INDIANA UNIVERSITY
BLOOMINGTON

Jefferson Lab

Hadronic Decays of the Tau Lepton

Emilie Passemar*
Indiana University/Jefferson Laboratory

International Workshop on e^+e^- collisions from Phi to Psi
Budker Institute of Nuclear Physics & Novosibirsk State U.
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Outline

1. Introduction and Motivation

2. $\tau \rightarrow K\pi\nu_\tau$ and V_{us} determination

3. Lepton Flavour Violation: $\tau \rightarrow \pi\pi\mu$

4. Conclusion and outlook

1.1 Hadronic τ-decays

- τ lepton discovered in 1976 by M. Perl et al. at SLAC-LBL

PDG'14

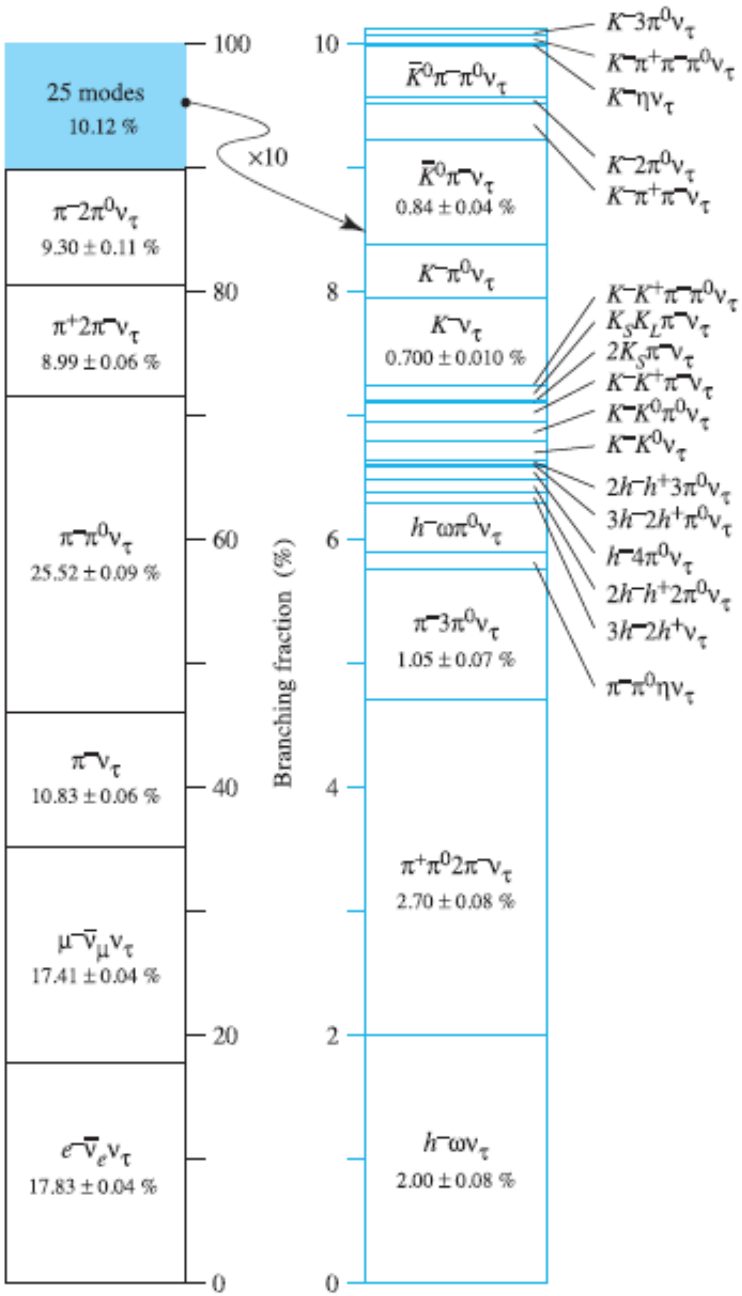
  - Mass : $\boxed{m_\tau = 1.77682(16)\ \text{GeV}}$
  - Lifetime : $\boxed{\tau_\tau = 2.096(10)\cdot 10^{-13}\ s}$

- The only lepton heavy enough to decay into hadrons : lots of semileptonic decays !

  ⇨ Very rich phenomenology

  Test of QCD and EW interactions

- For the tests:
  - Precise measurements needed
  - Hadronic uncertainties under control



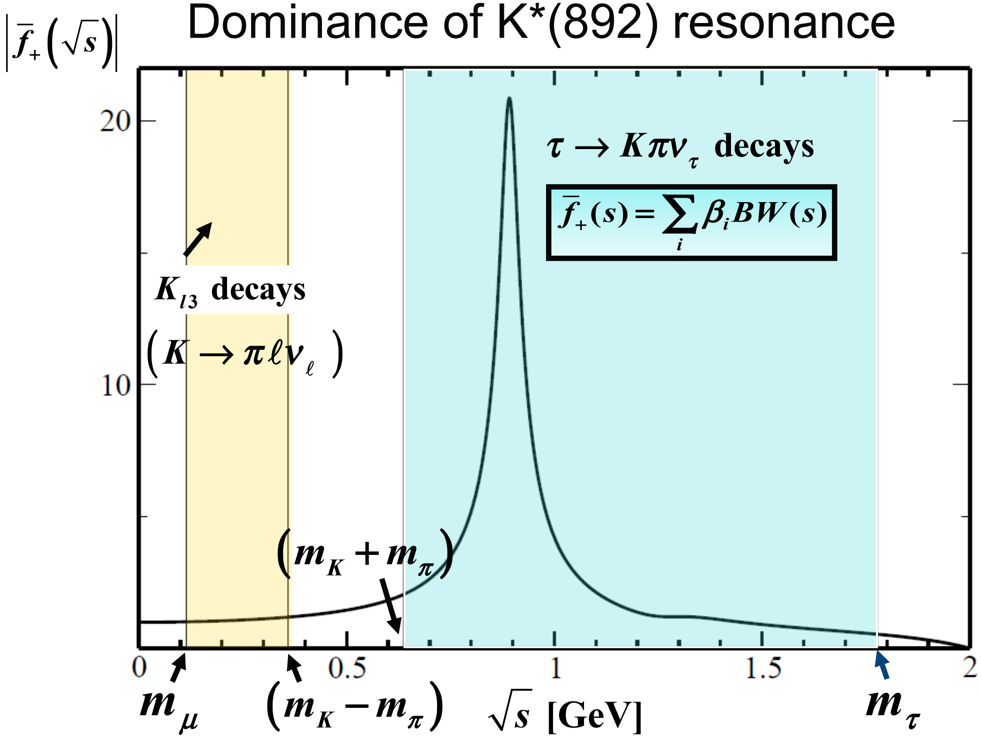
1.2 On the interest of using Dispersion Relations

- If E > 1 GeV: ChPT not valid anymore to describe dynamics of the process

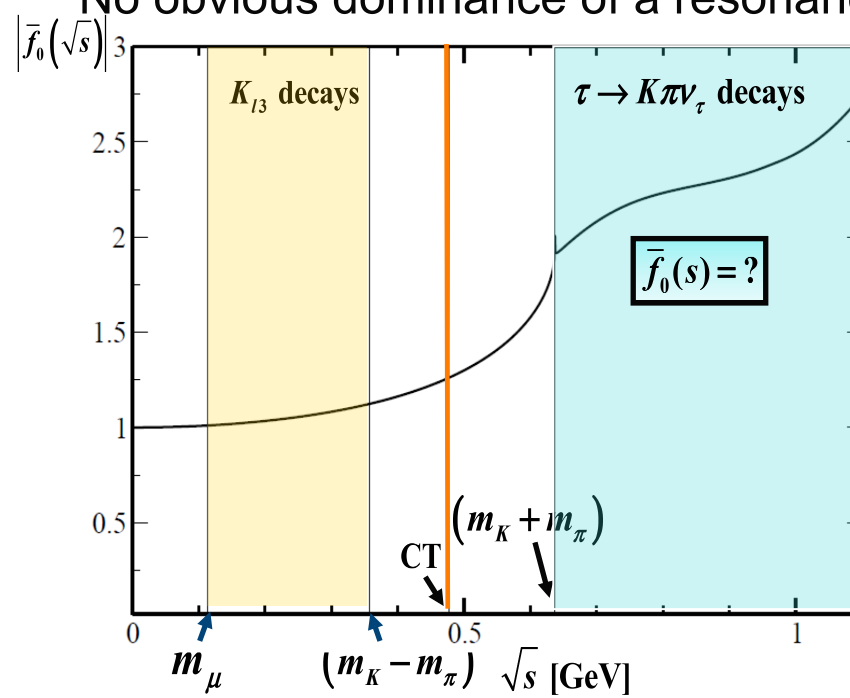
  ⇨ Resonances appear :

  - For ππ: *I=1*: ρ(770), ρ(1450), ρ(1700), …, *I=0*: "σ(~500)", f_0(980),…
  - For Kπ: *I=1*: K*(892), K*(1410), K*(1680), …, *I=0*: "κ(~800)", …

Kπ vector form factor:
Dominance of K*(892) resonance



Kπ scalar form factor:
No obvious dominance of a resonance



1.2 On the interest of using Dispersion Relations

- If E > 1 GeV: ChPT not valid anymore to describe dynamics of the process
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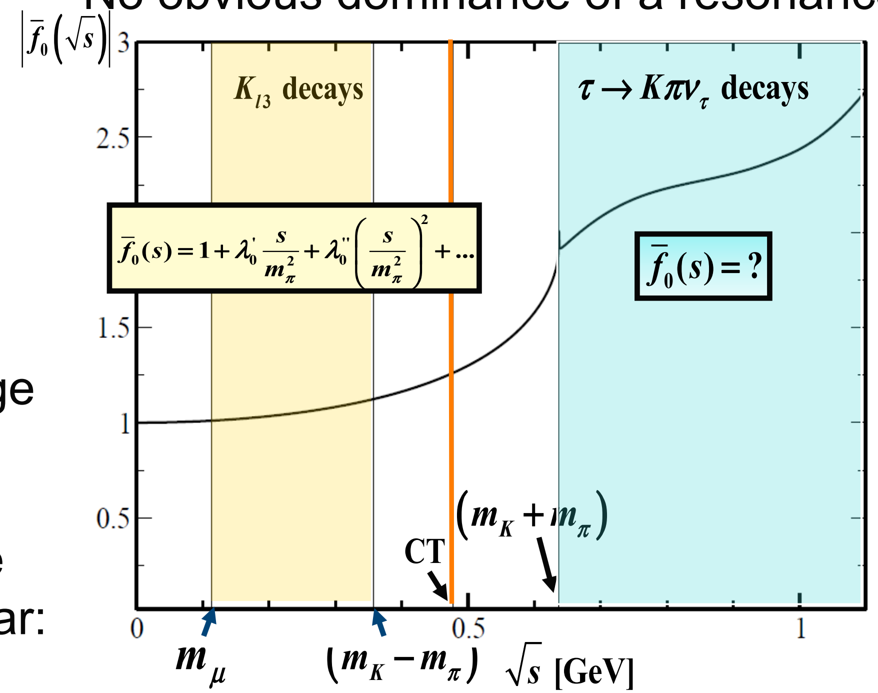
  - For ππ: *I=1*: ρ(770), ρ(1450), ρ(1700), …, *I=0*: "σ(~500)", f_0(980),…
  - For Kπ: *I=1*: K*(892), K*(1410), K*(1680), …, *I=0*: "κ(~800)", …

- With Dispersion Relation:
  - no need for making assumptions of a dominance of resonances
    - ⇨ directly given by the parametrization, phase shifts taken as inputs
  - Parametrization valid in a large range of energy:
    - ⇨ analyse several processes simultanously where the same quantity: FFs, amplitude appear: Ex: K_{l3} decays, $\tau \to K\pi\nu_\tau$

Kπ scalar form factor:
No obvious dominance of a resonance



2. $\tau \to K\pi\nu_\tau$ and V_{us} determination

2.1 $\tau \rightarrow K\pi\nu_\tau$

- Master formula for $\tau \rightarrow K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

$$\boxed{I_K^\tau = \int ds\, F\left(s, \overline{f}_+(s), \overline{f}_0(s)\right)}$$

Hadronic matrix element: Crossed channel from $K \rightarrow \pi l \nu_l$

$$\boxed{\langle \mathbf{K}\pi | \, \bar{\mathrm{s}}\gamma_\mu \mathrm{u} \, | 0 \rangle = \left[\left(p_K - p_\pi\right)_\mu + \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu \right] f_+(s) - \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu f_0(s)}$$

vector scalar

with $s = q^2 = (p_K + p_\pi)^2$, $\overline{f}_{0,+}(t) = \dfrac{f_{0,+}(t)}{f_+(0)}$

➡ Use a *dispersive parametrization* to combine with K_{l3} analysis

2.2 Dispersive representation for the form factors

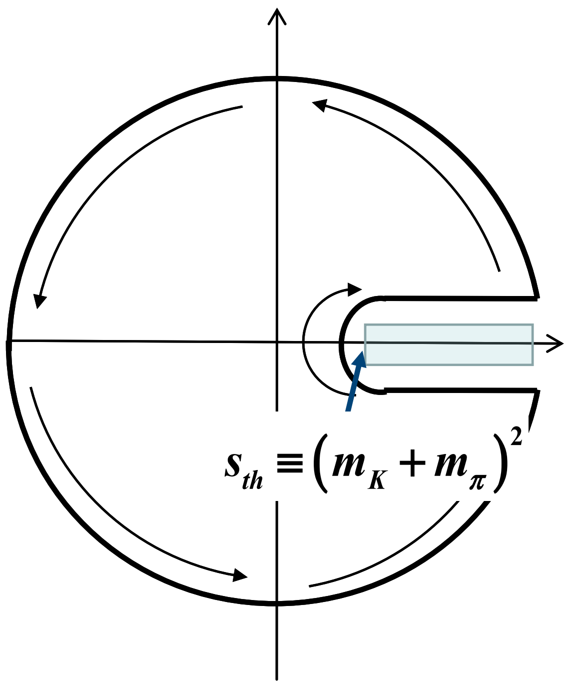
- Parametrization to analyse both K_{l3} and $\tau \to K\pi\nu_\tau$ ⇨ Use dispersion relations

Unitarity:

$$disc\left[\bar{f}_{0,+}(s)\right] \propto t_\ell^{I*}(s)\,\bar{f}_{0,+}(s)$$

- Omnès representation: ⇨

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$



$\phi_{+,0}(s)$: phase of the form factor

- $s < s_{in}$: $\phi_{+,0}(s) = \delta_{K\pi}(s)$

  Kπ scattering phase

- $s \geq s_{in}$: $\phi_{+,0}(s)$ unknown

  ⇨ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \quad \left(\bar{f}_{+,0}(s) \to 1/s\right)$

  Brodsky & Lepage

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied!

Determination of the Kπ FFs: Dispersive representation

Bernard, Boito, E.P.'11

- $\bar{f}_0(s)$: dispersion relation with 3 subtractions: 2 in s=0 and 1 in s = $(m_K+m_\pi)^2$

Callan-Treiman

$$\boxed{\bar{f}_0(s)=\exp\left[\frac{s}{\Delta_{K\pi}}\left(\ln C+\left(s-\Delta_{K\pi}\right)\left(\frac{\ln C}{\Delta_{K\pi}}-\frac{\lambda_0'}{m_\pi^2}\right)+\frac{\Delta_{K\pi}s\left(s-\Delta_{K\pi}\right)}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^2}\frac{\phi_0(s')}{\left(s'-\Delta_{K\pi}\right)\left(s'-s-i\varepsilon\right)}\right)\right]}$$

- $\bar{f}_+(s)$: dispersion relation with 3 subtractions in s=0

Boito, Escribano, Jamin'09,'10

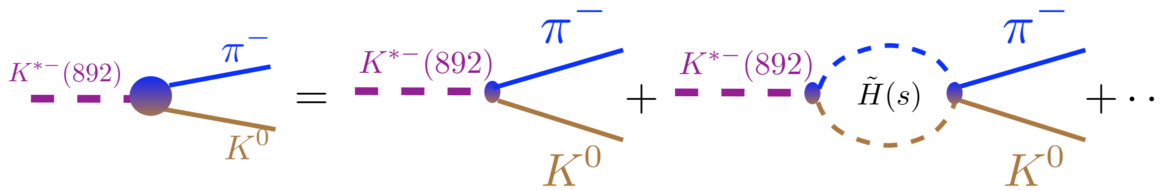
$$\boxed{\bar{f}_+(s)=\exp\left[\lambda_+'\frac{s}{m_\pi^2}+\frac{1}{2}\left(\lambda_+''-\lambda_+'^2\right)\left(\frac{s}{m_\pi^2}\right)^2+\frac{s^3}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^3}\frac{\phi_+(s')}{\left(s'-s-i\varepsilon\right)}\right]}$$

Extracted from a model including
2 resonances K*(892) and K*(1414)

Jamin, Pich, Portolés'08

Determination of the Kπ FFs: Dispersive representation

- Model for $\phi_+(s)$:



Boito, Escribano, Jamin'09,'10

$$\tilde{f}_+(s) = \left[\frac{m_{K^*}^2 - \kappa_{K^*}\left(\mathrm{Re}\,\tilde{H}_{K\pi}(0) + \mathrm{Re}\,\tilde{H}_{K\eta}(0)\right) + \beta s}{D\left(m_{K^*}, \Gamma_{K^*}\right)} - \frac{\beta s}{D\left(m_{K^{*\prime}}, \Gamma_{K^{*\prime}}\right)} \right]$$

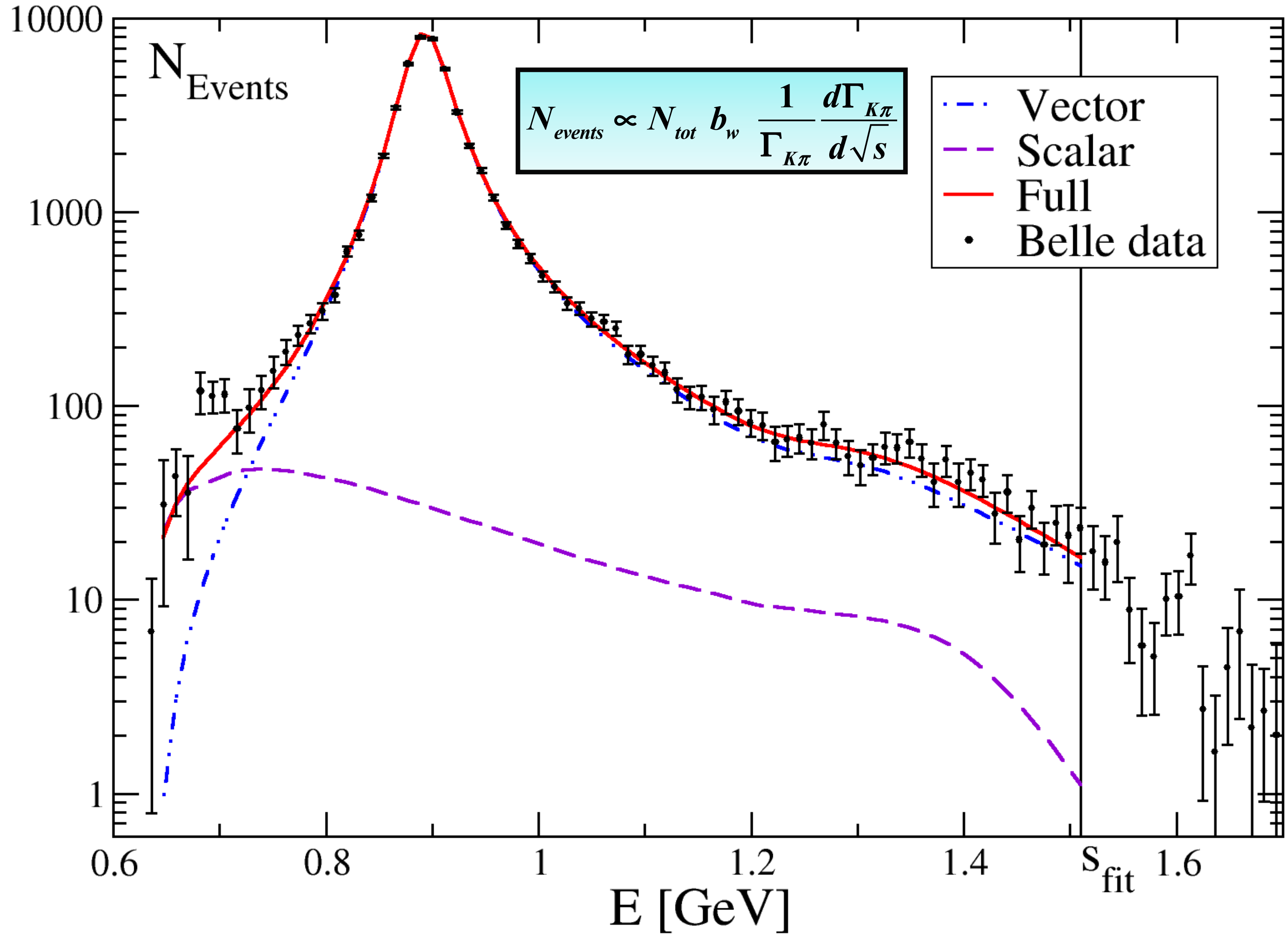
$K^*(892)$ $\qquad$ $K^*(1410)$

with $$D\left(m_n, \Gamma_n\right) = m_n^2 - s - \kappa_n \sum \mathrm{Re}\,\tilde{H} - i m_n \Gamma_n(s)$$

$$\Rightarrow \quad \tan \delta_{K\pi}^{P,1/2} = \frac{\mathrm{Im}\,\tilde{f}_+(s)}{\mathrm{Re}\,\tilde{f}_+(s)}$$

Fit to the $\tau \to K\pi\nu_\tau$ decay data + K_{l3} constraints

Bernard, Boito, E.P.'11

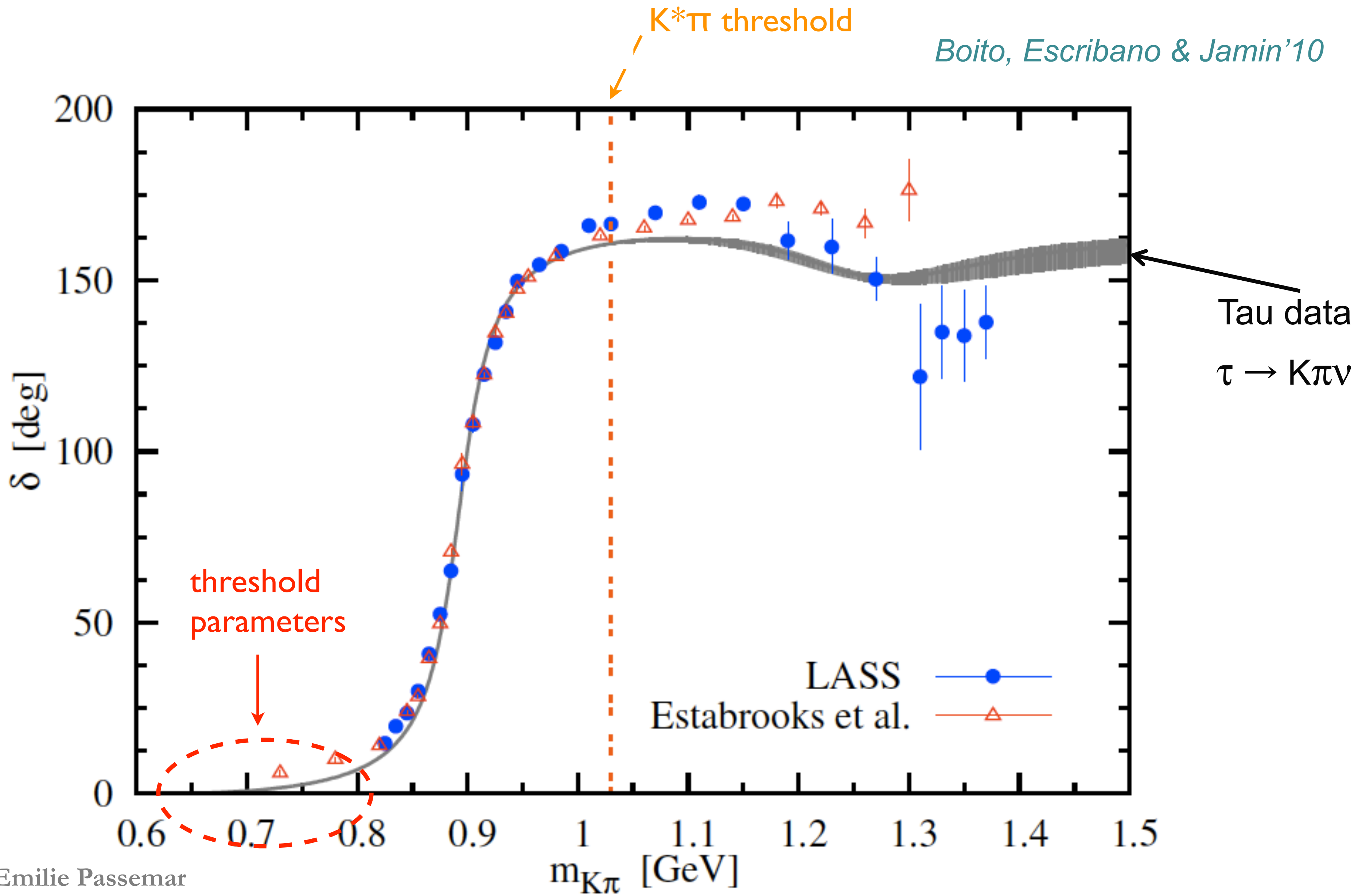


Fit to the $\tau \to K\pi\nu_\tau$ decay data + K_{l3} constraints

- Results:

	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$ Belle	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$ SuperB
$\ln C$	0.20193 ± 0.00892	0.20034 ± 0.00557
$\lambda'_0 \times 10^3$	13.139 ± 0.965	13.851 ± 0.592
m_{K^*} [MeV]	892.09 ± 0.22	892.01 ± 0.21
Γ_{K^*} [MeV]	46.287 ± 0.417	46.494 ± 0.436
$m_{K^{*\prime}}$ [MeV]	1292.5 ± 47.2	1259.8 ± 27.2
$\Gamma_{K^{*\prime}}$ [MeV]	171.64 ± 234.65	205.41 ± 10.27
β	-0.0204 ± 0.0289	-0.0350 ± 0.0229
$\lambda'_+ \times 10^3$	25.714 ± 0.332	25.655 ± 0.268
$\lambda''_+ \times 10^3$	1.1988 ± 0.0313	1.2176 ± 0.0089
$\chi^2/d.o.f$	$59.7/67$	$56.5/67$
I_K^τ	0.7655 ± 0.0416	0.7857 ± 0.0105
$f_+(0)V_{us}$	0.2134 ± 0.0061	0.21103 ± 0.0037

Kπ phase shift



2.3 Extraction of V$_{us}$

- Decay rate master formula

Antonelli, Cirigliano, Lusiani, E.P.'13

$$\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{EM}^{K\tau} + \cancel{\tilde{\delta}_{SU(2)}^{K\pi}}\right)^2$$

$$BR\left(\tau \to \overline{K^0}\pi^-\nu_\tau\right) = (0.416 \pm 0.008)\%$$

Belle'14

$$S_{ew} = 1.0201$$

Marciano & Sirlin'88, Braaten & Li'90, Erler'04

$$\delta_{EM}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\%$$

$$I_{K^0}^\tau = 0.50432 \pm 0.01721$$

$$f_+(0) = 0.9677(27)$$

FLAG'19
$N_f = 2+1$

$\Rightarrow$ $$f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\exp}$$ $\Rightarrow$ $$\left|V_{us}\right| = 0.2212 \pm 0.0027$$

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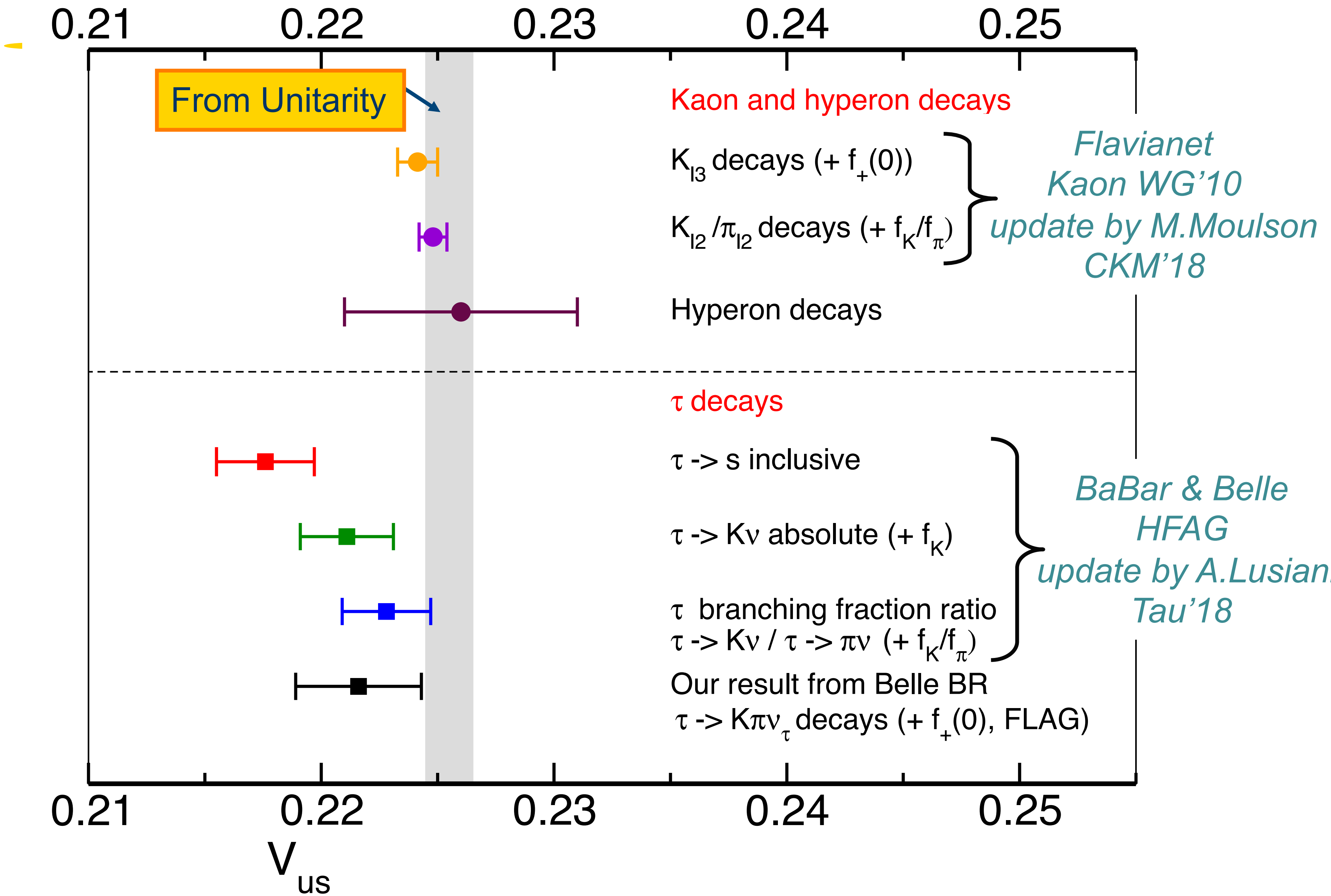
$$\Rightarrow \boxed{f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\mathrm{exp}}} \Rightarrow \boxed{\left|V_{us}\right| = 0.2212 \pm 0.0027}$$

- Result of fit to K_{l3} + $\tau \longrightarrow K\pi\nu_\tau$ and $K\pi$ scattering data including inelasticities in the dispersive FFs

$$\Rightarrow \boxed{f_+(0)\left|V_{us}\right| = 0.2163 \pm 0.0014}$$

Bernard'14


0.21
0.22
0.23
0.24
0.25
From Unitarity
Kaon and hyperon decays
K_{l3} decays (+ $f_+(0)$)
K_{l2}/π_{l2} decays (+ f_K/f_π)
Flavianet
Kaon WG'10
update by M.Moulson
CKM'18
Hyperon decays
τ decays
τ -> s inclusive
τ -> Kν absolute (+ f_K)
τ branching fraction ratio
τ -> Kν / τ -> $\pi\nu$ (+ f_K/f_π)
BaBar & Belle
HFAG
update by A.Lusiani
Tau'18
Our result from Belle BR
τ -> $K\pi\nu_\tau$ decays (+ $f_+(0)$, FLAG)
V_{us}

2.4 V_{us} using info on Kaon decays and $\tau \to K\pi\nu_\tau$

- Modes measured in the strange channel for $\tau \to s$:

HFAG'12

Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \overline{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$
$\Gamma_{40} = \pi^- \overline{K}^0 \pi^0 \nu_\tau$	$(0.3649 \pm 0.0108) \cdot 10^{-2}$
$\Gamma_{44} = \pi^- \overline{K}^0 \pi^0 \pi^0 \nu_\tau$	$(0.0269 \pm 0.0230) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^0 h^- h^- h^+ \nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^- \eta \nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

~70% of the decay modes crossed channels from Kaons!

2.4 V_{us} using info on Kaon decays and $\tau \to K\pi\nu_\tau$

Branching fraction	HFAG Winter 2012 fit		
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	➡	**(0.713 ± 0.003)%**
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	➡	**(0.471 ± 0.018)%**
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$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	➡	**(2.967 ± 0.060)%**

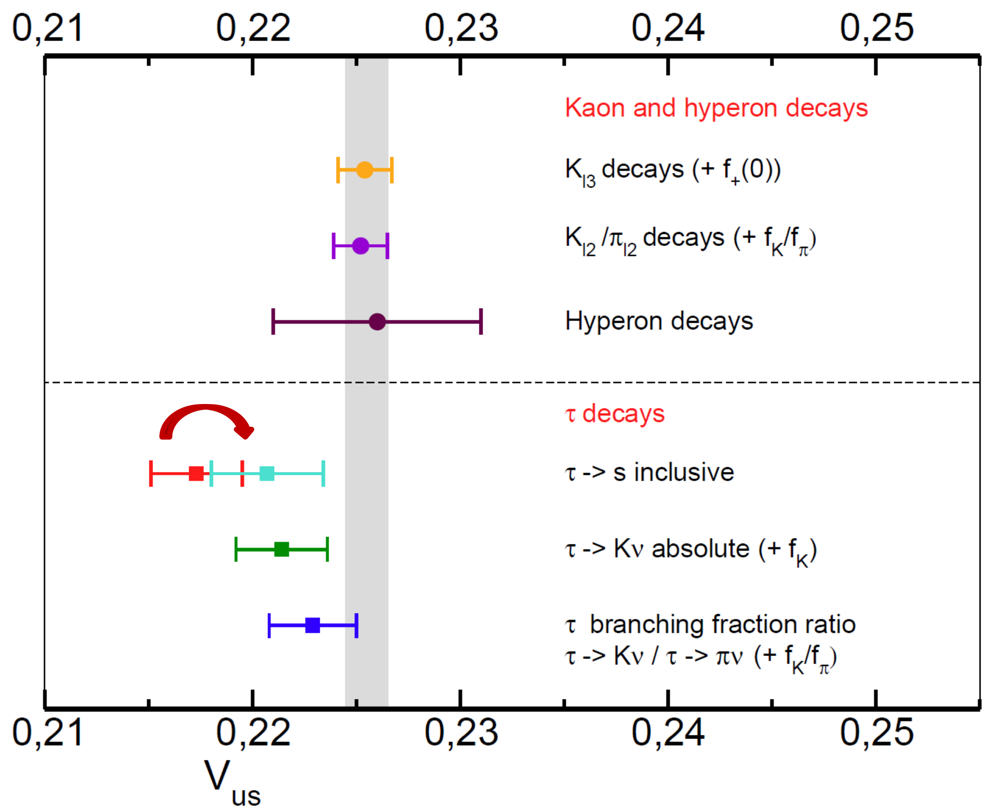
Antonelli, Cirigliano, Lusiani, E.P. '13

- Longstanding inconsistencies between τ and kaon decays in extraction of V_{us} seem to have been resolved !
  R. Hudspith, R. Lewis, K. Maltman, J. Zanotti'17

- Crucial input:
  $\tau \to K\pi\nu_\tau$ Br + spectrum

$$\left|V_{us}\right| = 0.2229 \pm 0.0022_{exp} \pm 0.0004_{theo}$$

➡ need new data



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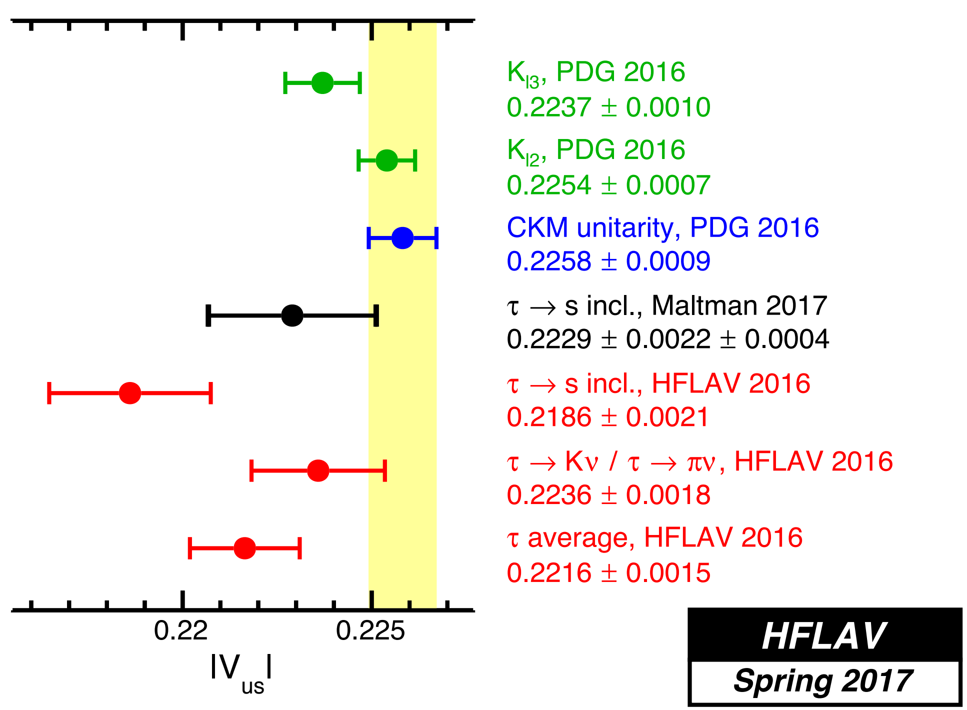
$\Rightarrow$ need new data



3. Lepton Flavour Violation: $\tau \rightarrow \pi\pi\mu$

Celis, Cirigliano, E.P.'14

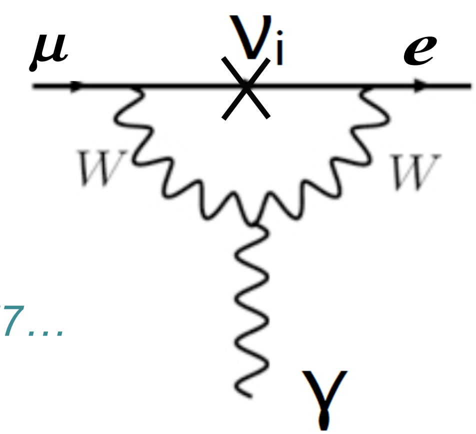
3.1 Introduction and Motivation

- Lepton Flavour Violation is an « accidental » symmetry of the SM (m_ν=0)

- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

e.g.: $\boxed{\mu \to e\gamma}$

$$\boxed{Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M_W^2}\right|^2 < 10^{-54}}$$



Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br(\tau \to \mu\gamma) < 10^{-40}\right]$$

- Extremely *clean probe of beyond SM physics*

2.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

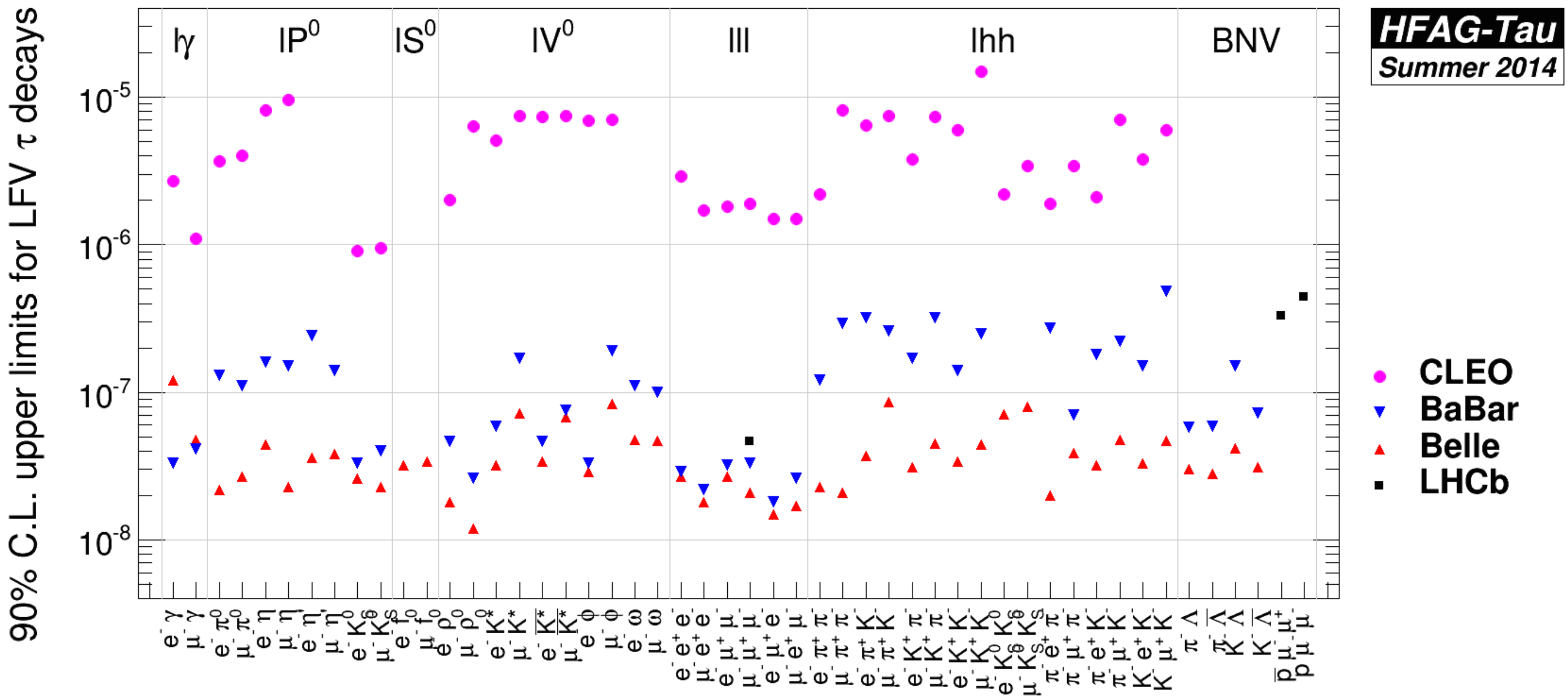
Talk by D. Hitlin @ CLFV2013

		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

3.2 CLFV processes: tau decays

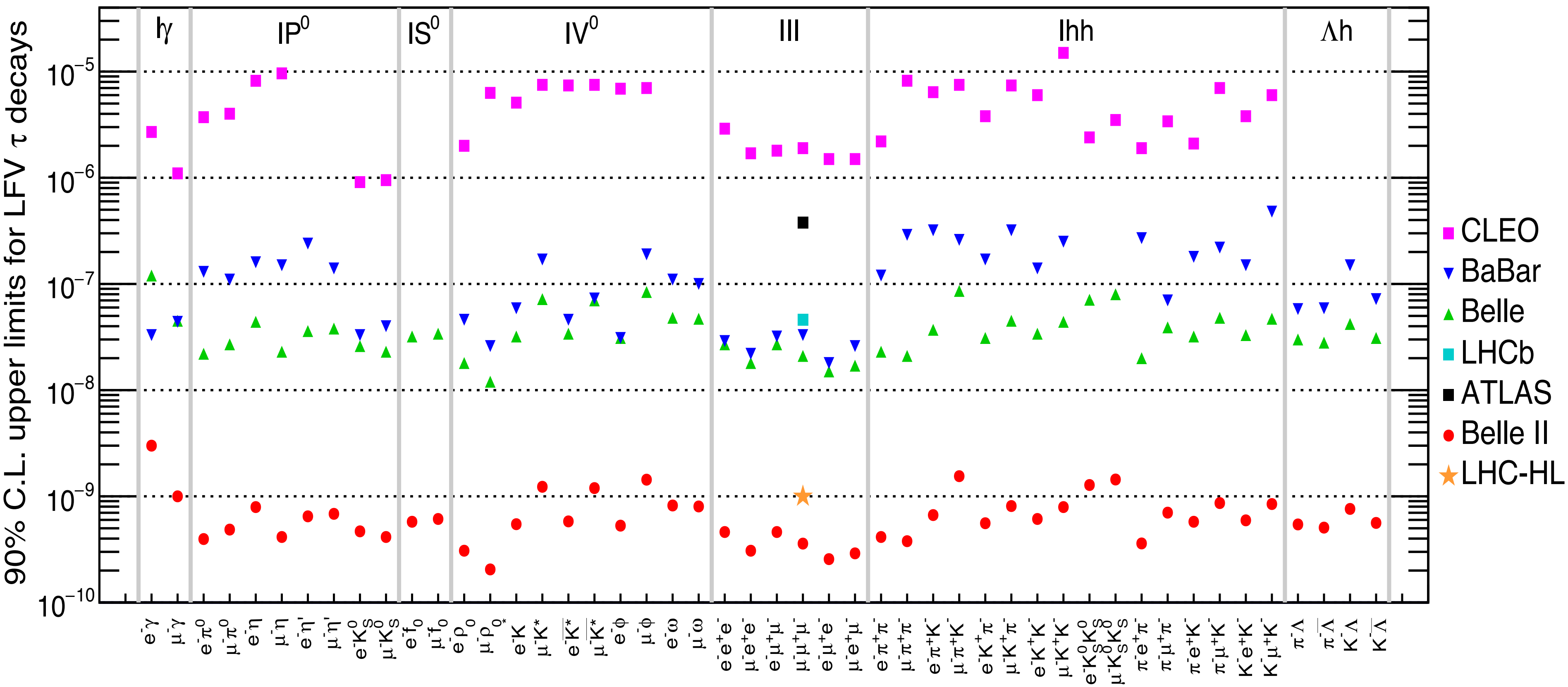
- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ $\leftarrow P,\ S,\ V,\ P\bar{P},\ldots$



- 48 LFV modes studied at Belle and BaBar

3.2 CLFV processes: tau decays

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \overline{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ↖ $P,\ S,\ V,\ P\overline{P},\ldots$



- Expected sensitivity 10^{-9} or better at *LHCb, Belle II, HL-LHC?*

3.3 Effective Field Theory approach

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger, Feldmann, Mannel, Turczyk'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R}\tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R}\tau\; \bar{q}\Gamma q}$
  - Lepton-gluon (Scalar, Pseudo-scalar): $\boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R}\tau\; G_{\mu\nu}^{a} G_a^{\mu\nu}}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R}\tau\; \bar{\mu}\, \Gamma P_{L,R}\mu}$

$\boxed{\Gamma \equiv 1\, , \gamma^\mu}$

- Each UV model generates a *specific pattern* of them

3.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau\to 3\mu$	$\tau\to\mu\gamma$	$\tau\to\mu\pi^+\pi^-$	$\tau\to\mu K\bar{K}$	$\tau\to\mu\pi$	$\tau\to\mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

3.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important ⇒ sensitive to large number of operators!

- But need reliable determinations of the hadronic part: form factors and *decay constants* (e.g. f_η, $f_{\eta'}$)

3.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O_V^q	–	–	✓ (I=1)	✓(I=0,1)	–	–
O_S^q	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O_A^q	–	–	–	–	✓ (I=1)	✓ (I=0)
O_P^q	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important ⇨ sensitive to large number of operators!

- But need reliable determinations of the hadronic part: form factors and *decay constants* (e.g. f_η, $f_{\eta'}$)

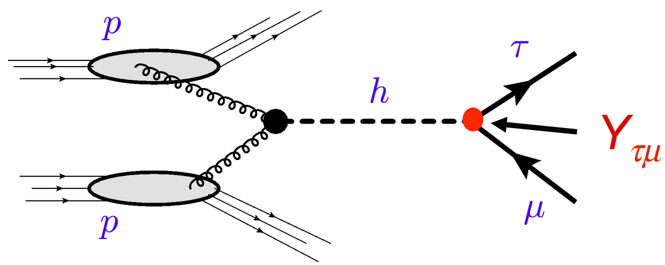
Ex: Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H} \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right)h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

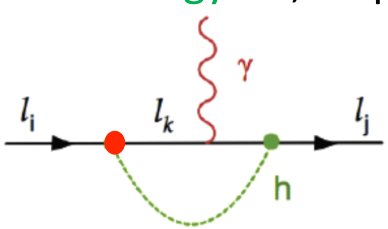
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

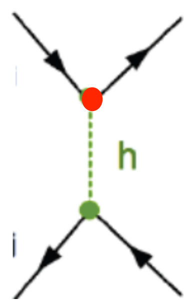
- High energy : LHC

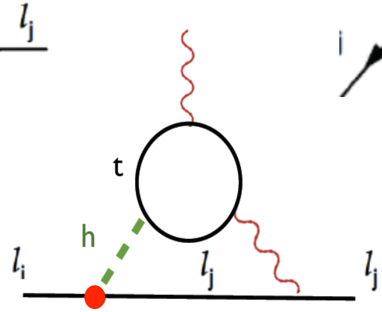


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







Ex: Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right)H^\dagger H} \Rightarrow \boxed{-Y_{ij}\left(\bar{f}_L^i f_R^j\right)h}$$

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{\mathrm{v}}\delta_{ij}$

Goudelis, Lebedev, Park'11
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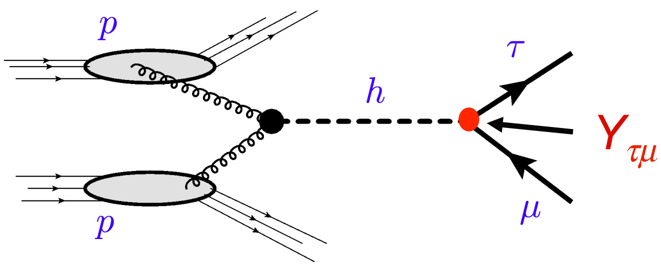
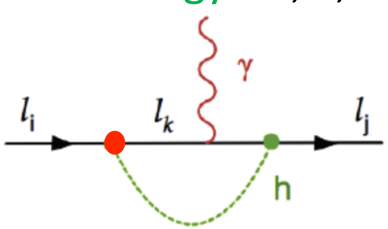
- High energy : LHC

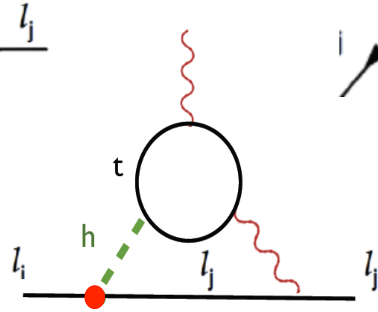


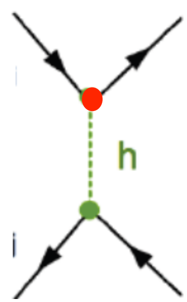
Hadronic part treated with perturbative QCD

Reverse the process

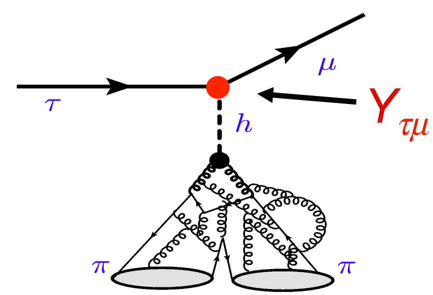
- Low energy : D, S, G operators







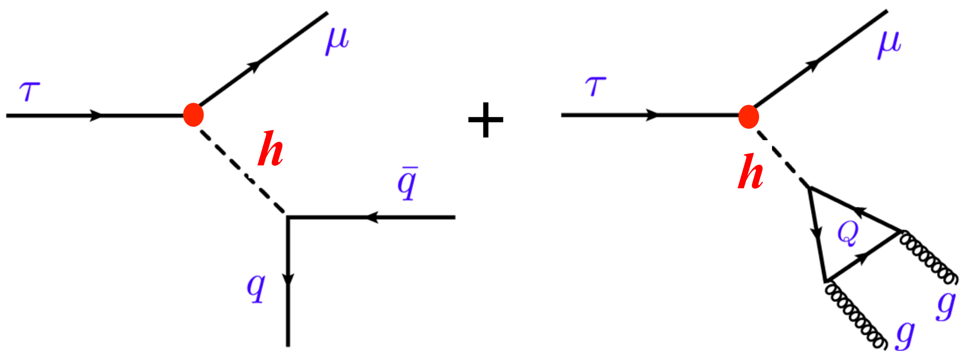
+

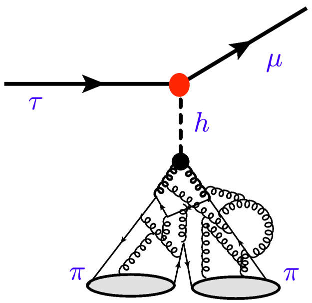


Hadronic part treated with non-perturbative QCD

3.5 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange





- Problem : Have the hadronic part under control, ChPT not valid at these energies!

  ⇨ Use *form factors* determined with *dispersion relations* matched at low energy to *CHPT*

  Daub, Dreiner, Hanhart, Kubis, Meissner'13

  Celis, Cirigliano, E.P.'14

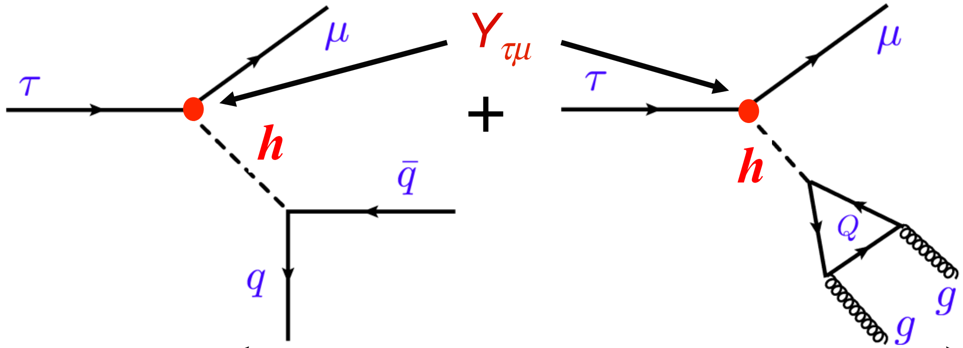
- Dispersion relations: based on unitarity, analyticity and crossing symmetry

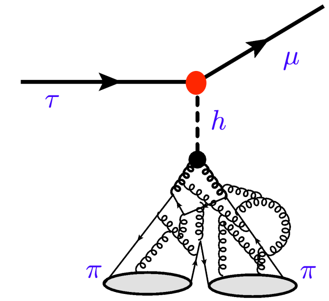
  ⇨ Take *all rescattering* effects into account

  $\pi\pi$ final state interactions important

3.5 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange





$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

$$\left\langle \pi^+\pi^- \left| m_u \bar{u}u + m_d \bar{d}d \right| 0 \right\rangle \equiv \Gamma_\pi(s) \qquad \left\langle \pi^+\pi^- \left| \theta^\mu_\mu \right| 0 \right\rangle \equiv \theta_\pi(s)$$

$$\left\langle \pi^+\pi^- \left| m_s \bar{s}s \right| 0 \right\rangle \equiv \Delta_\pi(s)$$

$$\boxed{\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q}$$

$$\boxed{\frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2\sqrt{s - 4m_\pi^2}}{256\pi^3 m_\tau^3} \frac{\left(|Y^h_{\tau\mu}|^2 + |Y^h_{\mu\tau}|^2\right)}{M_h^4 v^2} \left|\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)\right|^2}$$

$$f\left(y^h_q\right)$$

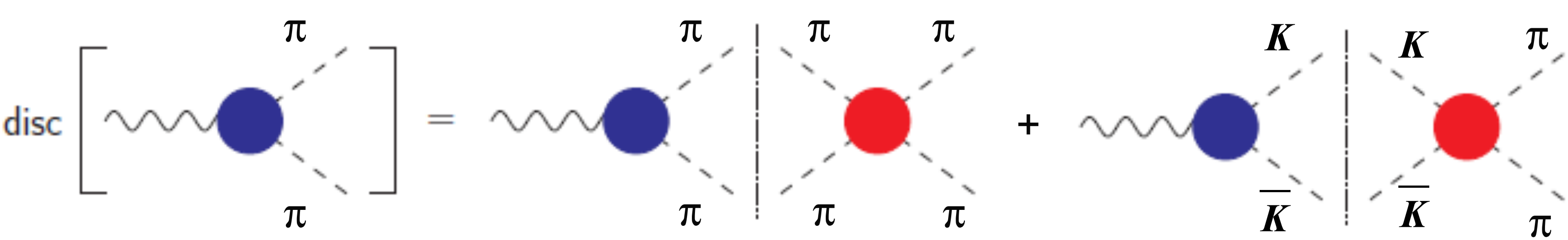
3.6 Unitarity

- Elastic approximation breaks down for the ππ S-wave at $K\bar{K}$ threshold due to the strong inelastic coupling involved in the region of *f_0(980)*

  Need to *solve a Coupled Channel* *Mushkhelishvili-Omnès* problem

Donoghue, Gasser, Leutwyler'90
Osset & Oller'98
Moussallam'99

- Unitarity the discontinuity of the form factor is known



$$\boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T_{nm}^*(s)\sigma_m(s)F_m(s)}$$

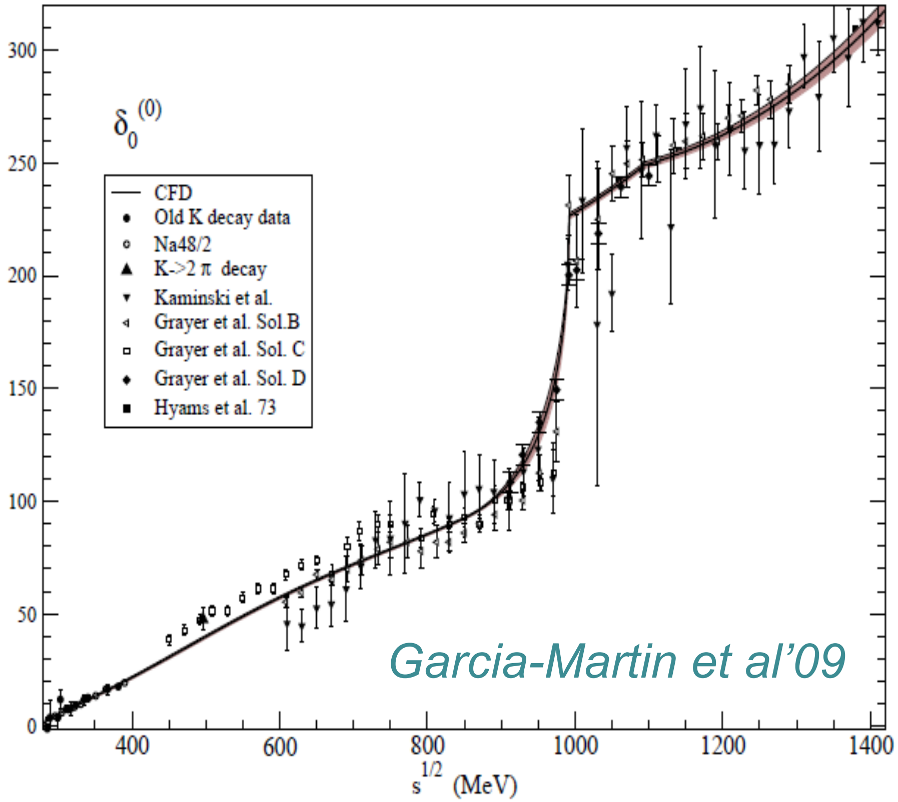
$$n = \pi\pi, K\bar{K}$$

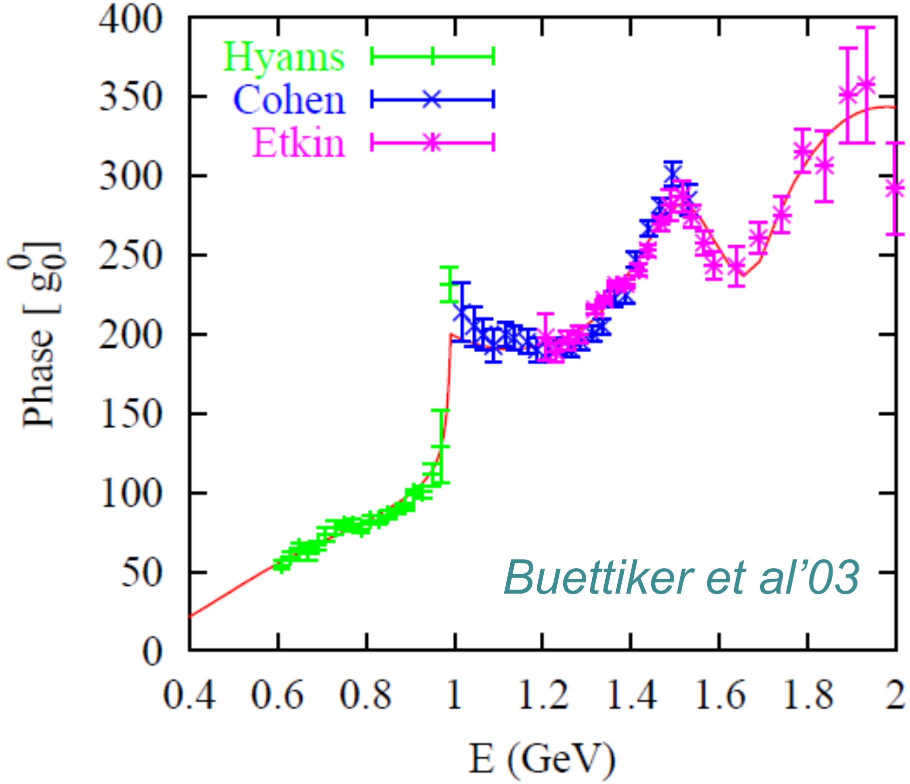
Scattering matrix:

$$\begin{pmatrix} \pi\pi \to \pi\pi, & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi, & K\bar{K} \to K\bar{K} \end{pmatrix}$$

Inputs for the coupled channel analysis

- Inputs : $\pi\pi \to \pi\pi, K\bar{K}$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buettiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

3.7 Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

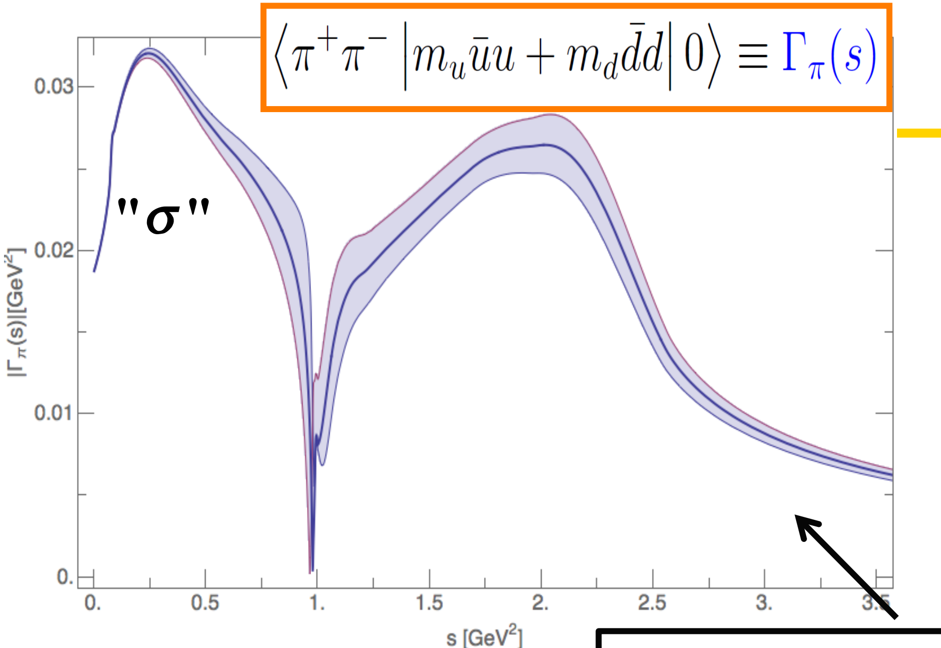
$$\boxed{\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}}$$

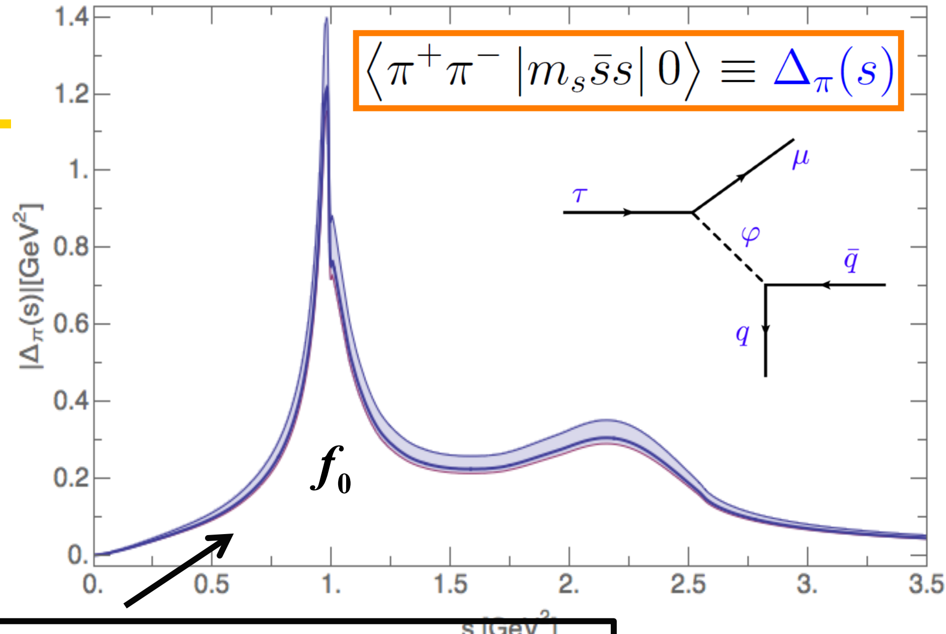
Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

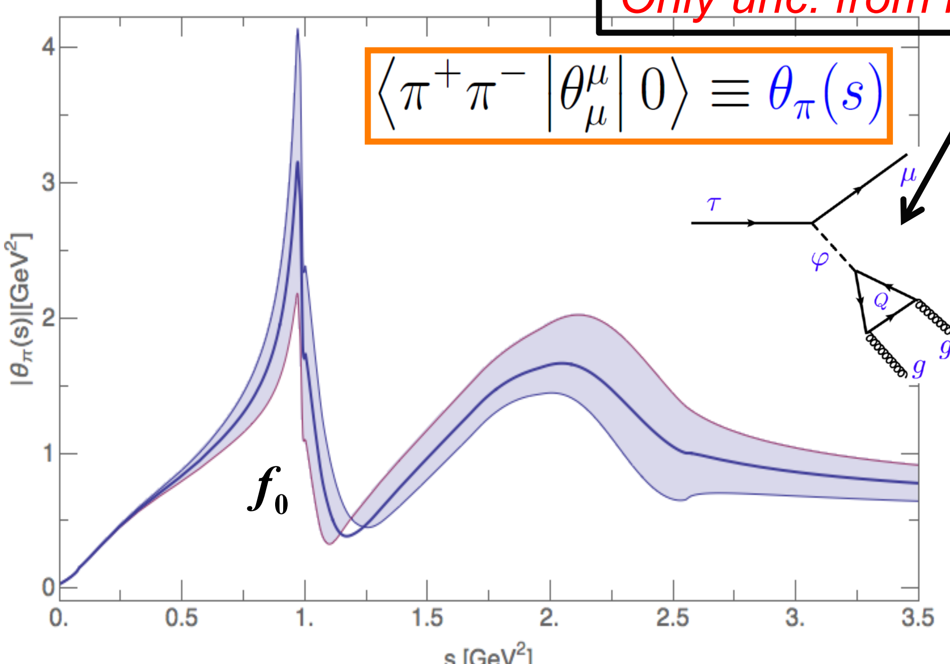
- Canonical solution found by solving dispersive integral equations iteratively starting with *Omnès functions* that are solutions of the one-channel unitary condition

$$\boxed{\Omega_{\pi,K}(s) \equiv \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{dt}{t}\frac{\delta_{\pi,K}(t)}{(t-s)}\right]}$$



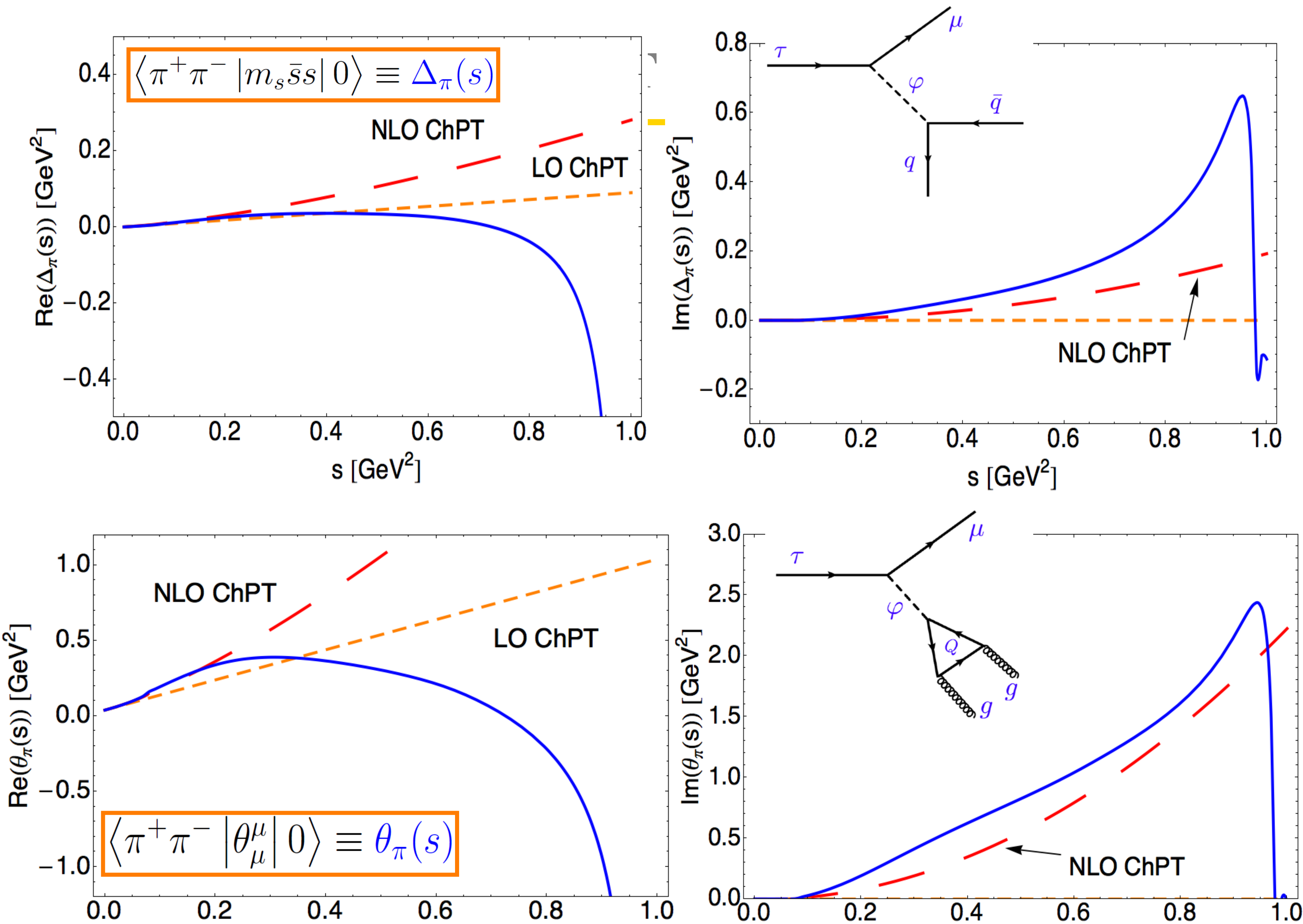


Only unc. from matching conditions here!



- Uncertainties:
  - Varying s_{cut} (1.4 GeV^2 - 1.8 GeV^2)
  - Varying the matching conditions
  - T matrix inputs

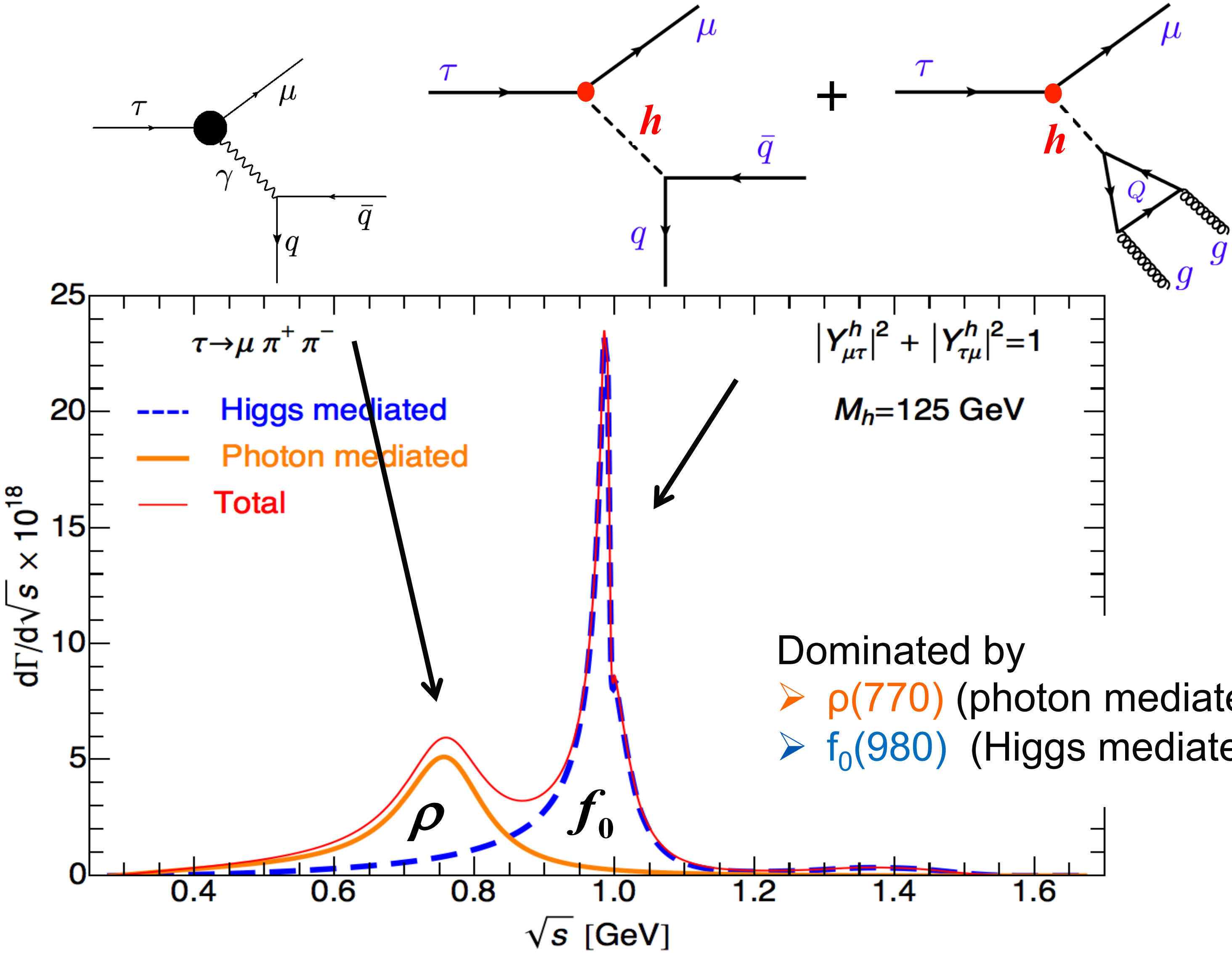
See also *Daub et al.'13*



$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$
NLO ChPT
LO ChPT
$\mathrm{Re}(\Delta_\pi(s))$ [GeV^2]
s [GeV^2]
τ
μ
φ
$\bar{q}$
q
$\mathrm{Im}(\Delta_\pi(s))$ [GeV^2]
NLO ChPT
s [GeV^2]
NLO ChPT
LO ChPT
$\mathrm{Re}(\theta_\pi(s))$ [GeV^2]
$\langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$
s [GeV^2]
τ
μ
φ
Q
g
g
$\mathrm{Im}(\theta_\pi(s))$ [GeV^2]
NLO ChPT
s [GeV^2]

3.8 Results: Spectrum

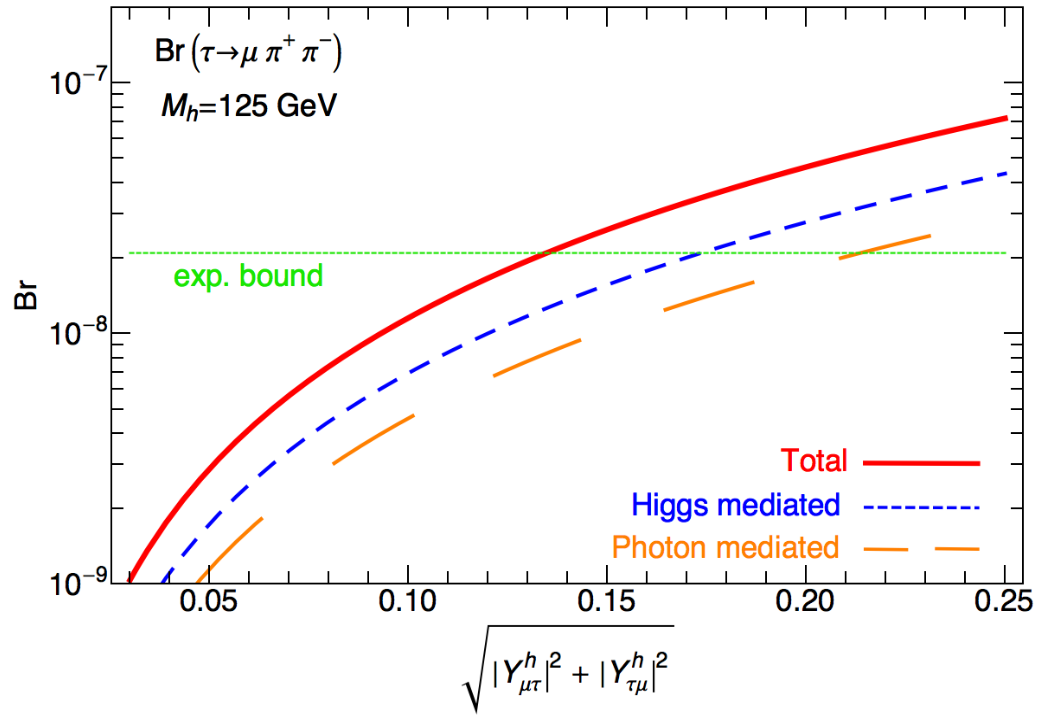
Celis, Cirigliano, E.P.'14



Dominated by
- $\rho(770)$ (photon mediated)
- $f_0(980)$ (Higgs mediated)

3.8 Results: Bounds

Celis, Cirigliano, E.P.'14



Bound:

$$\boxed{\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \leq 0.13}$$

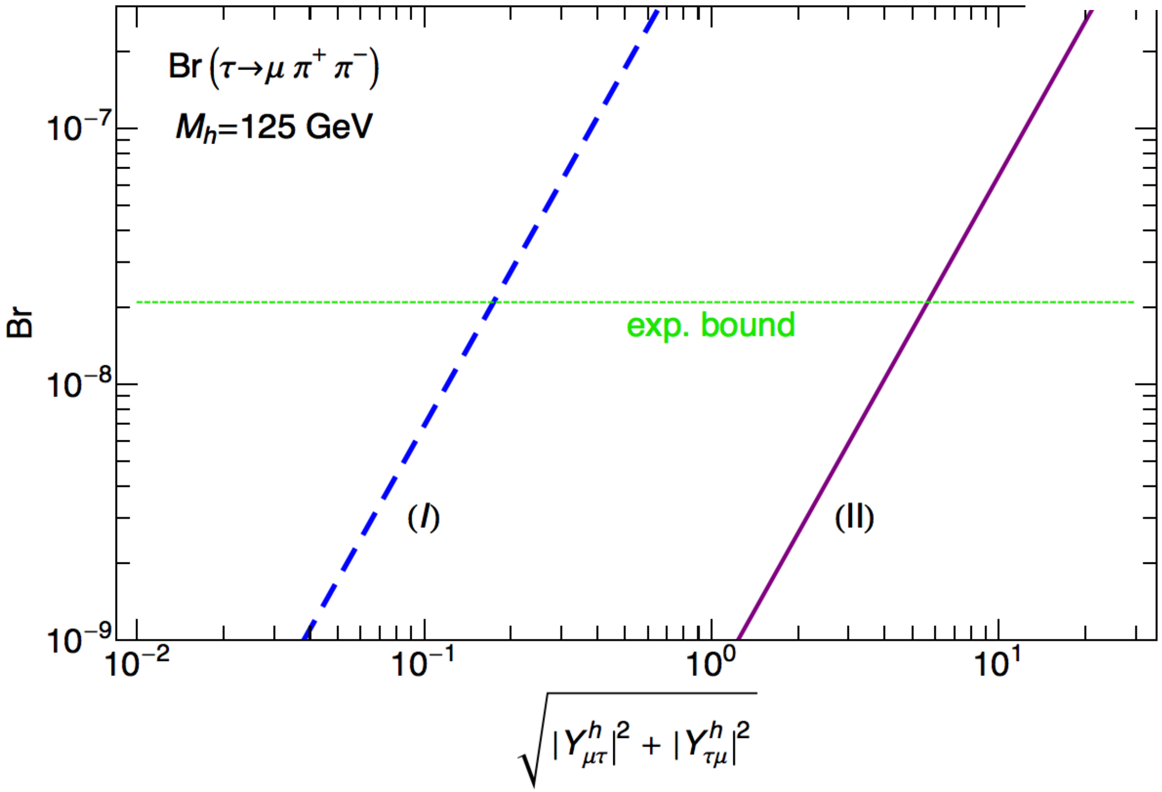
Less stringent but more robust handle on LFV Higgs couplings

Process	$(\mathrm{BR} \times 10^8)$ 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
? $\tau \to \mu\pi^0\pi^0$	$< 1.4 \times 10^3$ [87]	< 6.3	Scalar, Gluon

BaBar'10, Belle'10'11'13 except last from *CLEO'97*

3.9 Impact of our results

Celis, Cirigliano, E.P.'14

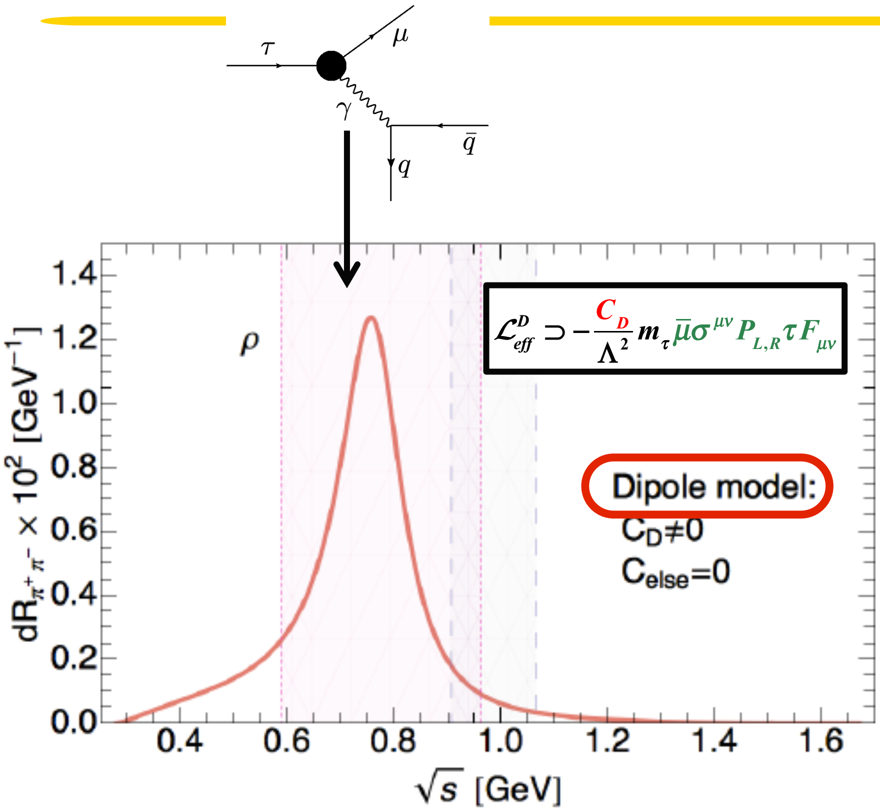


- Dispersive treatment of hadronic part ⇨ bound reduced by one order of magnitude!

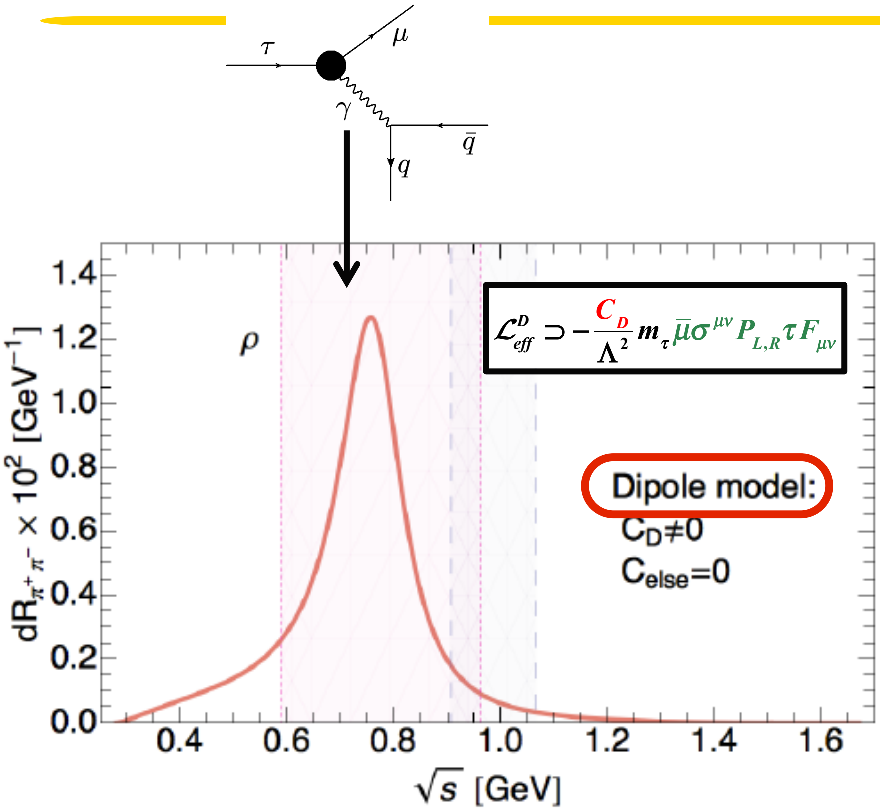
- ChPT, EFT only valid at low energy for $\boxed{\mathbf{p} \ll \Lambda = 4\pi f_\pi \sim 1\ \mathrm{GeV}}$

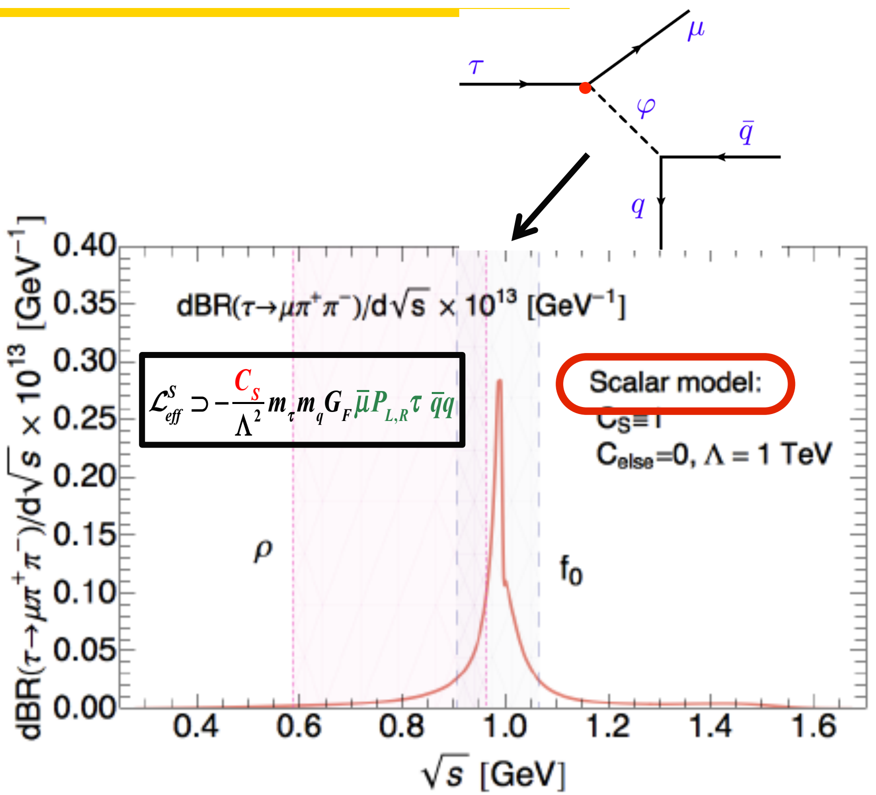
  ⇨ *not valid up to* $E = (m_\tau - m_\mu)$!

Discriminating power of $\tau \to \mu(e)\pi\pi$ decays

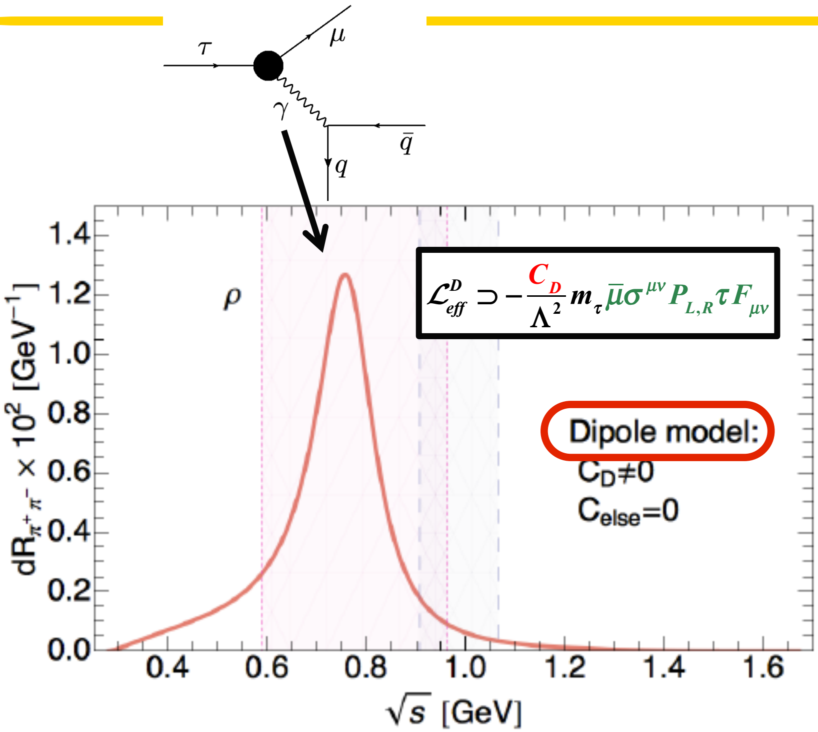


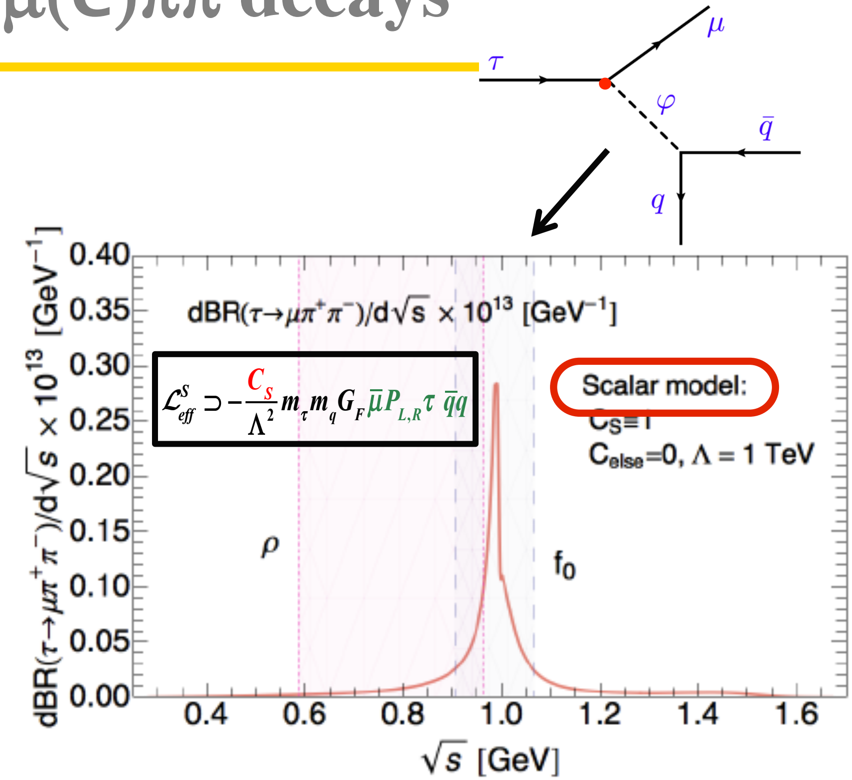
Discriminating power of $\tau \to \mu(e)\pi\pi$ decays

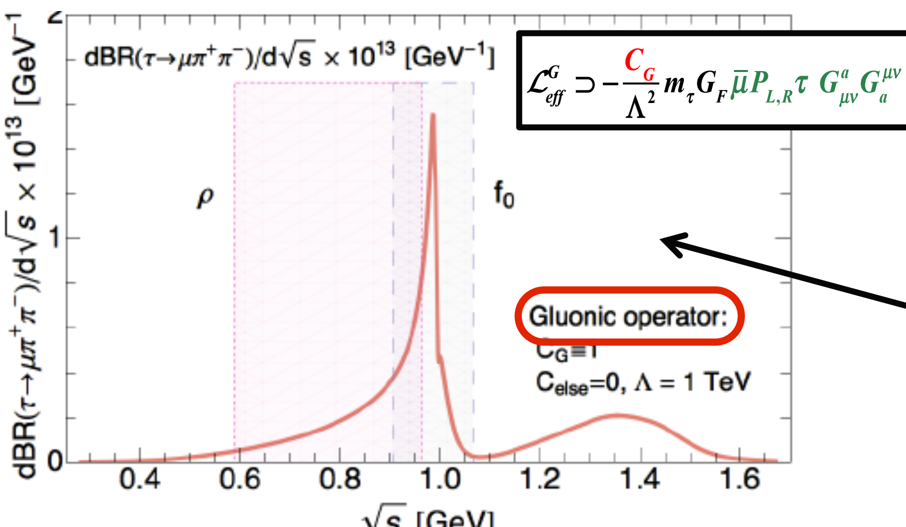




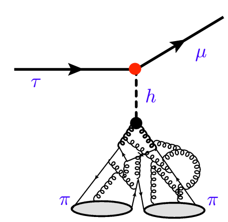
Discriminating power of $\tau \to \mu(e)\pi\pi$ decays







Very different distributions according to the *final hadronic state!*



4. Conclusion and outlook

Conclusion and outlook

- Hadronic tau decays are very important $\Rightarrow$ test of
  - QCD
  - EW effects: CPV in $\tau \to K\pi\nu_\tau$, V_{us}, Higgs LFV etc.

- We have looked at:
  - 2 body: FFs in $\tau \to K\pi\nu_\tau$ and search for LFV in in $\tau \to \pi\pi\mu$
  - 3 body effects in $\tau \to \pi\pi\pi\nu_\tau$ in progress

- Experimental activities: CLEO, Belle, BaBar, LHCb $\Rightarrow$ Belle II, BESIII, Tau-Charm factories

- Intense theoretical activities : QCD, new physics

- A lot of *very interesting physics* remains to be done in the tau sector!

- But we need more experimental measurements and accurate theoretical prediction until energies of $m_\tau \sim 1.8$ GeV

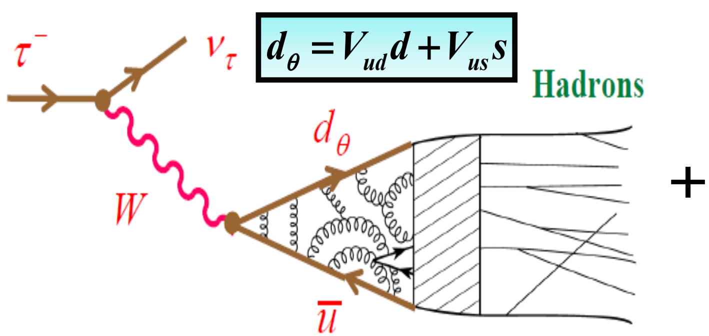
5. Back-up

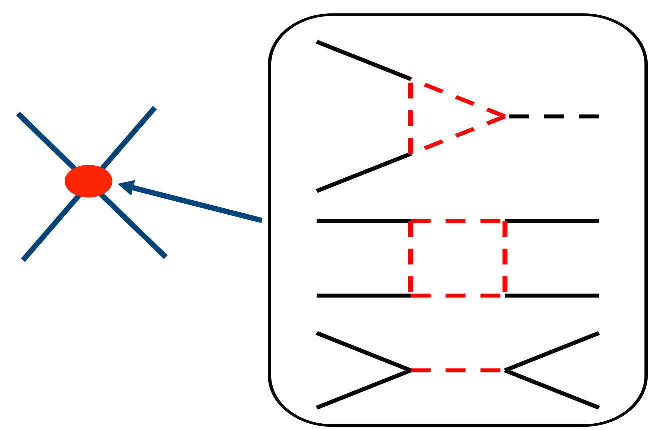
1.2 Hadronic Physics in tau decays

- Hadronic Physics: Interactions of quarks at low energy
  - Precise tests of the Standard Model:

    ⇨ Extraction of V_{us}, α_S, light quark masses…

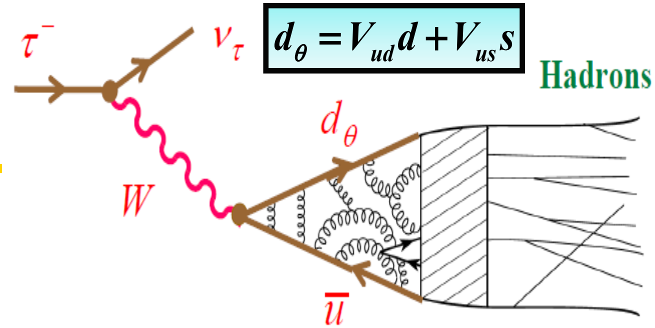
  - Look for physics beyond the Standard Model: High precision at low energy as a key to new physics?





SUSY loops
Z', Charged Higgs,
Right-Handed
Currents,….

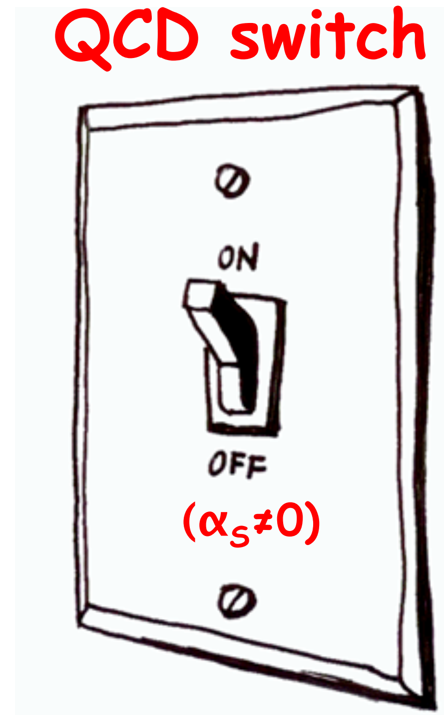
3.3 Determination of V_{us}



- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C$$

- Use OPE: $$R_\tau^{NS}\left(m_\tau^2\right) = N_C\, S_{EW} \left|V_{ud}\right|^2 \left(1 + \delta_P + \delta_{NP}^{ud}\right)$$

$$R_\tau^{S}\left(m_\tau^2\right) = N_C\, S_{EW} \left|V_{us}\right|^2 \left(1 + \delta_P + \delta_{NP}^{us}\right)$$



- From the measurement of R_τ^S ⇨ $\left|V_{us}\right|$

- Use instead : $$\delta R_\tau \equiv \frac{R_{\tau,NS}}{\left|V_{ud}\right|^2} - \frac{R_{\tau,S}}{\left|V_{us}\right|^2}$$

⇨ *SU(3) breaking* quantity: the flavour independent piece:
$\delta_P \sim 20\%$ cancels!

Results

$$|V_{us}|^2 = \frac{R_{\tau,S}}{\dfrac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

- $\delta R_{\tau,theo}$ determined from OPE (L+T) + phenomenology

$$\Rightarrow \quad \boxed{\delta R_{\tau,th} = \underbrace{(0.1544 \pm 0.0037)}_{J=0} + (9.3 \pm 3.4)\, m_s^2 + (0.0034 \pm 0.0028)}$$

Gamiz, Jamin, Pich, Prades, Schwab '07, Maltman '11

Input : m_s $\Rightarrow$ $m_s(2\text{ GeV}, \overline{\text{MS}}) = 93.9 \pm 1.1$ N_f=2+1+1 lattice average

FLAG'16

- Tau data : $R_{\tau,S} = 0.1646(23)$ and $R_{\tau,NS} = 3.4721(77)$ *HFLAV'16 + BaBar@ICHEP18*

- V_{ud} : $|V_{ud}| = 0.97425(22)$ *Towner & Hardy '08*

Results

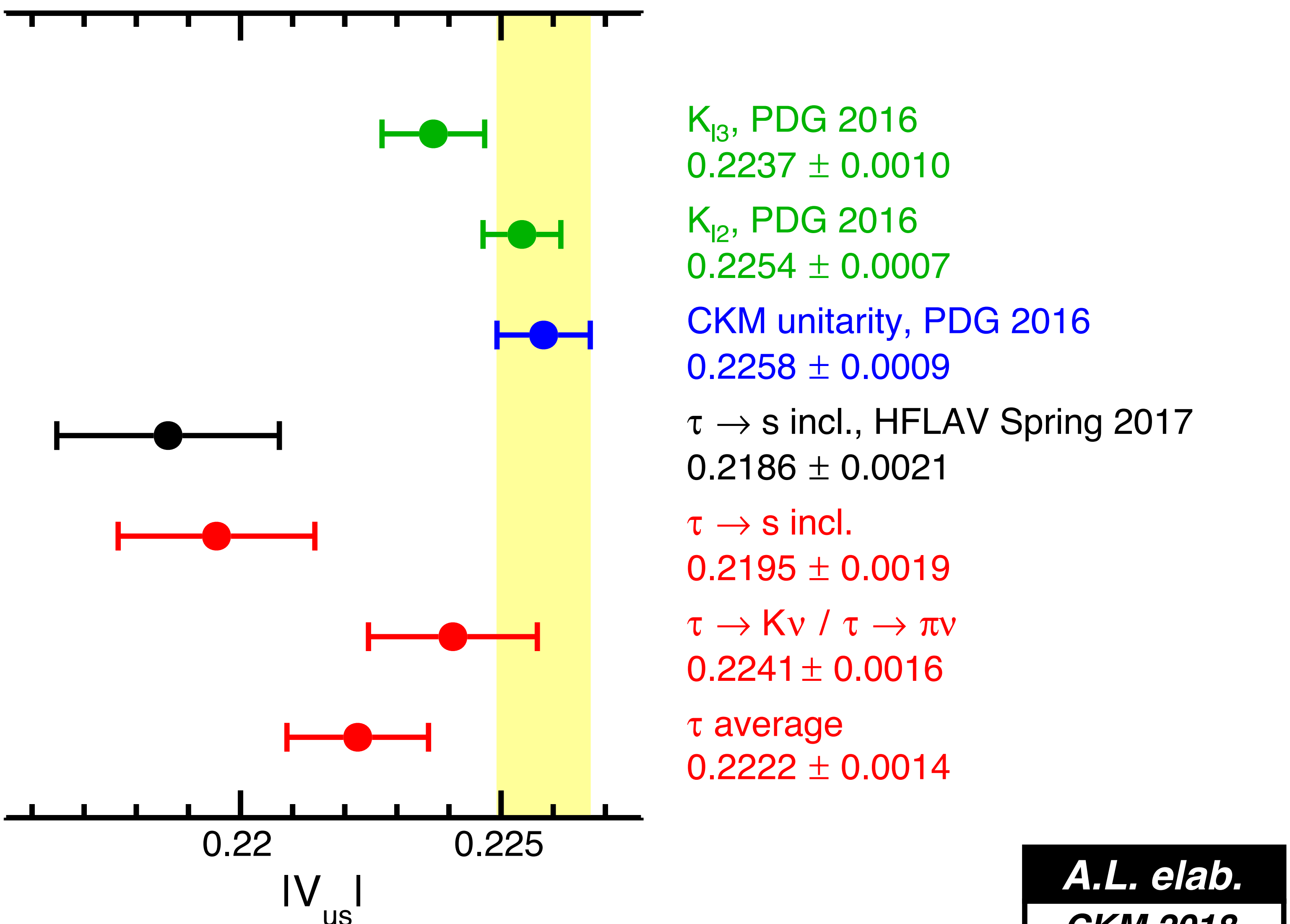
$$\boxed{|V_{us}|^2 = \frac{R_{\tau,S}}{\dfrac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

- $\delta R_{\tau,th} = 0.240(30)$

$\Rightarrow$ $\boxed{|V_{us}| = 0.2194 \pm 0.0016_{\text{exp}} \pm 0.0010_{\text{th}}}$

- Determination dominated by experimental uncertainties!
- 2.9σ away from unitarity!

3.3 Determination of V_{us}



3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- The angular CP asymmetry from Belle:

$$\frac{d\Gamma\left(\tau^- \to K\pi^-\nu_\tau\right)}{d\sqrt{Q^2}\, d\cos\theta\; d\cos\beta} = \left[A(Q^2) - B(Q^2)\;\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\right]\left|f_+(s)\right|^2$$
$$+\, m_\tau^2\left|\tilde{f}_0(s)\right|^2 - C(Q^2)\cos\psi\,\cos\beta\,\mathrm{Re}\left(f_+(s)\,\tilde{f}_0^*(s)\right)$$

CP violating term
S-P interference

- When integrating on the angle the interference term between scalar and vector vanishes

$$\frac{d\Gamma}{d\sqrt{Q^2}} = \frac{G_F^2 \sin^2\theta_c m_\tau^3}{3\times 2^5\times\pi^3 Q^2}\left(1-\frac{Q^2}{m_\tau^2}\right)^2\left(1+\frac{2Q^2}{m_\tau^2}\right)$$
$$\times q_1(Q^2)\left\{q_1(Q^2)^2\,|F_V|^2 + \frac{3}{4}\frac{Q^2}{\left(1+2Q^2/m_\tau^2\right)}\,|F_S|^2\right\}$$

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

Devi, Dhargyal, Sinha'14
Cirigliano, Crivellin, Hoferichter'17

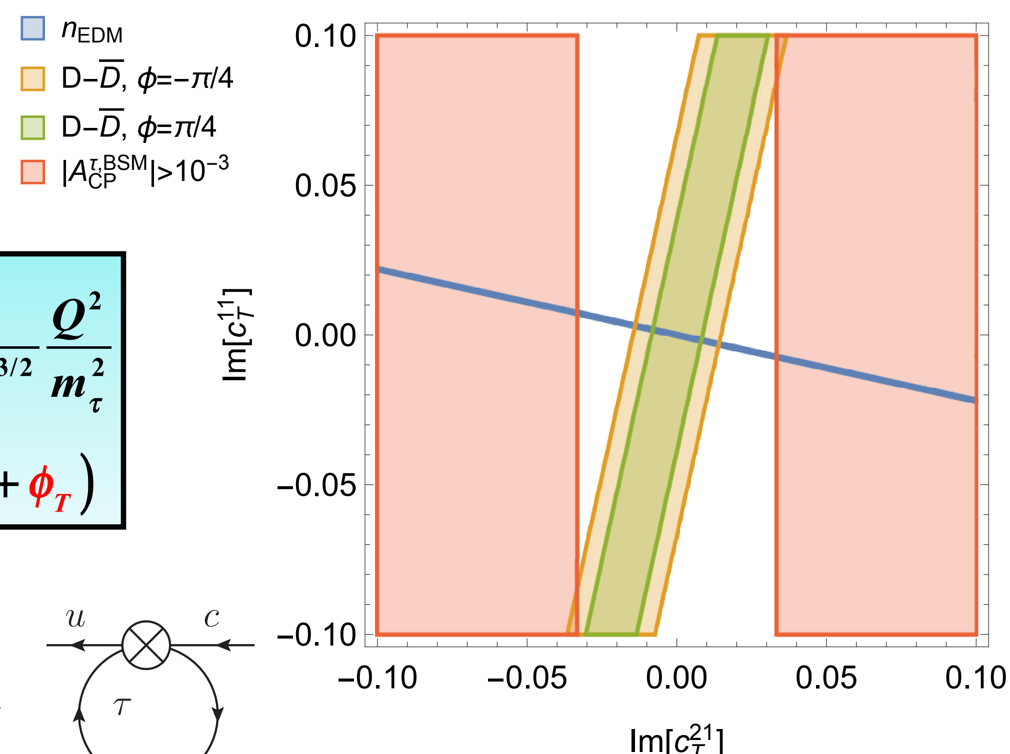
- We need a tensor interaction to get some interference:

$$\boxed{\mathcal{H}_T^{eff} \equiv G'(\bar{s}\sigma_{\mu\nu}u)(\bar{\nu}_\tau(1+\gamma_5)\sigma^{\mu\nu}\tau)}$$

with $G' = \frac{G_F}{\sqrt{2}} C_T, \quad C_T = |C_T| e^{i\phi_T}$

- When integrating the interference term between vector and tenson does not vanish:

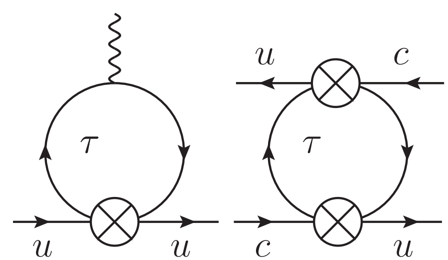
$$\frac{d\Gamma}{dQ^2} = \frac{d\Gamma_{SM}}{dQ^2} + \frac{d\Gamma_T}{dQ^2} + \frac{d\Gamma_{V-T}}{dQ^2}$$



$$\boxed{\frac{d\Gamma_{V-T}}{dQ^2} = G_F^2 \sin^2\theta_C \frac{m_\tau^3}{32\pi^3}\left(\frac{m_\tau^2 - Q^2}{m_\tau^2}\right)^2 \frac{q_1^3}{(Q^2)^{3/2}}\frac{Q^2}{m_\tau^2} \times |C_T||F_V(s)||F_T(s)|\cos\left(\delta_T(s) - \delta_V(s) + \phi_T\right)}$$

In conflict with bounds from neutron EDM and $D\bar{D}$ mixing

Cirigliano, Crivellin, Hoferichter'17



Extraction of V_{us}

- Decay rate master formula

Antonelli, Cirigliano, Lusiani, E.P.'13

$$\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \cancel{\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}}\right)^2$$

$$BR\left(\tau \to \overline{K^0}\pi^-\nu_\tau\right) = (0.416 \pm 0.008)\%$$

Belle'14

$$S_{\mathrm{ew}} = 1.0201$$

Marciano & Sirlin'88, Braaten & Li'90, Erler'04

$$\delta_{\mathrm{EM}}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\%$$

$$I_{K^0}^\tau = 0.50432 \pm 0.01721$$

$$f_+(0) = 0.9661(32)$$

$$\Rightarrow \quad f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\mathrm{exp}}$$

$$\Rightarrow \quad \left|V_{us}\right| = 0.2216 \pm 0.0027$$

Extraction of V$_{us}$

- Decay rate master formula

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$$\boxed{f_+(0) = 0.9661(32)}$$

FLAG'13

$$\Rightarrow \boxed{f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\mathrm{exp}}} \Rightarrow \boxed{\left|V_{us}\right| = 0.2216 \pm 0.0027}$$

- Result of fit to K_{l3} + $\tau \to K\pi\nu_\tau$ and $K\pi$ scattering data including inelasticities in the dispersive FFs

$$\Rightarrow \boxed{f_+(0)\left|V_{us}\right| = 0.2163 \pm 0.0014}$$

Bernard'14

4.4 Model discriminating power of Tau processes

- Summary table: *Celis, Cirigliano, E.P.'14*

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O_V^q	–	–	✓ (I=1)	✓(I=0,1)	–	–
O_S^q	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O_A^q	–	–	–	–	✓ (I=1)	✓ (I=0)
O_P^q	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- Recent progress in $\tau \to \mu(e)\pi\pi$ using *dispersive techniques* *Daub et al'13* *Celis, Cirigliano, E.P.'14*

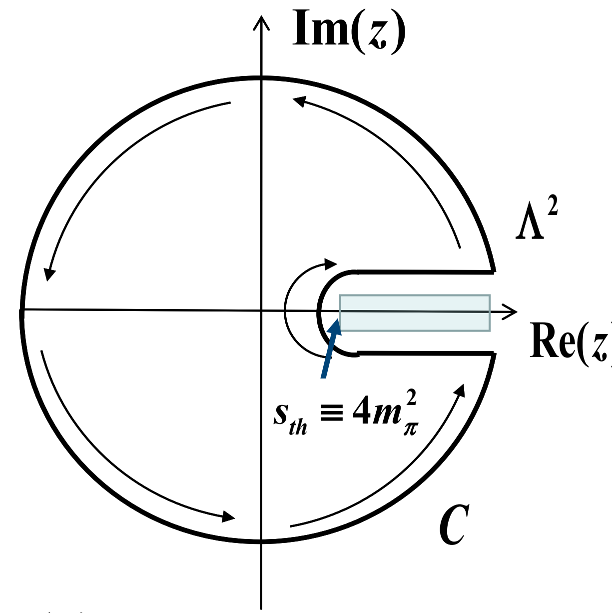
- Hadronic part: $\boxed{H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} |0\rangle = \left(Lorentz\ \mathrm{struct.}\right)_\mu^i F_i(s)}$ with $s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$

- Form factors determined by solving 2-channel unitarity condition, with I=0 s-wave ππ and KK scattering data as input

$$n = \pi\pi, K\bar{K} \qquad \boxed{\mathrm{Im} F_n(s) = \sum_{m=1}^{2} T_{nm}^*(s)\sigma_m(s)F_m(s)}$$

Canonical solution $X(s) = C(s), D(s)$:

- Knowing the discontinuity of X(s) ⇒ write a dispersion relation for it
- Analyticity of the FFs: X(z) is
  - real for $z < s_{th}$
  - has a branch cut for $z > s_{th}$
  - analytic for complex z



- Cauchy Theorem and Schwarz reflection principle:

$$X(s) = \frac{1}{\pi} \oint_C dz \frac{X(z)}{z-s}$$

$$= \frac{1}{2i\pi} \int_{s_{th}=4M_\pi^2}^{\Lambda^2} dz \frac{disc\left[F(z)\right]}{z-s-i\varepsilon} + \frac{1}{2i\pi} \int_{|z|=\Lambda^2} dz \frac{F(z)}{z-s}$$

$\Lambda \to \infty$

⇒ $$\boxed{X(s) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} dz \frac{\mathrm{Im}\left[X(z)\right]}{z-s-i\varepsilon}}$$

X(s) can be reconstructed everywhere from the knowledge of ImX(s)

T matrix parametrization

$$S_{mn} = \delta_{mn} + 2i\,\sqrt{\sigma_m \sigma_n}\,T_{mn}$$

$$S = \begin{pmatrix} \cos\gamma\ e^{2i\delta_\pi} & i\ \sin\gamma\ e^{i(\delta_\pi+\delta_K)} \\ i\ \sin\gamma\ e^{i(\delta_\pi+\delta_K)} & \cos\gamma\ e^{2i\delta_K} \end{pmatrix}$$

- Inelasticity: $\eta_0^0 \equiv \cos\ \gamma$.
- $\delta_\pi(s)$: ππ S wave phase shift
- $\delta_K(s)$: KK S wave phase shift

3.4 Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

$$\boxed{\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}}F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}}$$

Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$X(s) = C(s), D(s)$$

$$\boxed{\mathrm{Im}X_n^{(N+1)}(s) = \sum_{m=1}^{2} T^*_{mn}\sigma_m(s)X_m^{(N)}(s)} \longrightarrow \boxed{X_n^{(N+1)}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}X_n^{(N+1)}(s')}{s'-s}}$$

Determination of the polynomial

- Fix the polynomial with requiring $\boxed{F_P(s) \to 1/s}$ + ChPT:

Brodsky & Lepage'80

- Feynman-Hellmann theorem:

$$\boxed{\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2}$$

$$\boxed{\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2}$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_\mathrm{u} + m_\mathrm{d})\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_\mathrm{u} + m_\mathrm{s})\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_\mathrm{d} + m_\mathrm{s})\, B_0 + O(m^2)$$

Determination of the polynomial

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- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d)\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_u + m_s)\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_d + m_s)\, B_0 + O(m^2)$$

$\Rightarrow$

$$\boxed{\begin{aligned}
P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\
Q_\Gamma(s) &= \frac{2}{\sqrt{3}}\Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\
P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\
Q_\Delta(s) &= \frac{2}{\sqrt{3}}\Delta_K(0) = \frac{2}{\sqrt{3}}\left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots
\end{aligned}}$$

Determination of the polynomial

- At LO in ChPT:

$$M^2_{\pi^+} = (m_{\mathrm{u}} + m_{\mathrm{d}})\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_{\mathrm{u}} + m_{\mathrm{s}})\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_{\mathrm{d}} + m_{\mathrm{s}})\, B_0 + O(m^2)$$

- For the scalar FFs:

$$\boxed{\begin{aligned}
P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\
Q_\Gamma(s) &= \frac{2}{\sqrt{3}}\Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\
P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\
Q_\Delta(s) &= \frac{2}{\sqrt{3}}\Delta_K(0) = \frac{2}{\sqrt{3}}\left(M_K^2 - \frac{1}{2}M_\pi^2\right) + \cdots
\end{aligned}}$$

- Problem: large corrections in the case of the kaons!

  ⇨ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\ M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05}\,(M_K^2 - 1/2 M_\pi^2)$$

Daub, Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

  ⇨ Relax the constraints and match to ChPT

$$
\boxed{
\begin{aligned}
P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2\dot{C}_1 - \frac{4M_K^2}{\sqrt{3}}\dot{D}_1\right) s \\
Q_\theta(s) &= \frac{4}{\sqrt{3}}M_K^2 + \frac{2}{\sqrt{3}}\left(\dot{\theta}_K - \sqrt{3}M_\pi^2\dot{C}_2 - 2M_K^2\dot{D}_2\right) s
\end{aligned}
}
$$

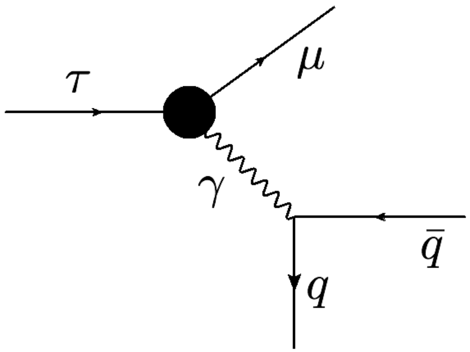
with $\dot{f} = \left(\dfrac{df}{ds}\right)_{s=0}$

- At LO ChPT: $\dot{\theta}_{\pi,K} = 1$

- Higher orders ⇨ $\boxed{\dot{\theta}_K = 1.15 \pm 0.1}$

3.1 Constraints from $\tau \to \mu\pi\pi$

- Contribution from dipole diagrams



$$\boxed{L_{eff} = c_L Q_{L\gamma} + c_R Q_{R\gamma} + h.c.}$$

with the dim-5 EM penguin operators :

$$\boxed{Q_{L\gamma,R\gamma} = \frac{e}{8\pi^2} m_\tau \left(\mu \sigma^{\alpha\beta} P_{L,R} \tau\right) F_{\alpha\beta}}$$

- $$\boxed{\frac{d\Gamma(\tau \to \ell\pi^+\pi^-)}{d\sqrt{s}} = \frac{\alpha^2 |F_V(s)|^2 \left(|c_L|^2 + |c_R|^2\right)}{768\pi^5 m_\tau} \frac{(s - 4m_\pi^2)^{3/2} (m_\tau^2 - s)^2 (s + 2m_\tau^2)}{s^2}}$$

with the vector form factor :

$$\boxed{C_{L,R} = f\left(Y_{\tau\mu}\right)}$$

$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) \middle| \tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) \middle| 0 \right\rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

- Diagram only there in the case of $\tau^- \to \mu^- \pi^+ \pi^-$ absent for $\tau^- \to \mu^- \pi^0 \pi^0$

  ⇨ neutral mode more model independent

Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements
    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

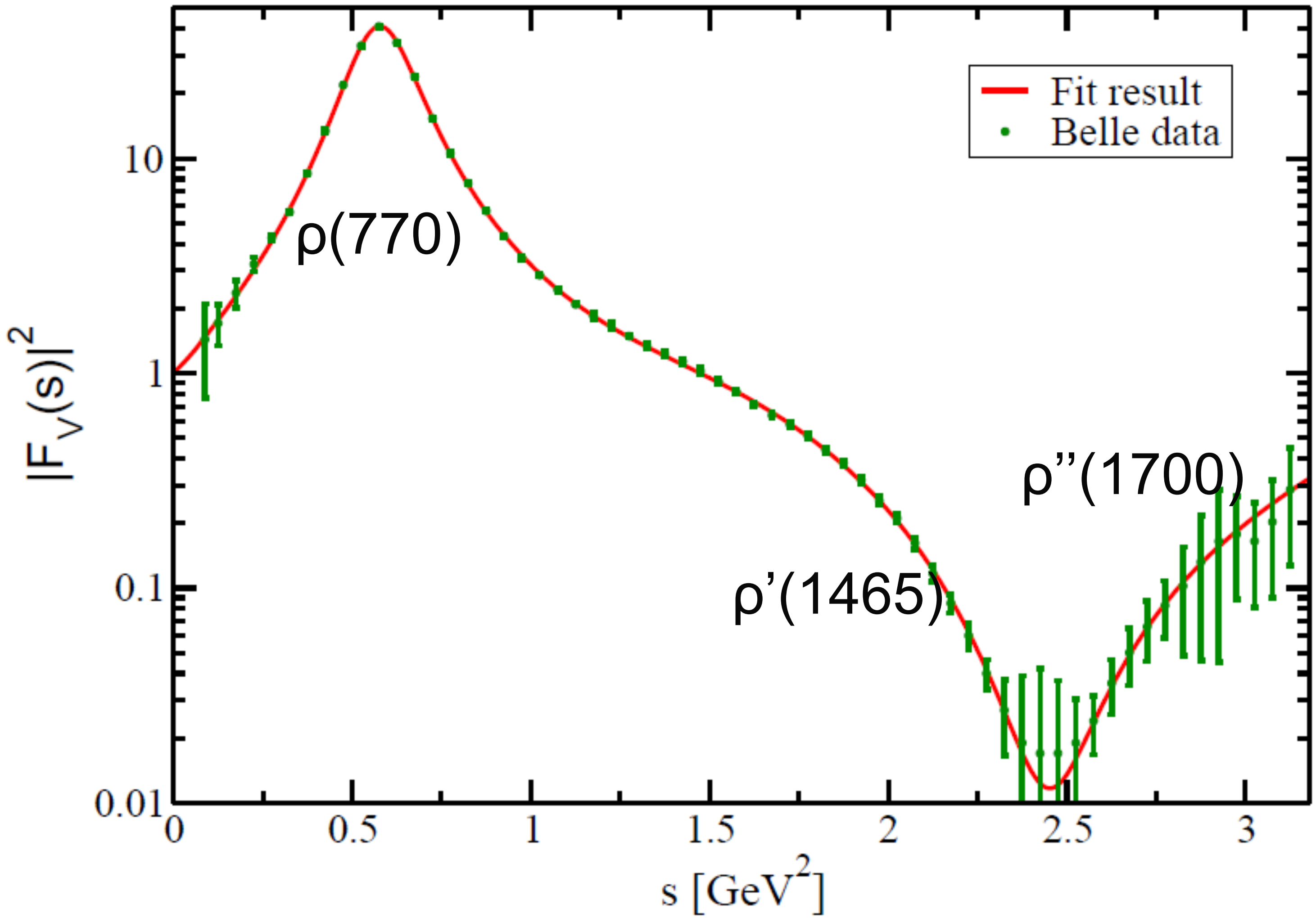
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$F_V(s) = \exp\left[\lambda'_V \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda''_V - \lambda'^2_V\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s'-s-i\varepsilon)}\right]$$

Extracted from a model including
3 resonances ρ(770), ρ'(1465)
and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the
    Belle data $\tau^- \to \pi^0\pi^-\nu_\tau$

Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!