyzed in detail four possible $K \bar{K}$ loop mechanisms (shown in the above figure) for the isospin breaking decay $f_1(1285) \rightarrow \pi^+\pi^-\pi^0$.

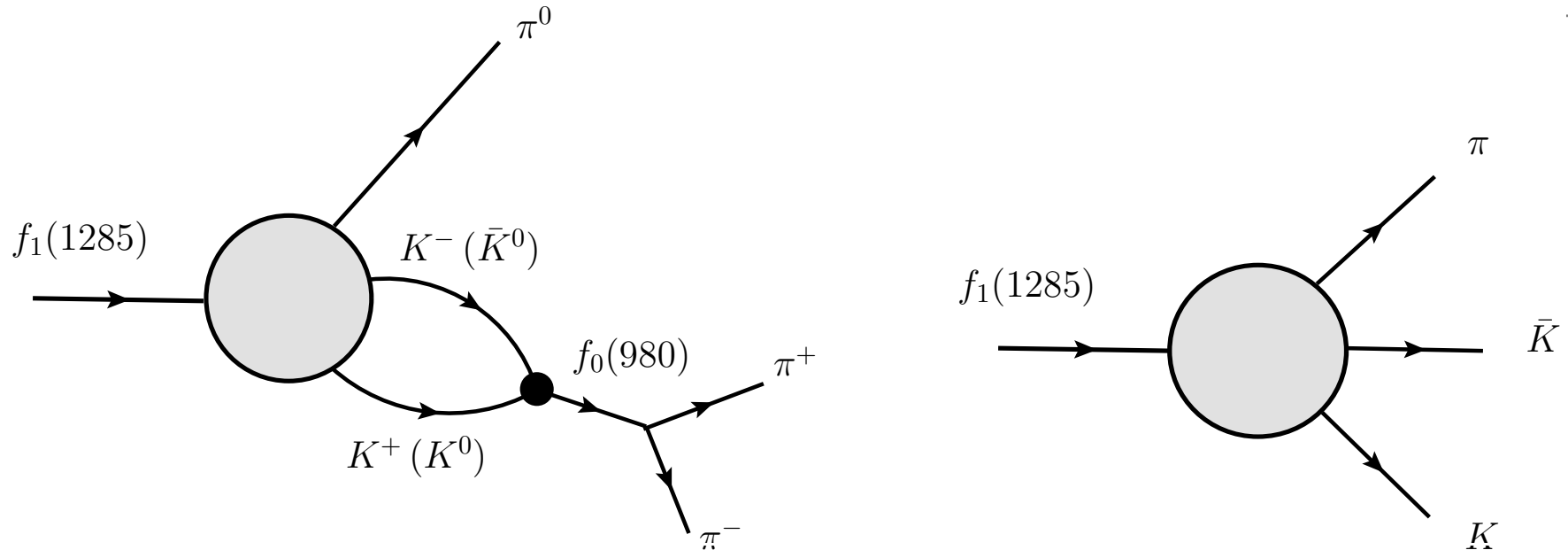
Decay $f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$

These mechanisms break the conservation of the isospin due to **the nonzero mass difference of the K^+ and K^0 mesons**. They result in the appearance of the narrow resonance structure in the $\pi^+\pi^-$ mass spectrum in the region of the $K\bar{K}$ thresholds, with the width $\approx 2m_{K^0} - 2m_{K^+} \approx 8$ MeV. The observation of such a structure in experiment is the direct indication on **the $K\bar{K}$ loop mechanism** of the breaking of the isotopic invariance.

We point out that existing data should be more precise, and it is difficult to explain them using the single specific mechanism from those listed above.

Taking the decay $f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ as an example, we also suggested the general approach to the description of the $K\bar{K}$ loop mechanism of the isotopic symmetry breaking (in the absence of logarithmic singularities in the amplitudes) in the form of some consistency condition between two sets of the experimental data.

Consistency condition



We keep in mind the relation of the type $\Gamma_{f_1(1285) \rightarrow f_0 \pi^0 \rightarrow \pi^+ \pi^- \pi^0} = |A_{f_1(1285) \rightarrow K \bar{K} \pi}(2m_{K^+})|^2 2.59 \times 10^{-6} \text{ GeV}^5$ between the descriptions of the $f_1(1285) \rightarrow \pi^+ \pi^- \pi^0$ and $f_1(1285) \rightarrow K \bar{K} \pi$ decays. Its comparison with the data on the decay $f_1(1285) \rightarrow \pi^+ \pi^- \pi^0$ permits one to verify their consistence with the data on the decay $f_1(1285) \rightarrow K^+ K^- \pi^0$ and with the idea of the breaking of isotopic invariance caused by the mass difference of K^+ and K^0 mesons.

Decay $\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$

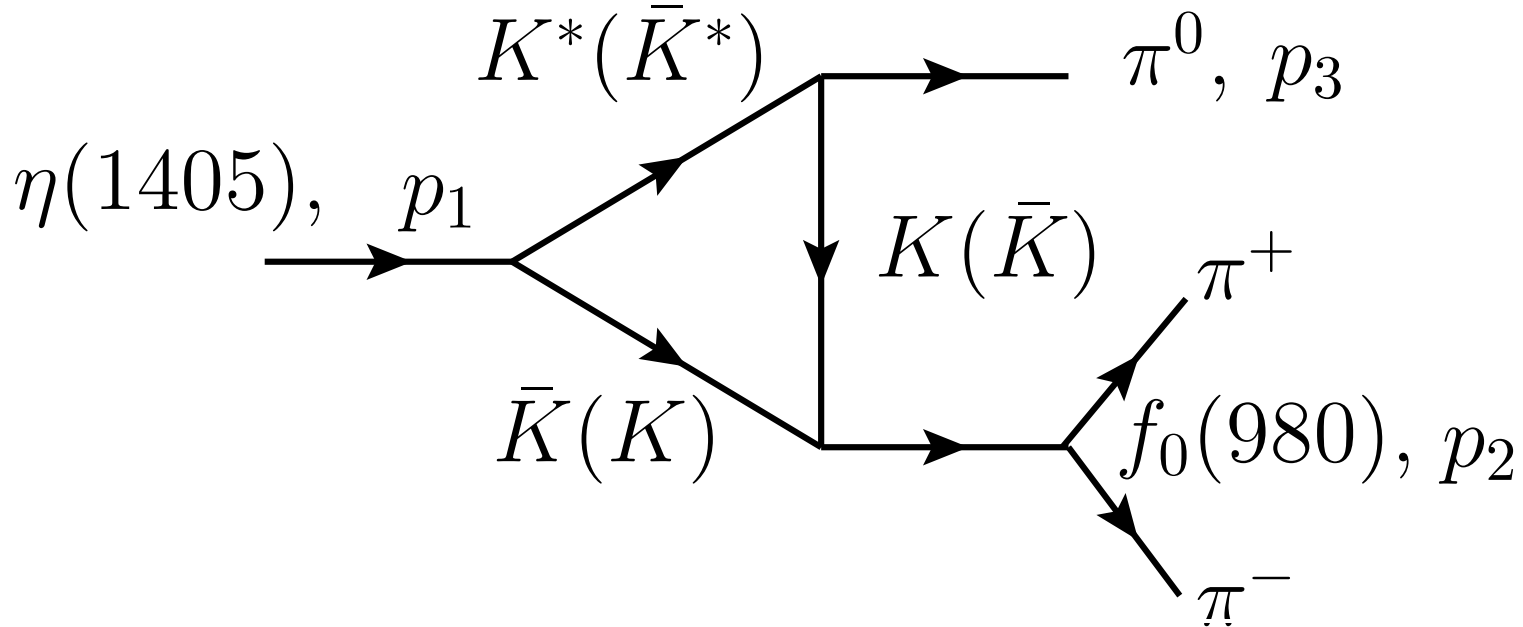
The BESIII Collaboration investigated the isospin breaking decay $\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ and measured the mass and width of the $\eta(1405)$ peak in this channel to be of 1409.0 ± 1.7 MeV and 48.3 ± 5.2 MeV, respectively, and the corresponding branching ratio $BR(J/\psi \rightarrow \gamma\eta(1405) \rightarrow \gamma f_0(980)\pi^0 \rightarrow \gamma\pi^+\pi^-\pi^0) = (1.50 \pm 0.16) \times 10^{-5}$. In addition, the BESIII gives the ratio

$$\frac{BR(\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{BR(\eta(1405) \rightarrow a_0^0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)} = (17.9 \pm 4.2)\%,$$

that rules out practically the explanation of the discovered effect by means of the $a_0(980) - f_0(980)$ mixing.

This large isospin breaking may be associated with manifestations of the anomalous Landau thresholds in the form of logarithmic triangle singularities, which are in the transition amplitude $\eta(1405) \rightarrow (K^*\bar{K} + \bar{K}^*K) \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$.

Decay $\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$



Indeed, in the region of the $\eta(1405)$ resonance all intermediate particles in the triangle loop of this diagram can lie on their mass shells. That is, in the hypothetical case of the stable $K^* = K^*(892)$ meson the logarithmic singularity appears in the imaginary part of the triangle diagram. **However, this effect can be correctly estimated only in view of the finite width of the K^* .**

Decay $\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$

Taking into account of the finite width of the K^* resonance ($\Gamma_{K^* \rightarrow K\pi} \approx 50$ MeV) we showed that the calculated width of the decay

$$\eta(1405) \rightarrow (K^*\bar{K} + \bar{K}^*K) \rightarrow K\bar{K}\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$$

is about a factor of 6 — 8 smaller than in the hypothetical case of the stable K^* ($\Gamma_{K^* \rightarrow K\pi} = 0$). Assuming the dominance of the decay

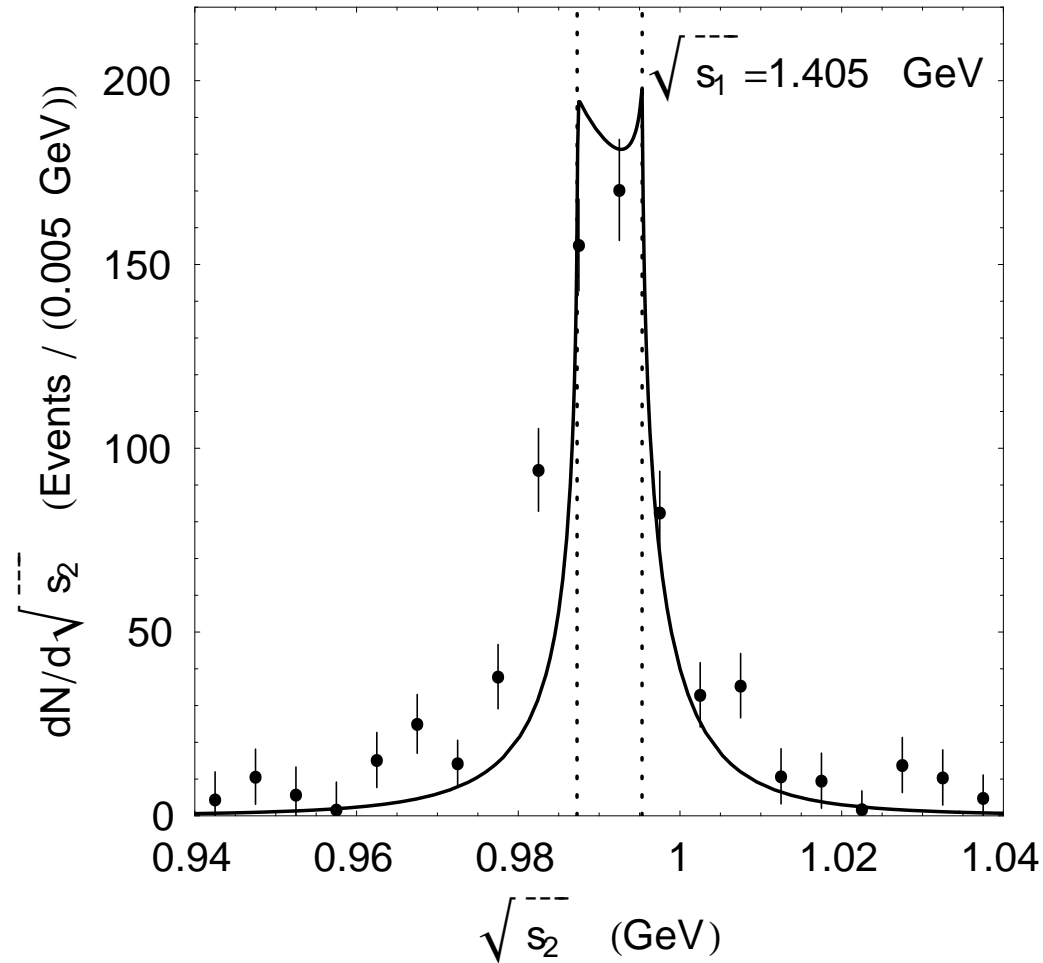
$$\eta(1405) \rightarrow (K^*\bar{K} + \bar{K}^*K) \rightarrow K\bar{K}\pi$$

we also obtained

$$BR(J/\psi \rightarrow \gamma\eta(1405) \rightarrow \gamma f_0(980)\pi^0 \rightarrow \gamma\pi^+\pi^-\pi^0) \approx 1.12 \times 10^{-5}$$

that agrees reasonably with experiment.

Decay $\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$



The shape of the $\pi^+\pi^-$ mass spectrum in the $\eta(1405) \rightarrow \pi^+\pi^-\pi^0$ decay calculated for the above triangle mechanism. The points are the BESIII data.

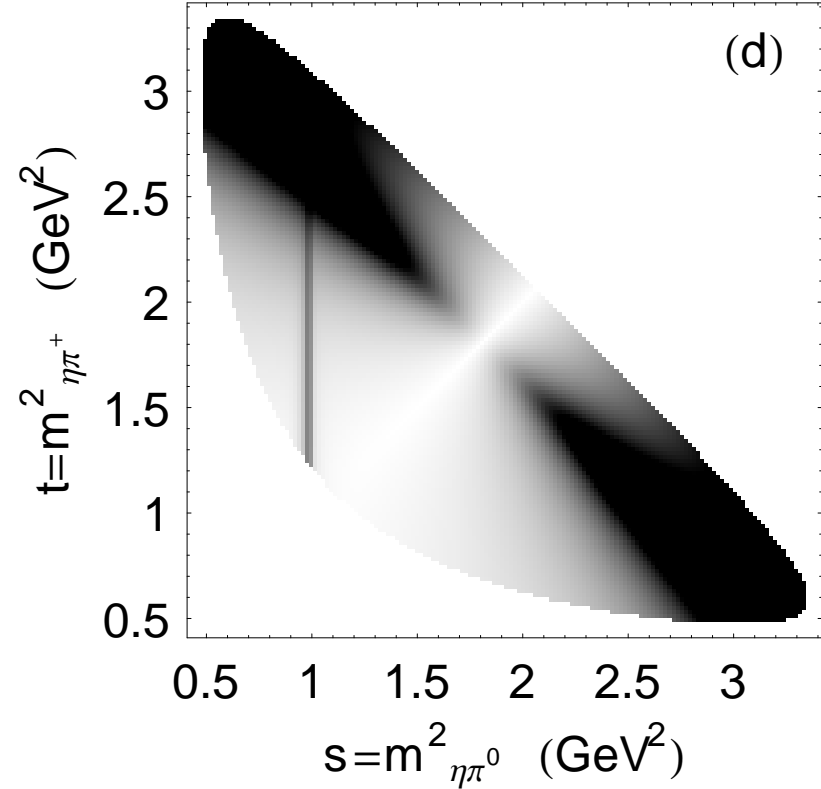
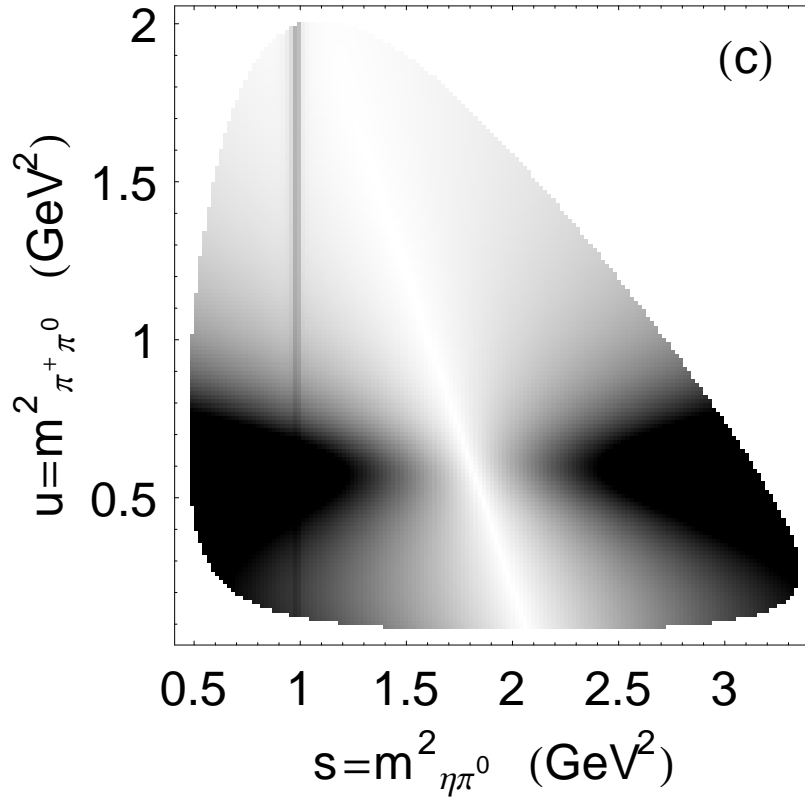
$a_0^0(980) - f_0(980)$ mixing in the D_s^+ , D^0 and Υ' decays

Light meson spectroscopy from hadronic heavy meson decays (in particular, study of the $a_0^0(980)$ and $f_0(980)$ resonances) is one of the lines of the BESIII, LHCb, and Belle programs on charm and beauty physics.

Recently we showed that the decays $D_s^+ \rightarrow \eta\pi^0\pi^+$, $D^0 \rightarrow K_S^0\pi^+\pi^-$, $D^0 \rightarrow K_S^0\eta\pi^0$, and $\Upsilon(10860) \rightarrow \Upsilon(1S)\eta\pi^0$ have potential for the $a_0^0(980) - f_0(980)$ mixing detection.

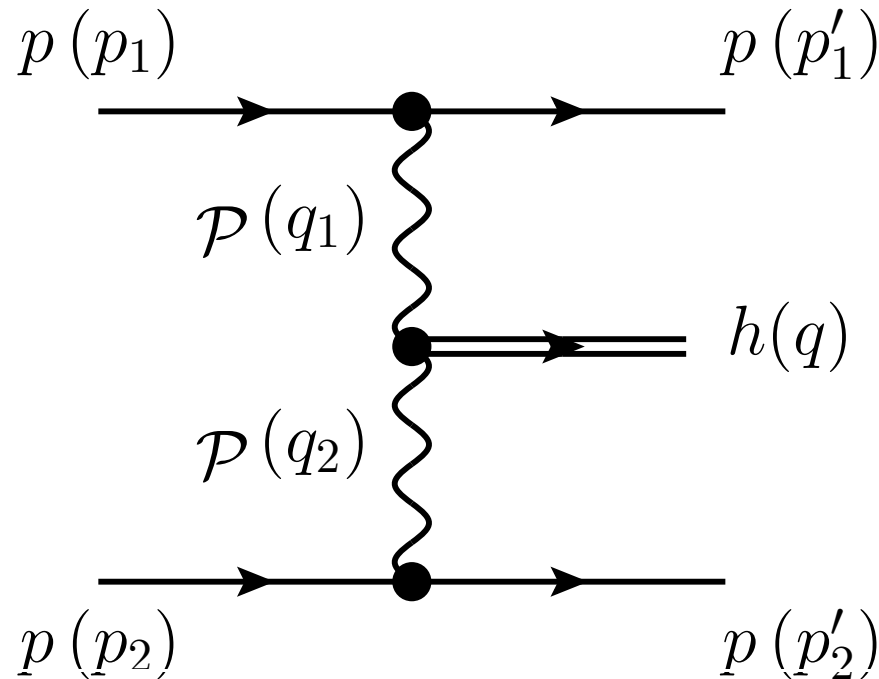
Moreover, the detection of the $a_0^0(980) - f_0(980)$ mixing in the $D_s^+ \rightarrow \eta\pi^0\pi^+$ channel can provide a unique opportunity to clarify the puzzling mechanisms of the $f_0(980)$ and $a_0^0(980)$ production in the three-body hadronic decays of $D_s^+ \rightarrow f_0(980)\pi^+ \rightarrow \pi^+\pi^-\pi^+$ and $D_s^+ \rightarrow K^+K^-\pi^+$.

$a_0^0(980) - f_0(980)$ mixing in the $D_s^+ \rightarrow \eta\pi^0\pi^+$ decay



The $D_s^+ \rightarrow [f_0(980) \rightarrow K\bar{K} \rightarrow a_0^0(980)]\pi^+ \rightarrow \eta\pi^0\pi^+$ transition, caused by the $a_0^0(980) - f_0(980)$ mixing, manifests itself as the vertical bands in the Dalitz plot distributions for the $D_s^+ \rightarrow \eta\pi^0\pi^+$ events against the main mechanism $D_s^+ \rightarrow \eta\rho^+ \rightarrow \eta\pi^0\pi^+$ with $\eta\rho^+$ in the intermediate state.

Isospin symmetry breaking in central diffractive production at the LHC



At very high energies, and in the central region, the double-Pomeron exchange mechanism gives the dominant contribution to the production of hadrons with the positive C parity and isospin $I = 0$. Therefore, the observation of resonances in the states with $I = 1$ will be indicative of their production or decay with the isotopic symmetry breaking.

Isospin symmetry breaking in central diffractive production at the LHC

Here, we bear in mind the cases of the anomalous breaking of the isotopic symmetry, i.e., when the cross section of the process breaking the isospin is not of the order of 10^{-4} of the cross section of the allowed process but of the order of 1%. We draw attention to the processes

$$pp \rightarrow p(f_1(1285))p \rightarrow p(\pi^+\pi^-\pi^0)p,$$

$$pp \rightarrow p(K\bar{K})p \rightarrow p(a_0^0(980))p \rightarrow p(\eta\pi^0)p$$

in which a similar situation can be realized. Note that there is no visible background in the $\pi^+\pi^-\pi^0$ and $\eta\pi^0$ channels.

Observation of the process $pp \rightarrow p(f_1(1285))p \rightarrow p(\pi^+\pi^-\pi^0)p$ would be a crucial confirmation of the first results from the VES and BESIII detectors, indicating the very large isospin breaking in the decay $f_1(1285) \rightarrow \pi^+\pi^-\pi^0$.

Conclusion

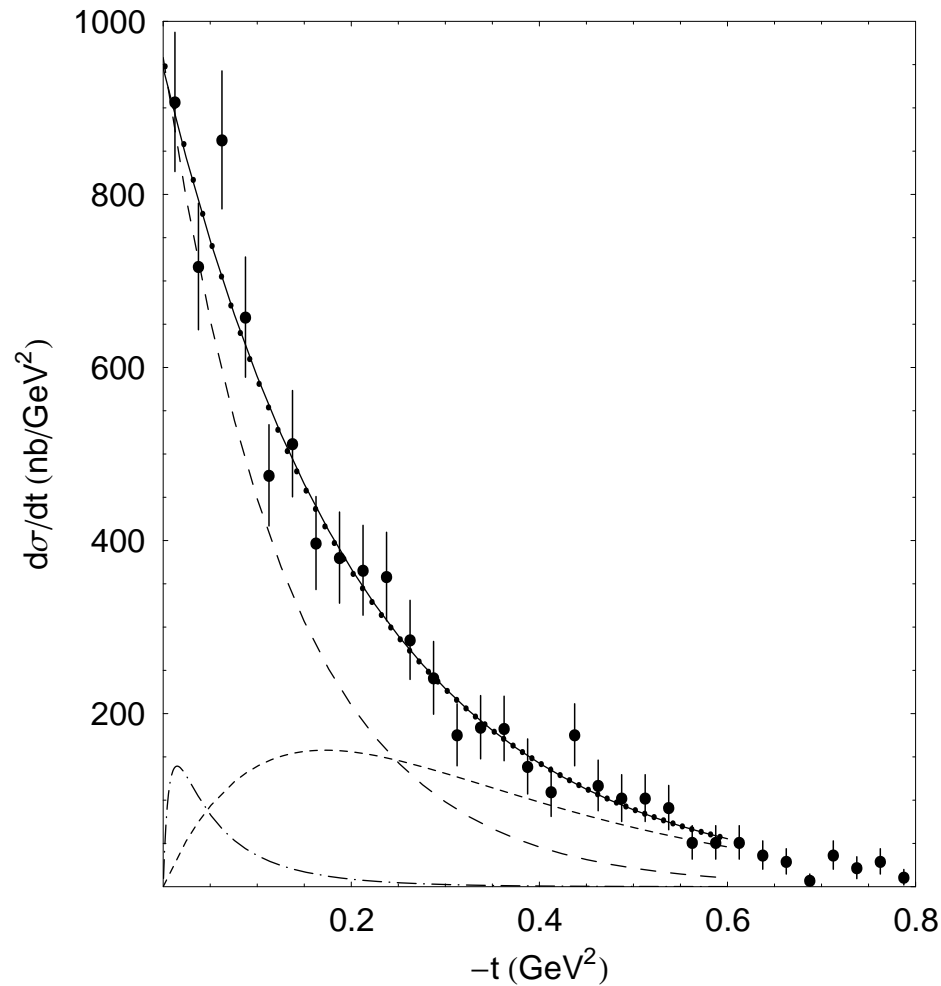
The mass differences for the charmed mesons D^+ , D^0 and D^{*+} , D^{*0} are approximately the same as for the K^+ and K^0 mesons. Therefore, various dynamic effects of the strong isotopic symmetry breaking may also be expected for charmonium states near thresholds of the corresponding decays.

Acknowledgments

The present work is partially supported by the program of fundamental scientific researches of the Siberian Branch of the Russian Academy of Sciences, project No. 0314-2019-0021.

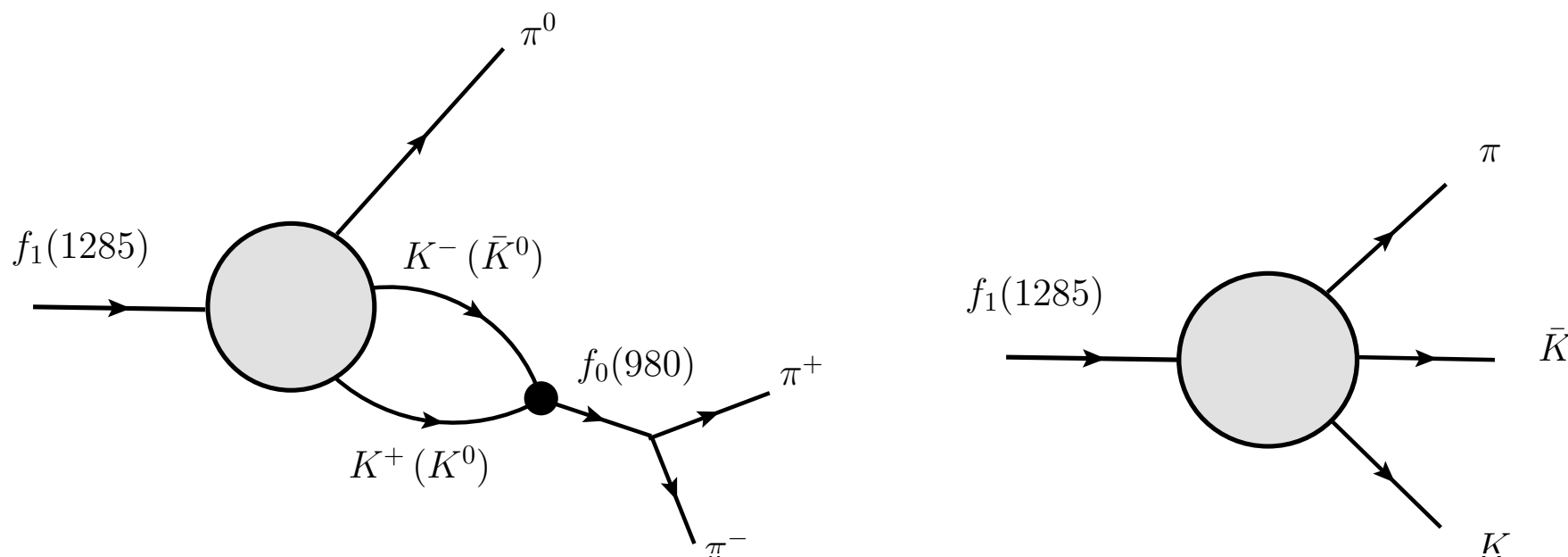
THANK YOU

Cross section at $P_{lab}^{\pi^-} = 18.3 \text{ GeV}$



The ρ_2 , b_1 and π Regge exchange contributions to the $\pi^- p \rightarrow (a_0^0(980) + \mathbf{f_0(980)}) n \rightarrow a_0^0(980) n \rightarrow \eta \pi^0 n$ reaction cross section.

Consistency condition



With a good accuracy $\mathcal{M}(f_1(1285) \rightarrow f_0(980)\pi^0; m) = g_{f_0 K^+ K^-} A(m) \times i[\rho_{K^+ K^-}(m) - \rho_{K^0 \bar{K}^0}(m)]$. The amplitude $A(m)$ is aware of all possible mechanisms of production of the $K \bar{K}$ pairs with isospin $I = 1$ in S wave in the process $f_1(1285) \rightarrow K \bar{K} \pi$. The information about $|A(m)|^2$ in the region of the $K^+ K^-$ and $K^0 \bar{K}^0$ thresholds can be extracted from the data on the $K \bar{K}$ mass spectra measured in the decays $f_1(1285) \rightarrow K \bar{K} \pi$.

Consistency condition

Fitting the data on $d\Gamma(f_1(1285) \rightarrow K^+ K^- \pi^0; m)/dm$, one can find the value $|A(2m_{K^+})|^2$ and obtain the following approximate estimate for the width

$$\Gamma_{f_1(1285) \rightarrow f_0 \pi^0 \rightarrow \pi^+ \pi^- \pi^0} = |A(2m_{K^+})|^2 2.59 \times 10^{-6} \text{ GeV}^5.$$

Its comparison with the data on the decay $f_1(1285) \rightarrow \pi^+ \pi^- \pi^0$ permits one to verify their consistence with the data on the decay $f_1(1285) \rightarrow K^+ K^- \pi^0$ and with the idea of the breaking of isotopic invariance caused by the mass difference of K^+ and K^0 mesons.

Decay $\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$

The taking into account of the finite width of the K^* resonance ($\Gamma_{K^* \rightarrow K\pi} \approx 50$ MeV), i.e., the averaging of the amplitude over the resonance Breit–Wigner distribution in accord with the spectral Källén–Lehmann representation for the propagator of the unstable K^* meson, smoothes the logarithmic singularities of the amplitude and hence makes the compensation of the contributions of the $K^{*+}K^- + K^{*-}K^+$ and $K^{*0}\bar{K}^0 + \bar{K}^{*0}K^0$ intermediate states more strong. This results in both the suppression of the calculated width of the decay $\eta(1405) \rightarrow \pi^+\pi^-\pi^0$ by the factor of 6–8 in comparison with the case of $\Gamma_{K^* \rightarrow K\pi} = 0$ and in the concentration of the main effect of the isospin breaking in the domain of the $\pi^+\pi^-$ invariant mass between the $K\bar{K}$ thresholds. Assuming the dominance of the $\eta(1405) \rightarrow (K^*\bar{K} + \bar{K}^*K) \rightarrow K\bar{K}\pi$ decay, one obtains

$$BR(J/\psi \rightarrow \gamma\eta(1405) \rightarrow \gamma f_0(980)\pi^0 \rightarrow \gamma\pi^+\pi^-\pi^0) \approx 1.12 \times 10^{-5},$$

that agrees reasonably with experiment.