Leptonic decays of the τ lepton

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UNIVERSITÄT
SIEGEN

$$\tau \to \ell\nu\bar{\nu}$$

$$\tau \to \ell\nu\bar{\nu} + \gamma$$

$$\tau \to \ell\nu\bar{\nu} + \ell'\ell'$$

Michel Parameters

$$\tau \to \ell \nu \bar{\nu}$$

$$\tau \to \ell \nu \bar{\nu} + \gamma$$

$$\tau \to \ell \nu \bar{\nu} + \ell' \ell'$$

$$\mathcal{L} = -\frac{4G_0}{\sqrt{2}} \sum_{\substack{\gamma = S,V,T \\ \epsilon,\omega = R,L}} g^{\gamma}_{\varepsilon\omega} \left(\bar{\ell}_{\epsilon} \Gamma^{\gamma} \nu_{\ell}\right) \left(\bar{\nu}_{\tau} \Gamma_{\gamma} \tau_{\omega}\right)$$

- $\tau \to \ell \nu \bar{\nu}$

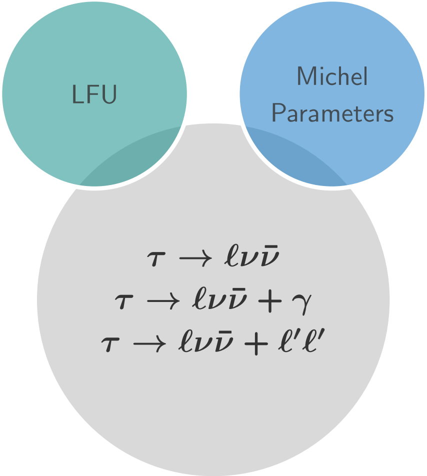
  Michel, Proc. Phys. Soc. A63 (1950) 514;
  Bouchiat, Michel, PR 106(1957) 170;
  Kinoshita, Sirlin, PR 107(1957) 593;
  Kinoshita, Sirlin, PR 108(1957) 844.

- $\tau \to \ell \gamma \nu \bar{\nu}$

  Arbuzov, Kopylova, JHEP 1609 (2016) 109;

- $\tau \to \ell \ell' \ell' \nu \bar{\nu}$

  Flores-Tlalpa, Lopez Castro, Roig, JHEP 1604 (2016) 185



LFU

Michel Parameters

$\tau \to \ell\nu\bar{\nu}$

$\tau \to \ell\nu\bar{\nu} + \gamma$

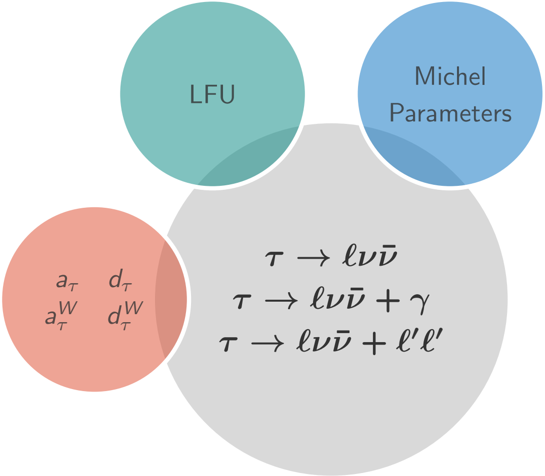
$\tau \to \ell\nu\bar{\nu} + \ell'\ell'$

$$R = \frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\tau \to \mu\nu\bar{\nu})}$$

$$R_\gamma = \frac{\Gamma(\tau \to e\gamma\nu\bar{\nu})}{\Gamma(\tau \to \mu\gamma\nu\bar{\nu})}$$

$$R_{ee} = \frac{\Gamma(\tau \to eee\nu\bar{\nu})}{\Gamma(\tau \to \mu ee\nu\bar{\nu})}$$

$$R_{\mu\mu} = \frac{\Gamma(\tau \to e\mu\mu\nu\bar{\nu})}{\Gamma(\tau \to \mu\mu\mu\bar{\nu})}$$



LFU

Michel Parameters

a_τ d_τ
a_τ^W d_τ^W

$$\tau \to \ell \nu \bar{\nu}$$
$$\tau \to \ell \nu \bar{\nu} + \gamma$$
$$\tau \to \ell \nu \bar{\nu} + \ell' \ell'$$

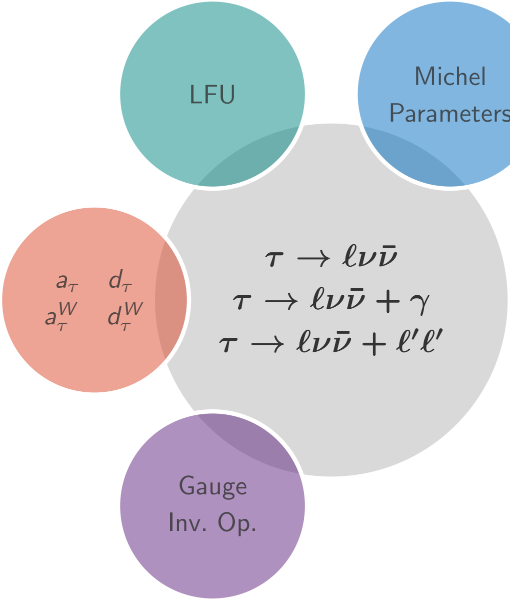
$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \left[C_{lW} \mathcal{O}_{lW} + C_{lB} \mathcal{O}_{lB} + \text{h.c.} \right]$$

Grzadkowski, Iskrzynski, Misiak, Rosiek, JHEP 1010 (2010) 085.

Dipole Moments:

$$\tilde{a}_\tau = \frac{2 m_\tau}{2} \frac{\sqrt{2} v}{\Lambda^2} \mathrm{Re} \left[\cos\theta_W C_{lB} - \sin\theta_W C_{lW} \right]$$
$$\tilde{d}_\tau = \frac{\sqrt{2} v}{\Lambda^2} \mathrm{Im} \left[\cos\theta_W C_{lB} - \sin\theta_W C_{lW} \right]$$

Eidelman, Epifanov, MF, Mercolli, Passera, JHEP 1603 (2016) 140.



LFU

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$\tau \to \ell \nu \bar{\nu}$

$\tau \to \ell \nu \bar{\nu} + \gamma$

$\tau \to \ell \nu \bar{\nu} + \ell' \ell'$

Gauge Inv. Op.

$$\mathcal{L}_{\rm eff} = \frac{1}{\Lambda^2}\left[C_{lW}\mathcal{O}_{lW} + C_{lB}\mathcal{O}_{lB} + \text{h.c.}\right]$$

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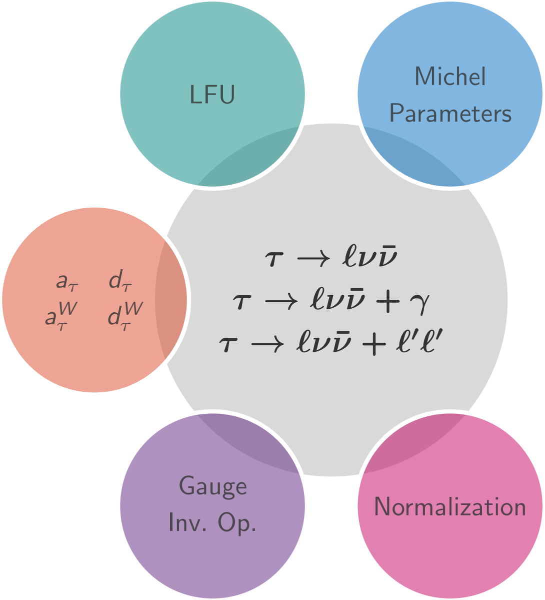


LFU

Michel Parameters

a_τ d_τ

a_τ^W d_τ^W

$$\tau \to \ell\nu\bar{\nu}$$

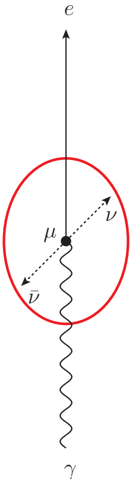
$$\tau \to \ell\nu\bar{\nu} + \gamma$$

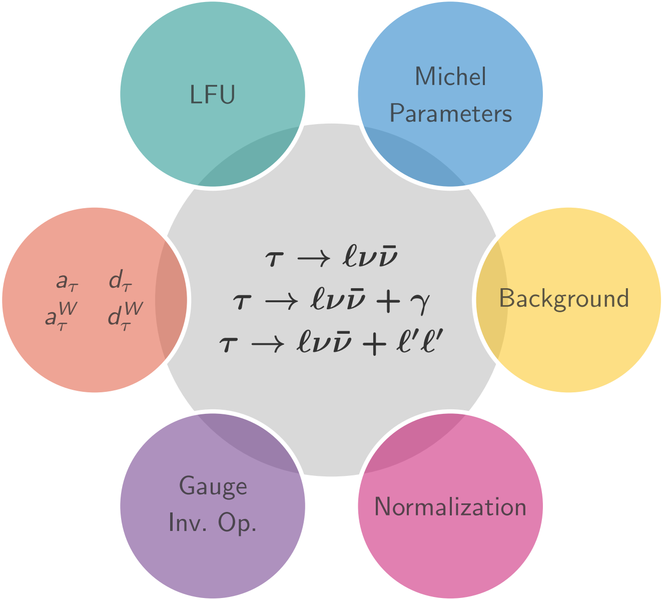
$$\tau \to \ell\nu\bar{\nu} + \ell'\ell'$$

Gauge Inv. Op.

Normalization

@ MEG





LFU

Michel Parameters

a_τ d_τ
a_τ^W d_τ^W

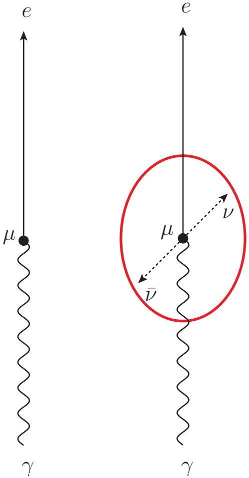
$$\tau \to \ell\nu\bar{\nu}$$
$$\tau \to \ell\nu\bar{\nu} + \gamma$$
$$\tau \to \ell\nu\bar{\nu} + \ell'\ell'$$

Background

Gauge Inv. Op.

Normalization

@ MEG



LFU

Michel Parameters

a_τ d_τ
a_τ^W d_τ^W

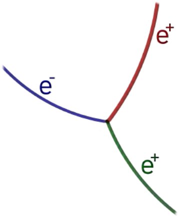
$$\tau \to \ell\nu\bar{\nu}$$
$$\tau \to \ell\nu\bar{\nu} + \gamma$$
$$\tau \to \ell\nu\bar{\nu} + \ell'\ell'$$

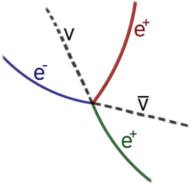
Background

Gauge Inv. Op.

Normalization

@ Mu3E





MonteCarlo @ NLO

- $\tau \to \ell\nu\bar{\nu}$:
  - Decay spectrum: Kinoshita and Sirlin, Phys. Rev. 113 (1959) 1652.
  - TAUOLA:

    M. Jezabek, Z. Was, S. Jadach, J. Kuhn, Comput.Phys.Commun. 70 (1992) 69.
- $\tau \to \ell\nu\bar{\nu} + \gamma$:
  - Polarized τ, lepton-photon spectrum, IR with photon mass:

    L. Mercolli, MF, M. Passera, JHEP 1507 (2015) 153.
  - Polarized τ, fully differential, IR with FKS subtraction:

    M. Pruna, A. Signer, Y. Ulrich, Phys.Lett. B772 (2017) 452.
  - Also:
    A. Fischer, T. Kurosu and F. Savatier, Phys. Rev. D 49, 3426;

    A.B. Arbuzov, E.S. Scherbakova, Phys.Lett. B597 (2004) 285.

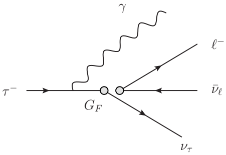
MonteCarlo @ NLO

- $\tau \to \ell\nu\bar{\nu} + \ell'\ell'$ @ LO
  - D. Yu. Bardin, T. G. Istatkov, and G. Mitselmakher, Sov.J.Nucl.Phys. 15 (1972) 161
  - P. M. Fishbane and K. J. F. Gaemers, PRD 33 (1986) 159
  - R. M. Djilkibaev and R. V. Konoplich, PRD 79 (2009) 073004
  - Flores-Tlalpa, Lopez Castro, Roig JHEP 1604 (2016) 185
  - Arroyo-Urena, Diaz, Meza-Aldama, Tavares-Velasco, Int.J.Mod.Phys. A32 (2017) 1750195
- Analytic LO Branching Ratio in the $m_e \to 0$ limit:
  - van Ritbergen, Stuart, NPB 564 (2000) 343
- $\tau \to \ell\nu\bar{\nu} + \ell'\ell'$ @ NLO:
  Original papers focus only on muon's rare decay:
  - C. Greub & MF, JHEP 1701 (2017) 084.
  - M. Pruna, A. Signer, Y. Ulrich, Phys.Lett. B765 (2017) 280.

Technical Ingredients

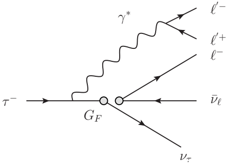
$$\mathcal{L} = \mathcal{L}_{\mathrm{QED}} + \mathcal{L}_{\mathrm{QCD}} - \frac{4G_F}{\sqrt{2}} \left(\bar{\psi}_{\nu_\mu} \gamma^\mu P_L \psi_\mu\right) \left(\bar{\psi}_e \gamma_\mu P_L \psi_{\nu_e}\right) + \mathrm{h.c.}$$

$$\tau \to \ell \nu \bar{\nu} + \gamma$$



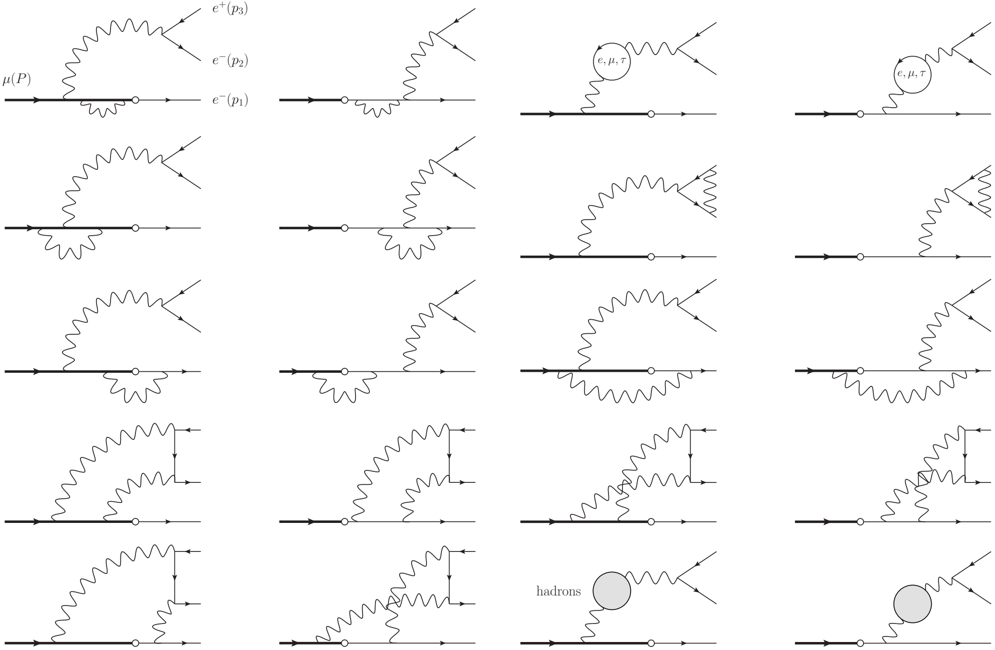
- virtual: 8 diagrams
- real: 6 diagrams

$$\tau \to \ell \nu \bar{\nu} + \ell' \ell'$$



- virtual: 22 diagrams + 2 had.
- real: 10 diagrams

(everything ×2 if $\ell = \ell'$).


$e^+(p_3)$
$e^-(p_2)$
$\mu(P)$
$e^-(p_1)$
e, μ, τ
e, μ, τ
hadrons

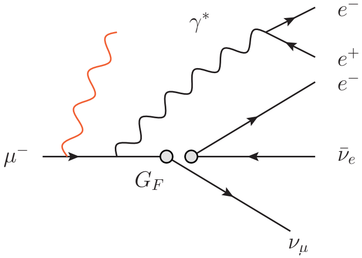
Technical Ingredients

The Montecarlo code:

- Full dependence on m_e, m_μ.
- `FORM` calculates and simplifies tree-level and one-loop diagrams
- `LoopTools` and `Collier` evaluates one-loop tensor coefficients.
  T. Hahn, M. Perez-Victoria, Comput.Phys.Commun. 118 (1999) 153;
  A. Denner, S. Dittmaier,L. Hofer, Comput.Phys.Commun. 212 (2017) 220.
- Very good numerical stability with `Collier` for $\tau \to ee e\nu\bar{\nu}$.
- $\Pi^{\rm had}(t)$ and $R^{\rm had}(z)$ provided by Jegerlehner's package `alphaQED`:
  `www-com.physik.hu-berlin.de/~fjeger/alphaQEDc17.tar.gz`

Real emission: IR regularization

- QED dipole subtraction:
  Catani, Seymour, Phys.Lett. B378 (1996) 287;
  S. Dittmaier, Nucl.Phys. B565 (2000) 69.



$$\int d\phi_{n+1} |\mathcal{M}_{\text{real}}|^2 = \int d\phi_{n+1} \left(|\mathcal{M}_{\text{real}}|^2 - |\mathcal{M}_{\text{sub}}|^2 \right) + \int d\phi_n d^3k |\mathcal{M}_{\text{sub}}|^2$$

where

$$|\mathcal{M}_{\text{sub}}|^2 = \sum_{i \neq j} g_{ij}(p_i, p_j, k) |\mathcal{M}_{\text{Born}}|^2$$

$$\tau \to \ell \nu \bar{\nu} \gamma$$

	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm LO}$	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}^{\rm Inc}_{\rm NLO}$	$1.728\,(10)_{\rm th}(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\rm Exc}_{\rm NLO}$	$1.645\,(19)_{\rm th}(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\rm th}(6)_\tau \times 10^{-3}$
K (Inc)	0.94	0.98
K (Exc)	0.90	0.97
Babar[†]	$(1.847 \pm 0.015 \pm 0.052) \times 10^{-2}$	$(3.69 \pm 0.03 \pm 0.10) \times 10^{-3}$
Belle[⋆]	$(1.79 \pm 0.02 \pm 0.10) \times 10^{-2}$	$(3.63 \pm 0.02 \pm 0.15) \times 10^{-3}$

[†] BABAR - PRD 91 (2015) 051103

[⋆] N. Shimizu - Belle - PTEP 2018 (2018) 023C01

- $E_\gamma \geq 10$ MeV
- Exclusive BR: $n = 1$ photon
- Inclusive BR: $n \geq 1$ photons

See also: L. Mercolli, MF, M. Passera JHEP 1507 (2015) 153

Acceptance cuts $\tau \to e\nu\bar{\nu}\gamma$

PDG benchmark value

$$\begin{cases} \cos\theta^*_{e\gamma} > 0.97 \\ 0.22\,\mathrm{GeV} \le E^*_\gamma \le 2.0\,\mathrm{GeV} \\ M_{e\gamma} \ge 0.14\,\mathrm{GeV} \end{cases} \longrightarrow \quad E^*_\gamma \ge 10\ \text{MeV}$$

$$\mathcal{B}_{\mathrm{exp}} = \epsilon_{\mathrm{det}} \cdot \epsilon_{\mathrm{th}} \cdot N_{\mathrm{obs}},$$

- ϵ_{det}: detector efficiencies
- $\epsilon_{\mathrm{th}} = \Gamma^{\mathrm{total}}/\Gamma^{\mathrm{with\,cuts}}$

	$\tau \to e\,\bar{\nu}\nu\gamma$	$\tau \to \mu\,\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm LO}$	$1.834(1) \cdot 10^{-2}$	$3.662(1) \cdot 10^{-3}$
$\mathcal{B}^{\sf excl}$	$1.645(1) \cdot 10^{-2}$	$3.571(1) \cdot 10^{-3}$
$\mathcal{B}^{\sf incl}$	$1.727(3) \cdot 10^{-2}$	$3.604(1) \cdot 10^{-3}$
$\mathcal{B}_{\sf exp}$	$1.847(54) \cdot 10^{-2}$	$3.69(10) \cdot 10^{-3}$
$\epsilon^{\rm th}_{\rm LO}$	$48.55(1)$	$4.966(1)$
$\epsilon^{\rm th}_{\rm NLO}$	$44.80(1)$	$4.911(1)$
$\epsilon' = \epsilon_{\rm NLO}/\epsilon_{\rm LO}$	$0.923(1)$	$0.989(1)$
$\epsilon' \cdot \mathcal{B}_{\sf exp}$	$1.704(50) \cdot 10^{-2}$	$3.65(10) \cdot 10^{-3}$

M. Pruna, A. Signer, Y. Ulrich Phys.Lett. B772 (2017) 452

$$\tau \to \ell \ell' \ell' \nu \bar{\nu}$$

	$\mathcal{B}_{\mathrm{LO}}$	$\delta\mathcal{B}_{\mathrm{NLO,QED}}$	$\delta\mathcal{B}_{\mathrm{NLO,had}}$	$\delta\mathcal{B}/\mathcal{B}$
$\tau \to ee e\nu\bar{\nu}$	$4.2488\,(4) \times 10^{-5}$	$-4.2\,(1) \times 10^{-8}$	-1.0×10^{-9}	-0.1%
$\tau \to \mu ee\nu\bar{\nu}$	$1.9891\,(1) \times 10^{-5}$	$4.4\,(1) \times 10^{-8}$	-6.6×10^{-10}	0.2%
$\tau \to e\mu\mu\nu\bar{\nu}$	$1.2513\,(6) \times 10^{-7}$	$2.70\,(1) \times 10^{-9}$	-3.6×10^{-10}	1.8%
$\tau \to \mu\mu\mu\nu\bar{\nu}$	$1.1837\,(1) \times 10^{-7}$	$2.276\,(2) \times 10^{-9}$	-3.5×10^{-10}	1.6%
$\mu \to eee\nu\bar{\nu}$	$3.6054\,(1) \times 10^{-5}$	$-6.69\,(5) \times 10^{-8}$	-1.8×10^{-11}	0.2%

Tau lifetime uncertainty:

$$\delta\tau_\tau/\tau_\tau = 1.7 \times 10^{-3}$$

Shift of the fine structure constant:

$$\Delta\alpha(4m_\mu^2) = 6 \times 10^{-3}$$

	$\mathcal{B}_{\rm LO}$	$\delta\mathcal{B}_{\rm NLO,QED}$	$\delta\mathcal{B}_{\rm NLO,had}$	$\delta\mathcal{B}/\mathcal{B}$
$\tau \to ee e\nu\bar{\nu}$	$4.2488\,(4) \times 10^{-5}$	$-4.2\,(1) \times 10^{-8}$	-1.0×10^{-9}	-0.1%
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$\mu \to eee\nu\bar{\nu}$	$3.6054\,(1) \times 10^{-5}$	$-6.69\,(5) \times 10^{-8}$	-1.8×10^{-11}	0.2%

$$\mathcal{B}_{\rm exp}(\mu^- \to e^+ e^- e^- \nu_\mu \bar{\nu}_e) = 3.4\,(4) \times 10^{-5}$$

SINDRUM, NPB 260 (1985) 1.

$$\mathcal{B}_{\rm exp}(\tau \to e e^- e^+ \nu\bar{\nu}) = 2.8\,(1.5) \times 10^{-5}$$

$$\mathcal{B}_{\rm exp}(\tau \to \mu e^- e^+ \nu\bar{\nu}) < 3.2 \times 10^{-5} \text{ at } 95\% \text{ CL}$$

CLEO, PRL 76 (1996) 2637.

Study of five-body leptonic decays of tau at Belle experiment

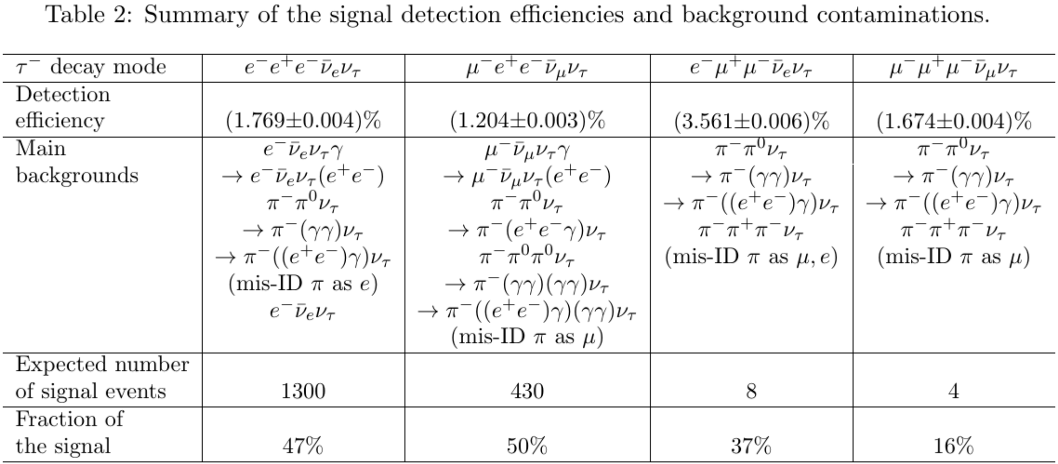


Table 2: Summary of the signal detection efficiencies and background contaminations.

τ^- decay mode	$e^-e^+e^-\bar{\nu}_e\nu_\tau$	$\mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$	$e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$	$\mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$
Detection efficiency	$(1.769\pm0.004)\%$	$(1.204\pm0.003)\%$	$(3.561\pm0.006)\%$	$(1.674\pm0.004)\%$
Main backgrounds	$e^-\bar{\nu}_e\nu_\tau\gamma$ $\rightarrow e^-\bar{\nu}_e\nu_\tau(e^+e^-)$ $\pi^-\pi^0\nu_\tau$ $\rightarrow \pi^-(\gamma\gamma)\nu_\tau$ $\rightarrow \pi^-((e^+e^-)\gamma)\nu_\tau$ (mis-ID π as e) $e^-\bar{\nu}_e\nu_\tau$	$\mu^-\bar{\nu}_\mu\nu_\tau\gamma$ $\rightarrow \mu^-\bar{\nu}_\mu\nu_\tau(e^+e^-)$ $\pi^-\pi^0\nu_\tau$ $\rightarrow \pi^-(e^+e^-\gamma)\nu_\tau$ $\pi^-\pi^0\pi^0\nu_\tau$ $\rightarrow \pi^-(\gamma\gamma)(\gamma\gamma)\nu_\tau$ $\rightarrow \pi^-((e^+e^-)\gamma)(\gamma\gamma)\nu_\tau$ (mis-ID π as μ)	$\pi^-\pi^0\nu_\tau$ $\rightarrow \pi^-(\gamma\gamma)\nu_\tau$ $\rightarrow \pi^-((e^+e^-)\gamma)\nu_\tau$ $\pi^-\pi^+\pi^-\nu_\tau$ (mis-ID π as μ, e)	$\pi^-\pi^0\nu_\tau$ $\rightarrow \pi^-(\gamma\gamma)\nu_\tau$ $\rightarrow \pi^-((e^+e^-)\gamma)\nu_\tau$ $\pi^-\pi^+\pi^-\nu_\tau$ (mis-ID π as μ)
Expected number of signal events	1300	430	8	4
Fraction of the signal	47%	50%	37%	16%

J. Sasaki (Belle) J.Phys.Conf.Ser. 912 (2017) 012002

Acceptance cuts $\tau \to ee e\nu\bar{\nu}$

$$\begin{cases} \sum_{i<j} \cos\theta_{ij} > 2.90 \\ n_\gamma \leq 1 \text{ with } \sum_i E^*_\gamma \leq 0.5\,\mathrm{GeV} \end{cases}$$

Acceptance cuts $\tau \to \mu e e\nu\bar{\nu}$

$$\begin{cases} \sum_{i<j} \cos\theta_{ij} > 2.93 \\ n_\gamma \leq 5 \text{ with } \sum_i E^*_\gamma \leq 0.3\,\mathrm{GeV} \end{cases}$$

J. Sasaki (Belle) J.Phys.Conf.Ser. 912 (2017) 012002

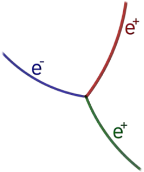
$$\mathcal{B}_{\mathrm{exp}} = \epsilon_{\mathrm{det}} \cdot \epsilon_{\mathrm{th}} \cdot N_{\mathrm{obs}},$$

- ϵ_{det}: detector efficiencies
- $\epsilon_{\mathrm{th}} = \Gamma^{\mathrm{total}}/\Gamma^{\mathrm{with\,cuts}}$

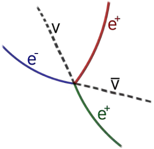
	$\tau \to eee\,\bar{\nu}\nu$	$\tau \to \mu ee\,\bar{\nu}\nu$
$\mathcal{B}_{\rm LO}$	$4.2488(4) \cdot 10^{-5}$	$1.9891(1) \cdot 10^{-5}$
$\mathcal{B}_{\rm NLO}$	$4.2445(4) \cdot 10^{-5}$	$1.9934(1) \cdot 10^{-5}$
$\epsilon^{\rm th}_{\rm LO}$	1.5145(2)	2.4370(2)
$\epsilon^{\rm th}_{\rm NLO}$	1.5492(9)	2.4571(5)
$\epsilon' = \epsilon_{\rm NLO}/\epsilon_{\rm LO}$	1.0229(6)	1.0082(2)

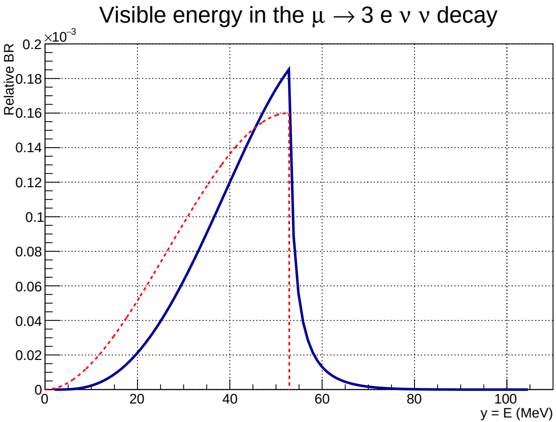
Searching for CLFV with Mu3e

Signal: $\mu \to eee$



Background: $\mu \to eee\nu\bar{\nu}$

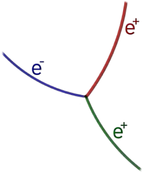




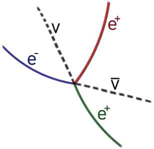
Calibbi, Signorelli, Riv.Nuovo Cim. 41 (2018) 1

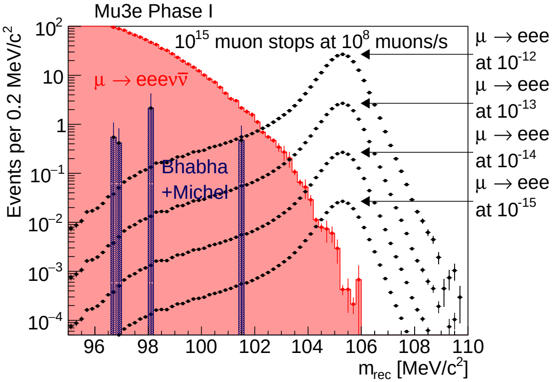
Searching for CLFV with Mu3e

Signal: $\mu \to eee$

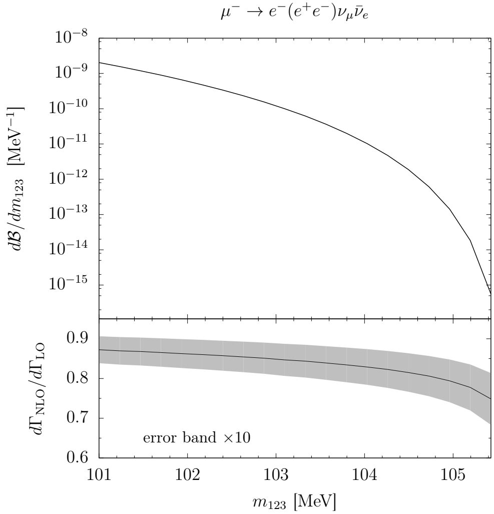


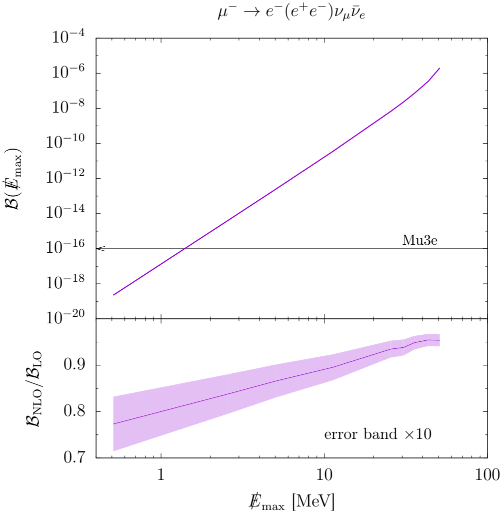
Background: $\mu \to eee\nu\bar{\nu}$





A. Perrevoort (Mu3e), 1802.09851 [physics.ins-det]



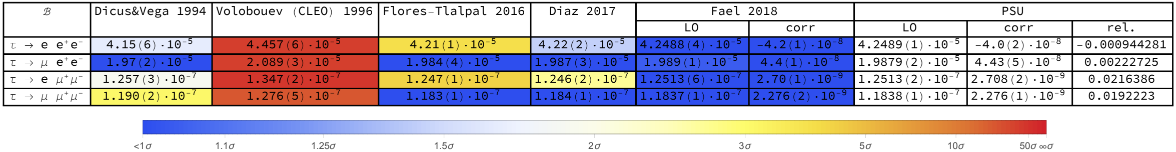


C. Greub & MF, JHEP 1701 (2017) 084.

Conclusions

- Two independent Monte Carlo programs are available for $\tau \to \ell\nu\bar{\nu}\gamma$ and $\tau \to \ell\nu\bar{\nu}\ell'\ell'$.
- Corrections to $\mathcal{B}(\tau \to \ell\nu\bar{\nu}\gamma)$ are of order $3 - 10\%$.
- Corrections to $\mathcal{B}(\tau \to \ell\nu\bar{\nu}\ell'\ell')$ are of order $0.1 - 1\%$.
- Detector acceptance or particularly stringent cuts can easily enhance radiative corrections at the 10 % level.
- Monte Carlo generators at NLO are mandatory to experimental analysis aiming at 1 % accuracy.

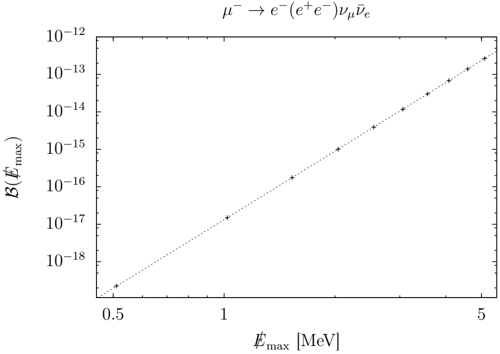
Backup



$\mathcal{B}$	Dicus&Vega 1994	Volobouev (CLEO) 1996	Flores-Tlalpal 2016	Diaz 2017	Fael 2018		PSU		
					LO	corr	LO	corr	rel.
$\tau \to e\ e^+e^-$	$4.15(6)\cdot10^{-5}$	$4.457(6)\cdot10^{-5}$	$4.21(1)\cdot10^{-5}$	$4.22(2)\cdot10^{-5}$	$4.2488(4)\cdot10^{-5}$	$-4.2(1)\cdot10^{-8}$	$4.2489(1)\cdot10^{-5}$	$-4.0(2)\cdot10^{-8}$	-0.000944281
$\tau \to \mu\ e^+e^-$	$1.97(2)\cdot10^{-5}$	$2.089(3)\cdot10^{-5}$	$1.984(4)\cdot10^{-5}$	$1.987(3)\cdot10^{-5}$	$1.989(1)\cdot10^{-5}$	$4.4(1)\cdot10^{-8}$	$1.9879(2)\cdot10^{-5}$	$4.43(5)\cdot10^{-8}$	0.00222725
$\tau \to e\ \mu^+\mu^-$	$1.257(3)\cdot10^{-7}$	$1.347(2)\cdot10^{-7}$	$1.247(1)\cdot10^{-7}$	$1.246(2)\cdot10^{-7}$	$1.2513(6)\cdot10^{-7}$	$2.70(1)\cdot10^{-9}$	$1.2513(2)\cdot10^{-7}$	$2.708(2)\cdot10^{-9}$	0.0216386
$\tau \to \mu\ \mu^+\mu^-$	$1.190(2)\cdot10^{-7}$	$1.276(5)\cdot10^{-7}$	$1.183(1)\cdot10^{-7}$	$1.184(1)\cdot10^{-7}$	$1.1837(1)\cdot10^{-7}$	$2.276(2)\cdot10^{-9}$	$1.1838(1)\cdot10^{-7}$	$2.276(1)\cdot10^{-9}$	0.0192223

table by Y. Ulrich

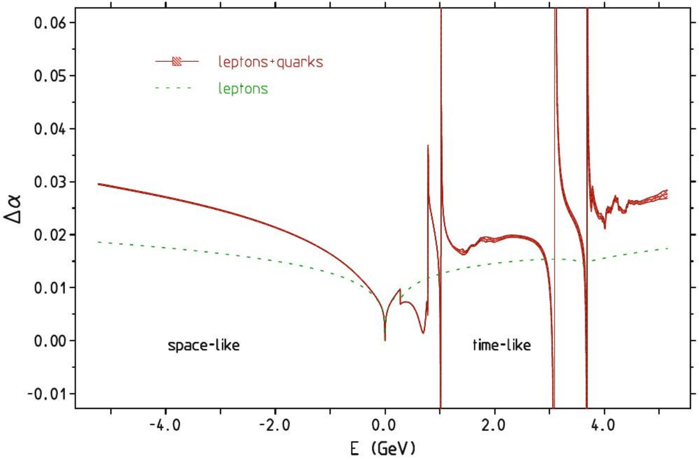
	MF,Greub	Pruna, Signer, Ulrich
Full mass dependence	✓	✓
Decaying μ	unpolarized	polarized
One-loop	LoopTools, Collier	GoSam
IR	PS slicing, dipoles	FKS
Phase space	analytic integration νs PS	fully differential
Had. corrections	✓	×



Fit:

$$\mathcal{B}(\not{E}_{\mathrm{max}}) = \kappa \left(\frac{\not{E}_{\mathrm{max}}}{m_e} \right)^{\gamma}$$

- $\kappa_{\mathrm{NLO}} = 2.217\,(2) \times 10^{-19}$
- $\gamma_{\mathrm{NLO}} = 6.0768\,(4)$



F. Jegerlehner, The anomalous magnetic moment of the muon (2nd Ed.), Springer.