Institute of High Energy Physics

The updated status of the pseudoscalar glueball studies

Qiang Zhao

Institute of High Energy Physics, CAS

and Theoretical Physics Center for Science Facilities (TPCSF), CAS

zhaoq@ihep.ac.cn

Outline

1. Controversial issues about the pseudoscalar glueball candidate η(1405)
2. Reconsile the the dynamical model calculations with LQCD simulations
3. The presence of the "triangle singularity"
4. Observables sensitive to the underlying dynamics
5. Brief summary

1. Controversial issues about the pseudoscalar glueball candidate $\eta(1405)$

Hadrons beyond the conventional QM and…

Exotics of Type-I:

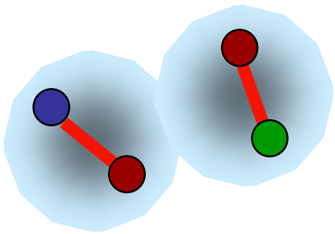
J^{PC} are not allowed by $Q\bar{Q}$ configurations, e.g. $0^{--}, 1^{-+}$ …

- **Direct observation**

Exotics of Type-II:

J^{PC} are the same as $Q\bar{Q}$ configurations

- **Outnumbering of conventional QM states?**
- **Peculiar properties?**



"Exotics" of Type-III:

Leading kinematic singularity can cause measurable effects, e.g. the triangle singularity.

- **What's the impact?**
- **How to distinguish a genuine state from kinematic effects?**

The arising of the E-ι puzzle:

E meson was first observed in 1965 in $p\bar{p} \rightarrow (K\bar{K}\pi)\pi^+\pi^-$.

Observation of ι(1440) at Mark II (left, 1980) and Crystal Ball (right, 1982)

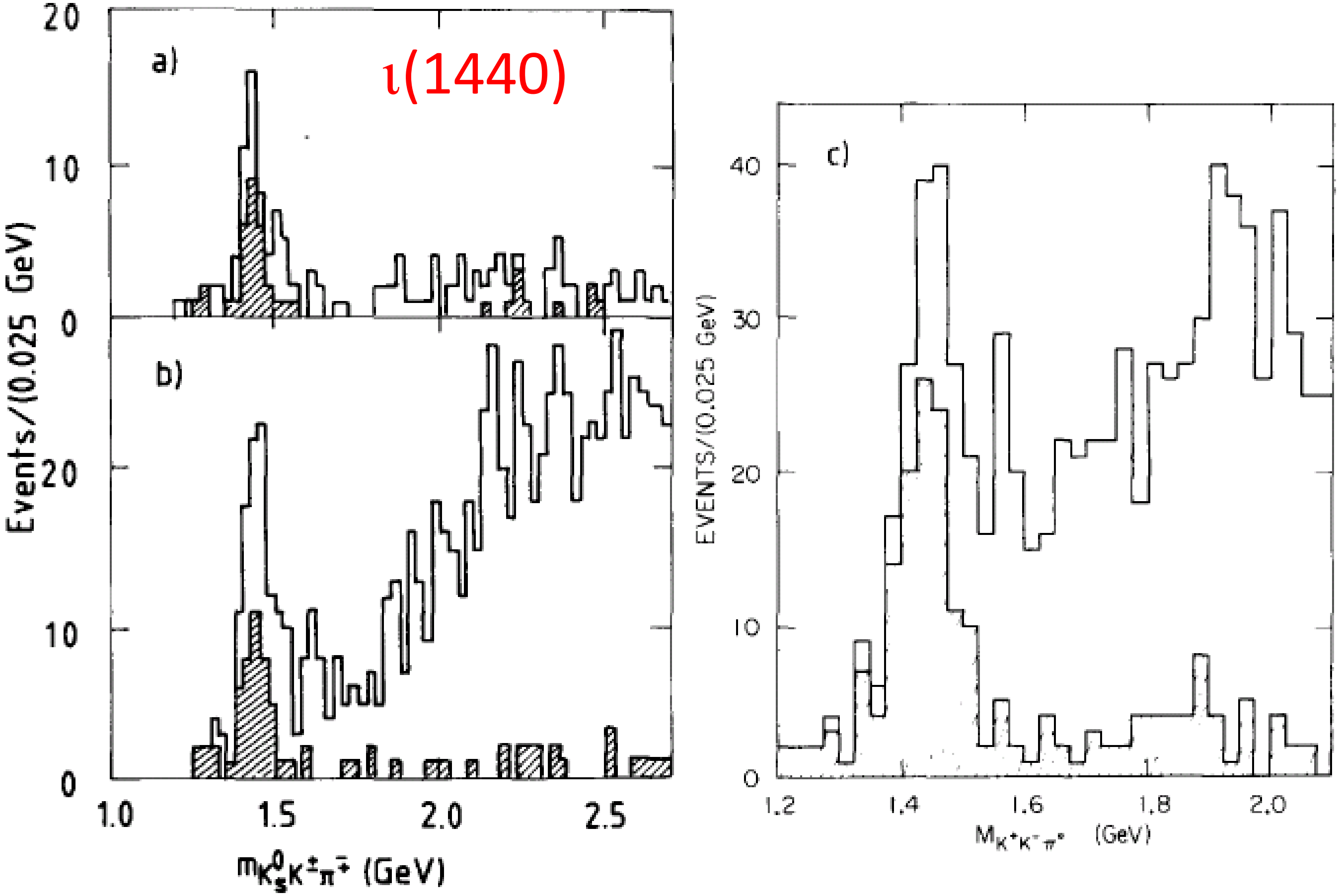
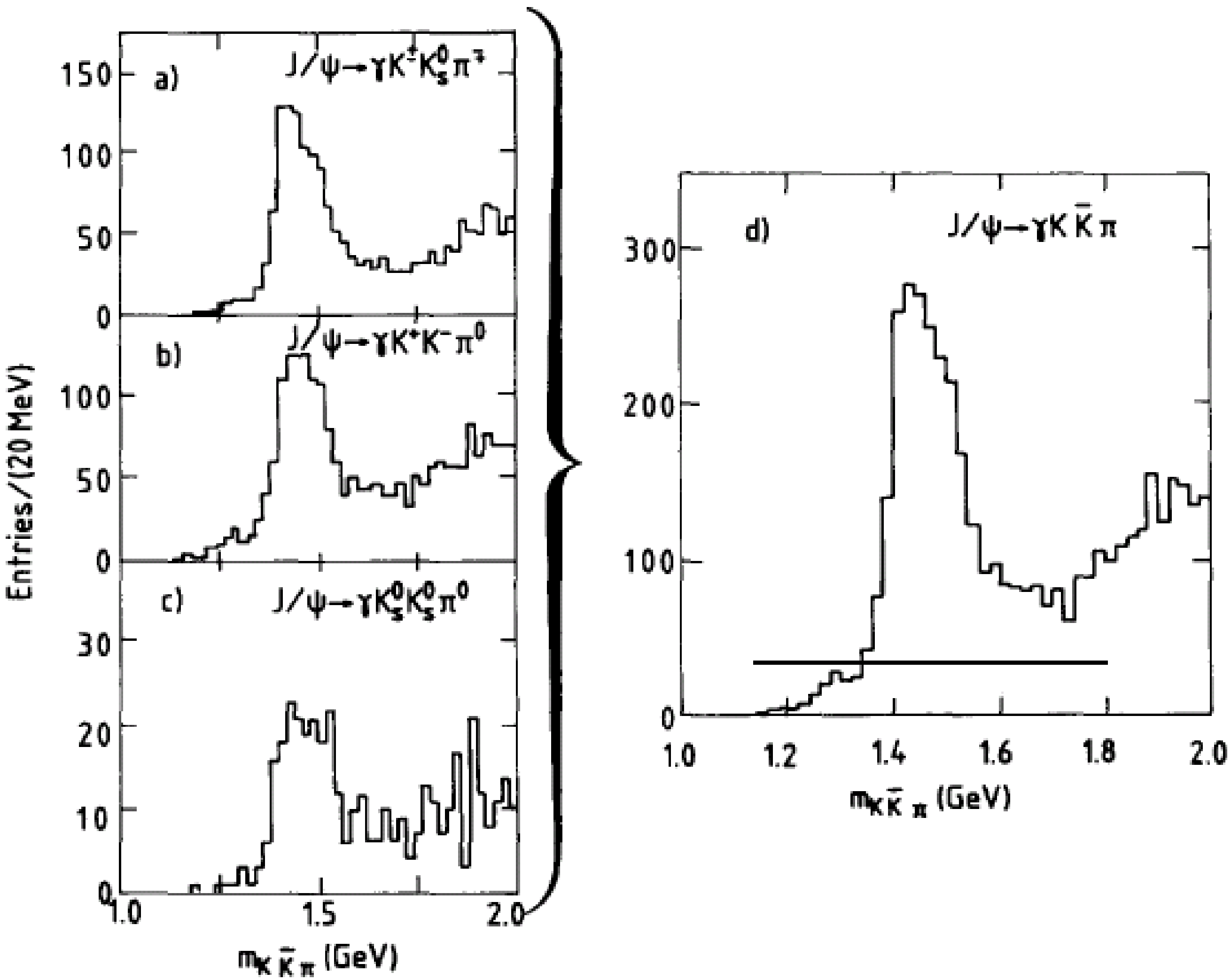
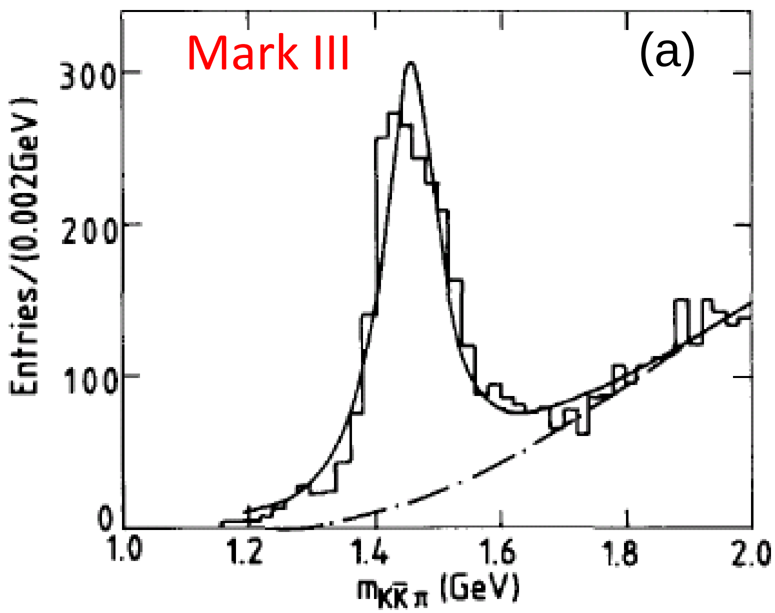


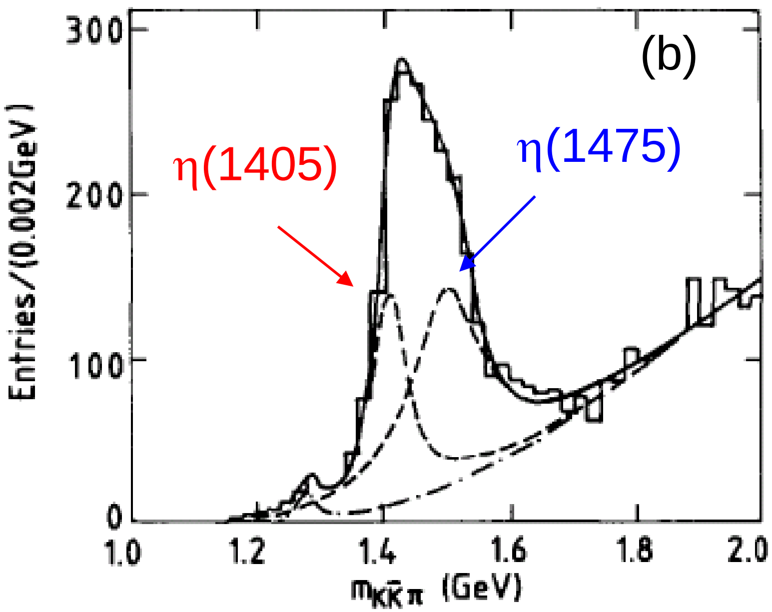
Fig. 69. Observation of the η(1440) by Mark II and Crystal Ball. (a) Mark II, radiative photon detection required, (b) Mark II, photon detection not required. The events in the shaded region have $m_{K\bar{K}} < 1.05$ GeV ("delta cut"). (c) Crystal Ball, events in the shaded region have $m_{K\bar{K}} < 1.125$ GeV.

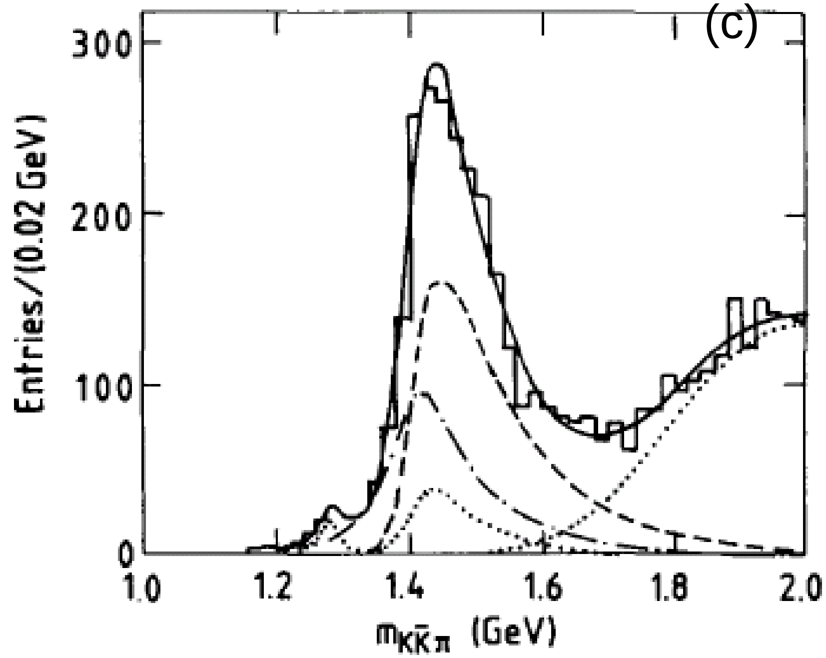
Confirmation of η(1440) at Mark III in 1987



Distorted lineshape?







(a) A single Breit-Wigner fit

(b) Two interfering B-W fit

(c) Coupled channel B-W fit

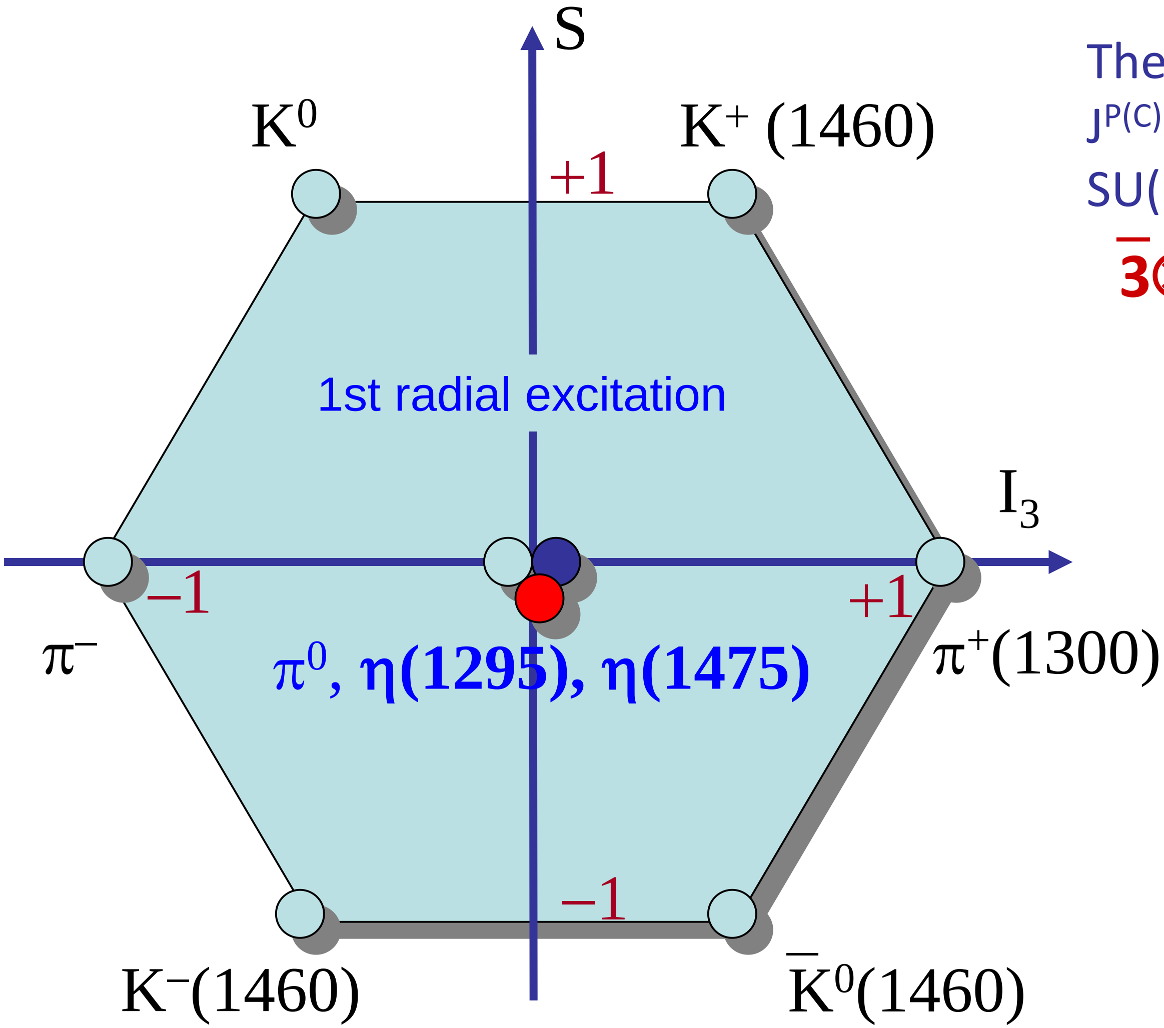
$$M = 1416 \pm 8^{+7}_{-5};\ \Gamma = 91^{+67}_{-31-38}{}^{+15}\ \mathrm{MeV}/c^2$$

$$M = 1490^{+14+3}_{-8-6};\ \Gamma = 54^{+37+13}_{-21-24}\ \mathrm{MeV}/c^2$$

Also “confirmed” by Obelix collaboration

Type-II exotics?

The abundance of 0^{-+} (I=0) states implies an exotic candidate

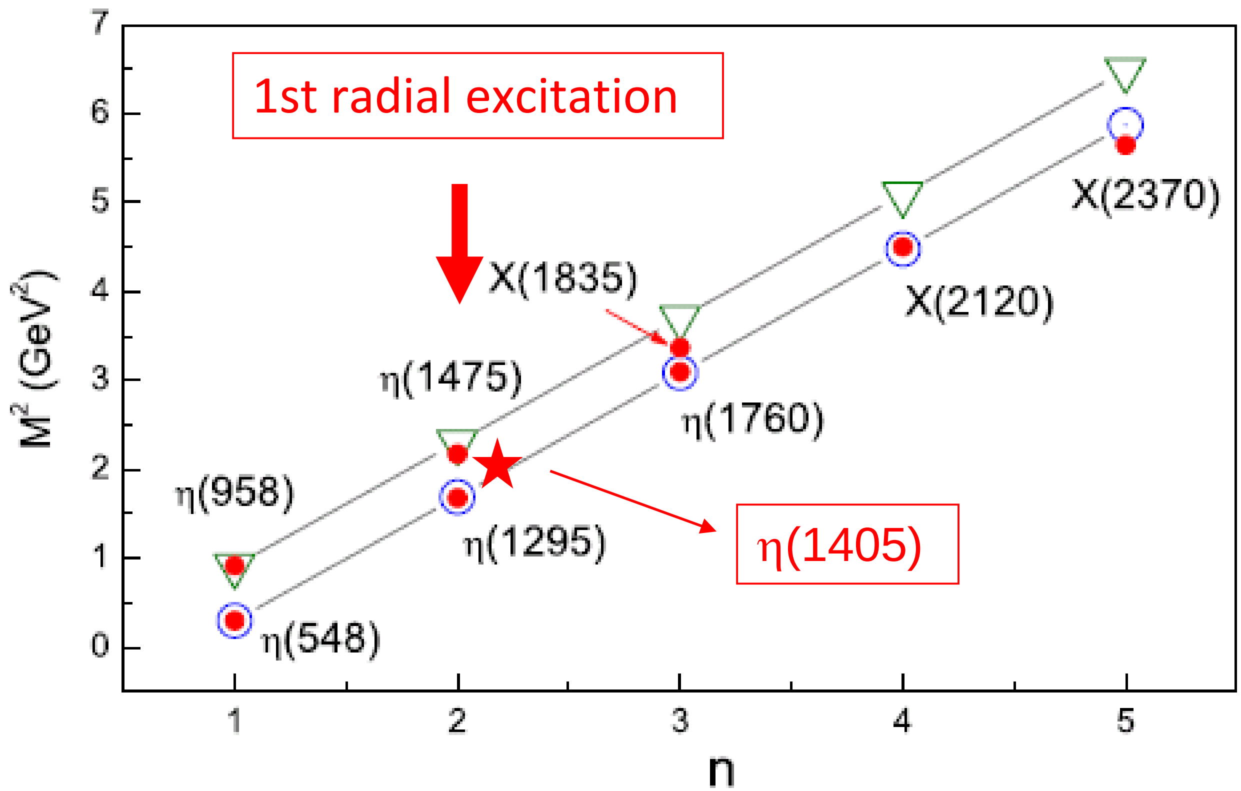


The first radial excitation of $J^{P(C)}=0^{-(+)}$ states also make a $\bar{q}q$ SU(3) flavor nonet:

$\bar{3}\otimes 3 = 1 \oplus 8$

Three η states have been listed by Particle Data Group around 1.2 ~ 1.5 GeV:
η(1295), η(1405), and **η(1475)**

- Regge trajectory for the η/η′ mass spectrum



- How to understand the presence of η(1405) ?

$$M^2 = M_0^2 + (n-1)\mu^2 \ (\mu^2 = 1.39 \text{ GeV})$$

J.S. Yu, Z.F. Sun, X. Liu, and Q. Z., PRD83, 114007 (2011)

The abundance of 0^{-+} (I=0) states implies a glueball candidate?

Positive:

- Flux tube model favors $M_G \cong 1.4$ GeV [1]
- A dynamical model based on $U_A(1)$ anomaly gives a similar mass [2].

Caveat:

- LQCD favors $M_G \cong 2.4 - 2.6$ GeV [3,4,5]

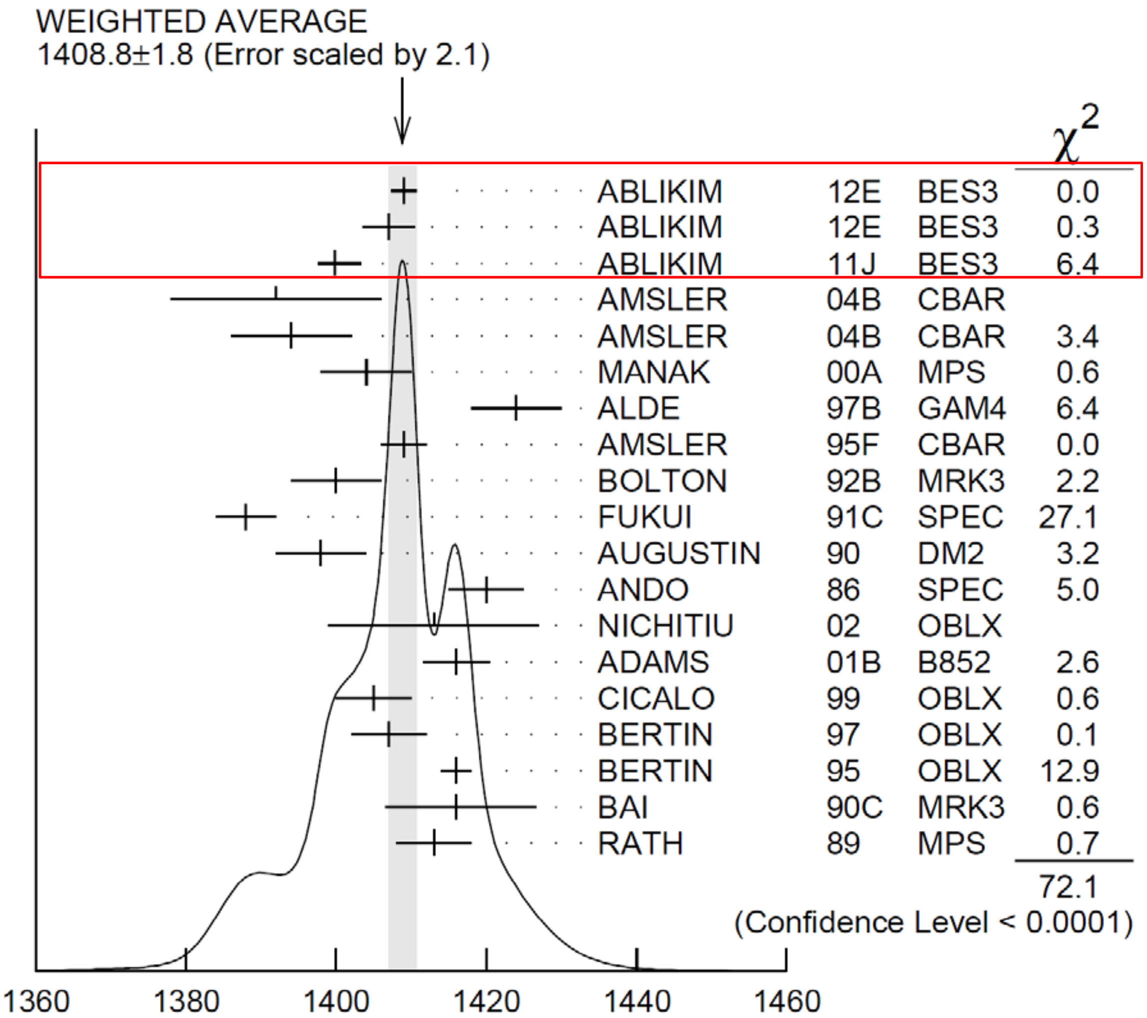
◆ **What can we learn from modern high-precision data? E.g. BESIII, Belle, LHCb…**

◆ **How to understand the HUGE difference between the dynamical calculations and LQCD results?**

[1] Faddeev, Niemi, and Wiedner, PRD70, 114033 (2004)
[2] H. Y. Cheng, H. n. Li, and K. F. Liu, Phys. Rev. D 79, 014024 (2009)
[3] Morningstar and Peardon, PRD60, 034509 (1999); Y. Chen et al., PRD73, 014516(2006)
[4] Richards, Irving, Gregory, and McNeile (UKQCD), PRD82, 034501 (2010)
[5] W. Sun et al. [CLQCD], arXiv:1702.08174[hep-lat]

$\eta(1405)$

$$I^G(J^{PC}) = 0^+(0^{-+})$$



$\eta(1405)$ mass (MeV)

$\eta(1405)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}\pi$	seen
Γ_2	$\eta\pi\pi$	seen
Γ_3	$a_0(980)\pi$	seen
Γ_4	$\eta(\pi\pi)_{S\text{-wave}}$	seen
Γ_5	$f_0(980)\eta$	seen
Γ_6	4π	seen
Γ_7	$\rho\rho$	<58 %
Γ_8	$\gamma\gamma$	
Γ_9	$\rho^0\gamma$	seen
Γ_{10}	$\phi\gamma$	
Γ_{11}	$K^*(892)K$	seen

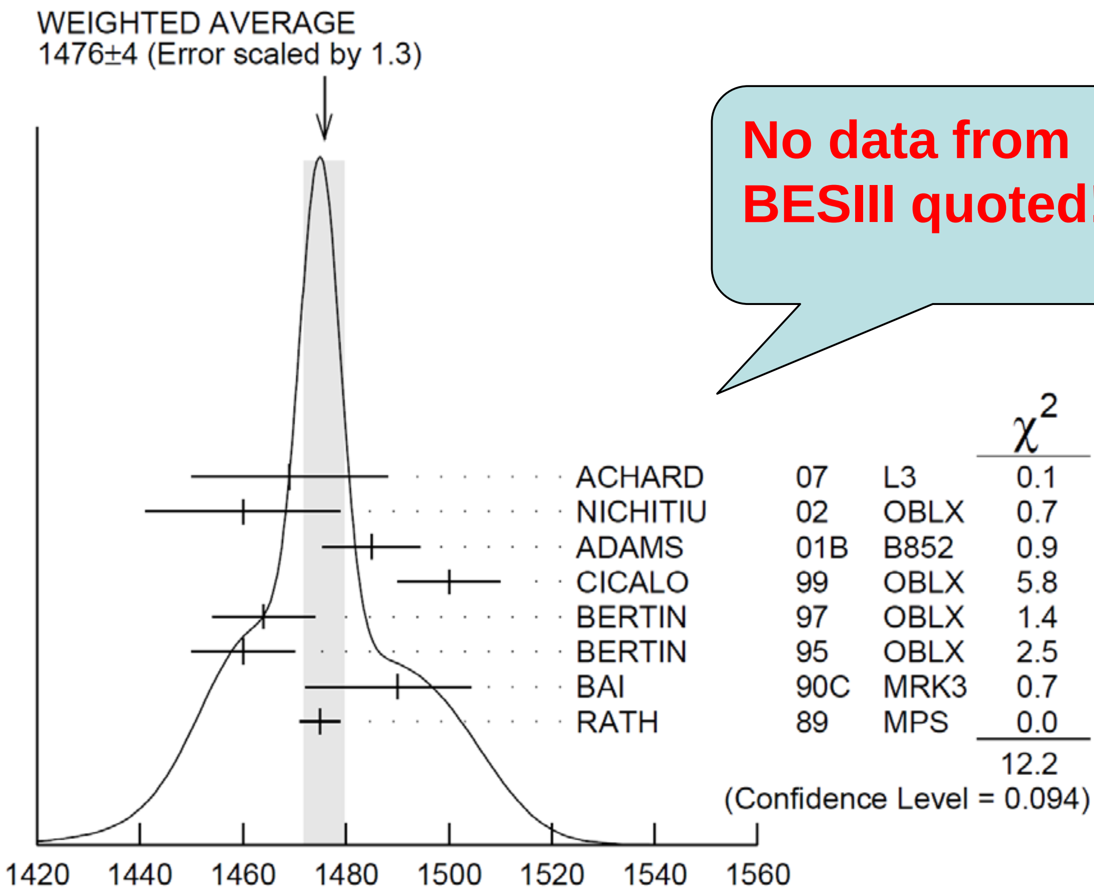
Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update

$\eta(1475)$

$$I^G(J^{PC}) = 0^+(0^{-+})$$

Apparent inconsistency between the analyses for η(1405) and η(1475)

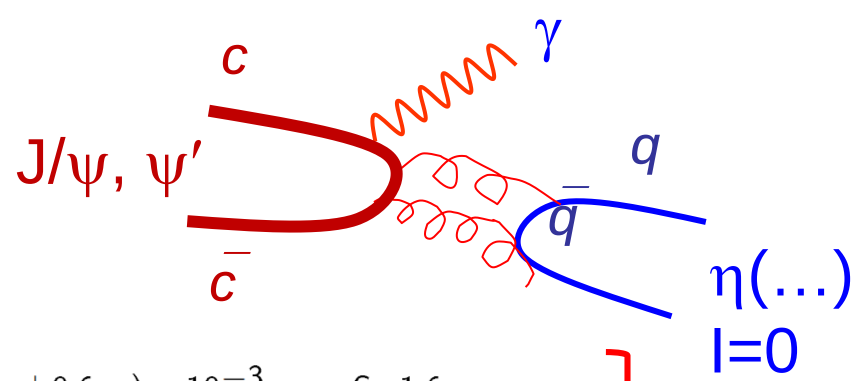
No data from BESIII quoted!!!



	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}\pi$	dominant
Γ_2	$K\overline{K}^*(892)+$ c.c.	seen
Γ_3	$a_0(980)\pi$	seen
Γ_4	$\gamma\gamma$	seen
Γ_5	$K^0_S K^0_S \eta$	possibly seen

BESIII measurements of η(...) states in J/ψ and ψ′ decays

Only a single state is observed in the J/ψ and ψ′ decays!



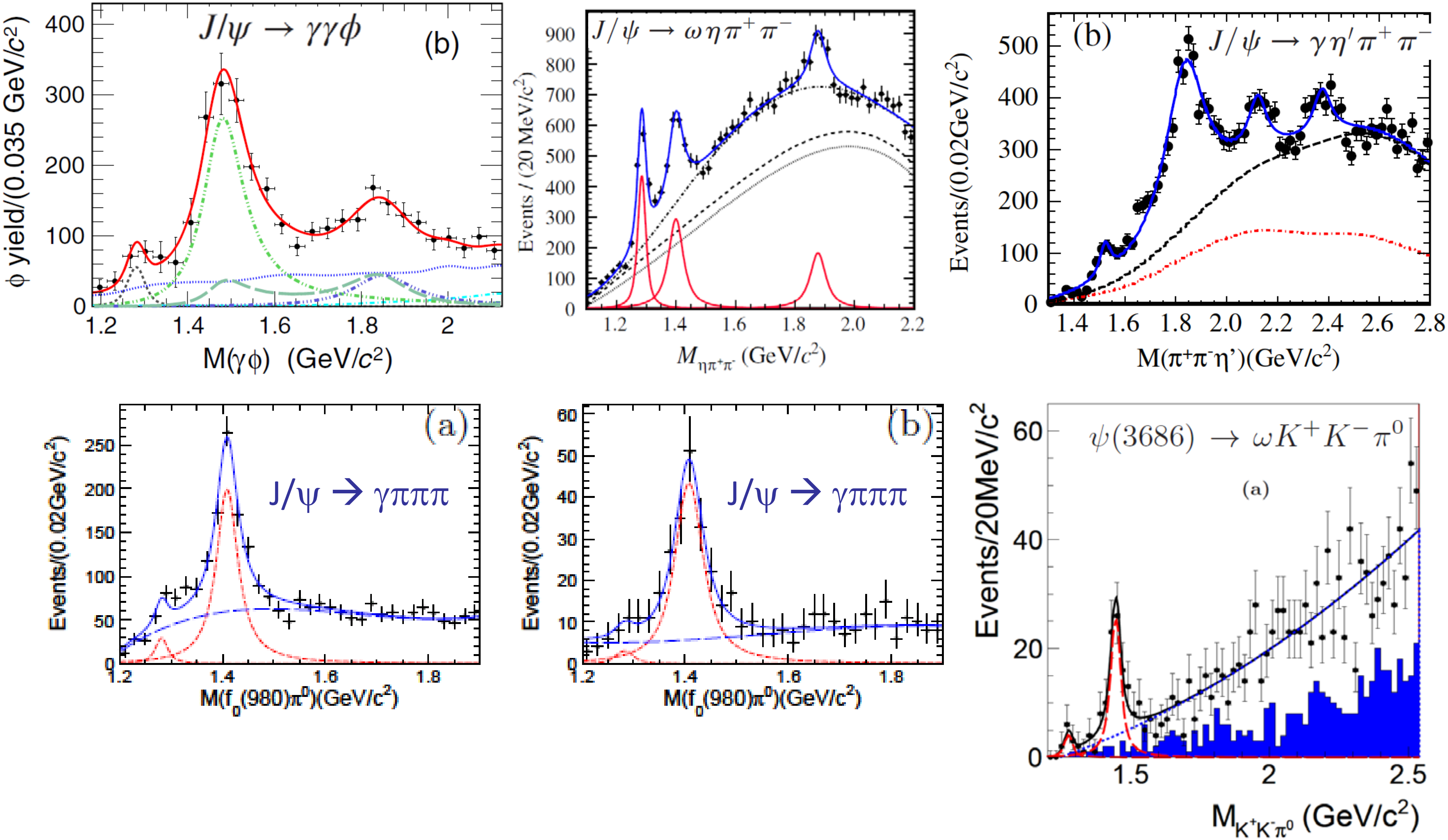
$J/\psi(1S)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

Γ_{151}	$\gamma\eta(1405/1475) \to \gamma K \bar{K} \pi$	[d]	$(2.8 \pm 0.6) \times 10^{-3}$	S=1.6	BES-III
Γ_{152}	$\gamma\eta(1405/1475) \to \gamma\gamma\rho^0$		$(7.8 \pm 2.0) \times 10^{-5}$	S=1.8	BES-III
Γ_{153}	$\gamma\eta(1405/1475) \to \gamma\eta\pi^+\pi^-$		$(3.0 \pm 0.5) \times 10^{-4}$		BES-III
Γ_{154}	$\gamma\eta(1405/1475) \to \gamma\gamma\phi$		$< 8.2 \times 10^{-5}$	CL=95%	BES-III
Γ_{165}	$\gamma\eta(1405/1475) \to \gamma\rho^0\rho^0$		$(1.7 \pm 0.4) \times 10^{-3}$	S=1.3	BES-III
Γ_{87}	$\phi\eta(1405) \to \phi\eta\pi^+\pi^-$		$(2.0 \pm 1.0) \times 10^{-5}$		BES-III

$\psi(2S)$

Γ_{94}	$\omega X(1440) \to \omega K^0_S K^- \pi^+ +$ c.c.	$(1.6 \pm 0.4) \times 10^{-5}$		BES-III
Γ_{95}	$\omega X(1440) \to \omega K^+ K^- \pi^0$	$(1.09 \pm 0.26) \times 10^{-5}$		BES-III
Γ_{155}	$\gamma\eta(1405)$			BES-II
Γ_{156}	$\gamma\eta(1405) \to \gamma K \bar{K} \pi$	$< 9 \times 10^{-5}$	CL=90%	BES-II
Γ_{157}	$\gamma\eta(1405) \to \eta\pi^+\pi^-$	$(3.6 \pm 2.5) \times 10^{-5}$		BES-II
Γ_{158}	$\gamma\eta(1475)$			BES-II
Γ_{159}	$\gamma\eta(1475) \to K \bar{K} \pi$	$< 1.4 \times 10^{-4}$	CL=90%	BES-II
Γ_{160}	$\gamma\eta(1475) \to \eta\pi^+\pi^-$	$< 8.8 \times 10^{-5}$	CL=90%	BES-II

Invariant mass spectra measured at BES-III



The $\eta(1405)$ and $\eta(1475)$ paradox:

- No experimental evidence for $\eta(1405)$ and $\eta(1475)$ to be present in the same decay channel!

  -- How to reconcile the conflicts between LQCD and phenomenological models?

  -- How to understand the low mass for the pseudoscalar glueball in phenomenological studies?

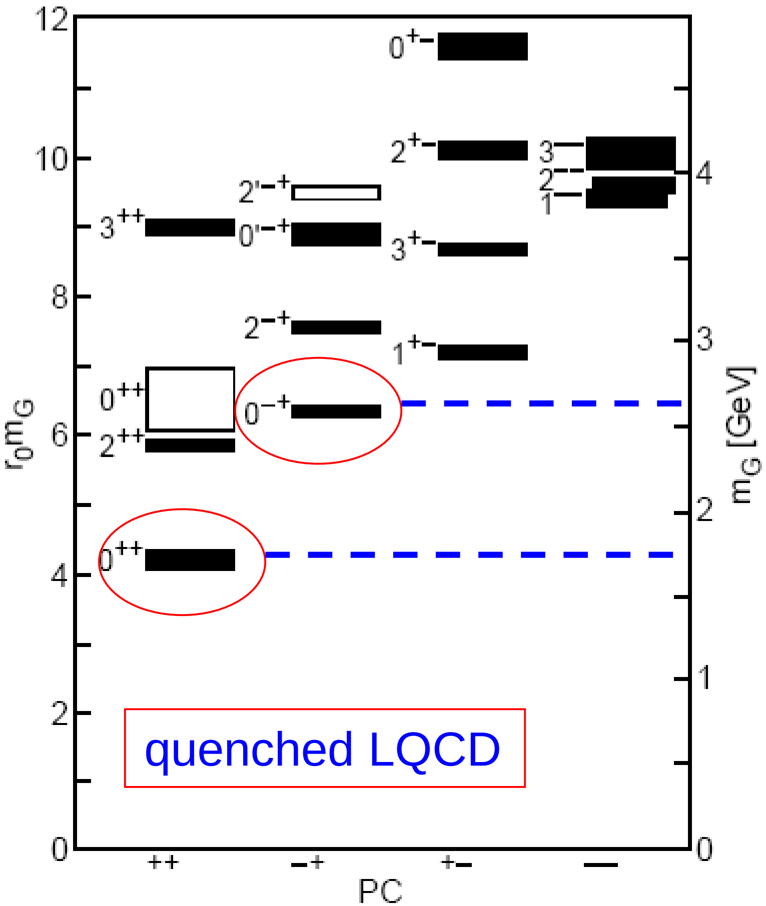
- The peak positions for the $\eta(1405/1475)$ are slightly different in different channels!

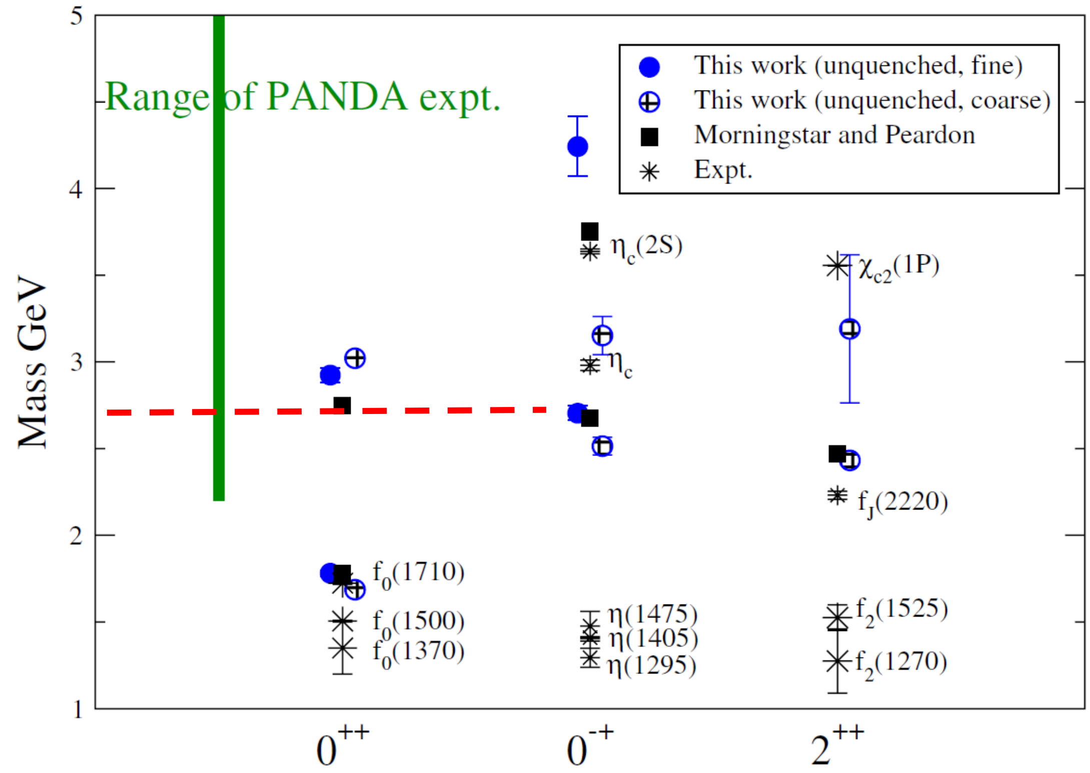
  -- How to understand different masses and lineshapes for $\eta(1405)$ and $\eta(1475)$ in different channels?

2. Reconsile the the dynamical model calculations with LQCD simulations

A brief status review: Qin, QZ, and Zhong, PRD 97, 096002 (2018)

Stable PG masses from LQCD simulations



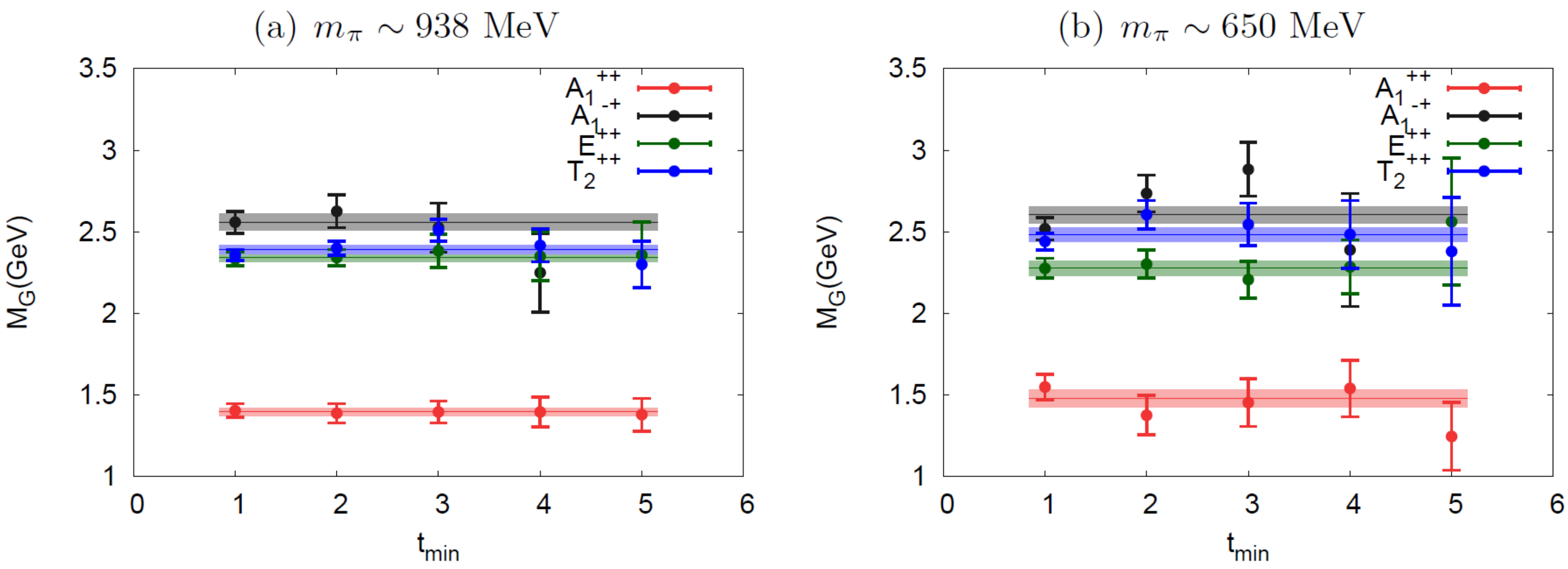


unquenched LQCD

Morningstar and Peardon, PRD60, 034509 (1999); Y. Chen et al., PRD73, 014516(2006)

Richards, Irving, Gregory, and McNeile (UKQCD), PRD82, 034501 (2010)

N_f = 2 LQCD study on anisotropic lattices



	m_π (MeV)	m_{0++} (MeV)	m_{2++} (MeV)	m_{0-+} (MeV)
$N_f = 2$	938	1397(25)	2367(35)	2559(50)
	650	1480(52)	2380(61)	2605(52)
$N_f = 2+1$ [13]	360	1795(60)	2620(50)	—
quenched [8]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [9]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

W. Sun et al. [CLQCD], arXiv:1702.08174[hep-lat]

Can mixing bring down the PG mass in a dynamical calc.?

- η(1295) and η(1475) are the 1st radial excitation between the flavor singlet and octet with I=0.

$$\begin{cases} \eta(1295) & = & \cos\alpha n\bar{n} - \sin\alpha s\bar{s} \\ \eta(1440) & = & \sin\alpha n\bar{n} + \cos\alpha s\bar{s} \end{cases}$$

- η(1405) is a pseudoscalar glueball candidate which favors to mix with the ground states η(547) and η′(958).
- **Caution:** Lattice QCD gives the pseudoscalar glueball mass of ~2.4 GeV.

$$\begin{pmatrix} \eta \\ \eta' \\ \eta'' \end{pmatrix} = U \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix}$$

- G. Li, Q. Zhao, C.H. Chang, JPG35, 055002 (2008); hep-ph/0701020
- C. Thomas, JHEP 0710:026, 2007
- R. Escribano, EPJC65, 467 (2010)
- H.Y. Cheng, H.n. Li and K.F. Liu, PRD79, 014024 (2009)
- … …

- **One can even include η_c ($\bar{c}c$) in the mixing scheme.**

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \\ |\eta_c\rangle \end{pmatrix} = U_{34}(\theta)U_{14}(\phi_G)U_{12}(\phi_Q)\begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix},$$

$M_G \cong 2.4$ GeV
$M\eta_c = 2.98$ GeV

$$U_{34}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad U_{14}(\phi_G) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_G & \sin\phi_G & 0 \\ 0 & -\sin\phi_G & \cos\phi_G & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_{12}(\phi_Q) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi_Q & \sin\phi_Q \\ 0 & 0 & -\sin\phi_Q & \cos\phi_Q \end{pmatrix}. \qquad \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix} = U_{34}(\theta_i)\begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix}$$

Constraints on the η and η′, but not strongly on a glueball candidate!

Y.-D. Tsai, H.-n. Li and Q.Z., PRD85, 034002 (2011)
Re-investigated in Qin, QZ, and Zhong, PRD 97, 096002 (2018)

Assuming that the decay constants in the flavor basis follow the same mixing pattern of the particle states, we have

$$\begin{pmatrix} f_\eta^q & f_\eta^s & f_\eta^c \\ f_{\eta'}^q & f_{\eta'}^s & f_{\eta'}^c \\ f_G^q & f_G^s & f_G^c \\ f_{\eta_c}^q & f_{\eta_c}^s & f_{\eta_c}^c \end{pmatrix} = U \begin{pmatrix} f_q & 0 & 0 \\ 0 & f_s & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f_c \end{pmatrix}$$

T. Feldmann, P. Kroll, and B. Stech, PRD 58, 114006 (1998); PLB 449, 339 (1999)

where

$$U(\theta,\phi_G,\phi_Q) = U_{34}(\theta)U_{14}(\phi_G)U_{12}(\phi_Q)U_{34}(\theta_i),$$

$$= \begin{pmatrix} c\theta c\theta_i - s\theta c\phi_G s\theta_i & -c\theta s\theta_i - s\theta c\phi_G c\theta_i & -s\theta s\phi_G c\phi_Q & -s\theta s\phi_G s\phi_Q \\ s\theta c\theta_i + c\theta c\phi_G s\theta_i & -s\theta s\theta_i + c\theta c\phi_G c\theta_i & c\theta s\phi_G c\phi_Q & c\theta s\phi_G s\phi_Q \\ -s\phi_G s\theta_i & -s\phi_G c\theta_i & c\phi_G c\phi_Q & c\phi_G s\phi_Q \\ 0 & 0 & -s\phi_Q & c\phi_Q \end{pmatrix}$$

The axial vector anomaly is given by the $U_A(1)$ Ward identity:

$$\partial^\mu J^j_{\mu 5} = \partial^\mu(\bar{j}\gamma_\mu\gamma_5 j) = 2m_j(\bar{j}i\gamma_5 j) + \frac{\alpha_s}{4\pi}G\tilde{G}$$

The axial vector anomaly can then relate the pseudoscalar meson masses to the flavor singlet pseudoscalar densities and the topological charge density:

$$\langle 0|\partial^\mu J^j_{\mu 5}|P\rangle = M_P^2 f_P^j$$

where

$$M_P^2 \equiv \begin{pmatrix} M_\eta^2 & 0 & 0 & 0 \\ 0 & M_{\eta'}^2 & 0 & 0 \\ 0 & 0 & M_G^2 & 0 \\ 0 & 0 & 0 & M_{\eta_c}^2 \end{pmatrix}$$

And

$$\mathcal{M}_{qsgc} = U^\dagger M_P^2 U \qquad \text{--- (A)}$$

Meanwhile, the axial vector anomaly gives:

$$\tilde{\mathcal{M}}_{qsgc} = \begin{pmatrix} m_{qq}^2 + \sqrt{2}G_q/f_q & m_{sq}^2 + G_q/f_s & m_{cq}^2 + G_q/f_c \\ m_{qs}^2 + \sqrt{2}G_s/f_q & m_{ss}^2 + G_s/f_s & m_{cs}^2 + G_s/f_c \\ m_{qg}^2 + \sqrt{2}G_g/f_q & m_{sg}^2 + G_g/f_s & m_{cg}^2 + G_g/f_c \\ m_{qc}^2 + \sqrt{2}G_c/f_q & m_{sc}^2 + G_c/f_s & m_{cc}^2 + G_c/f_c \end{pmatrix} \qquad \text{--- (B)}$$

The equivalence of Eqs. (A) and (B) gives:

$$U^\dagger \begin{pmatrix} M_\eta^2 & 0 & 0 & 0 \\ 0 & M_{\eta'}^2 & 0 & 0 \\ 0 & 0 & M_G^2 & 0 \\ 0 & 0 & 0 & M_{\eta_c}^2 \end{pmatrix} U \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} m_{qq}^2 + \sqrt{2}G_q/f_q & m_{sq}^2 + G_q/f_s & m_{cq}^2 + G_q/f_c \\ m_{qs}^2 + \sqrt{2}G_s/f_q & m_{ss}^2 + G_s/f_s & m_{cs}^2 + G_s/f_c \\ m_{qg}^2 + \sqrt{2}G_g/f_q & m_{sg}^2 + G_g/f_s & m_{cg}^2 + G_g/f_c \\ m_{qc}^2 + \sqrt{2}G_c/f_q & m_{sc}^2 + G_c/f_s & m_{cc}^2 + G_c/f_c \end{pmatrix}$$

with

$$\begin{cases} m_{qq,qs,qg,qc}^2 \equiv \dfrac{\sqrt{2}}{f_q}\langle 0|m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d|\eta_q, \eta_s, g, \eta_Q\rangle \\ m_{sq,ss,sg,sc}^2 \equiv \dfrac{2}{f_s}\langle 0|m_s \bar{s} i\gamma_5 s|\eta_q, \eta_s, g, \eta_Q\rangle, \\ m_{cq,cs,cg,cc}^2 \equiv \dfrac{2}{f_c}\langle 0|m_c \bar{c} i\gamma_5 c|\eta_q, \eta_s, g, \eta_Q\rangle, \\ G_{q,s,g,c} \equiv \dfrac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta_q, \eta_s, g, \eta_Q\rangle. \end{cases}$$

This allows a relation for the physical glueball mass and the topological charge density in association with the other constrained parameters:

$$\tilde{\mathcal{M}}^{31}_{qsgc} = m^2_{qg} + \sqrt{2}G_g/f_q$$
$$= -M^2_\eta(c\theta c\theta_i - s\theta c\phi_G s\theta_i)s\theta s\phi_G c\phi_Q + M^2_{\eta'}(s\theta c\theta_i + c\theta c\phi_G s\theta_i)c\theta s\phi_G c\phi_Q - M^2_G c\phi_G s\phi_G s\theta_i c\phi_Q,$$

$$\tilde{\mathcal{M}}^{32}_{qsgc} = m^2_{sg} + G_g/f_s$$
$$= M^2_\eta(c\theta s\theta_i + s\theta c\phi_G c\theta_i)s\theta s\phi_G c\phi_Q + M^2_{\eta'}(-s\theta s\theta_i + c\theta c\phi_G c\theta_i)c\theta s\phi_G c\phi_Q - M^2_G c\phi_G s\phi_G c\theta_i c\phi_Q.$$

$$\Rightarrow \quad \hat{R}_{31/32} \equiv \frac{\tilde{\mathcal{M}}^{31}_{qsgc}}{\tilde{\mathcal{M}}^{32}_{qsgc}} = \frac{m^2_{qg} + \sqrt{2}G_g/f_q}{m^2_{sg} + G_g/f_s}$$

$$\Rightarrow \quad M^2_G = -\frac{1}{\cos\phi_G \sin\theta_i \cos\phi_Q}\left\{\frac{\sqrt{2}G_g/f_q}{\sin\phi_G} - [-M^2_\eta(\cos\theta\cos\theta_i - \sin\theta\cos\phi_G\sin\theta_i)\sin\theta\cos\phi_Q + M^2_{\eta'}(\sin\theta\cos\theta_i + \cos\theta\cos\phi_G\sin\theta_i)\cos\theta\cos\phi_Q]\right\}.$$
$$\approx -\frac{1}{\sin\theta_i}\left\{\frac{\sqrt{2}G_g/f_q}{\sin\phi_G} - M^2_{\eta'}\sin\theta_i - (M^2_{\eta'} - M^2_\eta)\sin\theta\cos(\theta + \theta_i)\right\}$$

With the LQCD results for the topological charge density, we can fit the parameters:

TABLE I. The numerical values of all the parameters with $G_g = -0.054$ GeV^3 and $\phi_G = 12°$ fixed. The two quantities, m_{qc}^{2*} and m_{sc}^{2*} involve more complicated issues and are sensitive to m_{cc}^2 and ϕ_G. Further detailed discussions can be found in the context.

f_s/f_q	M_G (GeV)	m_{qq}^2 (GeV)^2	m_{ss}^2	m_{sg}^2	m_{cg}^2	m_{qc}^{2*}	m_{sc}^{2*}	m_{cq}^2	m_{cs}^2	G_q (GeV)^3	G_s	G_c
1.2	2.1	0.055	0.45	−0.041	−0.81	0.87	0.50	−0.24	−0.15	0.060	0.035	−0.092
1.3	2.1	0.0012	0.47	−0.067	−0.81	0.87	0.46	−0.25	−0.15	0.065	0.035	−0.092

where we have applied the condition: $m_{qs,sq}^2 \ll m_{qg}^2 \ll m_{qq}^2$

Note: $m_{qg}^2 \ll m_{sg}^2$

$$\hat{R}_{31/32} \equiv \frac{\tilde{\mathcal{M}}_{qsgc}^{31}}{\tilde{\mathcal{M}}_{qsgc}^{32}} = \frac{m_{qg}^2 + \sqrt{2}G_g/f_q}{m_{sg}^2 + G_g/f_s}$$

If $m_{qg}^2 \sim m_{sg}^2 \ll G_g/f_q \sim G_g/f_s$ ⇒ $\hat{R}_{31/32} \simeq \sqrt{2} f_s/f_q$

⇒ $M_G \sim 1.4$ GeV !

Inappropriate approx. made in

H.-Y. Cheng, H.-n. Li and K.-F. Liu, PRD79, 014024 (2009)

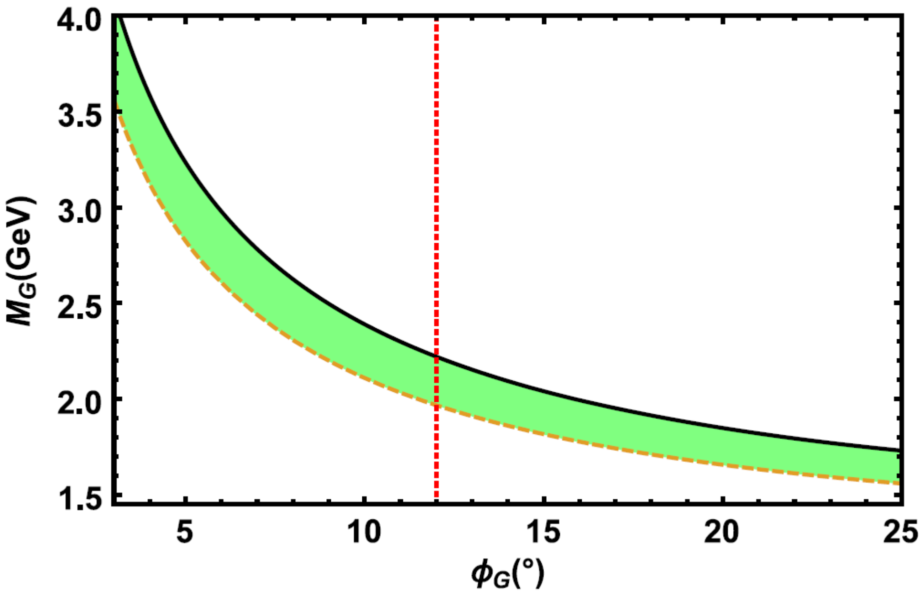
Y.-D. Tsai, H.-n. Li and Q.Z., PRD85, 034002 (2011)

With the LQCD results for the topological charge density, we can fit the parameters:

TABLE I. The numerical values of all the parameters with $G_g = -0.054$ GeV^3 and $\phi_G = 12°$ fixed. The two quantities, m_{qc}^{2*} and m_{sc}^{2*} involve more complicated issues and are sensitive to m_{cc}^2 and ϕ_G. Further detailed discussions can be found in the context.

f_s/f_q	M_G (GeV)	m_{qq}^2 (GeV)^2	m_{ss}^2	m_{sg}^2	m_{cg}^2	m_{qc}^{2*}	m_{sc}^{2*}	m_{cq}^2	m_{cs}^2	G_q (GeV)^3	G_s	G_c
1.2	2.1	0.055	0.45	−0.041	−0.81	0.87	0.50	−0.24	−0.15	0.060	0.035	−0.092
1.3	2.1	0.0012	0.47	−0.067	−0.81	0.87	0.46	−0.25	−0.15	0.065	0.035	−0.092

where we have applied the condition: $m_{qs,sq}^2 \ll m_{qg}^2 \ll m_{qq}^2$

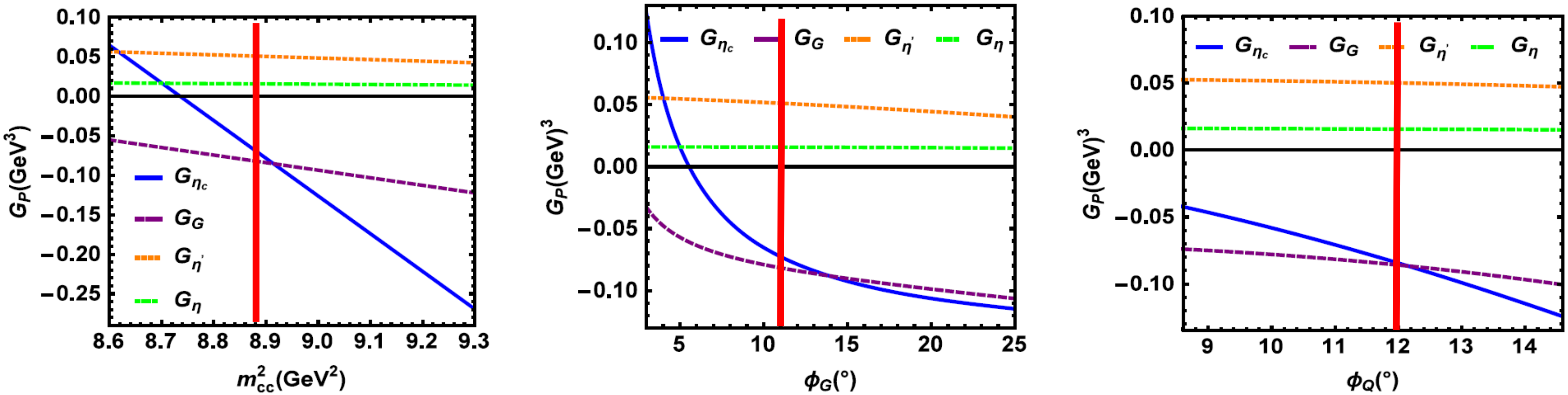


$$\hat{R}_{31/32} \equiv \frac{\tilde{\mathcal{M}}_{qsgc}^{31}}{\tilde{\mathcal{M}}_{qsgc}^{32}} = \frac{m_{qg}^2 + \sqrt{2}G_g/f_q}{m_{sg}^2 + G_g/f_s}$$

FIG. 1. The physical glueball mass M_G varies with $\phi_G \in (3\text{–}25)°$, with $\theta = -11°$, $\phi_Q = 11.6°$, and $f_q = 131$ MeV.

The dependence of G_P on m_{cc}^2, ϕ_G, and ϕ_Q

$\phi_G = 12°$ and $\phi_Q = 11.6°$ | $m_{cc}^2 = M_{\eta_c}^2$, and $\phi_Q = 11.6°$ | $\phi_G = 12°$ and $m_{cc}^2 = M_{\eta_c}^2$



The topological susceptibility can be extracted for the pseudoscalar mesons:

$$\begin{cases} \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta\rangle = 0.016\ \mathrm{GeV}^3, \\ \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle = 0.051\ \mathrm{GeV}^3, \\ \langle 0|\alpha_s G\tilde{G}/(4\pi)|G\rangle = -0.084\ \mathrm{GeV}^3, \\ \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_c\rangle = -0.079\ \mathrm{GeV}^3, \end{cases}$$

LQCD results:

$$\begin{cases} \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta\rangle \approx 0.021\ \mathrm{GeV}^3 \\ \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle \approx 0.035\ \mathrm{GeV}^3 \\ G_g = -(0.054 \pm 0.008)\ \mathrm{GeV}^3 \end{cases}$$

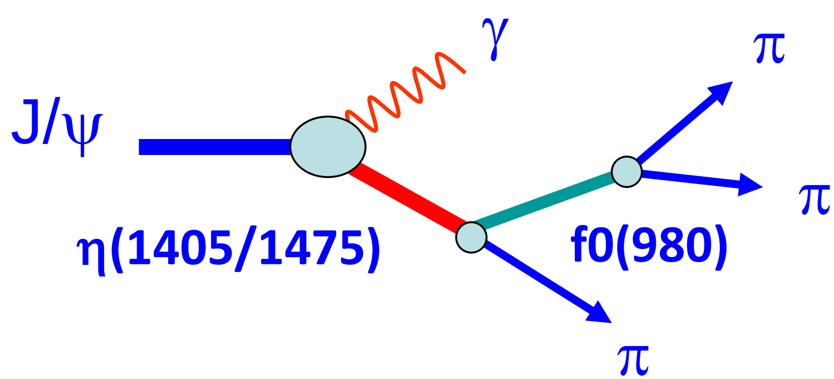
- **Low mass pseudoscalar glueball is unlikely to be favored!**
- **Similar conclusion from V. Vento et al.**

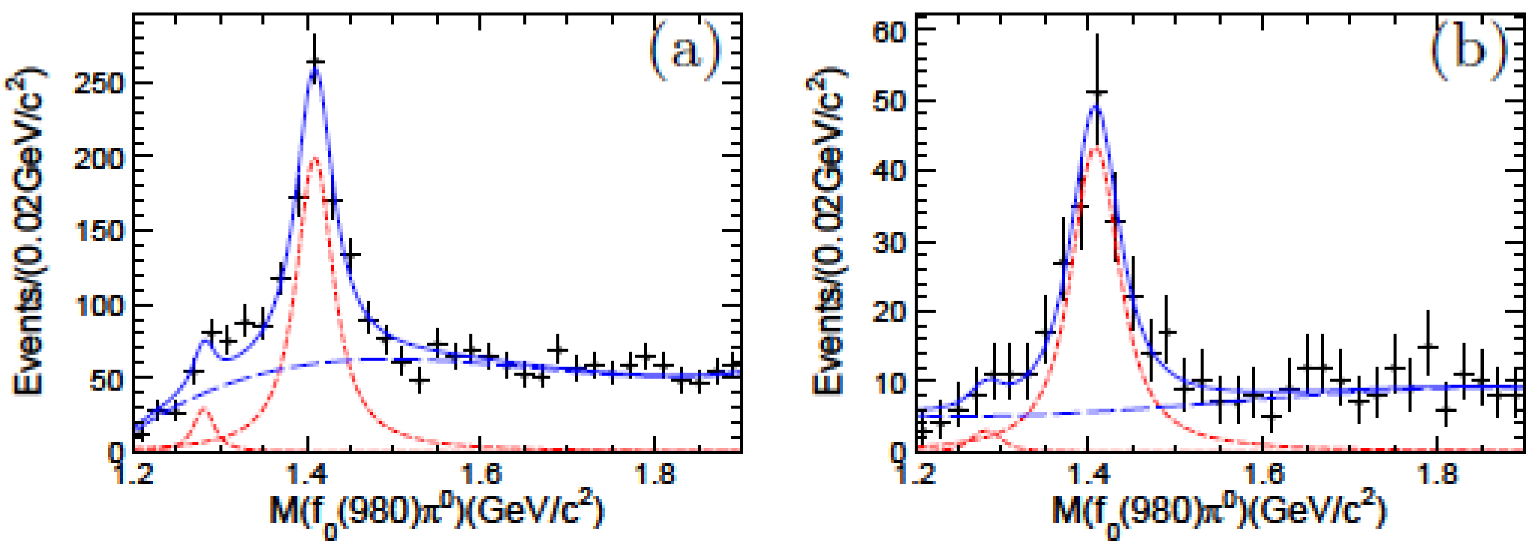
3. The presence of the "triangle singularity"

PRL **108,** 182001 (2012) PHYSICAL REVIEW LETTERS week ending 4 MAY 2012

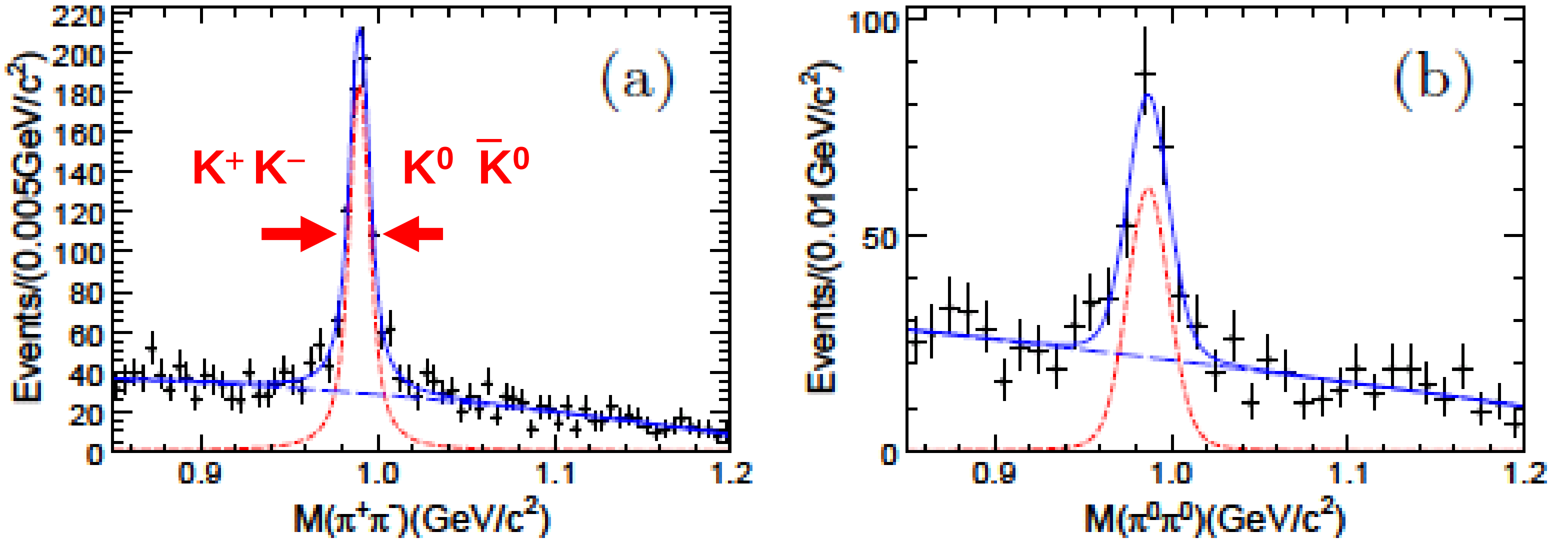
First Observation of $\eta(1405)$ Decays into $f_0(980)\pi^0$

Isospin-violating decay of J/ψ → γ η(1405) → γπππ





BES-III Collaboration, Phys. Rev. Lett. 108, 182001 (2012)



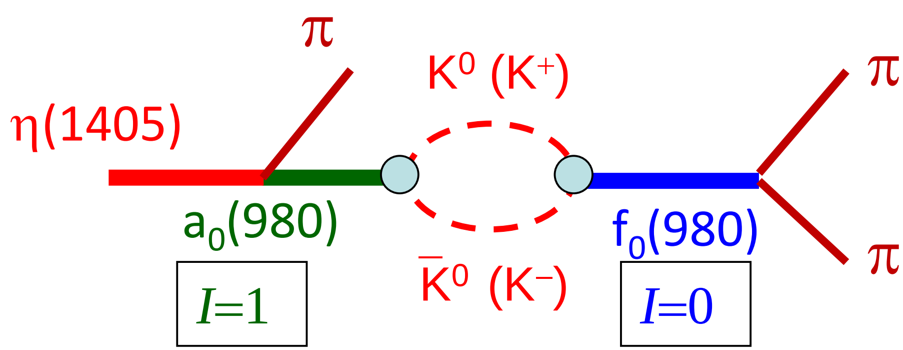
- **$f_0(980)$ is extremely narrow: $\Gamma \cong 10$ MeV !**

PDG: $\Gamma \cong 40\sim100$ MeV.

- **Anomalously large isospin violation!**

$$\frac{Br(\eta(1405) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{Br(\eta(1405) \to a_0^0(980)\pi^0 \to \eta\pi^0\pi^0)} \cong (17.9 \pm 4.2)\%$$

"$a_0(980)$-$f_0(980)$ mixing" gives only ~1% isospin violation effects !

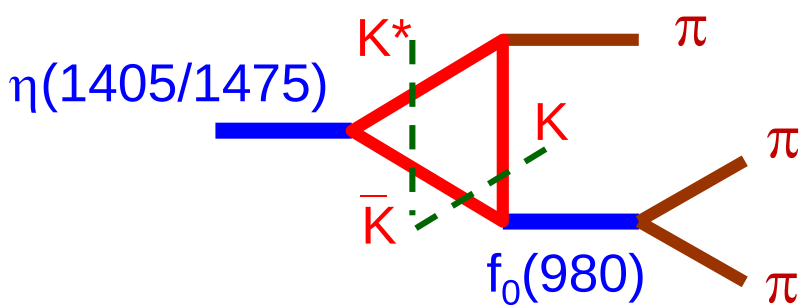


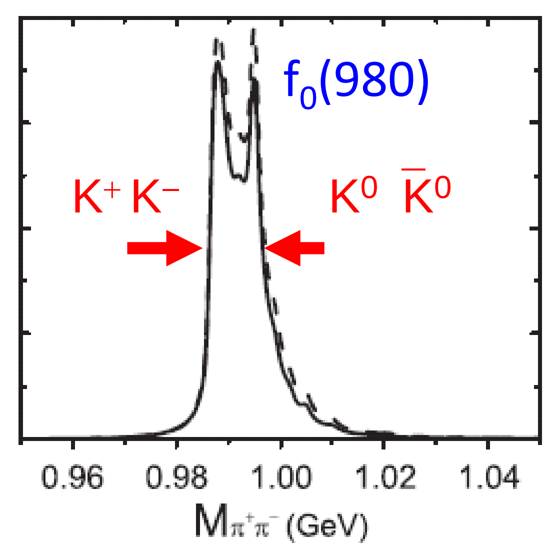
$$g(a_0K^+K^-)\, g(f_0K^+K^-) = -g(a_0K^0\bar{K}^0)\, g(f_0K^0\bar{K}^0)$$

$$M(K^0)-M(K^\pm) = m_d - m_u$$

"Triangle singularity"

Internal $\bar{K}K^*(K)$ approach the on-shell condition simultaneously!





Manifestation of Landau singularity!

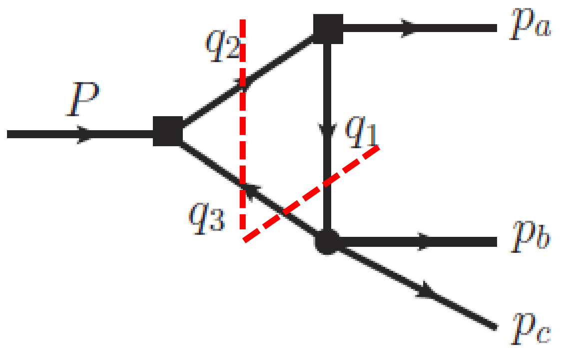
J.J. Wu, X.H. Liu, Q.Z. and B.S. Zou, PRL(2012);

X.G. Wu, J.-J. Wu, Q. Z., and B.-S. Zou, PRD87, 014023 (2013)

“Exotics” of Type-III:
Peak structures caused by kinematic effects, in particular, by triangle singularity.

$$\Gamma_3(s_1,s_2,s_3) = \frac{1}{i(2\pi)^4}\int \frac{d^4q_1}{(q_1^2-m_1^2+i\epsilon)(q_2^2-m_2^2+i\epsilon)(q_3^2-m_3^2+i\epsilon)} = \frac{-1}{16\pi^2}\int_0^1\int_0^1\int_0^1 da_1\,da_2\,da_3\,\frac{\delta(1-a_1-a_2-a_3)}{D-i\epsilon},$$

$$D \equiv \sum_{i,j=1}^{3} a_i a_j Y_{ij},\quad Y_{ij} = \frac{1}{2}\left[m_i^2+m_j^2-(q_i-q_j)^2\right]$$



The TS occurs when all the three internal particles can approach their on-shell condition simultaneously:

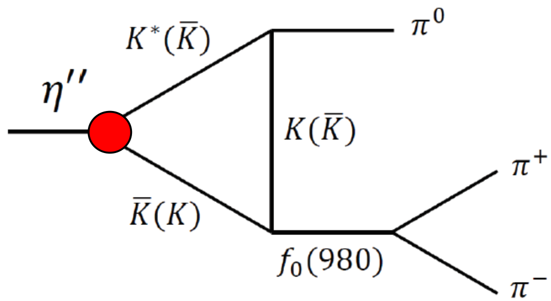
$\partial D/\partial a_j = 0$ for all j=1,2,3. ⇨ $\det[Y_{ij}] = 0$

L. D. Landau, Nucl. Phys. 13, 181 (1959);
J.J. Wu, X.-H. Liu, Q. Zhao, B.-S. Zou, Phys. Rev. Lett. 108, 081003 (2012);
Q. Wang, C. Hanhart, Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013); Phys. Lett. B 725, 106 (2013)
X.-H. Liu, M. Oka and Q. Zhao, PLB753, 297(2016);
F.-K. Guo, C. Hanhart, U.-G. Meissner, Q. Wang, Q. Zhao, B.-S. Zou, arXiv:1705.00141[hep-ph], Rev. Mod. Phys. 90, 015004 (2018)

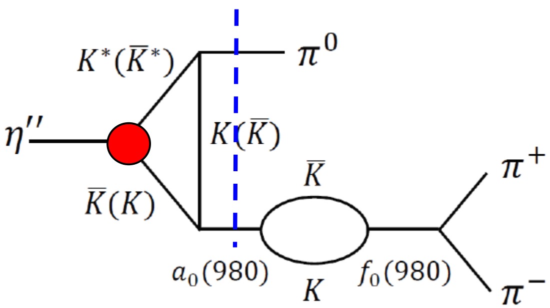
Manifestations of TS in various processes:

F. Aceti, W.H. Liang, E. Oset, J.J. Wu and B. S. Zou, PRD86, 114007 (2012)
Q. Wang, C. Hanhart and Q. Zhao, PRL111, no. 13, 132003 (2013); PLB725 (2013) 106
X.-G. Wu, C. Hanhart, Q. Wang and Q. Zhao, PRD 89, 054038 (2014)
X.-H. Liu and G. Li, PRD88, 014013 (2013);
X.-H. Liu, PRD90, 074004 (2014)
F. Aceti, J.M. Dias and E. Oset, EPJA51, no. 4, 48 (2015)
F. Aceti, J.J. Xie and E. Oset, PLB750, 609 (2015)
X.-H. Liu, M. Oka and Q. Zhao, PLB753, 297 (2016)
F.-K. Guo, U.-G. Meißner, W. Wang and Z. Yang, PRD92, 071502 (2015)
X.-H. Liu, Q. Wang and Q. Zhao, PLB757, 231 (2016)
W. Qin, S. R. Xue and Q. Zhao, PRD 94, no. 5, 054035 (2016)
M. Mikhasenko, arXiv:1507.06552 [hep-ph].
N.N. Achasov, A.A. Kozhevnikov and G.N. Shestakov, PRD92, no. 3, 036003 (2015)
N.N. Achasov, A.A. Kozhevnikov and G.N. Shestakov, PRD93, no. 11, 114027 (2016)
N.N. Achasov and G.N. Shestakov, Nucl. Part. Phys. Proc. 287-288, 89 (2017)
N.N. Achasov and G.N. Shestakov, JETP Lett. 107, no. 5, 276 (2018)
M. Bayar, F. Aceti, F.-K. Guo and E. Oset, PRD94, no. 7, 074039 (2016)
M. Albaladejo, J.T. Daub, C. Hanhart, B. Kubis and B. Moussallam, JHEP1704, 010 (2017)
L. Roca and E. Oset, PRC95, no. 6, 065211 (2017)
V.R. Debastiani, S. Sakai and E. Oset, PRC96, no. 2, 025201 (2017)
X.-H. Liu and U.-G. Meißner, EPJC77, no. 12, 816 (2017)
S. Sakai, E. Oset and A. Ramos, EPJA54, no. 1, 10 (2018)
S.R. Xue, H.J. Jing, F.K. Guo and Q. Zhao, PLB779, 402 (2018)
……

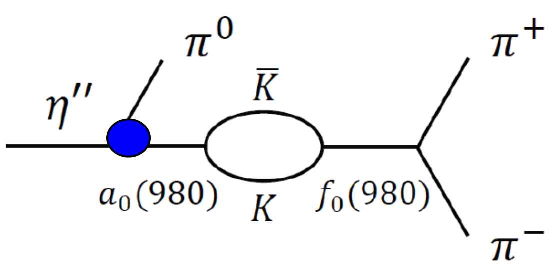
Updated study of η(1405/1475)→ 3π, K ‾ππ, ηππ with width effects



(a)



(b)



(c)

- **Direct isospin breaking via the TS mechanism**

- **a0-f0 mixing enhanced by the TS mechanism**

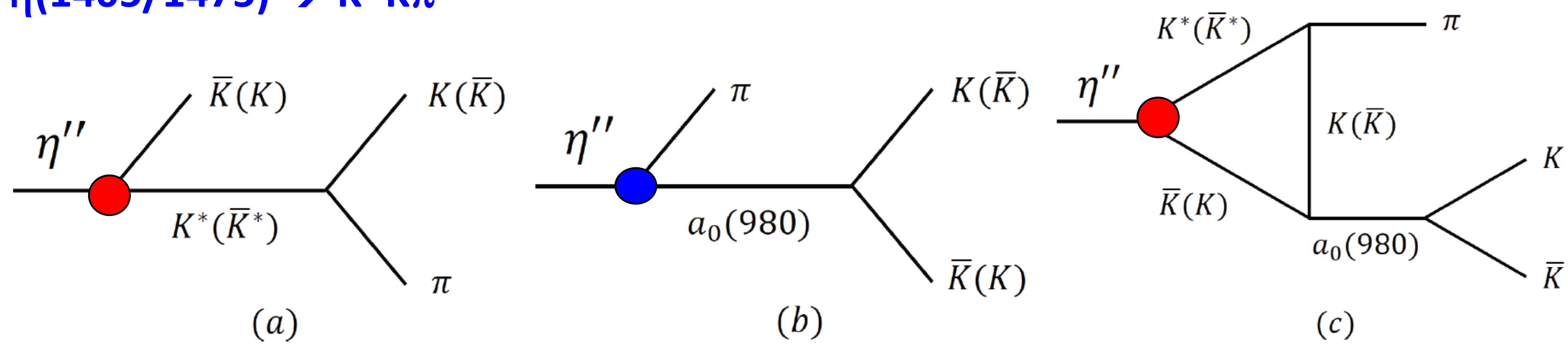
-- Unitarized treatment for a0 and f0;
-- To separate (b) and (c) allows a self-contained evaluation of the TS and a0-f0 mixing contributions.

- **a0-f0 mixing at tree level**

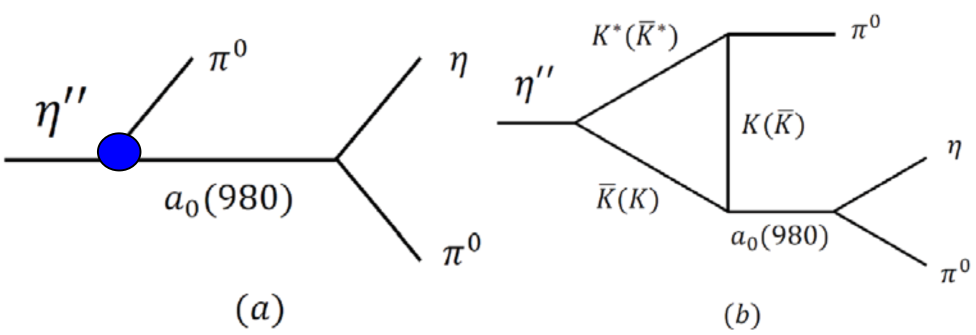
M.C. Du and Q.Z., to be submitted.
See also N.N. Achasov et al., PRD92, 036003 (2015)

Updated study of η(1405/1475)→ 3π, K $\bar{\pi}\pi$, ηππ with width effects

η(1405/1475) → K $\bar{K}\pi$



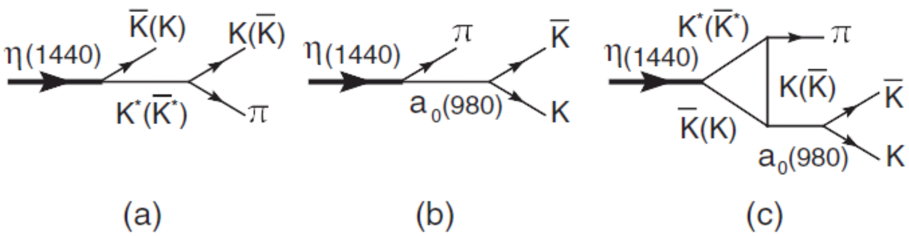
η(1405/1475) → ηππ



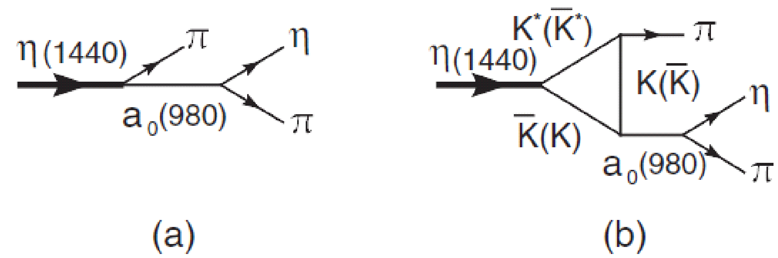
M.C. Du and Q.Z., to be submitted.

Interferences from the TS mechanism

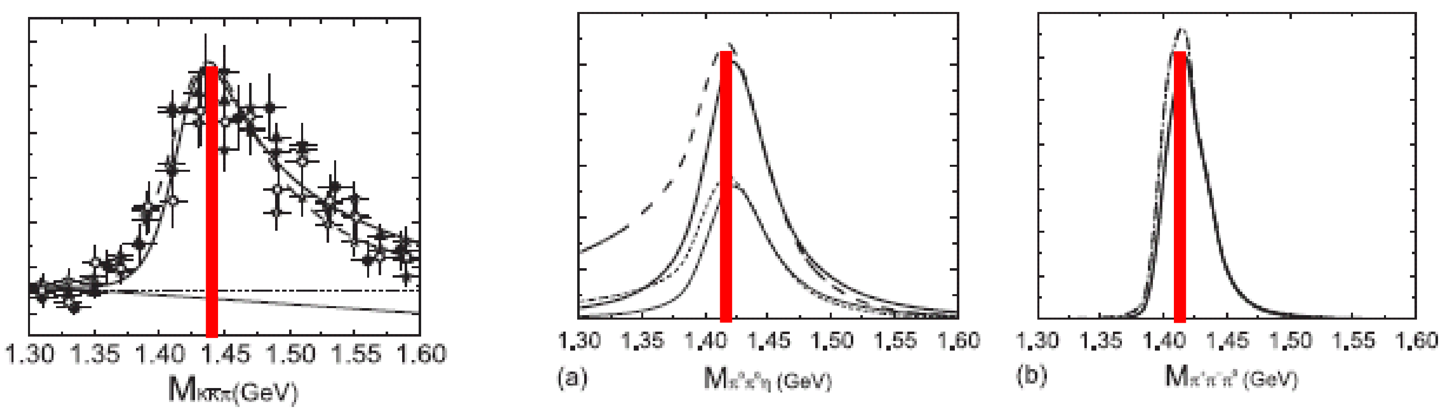
$\eta(1405/1475) \rightarrow K\bar{K}\pi$



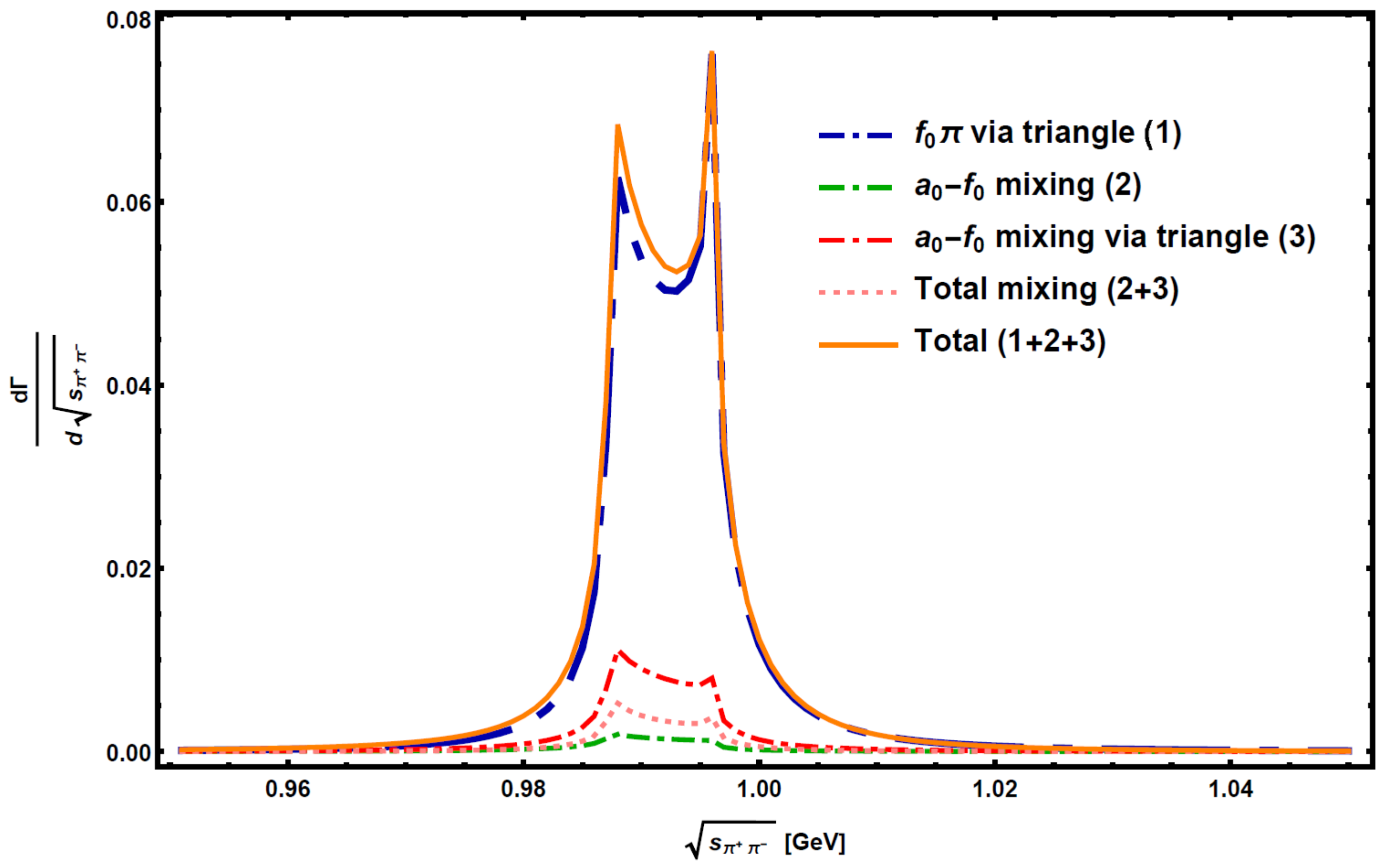
$\eta(1405/1475) \rightarrow \eta\pi\pi$



- **The "Triangle Singularity" mechanism can shift the peak positions exclusive channels.**
- **Different lineshapes in difference channels, i.e. $K\bar{K}\pi$, $\eta\pi\pi$, and 3π.**
- **No obvious need for two independent states, $\eta(1405)$ and $\eta(1475)$!**

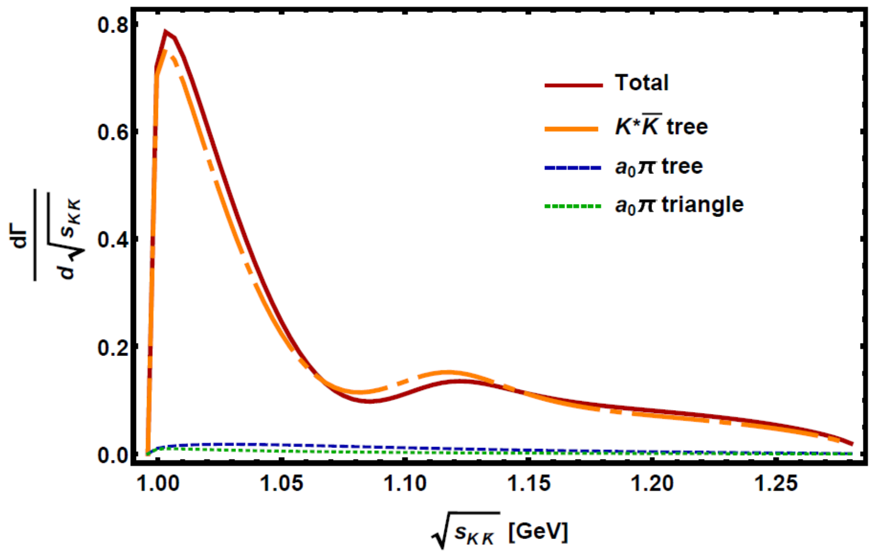


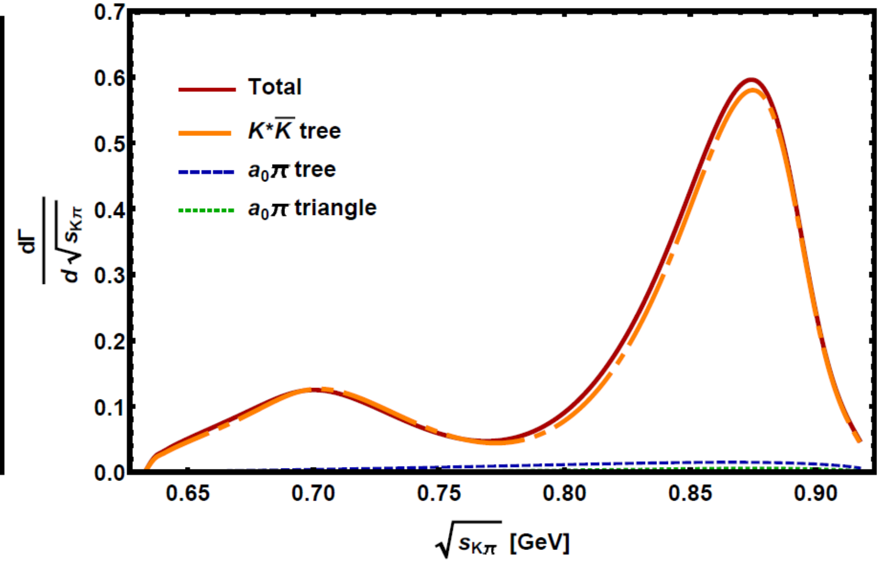
Still the dominance of the TS is present in η(1405/1475) → 3π with the width effects



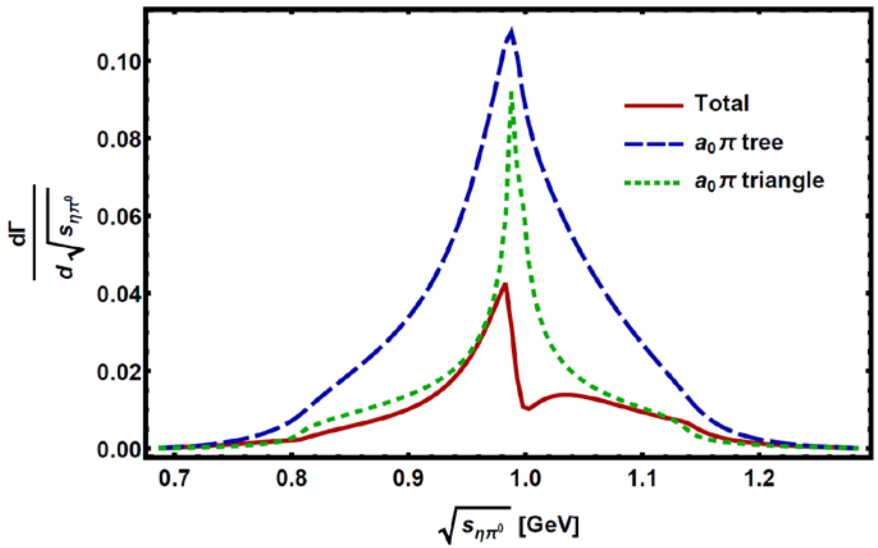
M.C. Du and Q.Z., to be submitted.

$\eta(1405/1475) \rightarrow K\bar{K}\pi$





$\eta(1405/1475) \rightarrow \eta\pi\pi$



M.C. Du and Q.Z., to be submitted.

4. Observables sensitive to the underlying dynamics

Backup slides: The radiative decays of $\eta(1405/1475) \to \gamma V$ with $V=\rho, \omega, \phi$

5. Brief summary

We have to alter our view of the pseudoscalar spectrum dramatically even for the 1st radial excitation! **(A brief status review: Qin, QZ, and Zhong, PRD 97, 096002 (2018))**

- **The $\eta(1405)$ puzzle is originated from the triangle singularity mechanism.**
- **The dynamical calculations of the PG mass are consistent with the LQCD expectations if an inappropriate approx. is corrected.**

- **Where to look for the pseudoscalar glueball candidate?**

Isoscalar pseudoscalars with higher masses above 2 GeV, e.g. X(2120), X(2370) …

中国科学院高能物理研究所
Institute of High Energy Physics

中國科學院
CHINESE ACADEMY OF SCIENCES

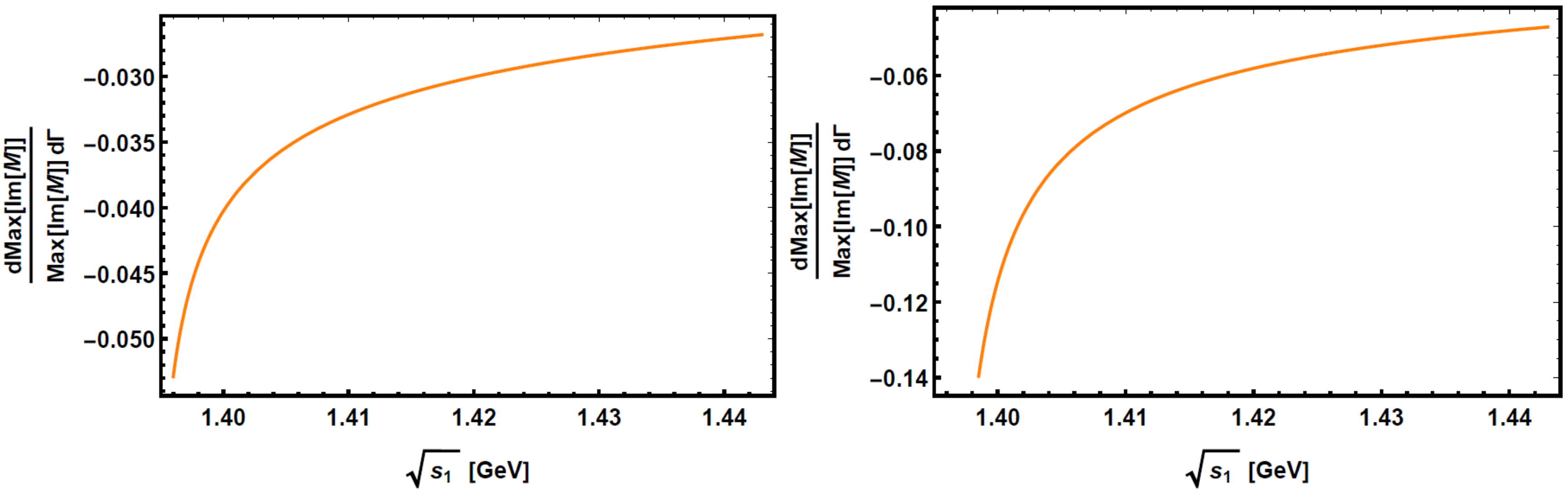
Thanks for your attention!

Decompose the triangle loop integral:

$$\begin{aligned}
M &= -i\int\frac{d^4q}{(2\pi)^4}\frac{(2p_1-q)_\mu(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2})(q-2p_2)_\nu}{D_1D_2D_3}\\
&= -i(s_1-m_1^2+im_1\Gamma_1+s_2-2s_3+2m_K^2-\frac{(s_1-m_K^2)(s_2-m_K^2)}{m_1^2-im_1\Gamma_1})\int\frac{d^4q}{(2\pi)^4}\frac{1}{D_1D_2D_3}\\
&- i\frac{(s_1-m_K^2)(s_2-m_K^2)}{m_1^2-im_1\Gamma_1})\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2D_2D_3}-i(1+\frac{s_1-m_K^2}{m_1^2-im_1\Gamma_1})\int\frac{d^4q}{(2\pi)^4}\frac{1}{D_1D_3}\\
&- i(1+\frac{s_2-m_K^2}{m_1^2-im_1\Gamma_1})\int\frac{d^4q}{(2\pi)^4}\frac{1}{D_1D_2}+i\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2D_1}-i\frac{m_K^2-s_1}{m_1^2-im_1\Gamma_1}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2D_3}-i\frac{m_K^2-s_2}{m_1^2-im_1\Gamma_1}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2D_2}\\
&+ i\int\frac{d^4q}{(2\pi)^4}\frac{1}{D_2D_3}
\end{aligned}$$

where $D_1=q^2-m_1^2+im_1\Gamma_1,\ D_2=(q-p_2)^2-m_2^2,\ D_3=(p_1-q)^2-m_3^2$ and $s_2=p_2^2=m_\pi^2$.

K* width effects are treated by the G.'t Hooft and M. Veltman formulation with complex mass [G.'t Hooft and M. Veltman, Nucl. Phys. B **153**, 365 (1979)]. To see whether the amplitude is sensitive to Γ as it varies a small value in the vicinity around its physical value, we can calculate the logarithmic derivative of $\Im M$,



4. Observables sensitive to the underlying dynamics

- **Radiative decay patterns are out of intuition**

Immediate crucial questions:

i) If $\eta(1440)$ is assigned as the $(s\bar{s})$ partner of $\eta(1295)$, can we understand that $\eta(1440) \to \phi\ (s\bar{s})\ \gamma$ is much smaller than $\eta(1440) \to \rho^0\ (n\bar{n})\ \gamma$?

$$\begin{aligned} \eta(1295) &= \cos\alpha n\bar{n} - \sin\alpha s\bar{s} \\ \eta(1440) &= \sin\alpha n\bar{n} + \cos\alpha s\bar{s} \end{aligned}$$

ii) Why $J/\psi \to \gamma\eta(1440)$ is so much stronger than $J/\psi \to \gamma\eta(1295)$?

Particle Data Group 2012:

$BR(J/\psi \to \gamma\eta(1295))/BR(J/\psi \to \gamma\eta(1440)) \leq 0.1$

Answer to question (i):

By assigning $\eta(1295)$ and $\eta(1440)$ as the first radial excitation of η and η', we can organize them as the following mixtures between $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$:

$$\begin{aligned} \eta(1295) &= \cos\alpha\, n\bar{n} - \sin\alpha\, s\bar{s} \\ \eta(1440) &= \sin\alpha\, n\bar{n} + \cos\alpha\, s\bar{s} \ , \end{aligned} \tag{1}$$

where α is the mixing angle.

In the J/ψ radiative decays, it is a good approximation that the photon is radiated by the charm (anti-)quark, and the light $q\bar{q}$ of 0^{-+} is produced by the gluon radiation. By defining the production strength for the $q\bar{q}$ of 0^{-+} as the following:

$$g_0 \equiv \langle q\bar{q}|\hat{H}|J/\psi, \gamma\rangle \ , \tag{2}$$

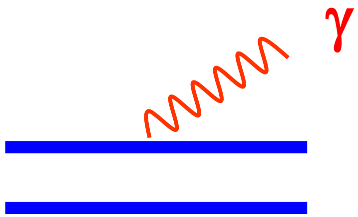
one can express the production amplitudes for $\eta(1295)$ and $\eta(1440)$ as

$$\begin{aligned} \mathcal{M}(\eta(1295)) &= (\sqrt{2}\cos\alpha - R\sin\alpha)g_0 \ , \\ \mathcal{M}(\eta(1440)) &= (\sqrt{2}\sin\alpha + R\cos\alpha)g_0 \ , \end{aligned} \tag{3}$$

$$\frac{B.R.(J/\psi \to \gamma\eta(1440))}{B.R.(J/\psi \to \gamma\eta(1295))} = \left(\frac{q_{\eta(1440)}}{q_{\eta(1295)}}\right)^3 \left(\frac{\sqrt{2}\sin\alpha + R\cos\alpha}{\sqrt{2}\cos\alpha - R\sin\alpha}\right)^2 \simeq 10$$

with $R \doteq 1$, one has $\alpha \simeq 38°$

Answer to question (ii):



Magnetic dipole transition operator:

$$\hat{H}_{em} \equiv \langle \phi_A \chi_S | \sum_i^2 e_i \mu_i \overrightarrow{\sigma}_i \cdot \overrightarrow{\epsilon}_\gamma | \phi_S \chi_A \rangle$$

The flavor and spin wavefunction for the pseudoscalar:

$$\begin{aligned} \phi_S(s\bar{s}) &\equiv (s\bar{s} + \bar{s}s)/\sqrt{2} \,, \\ \phi_S(n\bar{n}) &\equiv (n\bar{n} + \bar{n}n)/\sqrt{2} \,, \\ \chi_A &\equiv (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2} \end{aligned}$$

The flavor and spin wavefunction for the vector:

$$\begin{aligned} \phi_A(\phi) &\equiv (s\bar{s} - \bar{s}s)/\sqrt{2} \,, \\ \phi_A(\rho^0) &\equiv ((u\bar{u} - \bar{u}u) - (d\bar{d} - \bar{d}d))/2 \,, \\ \phi_A(\omega) &\equiv ((u\bar{u} - \bar{u}u) + (d\bar{d} - \bar{d}d))/2 \,, \\ \chi_S &\equiv \uparrow\uparrow,\ \downarrow\downarrow,\ (\uparrow\downarrow + \downarrow\uparrow)/\sqrt{2} \,, \end{aligned}$$

The M1 transition amplitudes for η(1440) → γ V:

$$h_{\phi\gamma} = -\frac{e}{3m_s}\cos\alpha \ ,$$
$$h_{\rho^0\gamma} = \frac{e}{2m_q}\sin\alpha \ ,$$
$$h_{\omega\gamma} = \frac{e}{6m_q}\sin\alpha \ ,$$

where $m_q = m_u = m_d$ and $m_s \simeq 5m_q/3$.

$$\Rightarrow \quad B.R.(\gamma\phi) : B.R.(\gamma\rho^0) : B.R.(\gamma\omega) \simeq \frac{\cos^2\alpha}{25} : \frac{\sin^2\alpha}{4} : \frac{\sin^2\alpha}{36} \ .$$
$$\simeq 1 : 3.8 : 0.42.$$

with $\alpha \simeq 38°$

So far, there is no obvious difficulty for having only one η(1440) to cope with the existing observables!

Study of $\eta(1475)$ and $X(1835)$ in radiative J/ψ decays to $\gamma\phi$

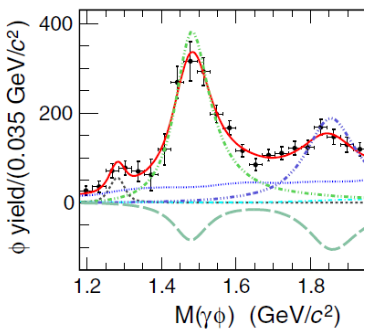
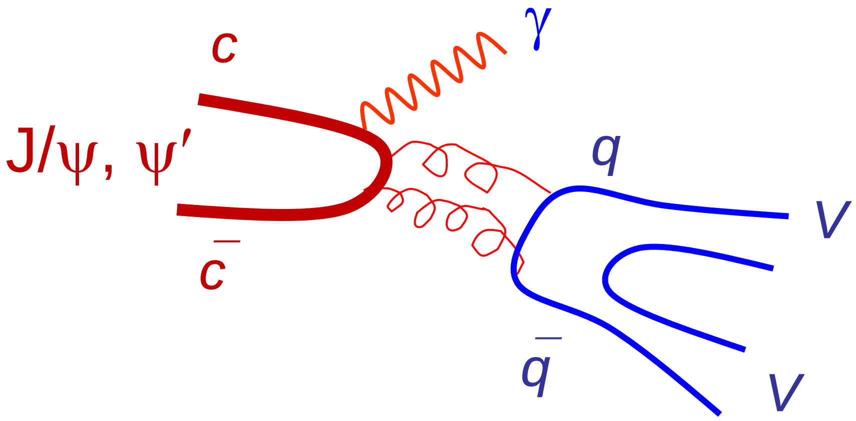


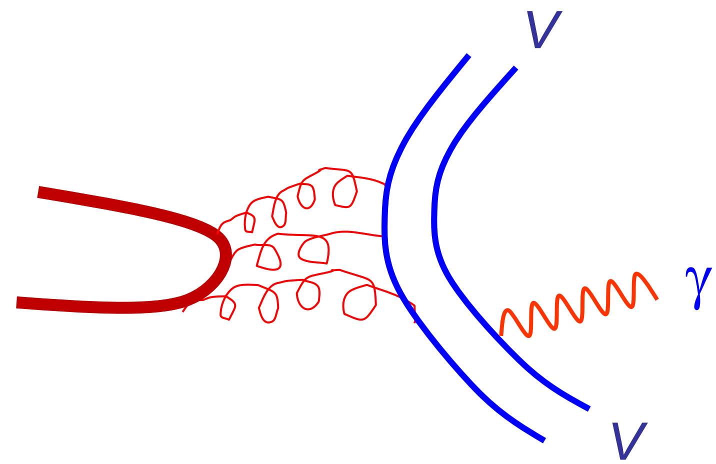
FIG. 3. Fits to the $M_{\gamma\phi}$
event) for the case of
interference. The dots w
solid, (green) dashed dou
(black) dashed, (blue) do
results, the structures arou
grounds and interference

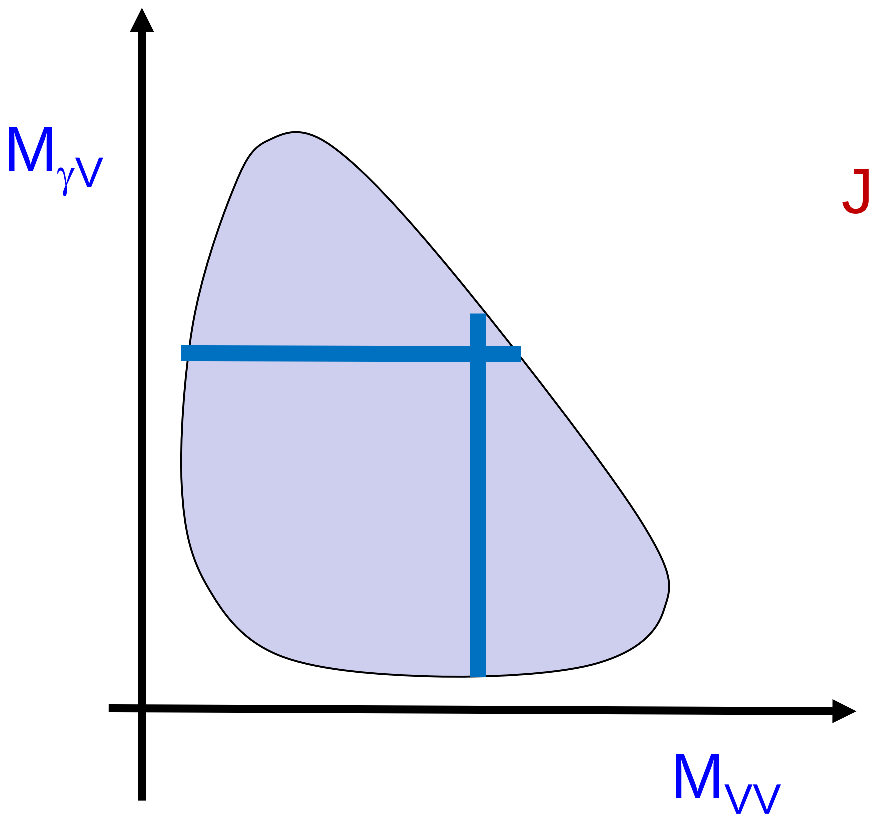
The partial width ratio of $(\Gamma_{\eta(1405/1475)\to\gamma\rho} : \Gamma_{\eta(1405/1475)\to\gamma\phi})$ is calculated to be (11.10 ± 3.50): 1 for the case of destructive interference and (7.53 ± 2.49): 1 for constructive interference, where the branching fraction of $J/\psi\to\gamma\eta(1405/1475)\to\gamma\gamma\rho$ is taken from the BES measurement [3]. The ratio is slightly larger than the prediction of 3.8: 1 in Ref. [10] for the case of a single pseudoscalar state. On the other hand, if the $\eta(1405)$ and the $\eta(1475)$ are different states, the observation of the $\eta(1475)$ decaying into $\gamma\phi$ final state suggests that the $\eta(1475)$ contains a sizable $s\bar{s}$ component and, if so, should be the radial excitation of the η' [6]. The observation

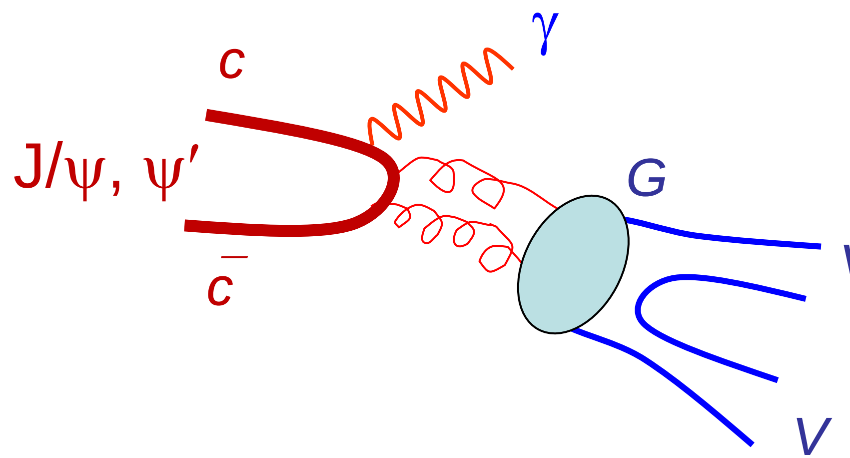
[6] L. Faddeev, A. J. Niemi, and U. Wiedner, Phys. Rev. D **70**, 114033 (2004).

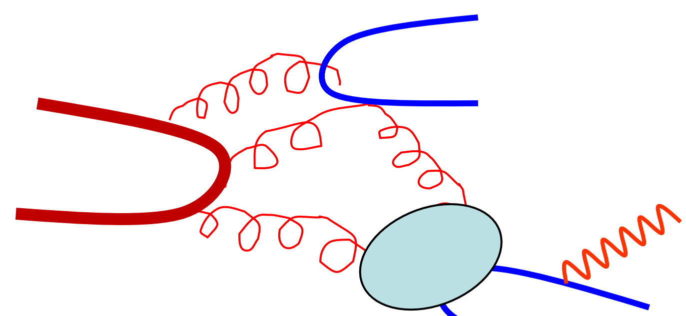
[10] X. G. Wu, J. J. Wu, Q. Zhao, and B. S. Zou, Phys. Rev. D **87**, 014023 (2013).

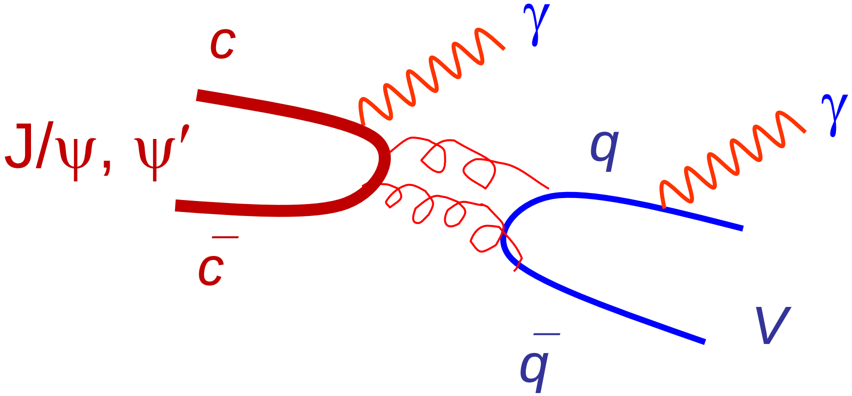

c
γ
J/ψ, ψ′
q
V
c̄
q̄
V

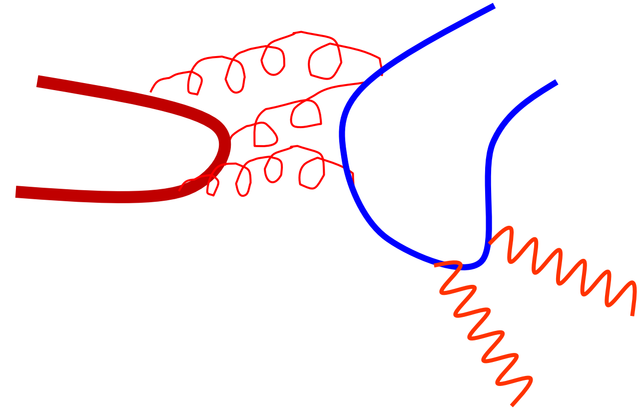

V
γ
V

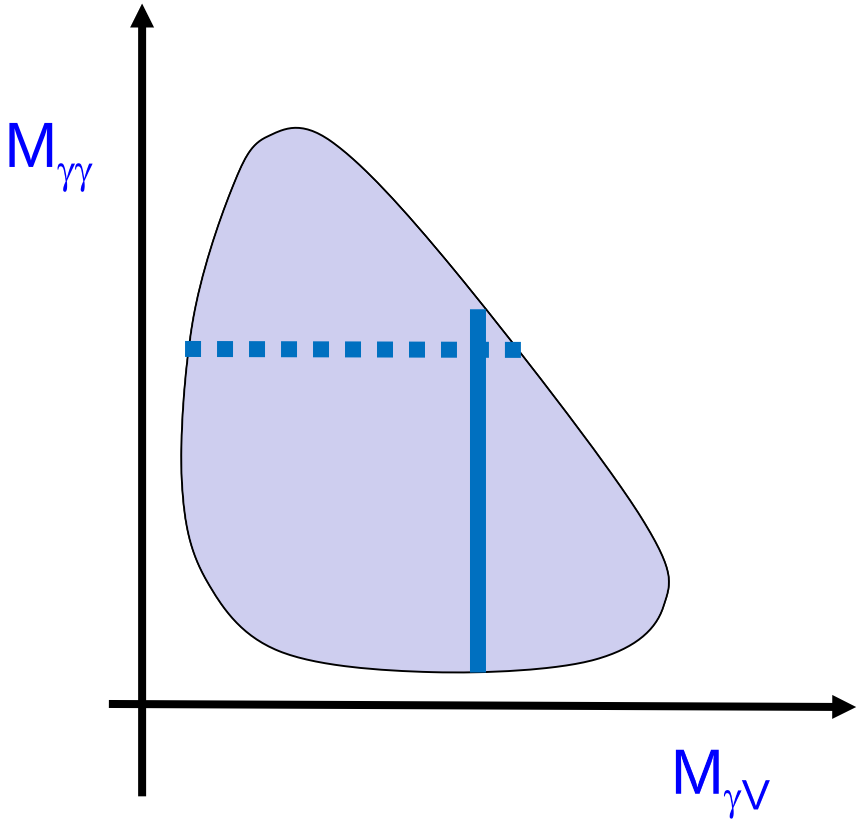

M$_{\gamma V}$
M$_{VV}$

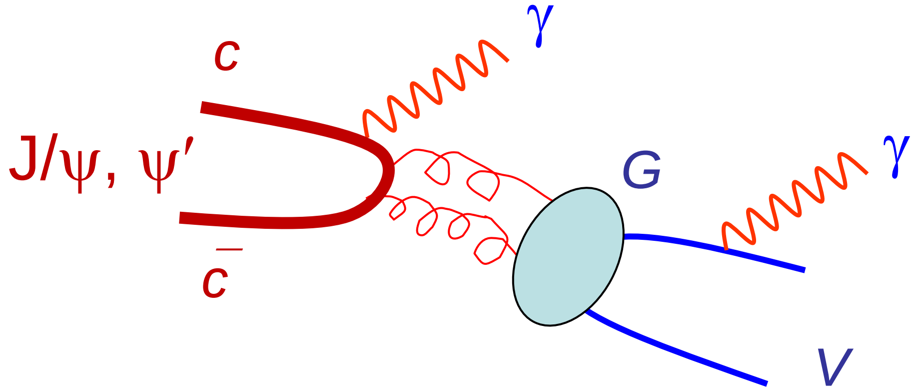

γ
c
J/ψ, ψ′
G
V
c̄
V

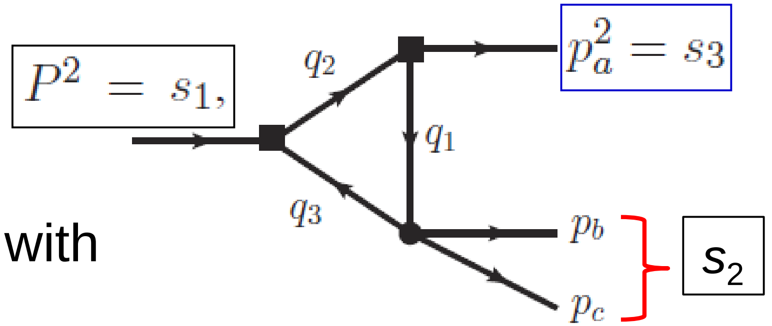



c
γ
J/ψ, ψ′
γ
q
$\bar{c}$
V
$\bar{q}$




$M_{\gamma\gamma}$
$M_{\gamma V}$


γ
c
γ
J/ψ, ψ′
G
$\bar{c}$
V

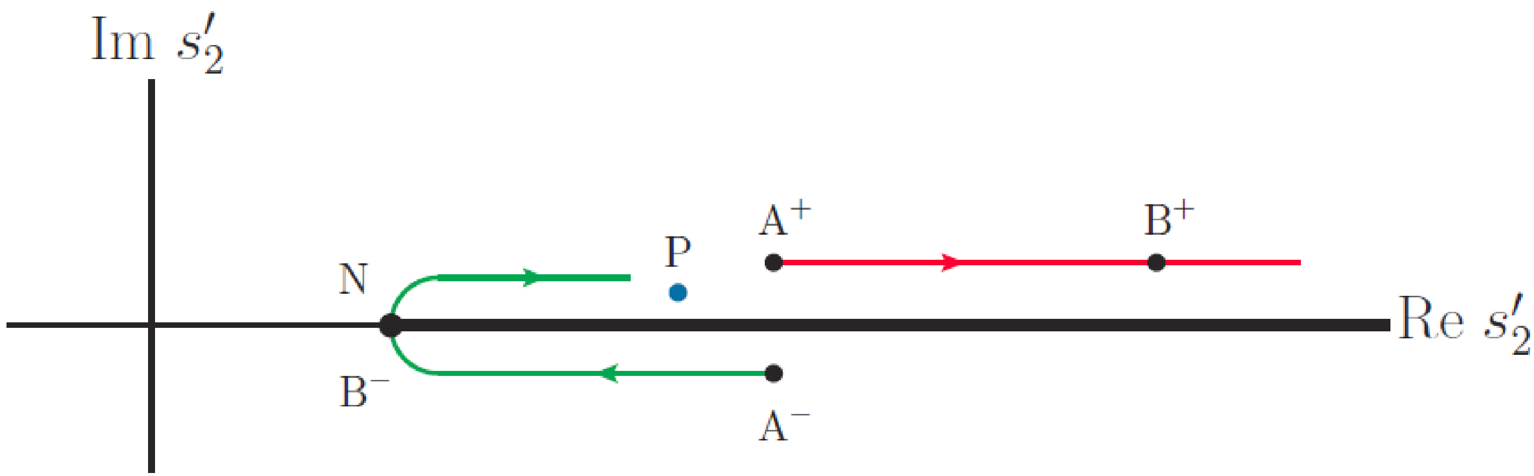


Trajectories of $s^{\pm}_2$ in the complex s'_2-plane with s_1 increasing from $s_{1N} \rightarrow \infty$:

A⁺ :(s_1=s_{1N}, s_2^+=s_{2C} +$i\varepsilon$) → **B⁺** :(s_1=s_{1C}, s_2^+=s_{2N}+m_3 $\lambda(s_3,m_1^2,m_2^2)/(m_1m_2)$+$i\varepsilon$)

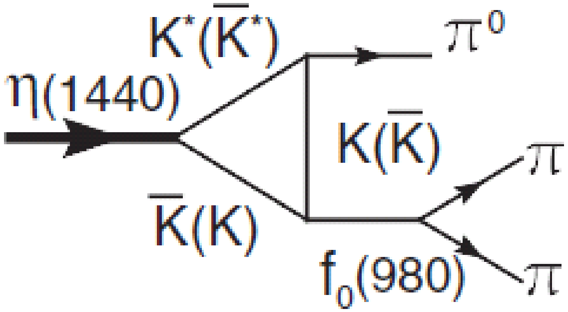
A⁻ :(s_1=s_{1N}, s_2^-=s_{2C} −$i\varepsilon$) → **B⁻** :(s_1=s_{1C}, s_2^-=s_{2N})

P : $s_2 + i\epsilon.$



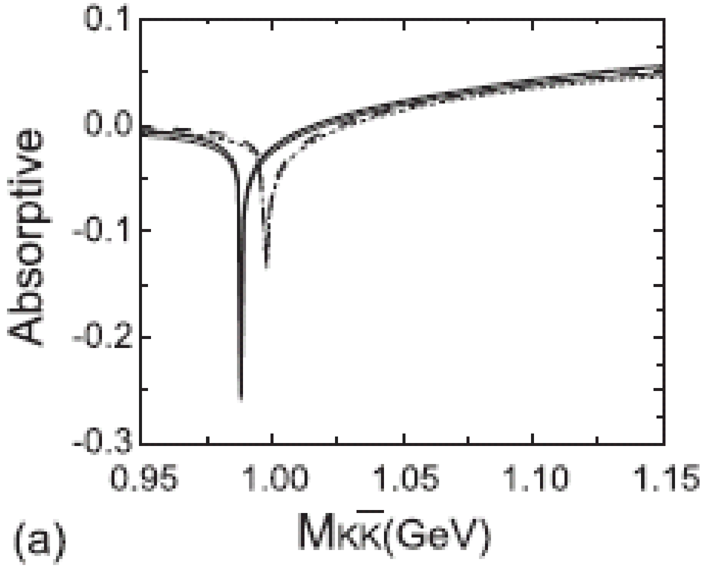
$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{\pi} \int_{(m_1+m_3)^2}^{\infty} \frac{ds'_2}{s'_2 - s_2 - i\epsilon} \sigma(s_1, s'_2, s_3)$$

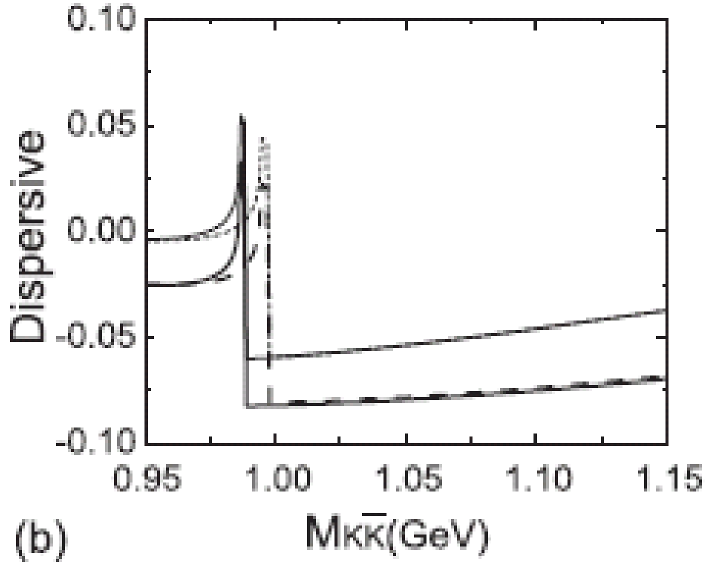
Triangle loop amplitudes:



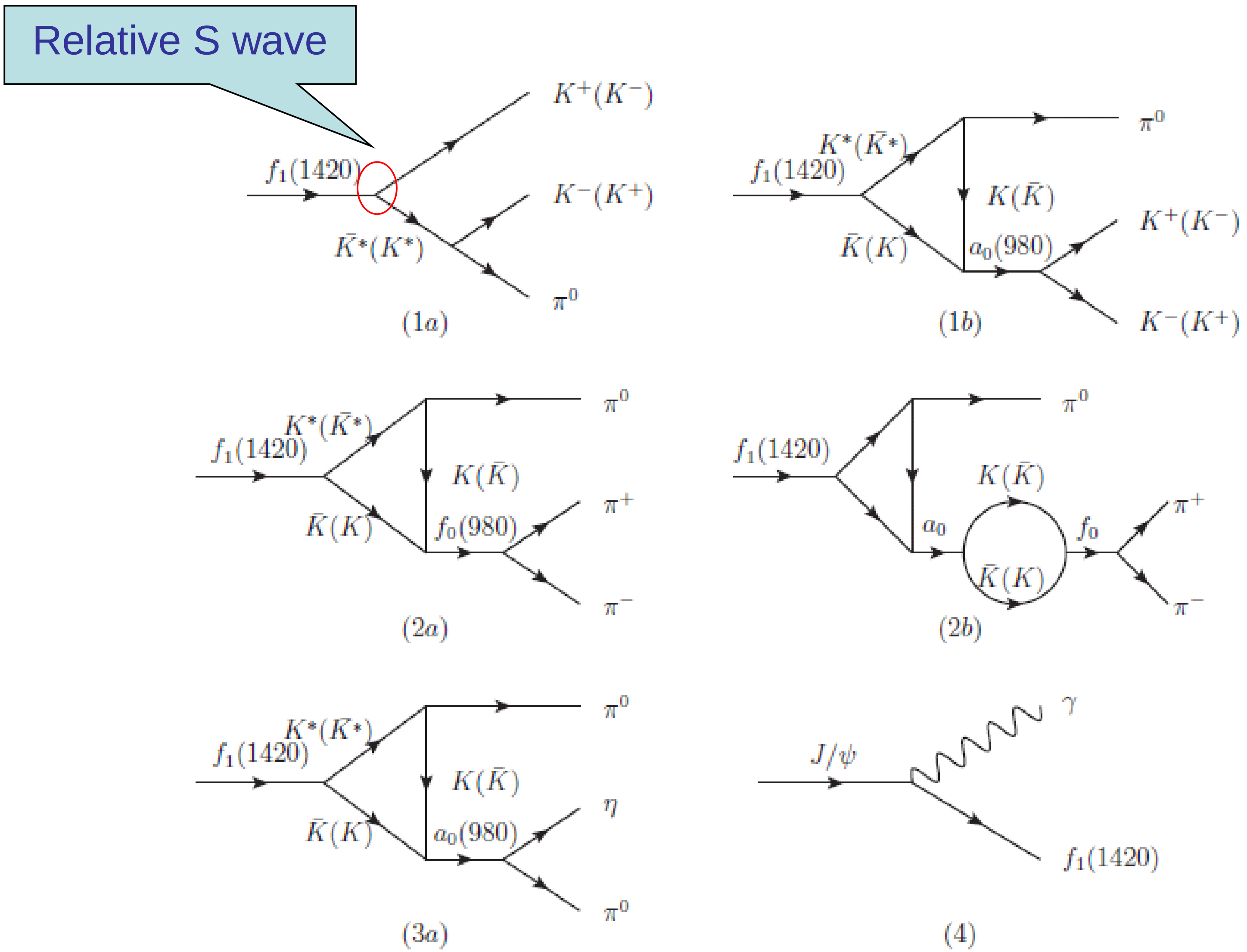
Absorptive amplitudes

Dispersive amplitudes

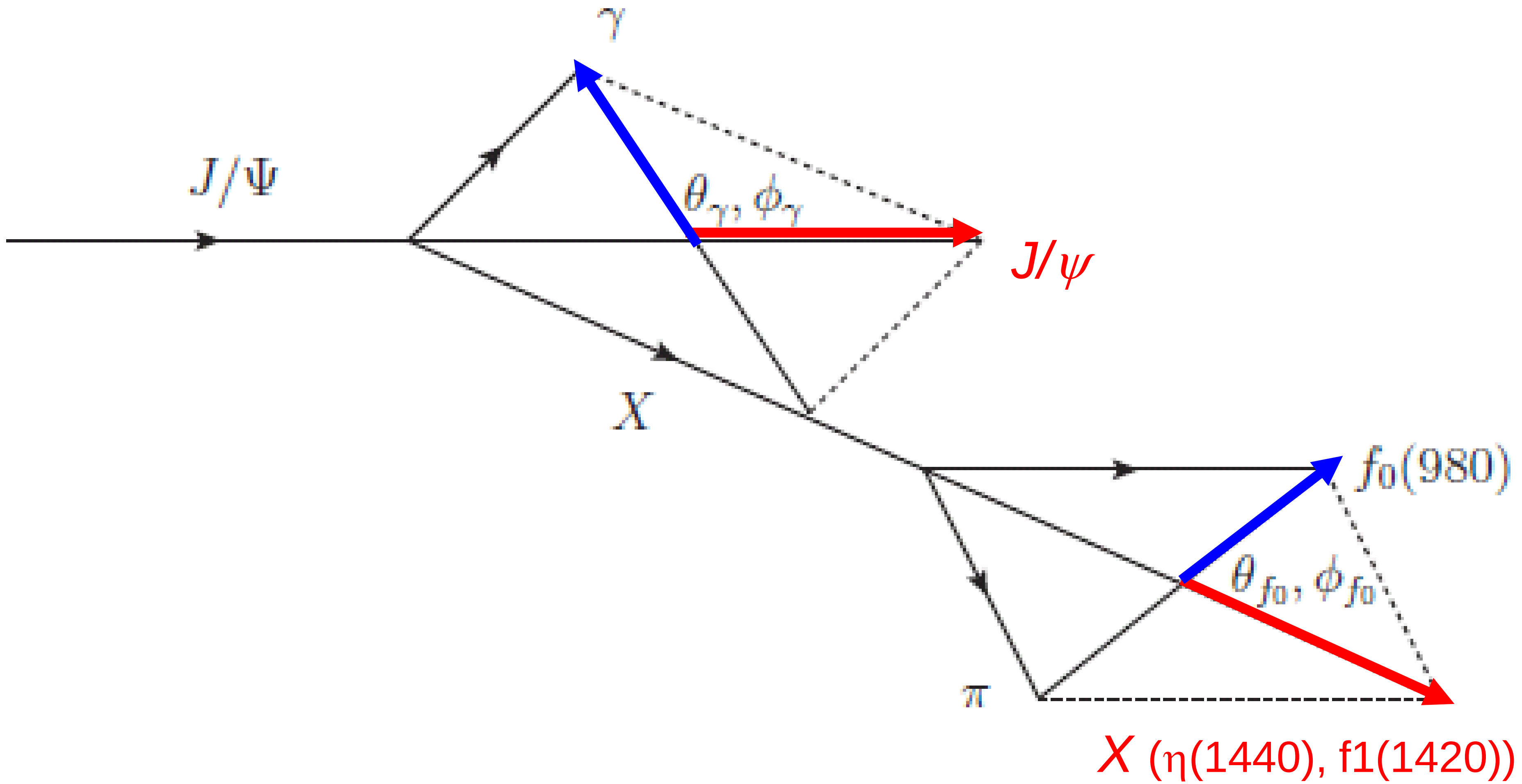


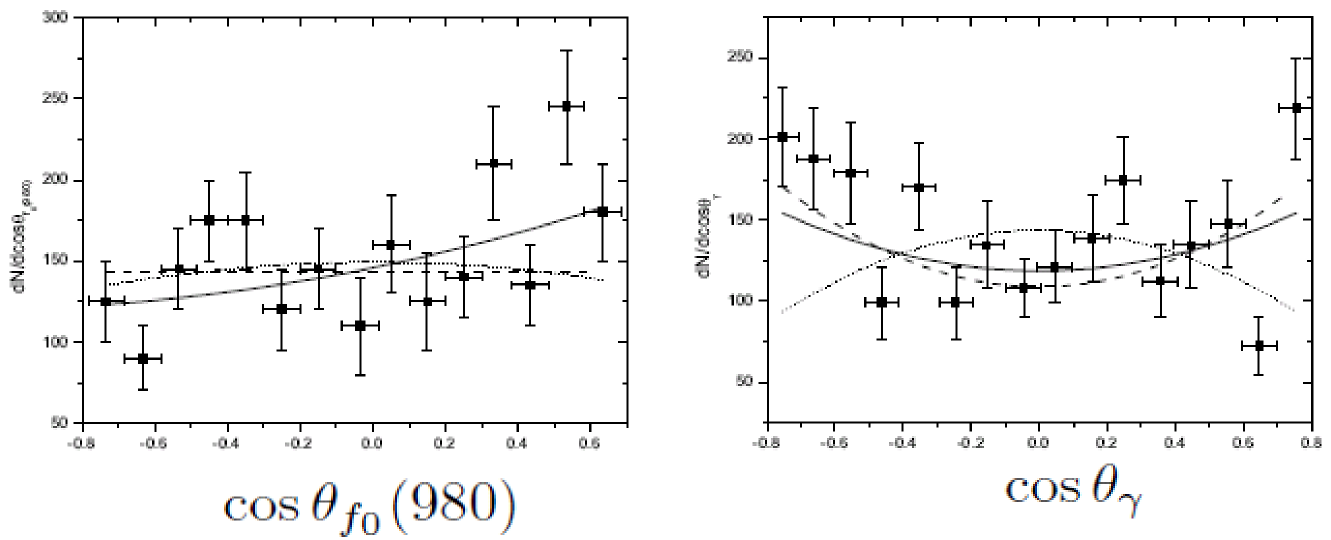


Contributions from $f_1(1420)$



Kinematics:

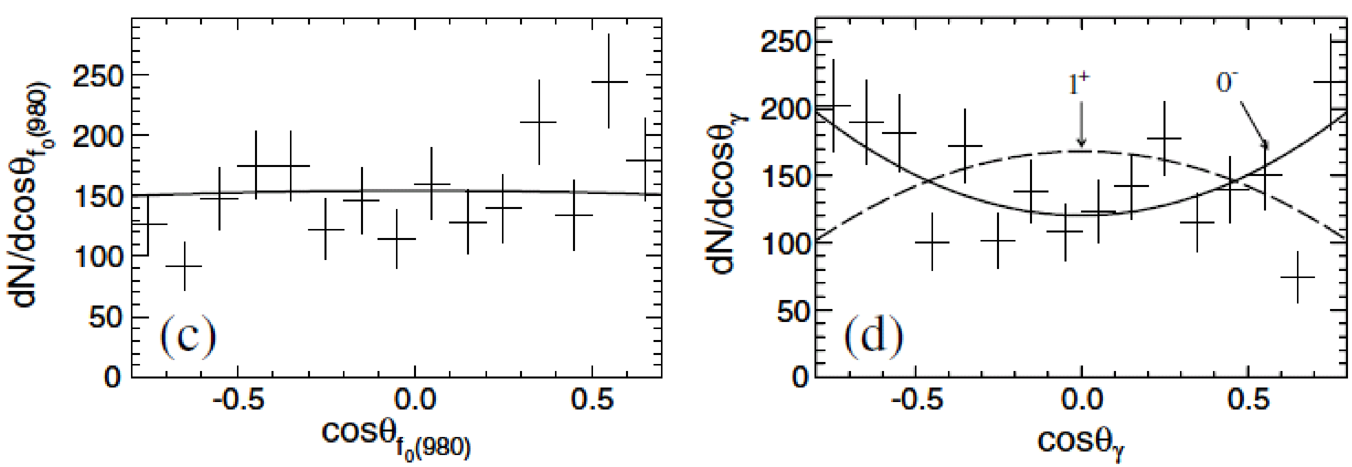




Dashed: eta(1440)
Dotted: f1(1420)
Solid: eta(1440) + f1

$$\chi^2/d.o.f = 38.3/14; \quad b_\gamma = 118.5 \pm 8.8,\ c = 0.538 \pm 0.312$$

$$\chi^2/d.o.f = 19.8/12; \quad b_{f_0} = 145.7 \pm 10.7,\ c_1 = 0.314 \pm 0.128,\ c_2 = 0.141 \pm 0.317$$



BESIII results:

immediate states	$\chi^2/d.o.f$ for $\cos\theta_\gamma$	$\chi^2/d.o.f$ for $\cos\theta_{f_0}$
$\eta(1440)$	40.2/15	26.8/14
$f_1(1420)$	59.0/15	26.4/13
$\eta(1440)$ and $f_1(1420)$	38.3/14	19.8/12