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Strange resonances from analyticity and dispersion relations

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In collaboration with
A. Rodas, J. Ruiz de Elvira

Phys.Rev. D93 (2016) no.7, 074025
Eur.Phys.J. C77 (2017) no.2, 91
Eur.Phys.J. C78 (2018) no.11, 897
and work in preparation

International Workshop on e+e- collisions from Phi to Psi 25th-February 1st-March, 2019

Motivation to study πK scattering with Dispersion Relations

- π,K appear as final products of almost all hadronic strange processes:
  B,D, decays, CP violation studies… many examples in this workshop
- π,K are Goldstone Bosons of QCD → Test Chiral Symmetry Breaking
- Main or relevant source for PDG parameters of:
  K_0^* (700), $K_0^*(1430)$,$K_1^*(892)$,$K_1^*(1410)$,$K_2^*(1410)$,$K_3^*(1780)$
- Extracted frequently with strong model dependences (Breit Wigners,….)

Analytic Methods reduce model independence
Dispersion Relations model **in**dependent

Particularly controversial:
κ/ K_0^* (700) light scalar meson. "needs confirmation" @PDG.
Light scalar mesons **longstanding candidates for non-ordinary mesons**.
Needed to identify the lightest scalar nonet

Was $K_0^*(800)$ until last 2018 PDG revision!
Partly triggered by our 2017 work

Overview of the $K_0^*(700)$ or "kappa" meson until 2018 @PDG

- Omittted from the 2018PDG summary table since, "needs confirmation"

  All descriptions of data respecting unitarity and chiral symmetry find a pole at M=650-770 MeV and Γ~550 MeV or larger.

  Best determination comes from a SOLUTION of a Roy-Steiner dispersive formalism, consistent with UChPT

  Decotes Genon et al 2006

2017PDG $\mathbf{K_0^*(700)}$ dominated by such a SOLUTION

$M - i\,\Gamma/2 = (682 \pm 29) - i(273 \pm i12)$ MeV

New PDG2018:

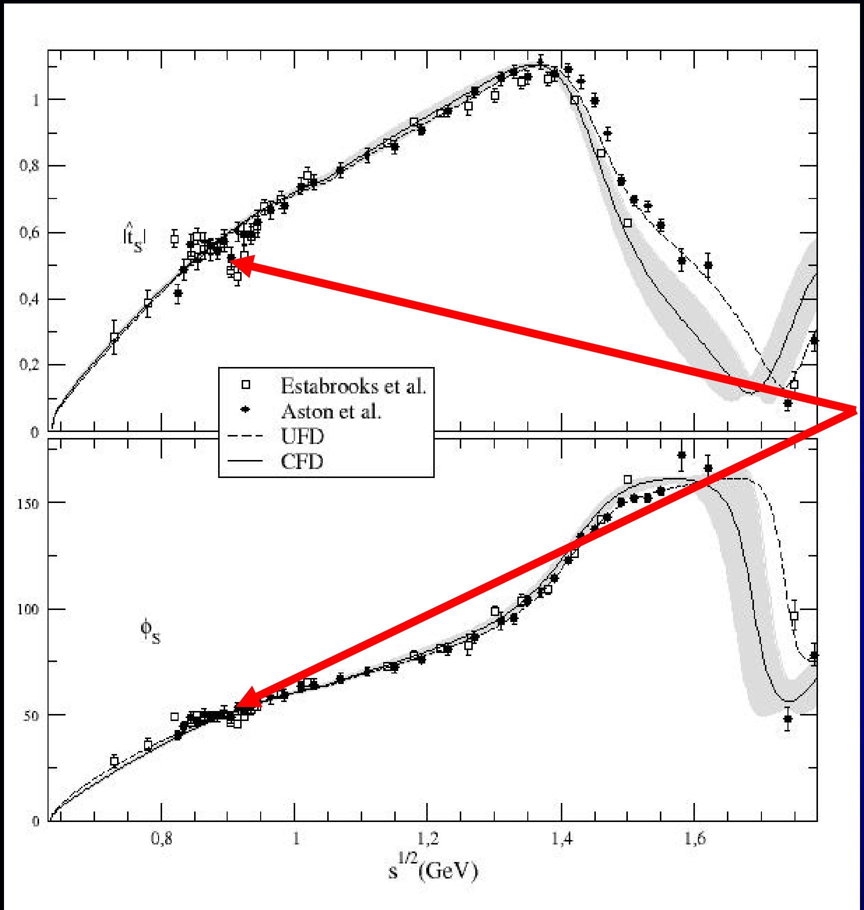
(630-730)-i(260-340) MeV name changed to $\mathbf{K_0^*(700)}$

But still "Needs Confirmation"

PDG may reconsider situation.. **if** additional independent dispersive DATA analysis.

We were encouraged by PDG members to do it.

Data on πK scattering: S-channel



Most reliable sets:
Estabrooks et al. 78 (SLAC)
Aston et al.88 (SLAC-LASS)

I=1/2 and 3/2 combination

No clear "peak" or phase movement of $\kappa/K_0^*(700)$ resonance

Definitely NO BREIT-WIGNER shape

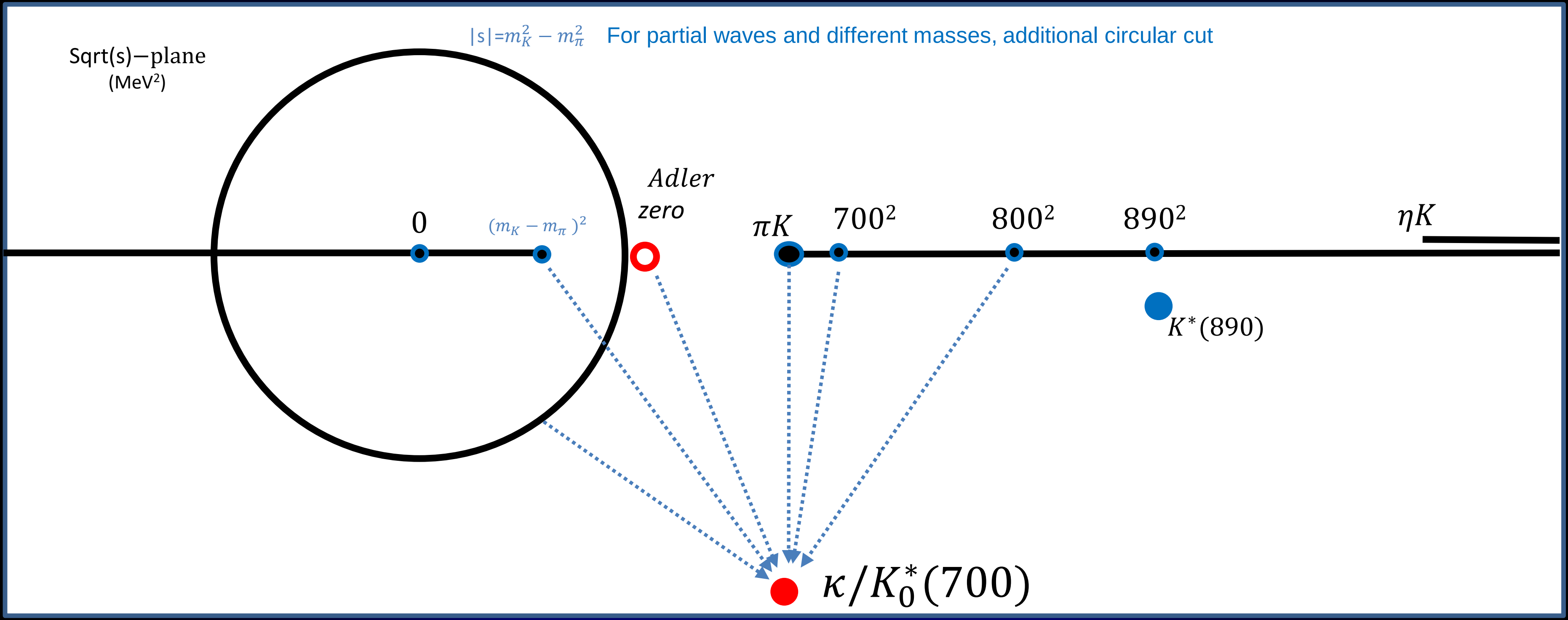
Mathematically correct to use POLES

Strong support for K_0*(700) from decays of heavier mesons, but rigorous model-independent extractions absent. Often inadequate Breit-Wigner formalism

POLE extraction rigorous when using Dispersion Relations or complex-analyticity properties

Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

Analyticity is expressed in the *s*-variable, not in Sqrt(s)



Important for the $\kappa/K_0^*(700)$

- Threshold behavior (chiral symmetry)
- Subthreshold behavior (chiral symmetry →Adler zeros)
- Other cuts (Left & circular)
- Avoid spurious singularities

Less important for other resonances…

Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

- As πK checks: Small inconsistencies.

Forward dispersion relations for K π scattering.

Since interested in the resonance region, we use minimal number of subtractions

Defining the s↔u symmetric and anti-symmetric amplitudes at t=0

$$T^{+}(s)=\frac{T^{1/2}(s)+2T^{3/2}(s)}{3}=\frac{T^{I_t=0}(s)}{\sqrt{6}},$$
$$T^{-}(s)=\frac{T^{1/2}(s)-T^{3/2}(s)}{3}=\frac{T^{I_t=1}(s)}{2}.$$

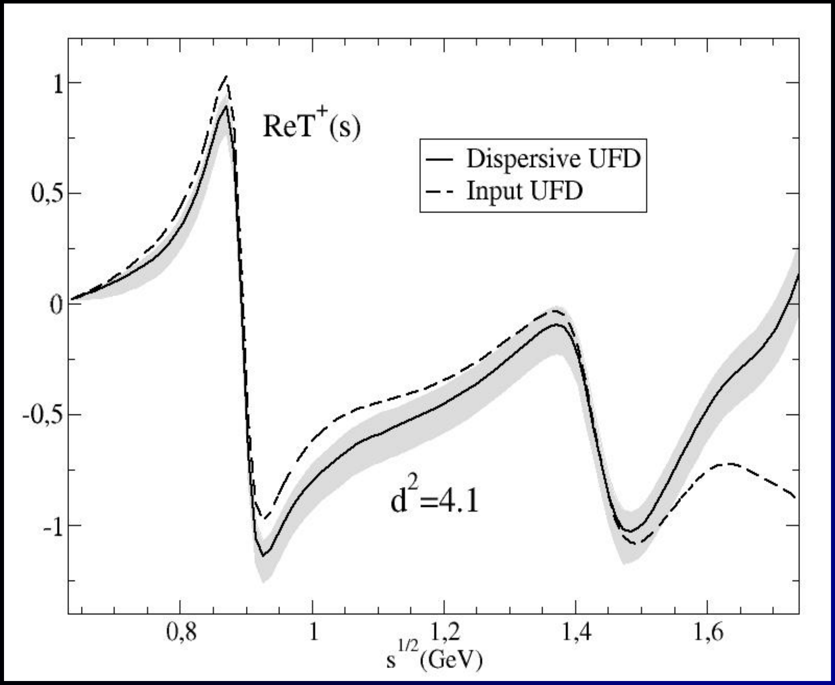
We need one subtraction for the symmetric amplitude

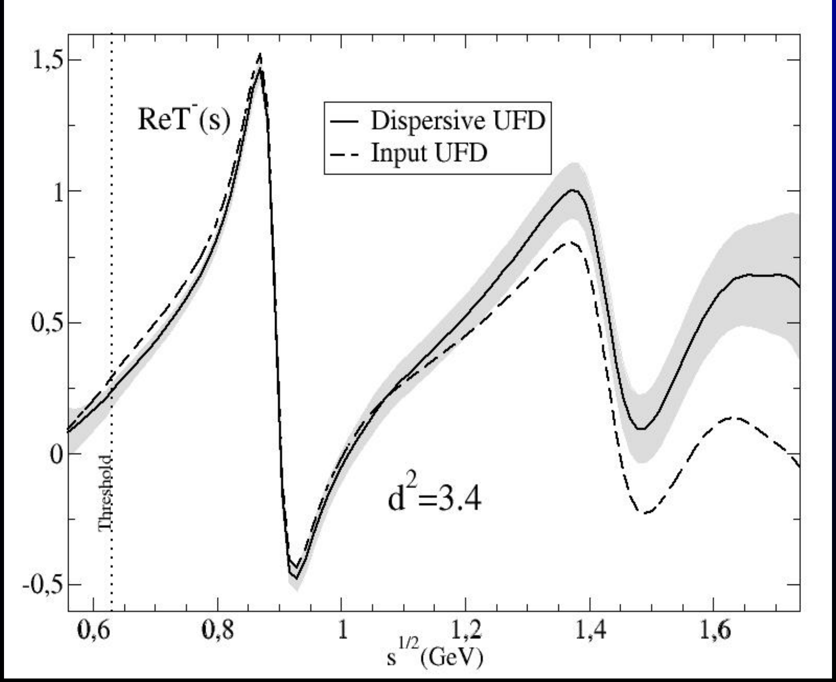
$$\mathrm{Re}T^{+}(s)=T^{+}(s_{\rm th})+\frac{(s-s_{\rm th})}{\pi}P\int_{s_{\rm th}}^{\infty}ds'\left[\frac{\mathrm{Im}T^{+}(s')}{(s'-s)(s'-s_{\rm th})}-\frac{\mathrm{Im}T^{+}(s')}{(s'+s-2\Sigma_{\pi K})(s'+s_{\rm th}-2\Sigma_{\pi K})}\right],$$

And none for the antisymmetric

$$\mathrm{Re}T^{-}(s)=\frac{(2s-2\Sigma_{\pi K})}{\pi}P\int_{s_{\rm th}}^{\infty}ds'\frac{\mathrm{Im}T^{-}(s')}{(s'-s)(s'+s-2\Sigma_{\pi K})}.$$

where $\Sigma_{\pi K}=m_\pi^2+m_K^2$





Forward Dispersion Relation analysis of πK scattering DATA up to 1.6 GeV

(**not a solution** of dispersión relations, but a constrained fit)

A.Rodas & JRP, PRD93,074025 (2016)

First observation:
Forward Dispersion relations
Not well satisfied by data
Particularly at high energies

So we use
Forward Dispersion Relations
as CONSTRAINTS on fits

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- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)

How well Forward Dispersion Relations are satisfied by unconstrained fits

Every 22 MeV calculate the difference between both sides of the DR /uncertainty

Define an averaged χ^2 over these points, that we call d^2

d^2 close to 1 means that the relation is well satisfied

$d^2 \gg 1$ means the data set is inconsistent with the relation.

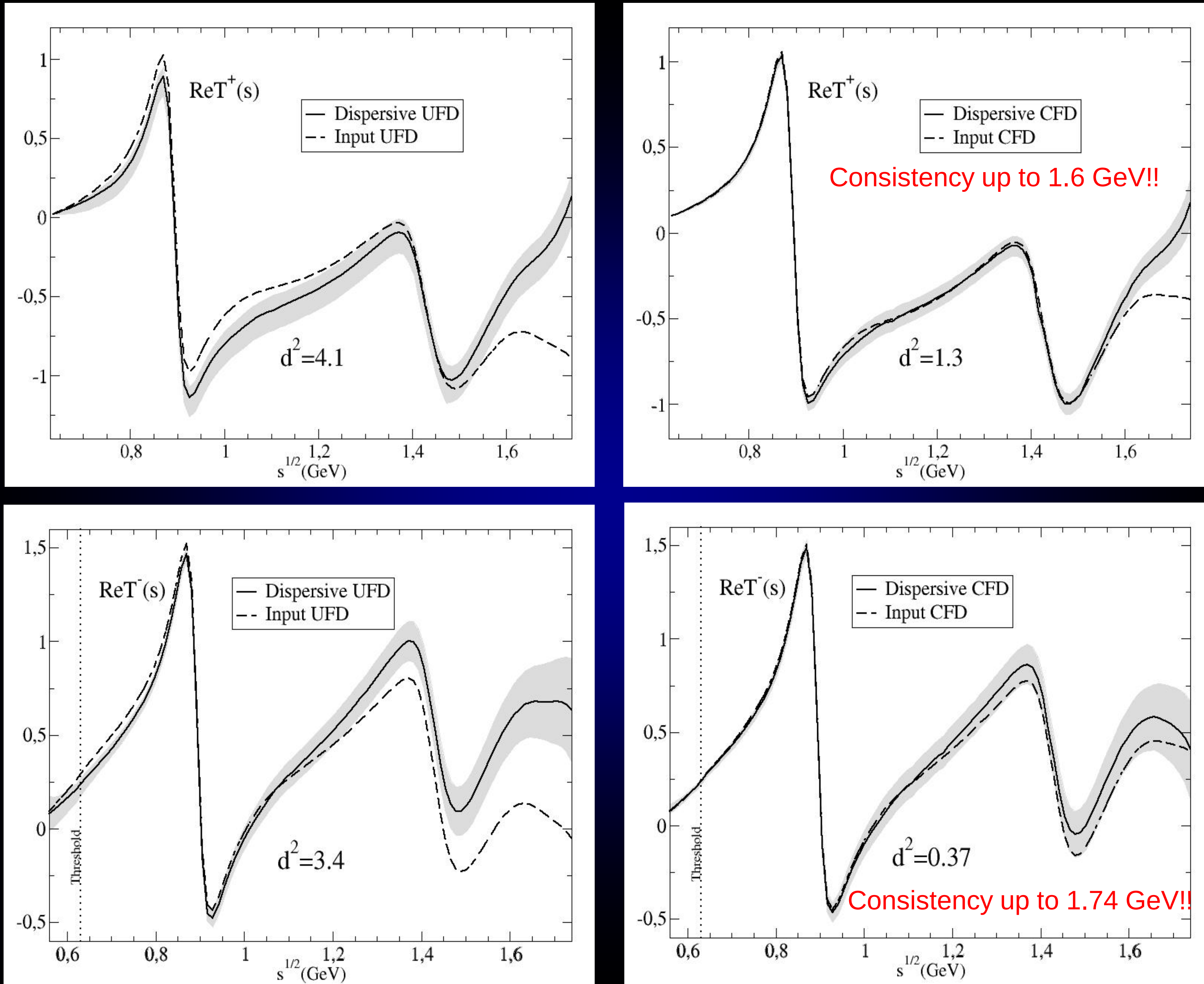
This can be used to check DR

To obtain CONSTRAINED FITS TO DATA (CFD) we minimize:

$$W^2(d^2_{I^+} + d^2_{I^-}) + \sum_{I=\frac{1}{2},\frac{3}{2}} \left(\frac{\Delta_I}{\delta\Delta_I}\right)^2 + \sum_k \left(\frac{p_k^{UFD} - p_k}{\delta p_k^{UFD}}\right)^2,$$

2 FDR's — Sum Rules threshold — Parameters of the unconstrained data fits

W roughly counts the number
of effective degrees of freedom
(sometimes we add weight on certain energy regions)

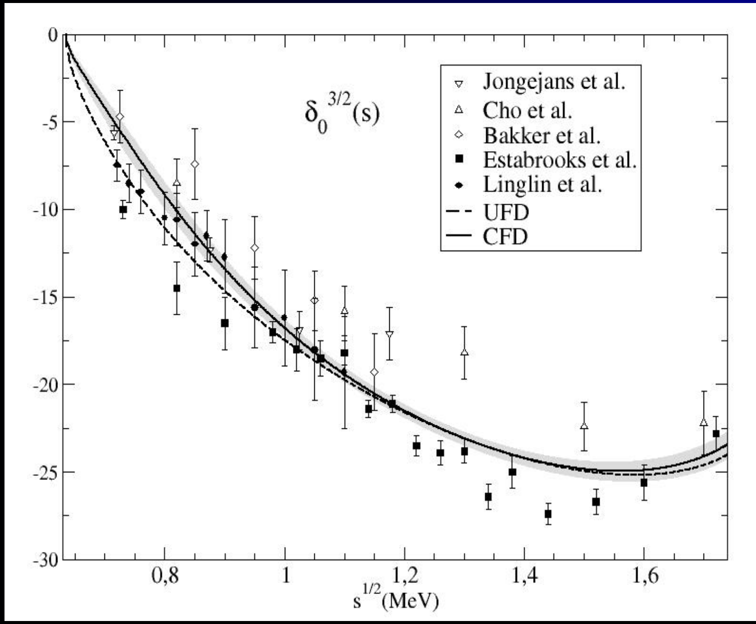

ReT+(s)
Dispersive UFD
Input UFD
d2=4.1
s1/2(GeV)
ReT+(s)
Dispersive CFD
Input CFD
Consistency up to 1.6 GeV!!
d2=1.3
s1/2(GeV)
ReT-(s)
Dispersive UFD
Input UFD
Threshold
d2=3.4
s1/2(GeV)
ReT-(s)
Dispersive CFD
Input CFD
Threshold
d2=0.37
Consistency up to 1.74 GeV!!
s1/2(GeV)

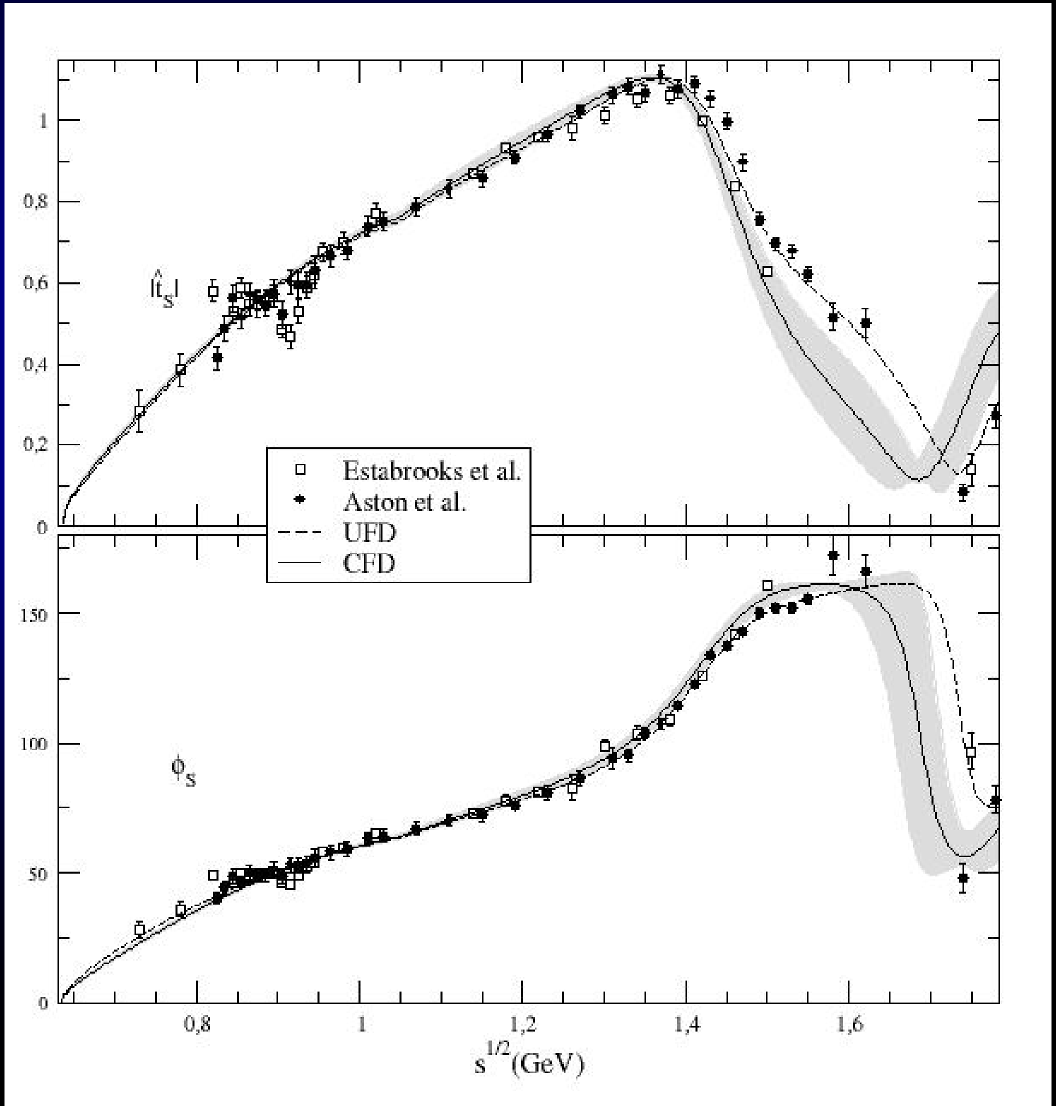
From Unconstrained (UFD) to Constrained Fits to data (CFD)

S-waves. The most interesting for the K_0^* resonances

Largest changes from UFD to CFD

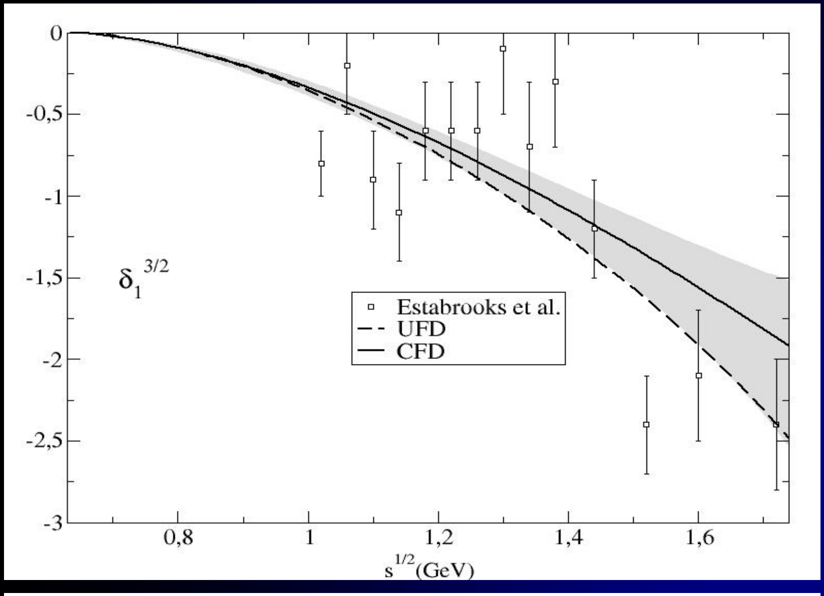
at higher energies

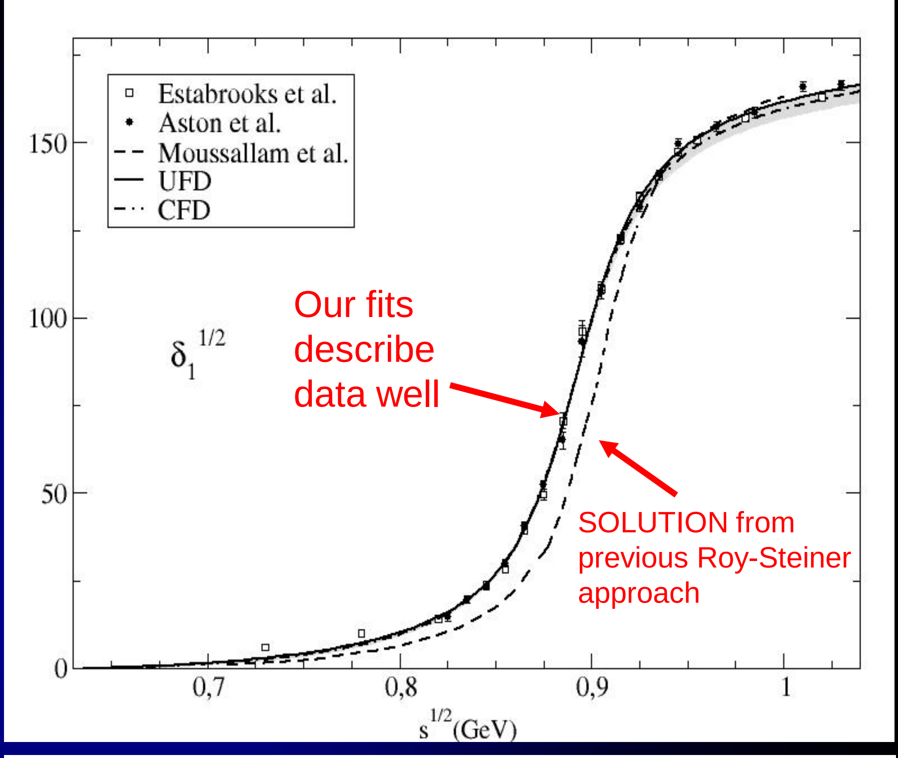


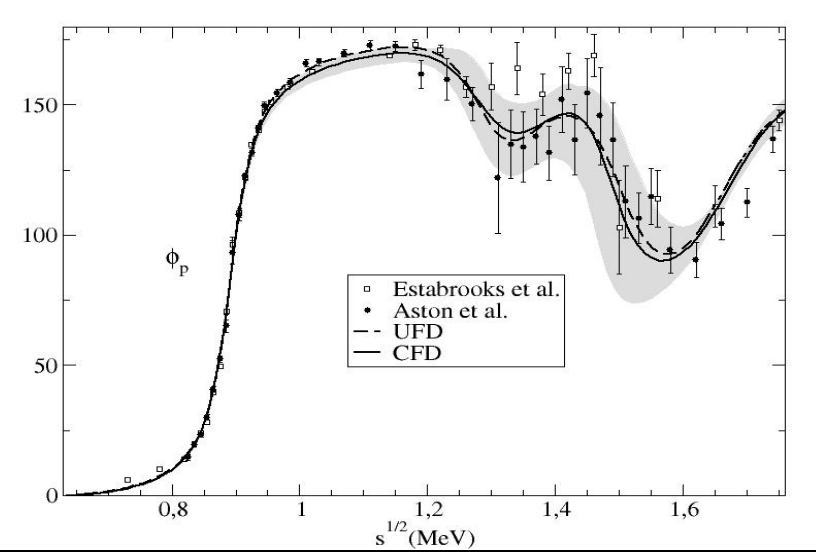


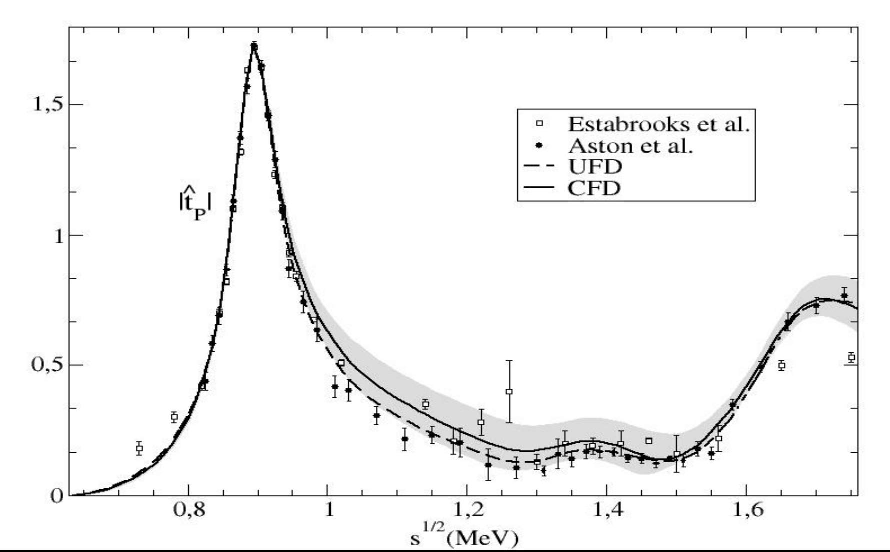
From Unconstrained (UFD) to Constrained Fits to data (CFD)

P-waves: Small changes



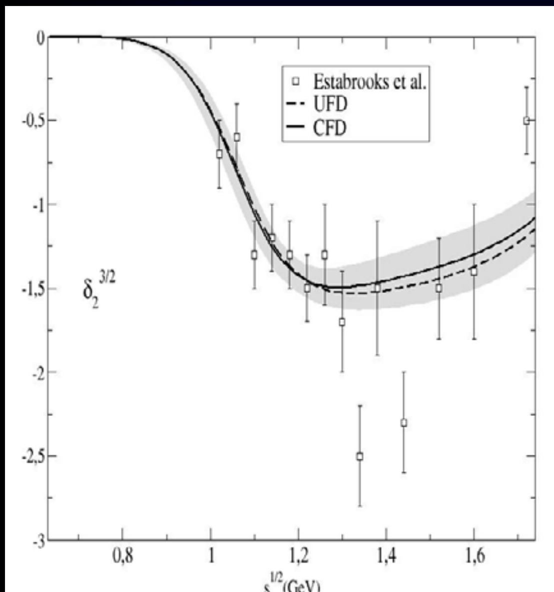


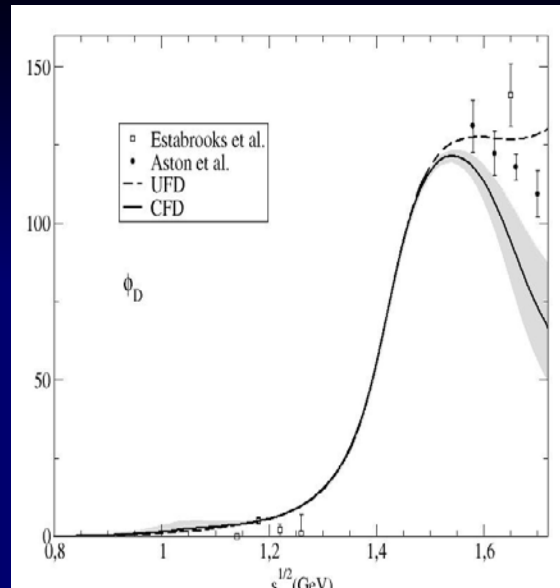


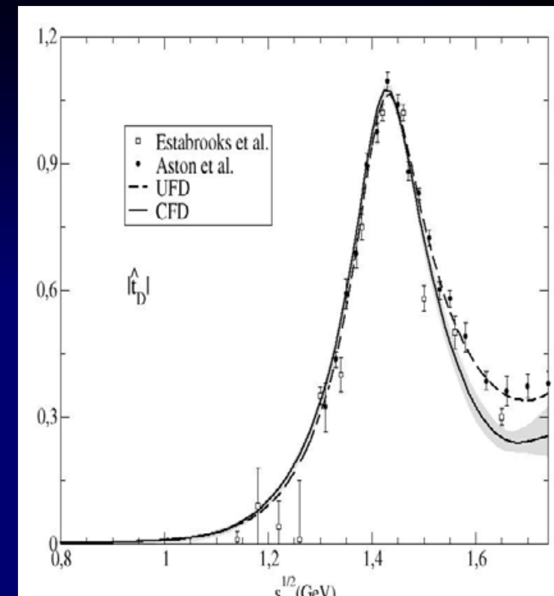


From Unconstrained (UFD) to Constrained Fits to data (CFD)

D-waves: Largest changes of all, but at very high energies

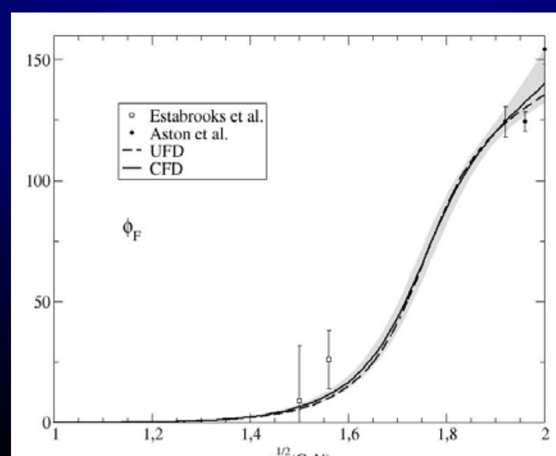


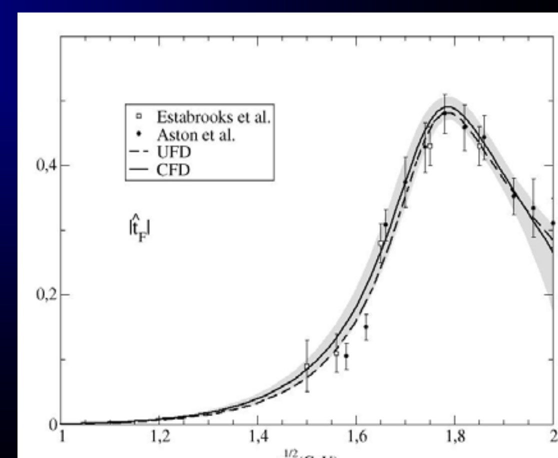




F-waves:

Imperceptible changes





Regge parameterizations allowed to vary: Only πK-ρ residue changes by 1.4 deviations

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  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

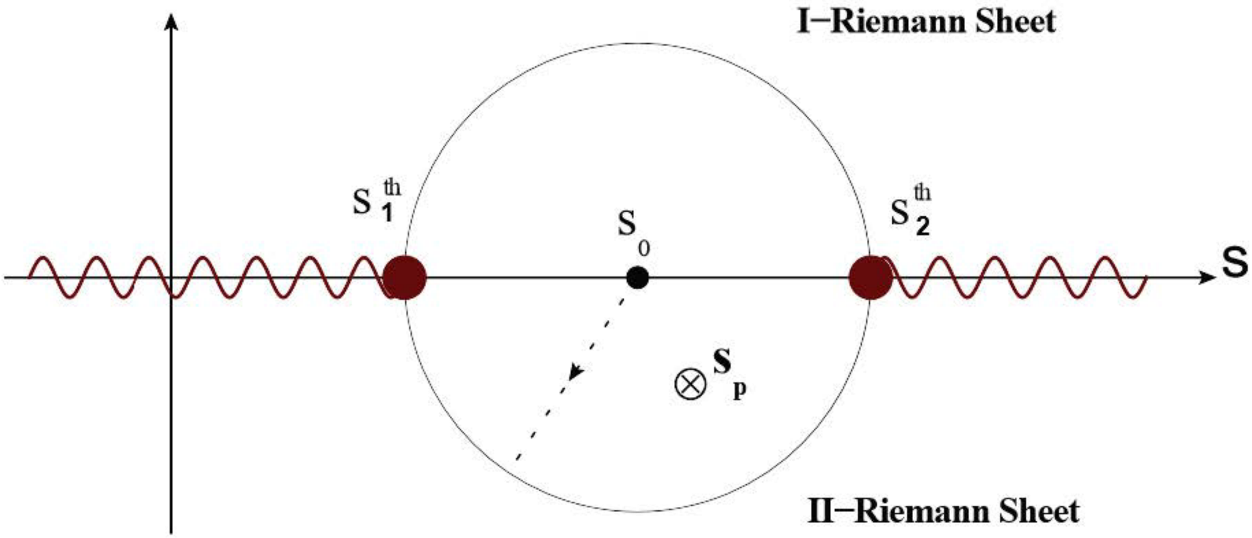
Strange resonance poles from CFD: Using Padé sequences

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

Almost model independent: Does not assume any particular functional form
(but local determination)

Based on previous works by P.Masjuan, J.J. Sanz Cillero, I. Caprini, J.Ruiz de ELvira

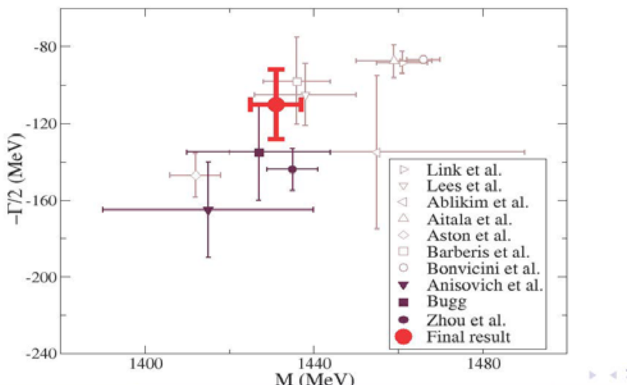
- The method is suitable for the calculation of both elastic and inelastic resonances.
- The Padé sequence gives us the continuation to the continuous Riemann Sheet.
- We take care of the calculation of the errors. Apart from the experimental and systematic errors of each parameterization we also include different fits.



The method can be used for inelastic resonances too. Provides resonance parameters WITHOUT ASSUMING SPECIFIC FUNCTIONAL FORM

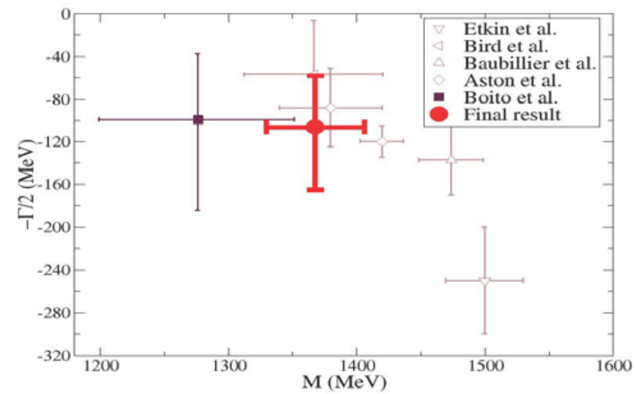
- For the $K_0^*(1430)$ we find

$$\sqrt{s_p} = (1431 \pm 6) - i(110 \pm 19) MeV$$
$$\sqrt{s_p} = (1425 \pm 50) - i(135 \pm 40) MeV (PDG)$$



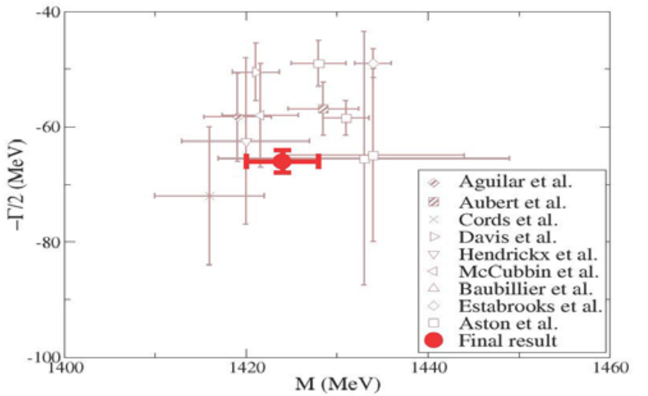
- For the $K_1^*(1410)$ we find

$$\sqrt{s_p} = (1368 \pm 38) - i(106^{+48}_{-59}) MeV$$
$$\sqrt{s_p} = (1414 \pm 15) - i(116 \pm 10) MeV (PDG)$$



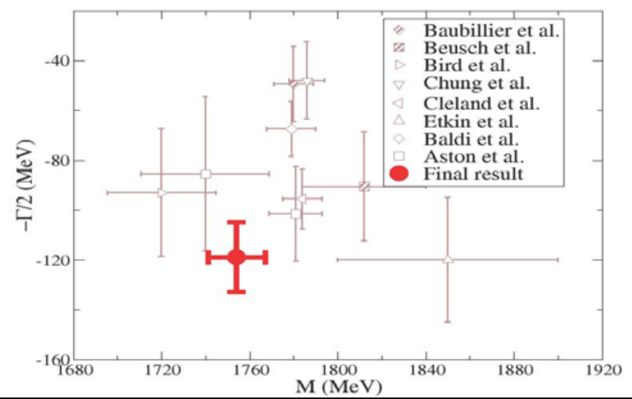
- For the $K_2^*(1430)$ we find

$$\sqrt{s_p} = (1424 \pm 4) - i(66 \pm 2) MeV$$
$$\sqrt{s_p} = (1432.4 \pm 1.3) - i(55 \pm 3) MeV (PDG)$$



- For the $K_3^*(1780)$ we find

$$\sqrt{s_p} = (1754 \pm 13) - i(119 \pm 14) MeV$$
$$\sqrt{s_p} = (1776 \pm 7) - i(80 \pm 11) MeV (PDG)$$



Kappa pole analytic determinations from constrained fits

1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

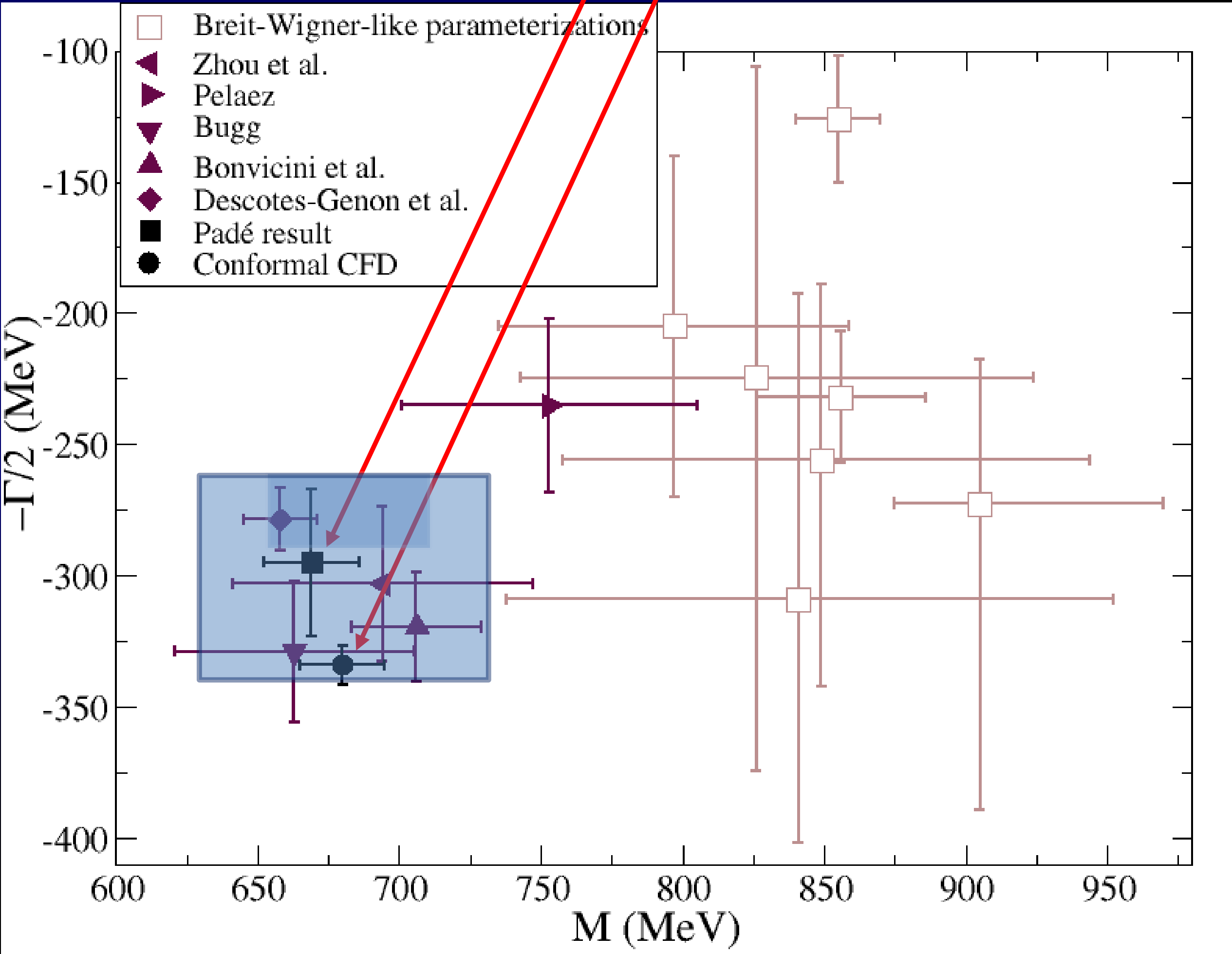
Fantastic analyticity properties, but not model independent

(680±15)-i(334±7.5) MeV

2) Using Padé Sequences…

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017) 77:91

(670±18)-i(295± 28) MeV



Compare to PDG2017:
(682±29)-i(273±12) MeV

New PDG2018:
(630-730)-i(260-340) MeV
And name changed
$K_0^*(700)$
Still "Needs Confirmation"

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  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits up to 1.5 GeV
  JRP, A.Rodas, Eur.Phys.J. C78 (2018)

ππ→KK HDR

g^I_J =ππ → KK partial waves. We study (I,J)=(0,0),(1,1),(0,2)

f^I_J = Kπ → Kπ partial waves. Taken from previous dispersive study

JRP, A. Rodas PRD 2016

$$g_0^0(t) = \frac{\sqrt{3}}{2} m_+ a_0^+ + \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{Im}\, g_0^0(t')}{t'(t'-t)}dt' + \frac{t}{\pi}\sum_{\ell\geq 2}\int_{4m_\pi^2}^\infty \frac{dt'}{t'} G^0_{0,2\ell-2}(t,t')\mathrm{Im}\, g^0_{2\ell-2}(t') + \sum_\ell \int_{m_+^2}^\infty ds' G^+_{0,\ell}(t,s')\mathrm{Im}\, f_\ell^+(s'),$$

$$g_1^1(t) = \frac{1}{\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{Im}\, g_1^1(t')}{t'-t}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^\infty dt' G^1_{1,2\ell-1}(t,t')\mathrm{Im}\, g^1_{2\ell-1}(t') + \sum_\ell \int_{m_+^2}^\infty ds' G^-_{1,\ell}(t,s')\mathrm{Im}\, f_\ell^-(s'),$$

$$g_2^0(t) = \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{Im}\, g_2^0(t')}{t'(t'-t)}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^\infty \frac{dt'}{t'} G'^0_{2,4\ell-2}(t,t')\mathrm{Im}\, g^0_{4\ell-2}(t') + \sum_\ell \int_{m_+^2}^\infty ds' G'^+_{2,\ell}(t,s')\mathrm{Im}\, f_\ell^+(s'). \quad (39)$$

$G^I_{J,J'}$(t,t') =integral kernels, depend on a parameter

Lowest # of subtractions. Odd pw decouple from even pw.

$$g_\ell^0(t) = \Delta_\ell^0(t) + \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{dt'}{t'}\frac{\mathrm{Im}\, g_\ell^0(t)}{t'-t}, \quad \ell = 0,2,$$

$$g_1^1(t) = \Delta_1^1(t) + \frac{1}{\pi}\int_{4m_\pi^2}^\infty dt' \frac{\mathrm{Im}\, g_1^1(t)}{t'-t}, \quad (40)$$

Δ(t) depend on higher waves or on Kπ→Kπ.

Integrals from 2π threshold !

Solve in descending J order

We have used models for higher waves, but give very small contributions

For unphysical region below KK threshold, we used Omnés function

$$\Omega_\ell^I(t) = \exp\left(\frac{t}{\pi}\int_{4m_\pi^2}^{t_m} \frac{\phi_\ell^I(t')dt'}{t'(t'-t)}\right),$$

$$\Omega_\ell^I(t) \equiv \Omega_{l,R}^I(t) e^{i\phi_\ell^I(t)\theta(t-4m_\pi^2)\theta(t_m-t)},$$

This is the form of our HDR: Roy-Steiner+Omnés formalism

$$g_0^0(t) = \Delta_0^0(t) + \frac{t\Omega_0^0(t)}{t_m - t}\left[\alpha + \frac{t}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{(t_m - t')\Delta_0^0(t')\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)} + \frac{t}{\pi}\int_{t_m}^{\infty} dt' \frac{(t_m - t')|g_0^0(t')|\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)}\right]$$

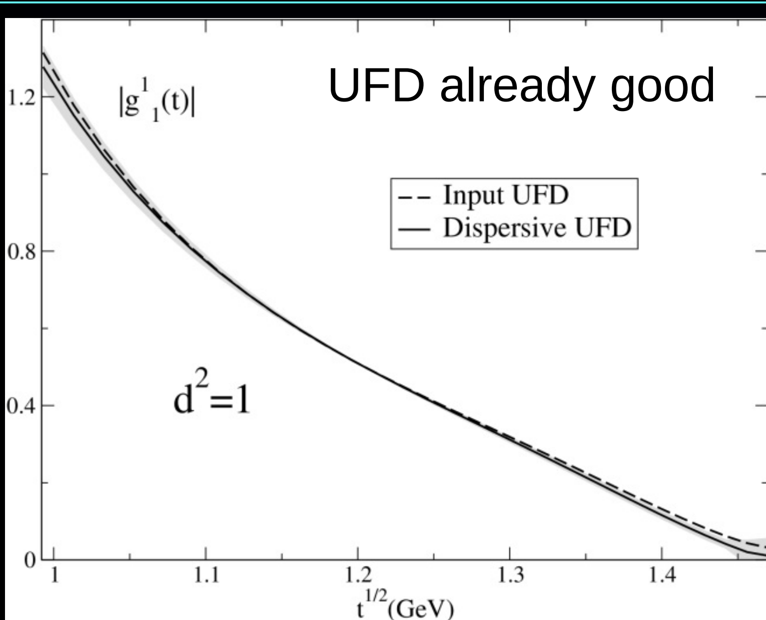
$$g_1^1(t) = \Delta_1^1(t) + \Omega_1^1(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_1^1(t')\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_1^1(t')|\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)}\right],$$

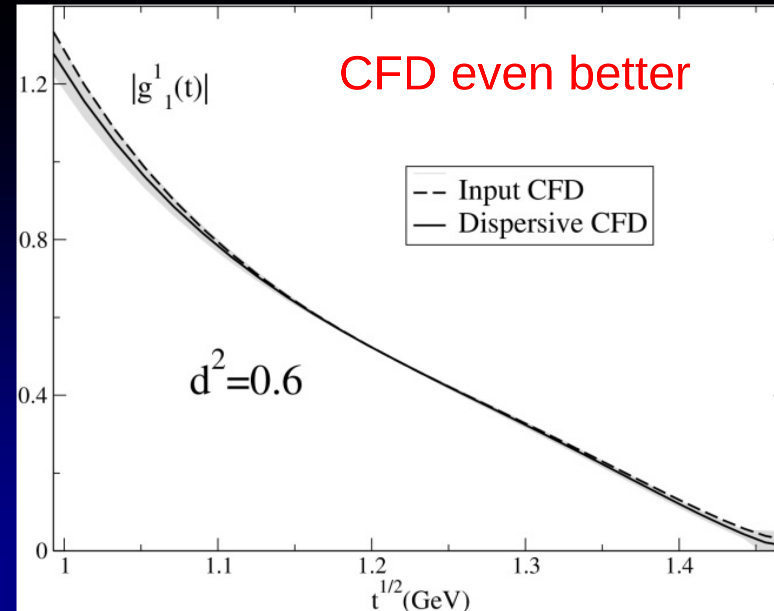
$$g_2^0(t) = \Delta_2^0(t) + t\Omega_2^0(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_2^0(t')\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_2^0(t')|\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)}\right].$$

We can now check how well these HDR are satisfied

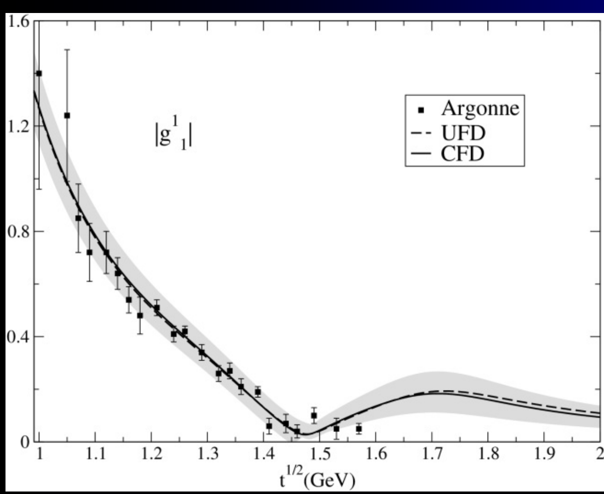
ππ→KK Hiperbolic Dispersion Relations I=1,J=1, UFD vs.CFD

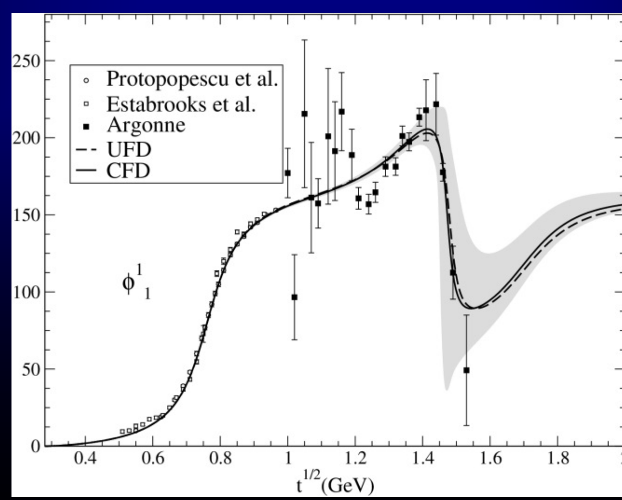
JRP, A.Rodas, Eur.Phys.J. C78 (2018)

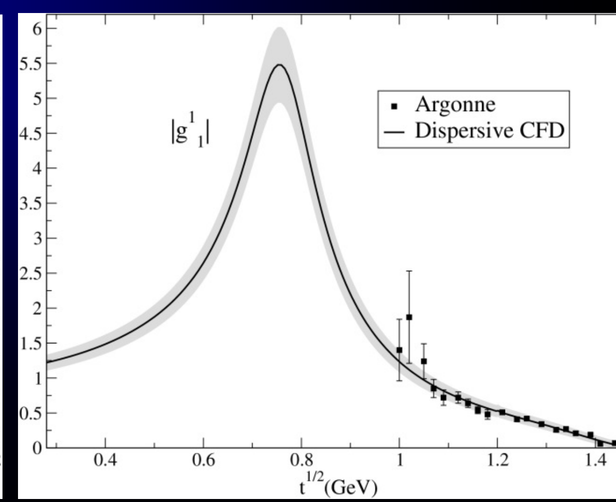




Requires almost imperceptible change from UFD to CFD

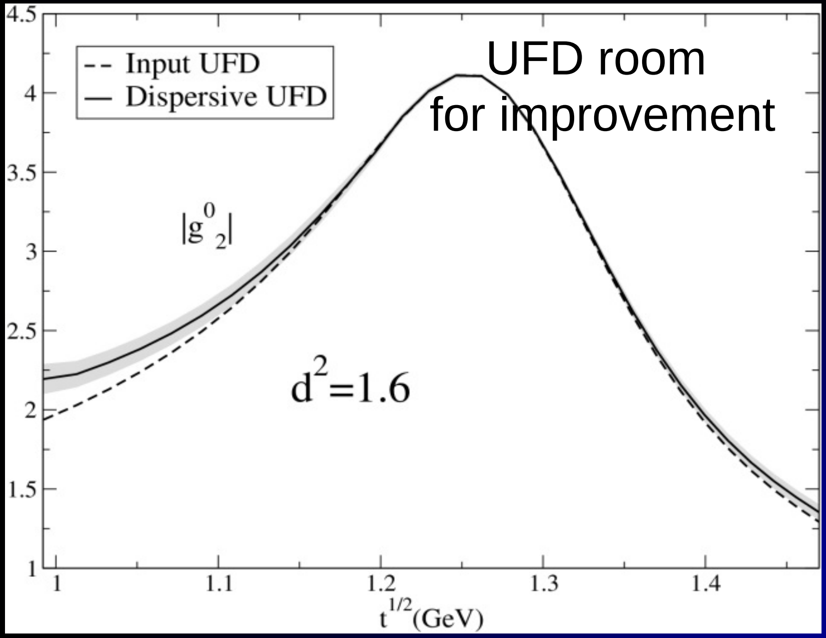


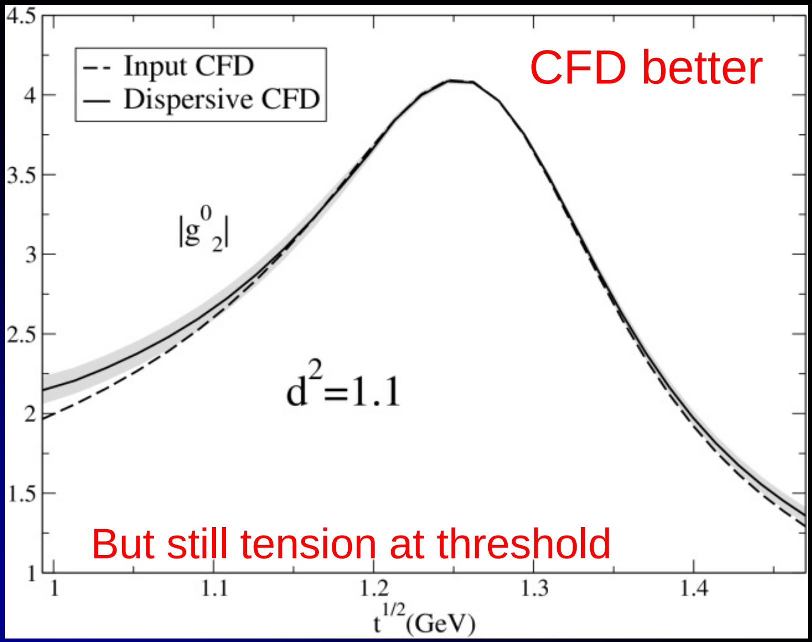




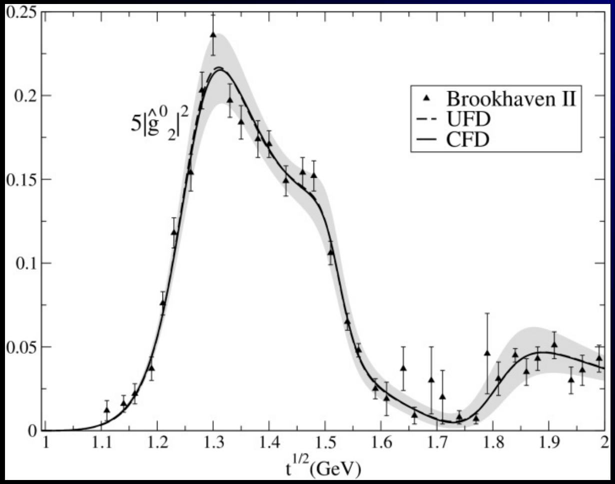
ππ→KK Hiperbolic Dispersion Relations I=2,J=2, UFD vs. CFD

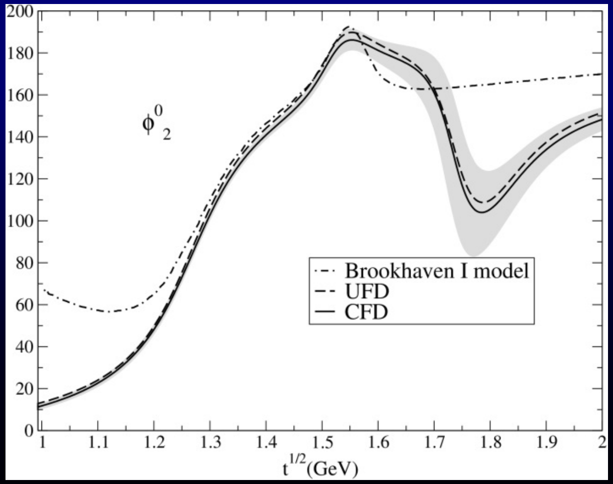
JRP, A.Rodas, Eur.Phys.J. C78 (2018)





Very small change from UFD to CFD. Only significant at threshold and high energies



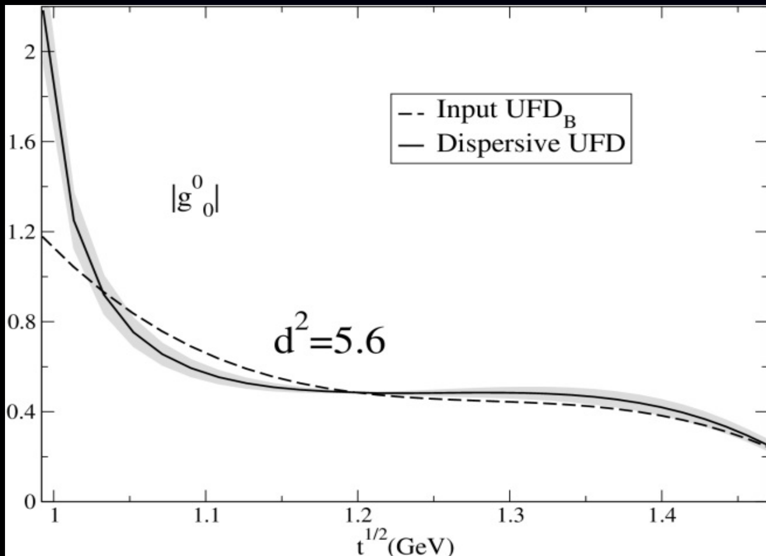


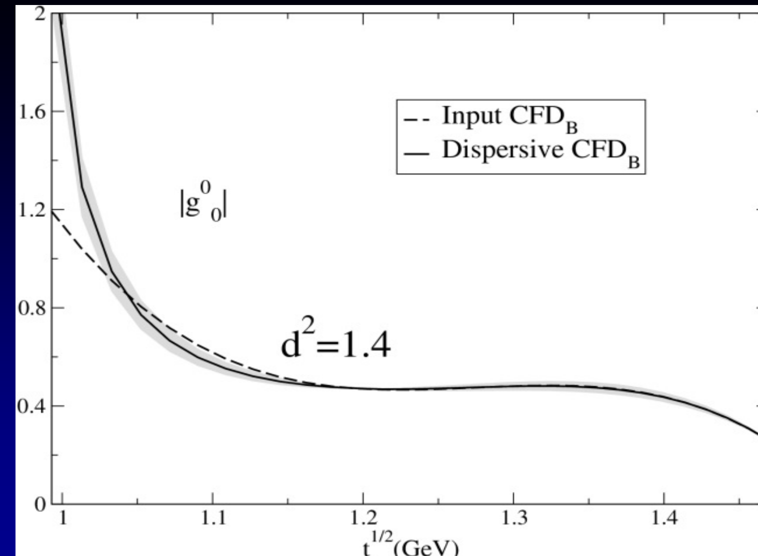
Other parameterizations (BW…), worse.

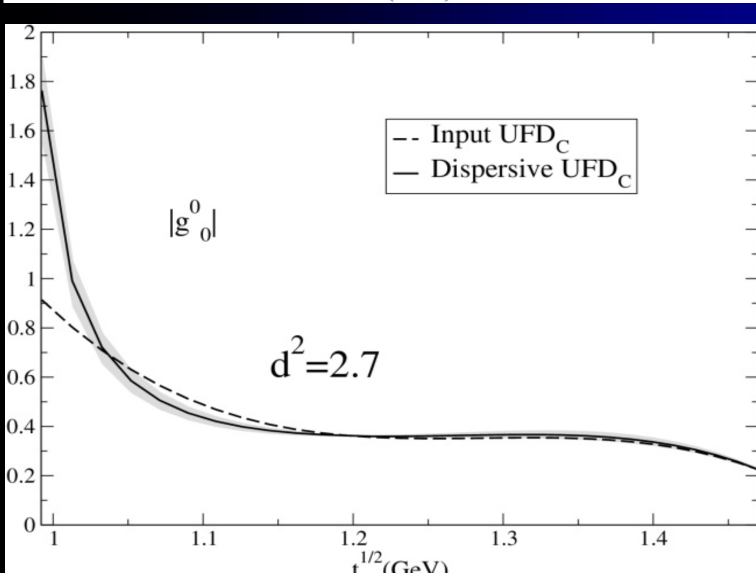
 JRP, A.Rodas, Eur.Phys.J. C78 (2018)

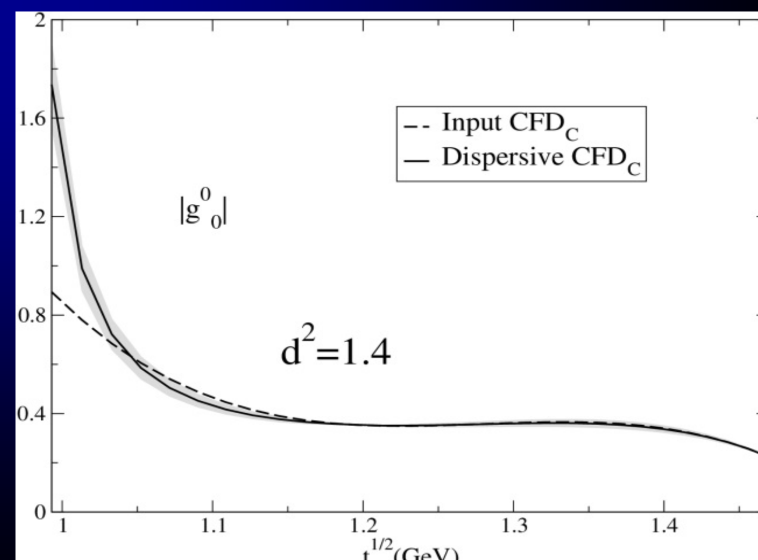
Two possible sets of data

We use I=0,J=2 CFD as input.



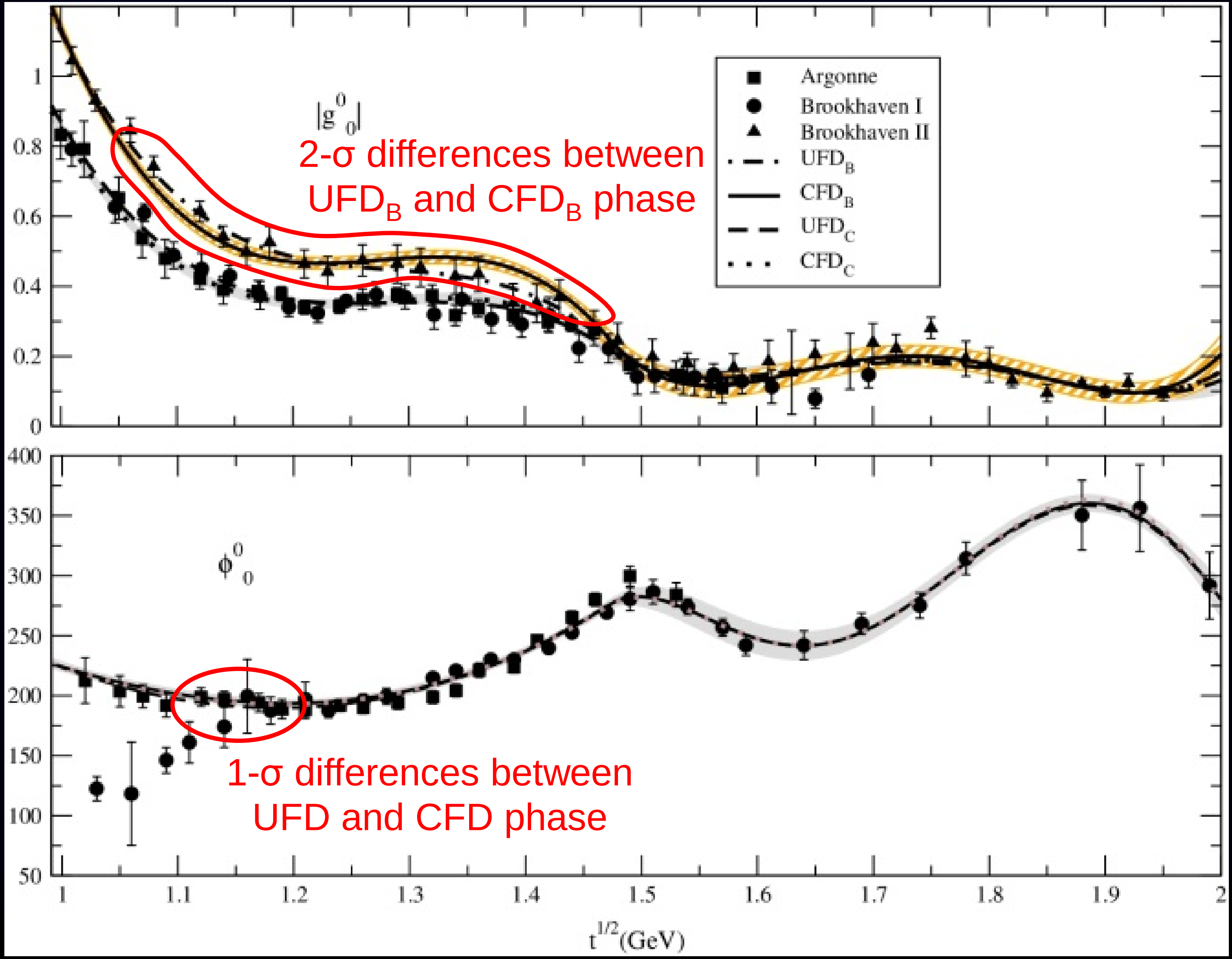






Remarkable improvement from UFD to CFD, except at threshold.
Both data sets equally acceptable now.

Some 2-σ level differences between UFD_B and CFD_B between 1.05 and 1.45 GeV
CFD_C consistent within 1-σ band of UFD_C



Our Dispersive/Analytic Approach for πK and strange resonances

Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

- As πK checks: Small inconsistencies.
- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)
- Analytic methods to extract poles: reduced model dependence on strange resonances
  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits up to 1.5 GeV
  JRP, A.Rodas, Eur.Phys.J. C78 (2018)

- From fixed-t DR:
  ππ→KK influence small.
  κ/K_0^*(700) out of reach
- From Hyperbolic DR:
  ππ→KK influence important.
  JRP, A.Rodas, in progress. PRELIMINARY results shown here

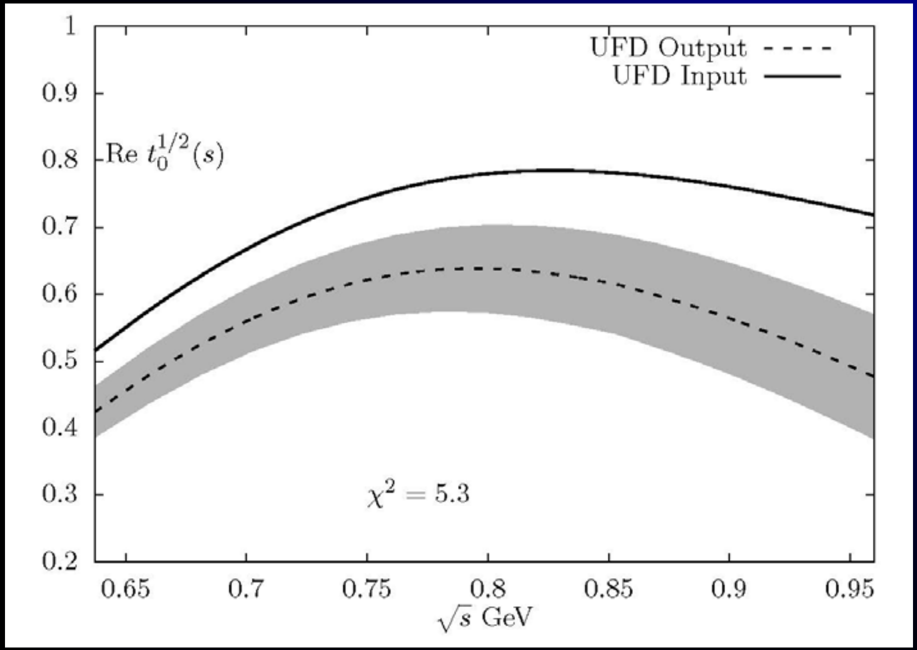
- As πK Checks: Large inconsistencies.
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV

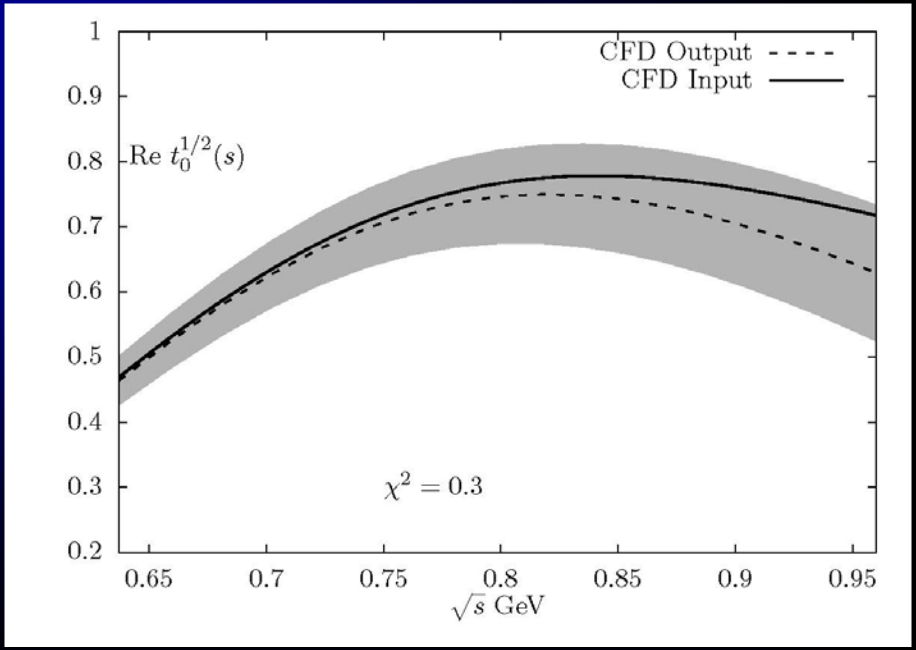
πK Hiperbolic Dispersion Relations I=1/2, J=0

Preliminary!!

LARGE inconsistencies of unconstrained fits with the minimal number of subtractions (shown here)
Fairly consistent with one more subtraction for F^-

Consistent within uncertainties
if we use the DR as constraints



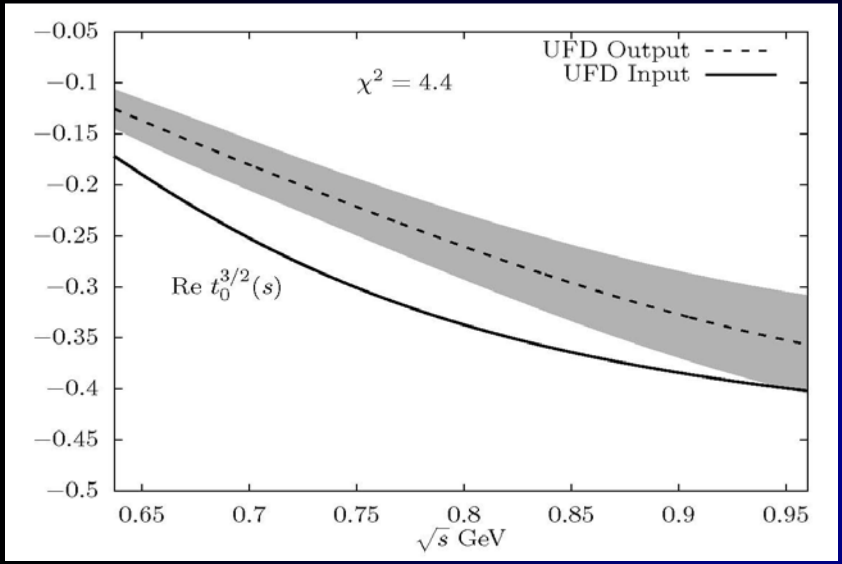


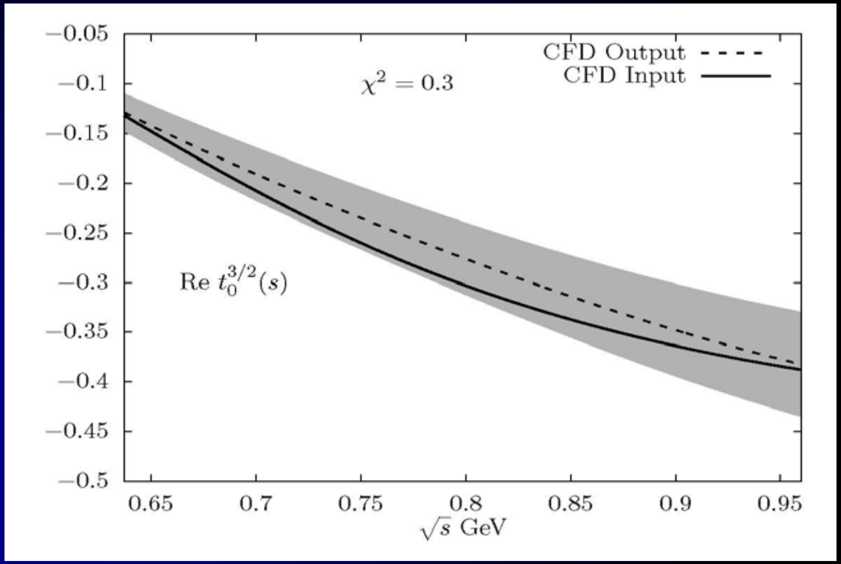
πK Hiperbolic Dispersion Relations I=3/2, J=0 and I=1/2, J=0

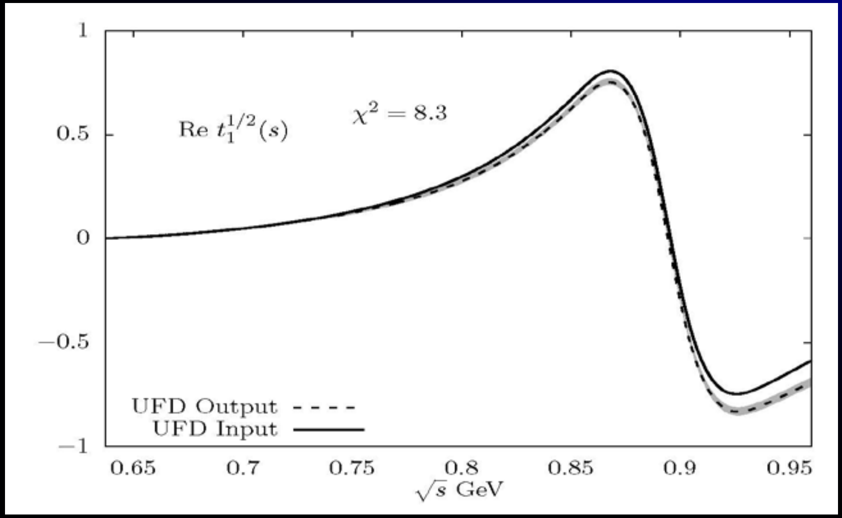
Preliminary!!

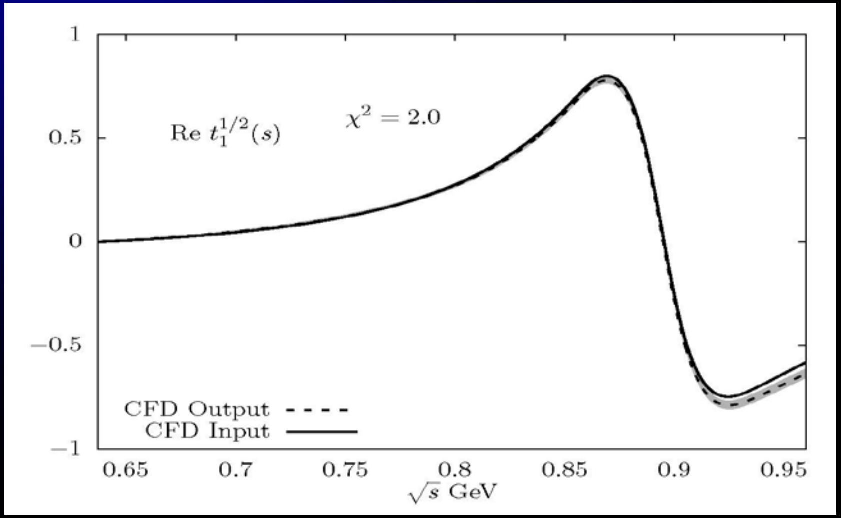
SIZABLE inconsistencies of unconstrained fits with the minimal number of subtractions (shown here). Fairly consistent with one more subtraction for F^-

Made consistent within uncertainties when we use the DR as constraints





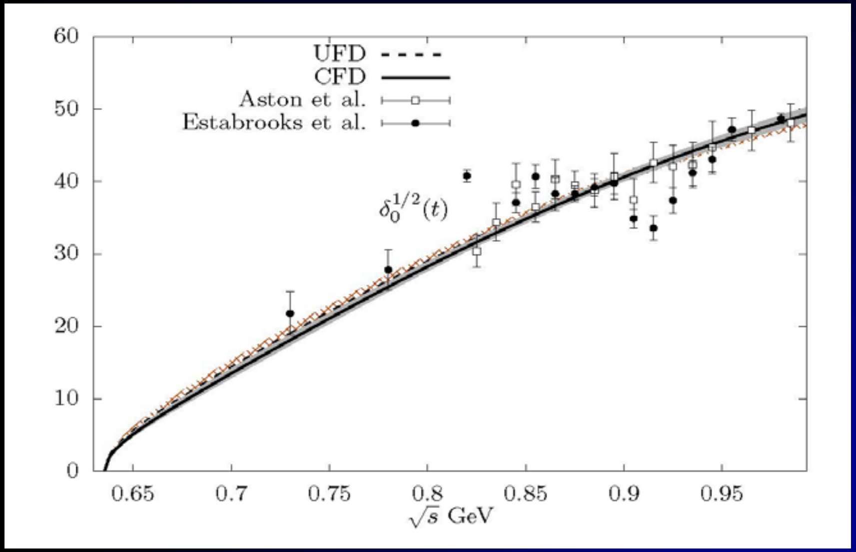


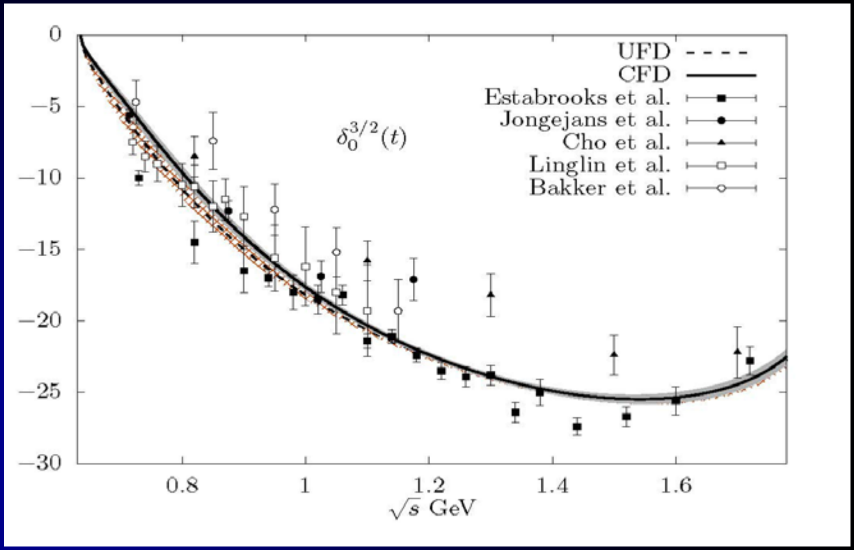


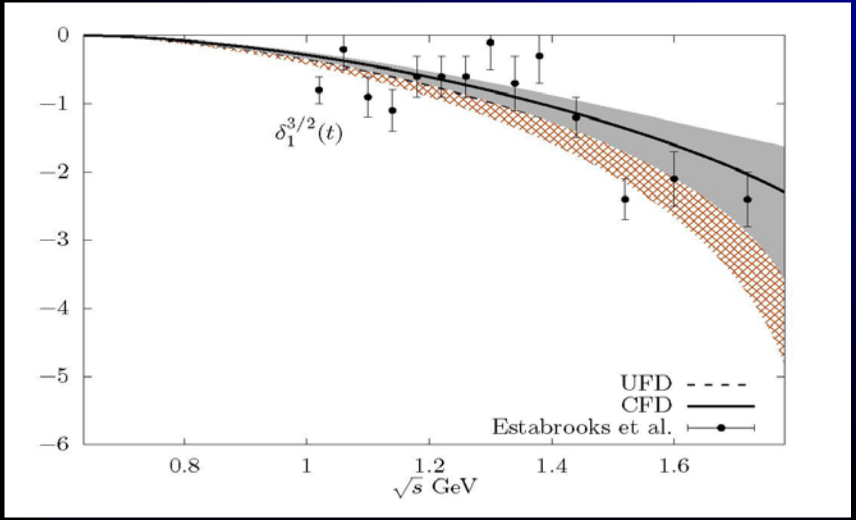
πK CFD vs. UFD

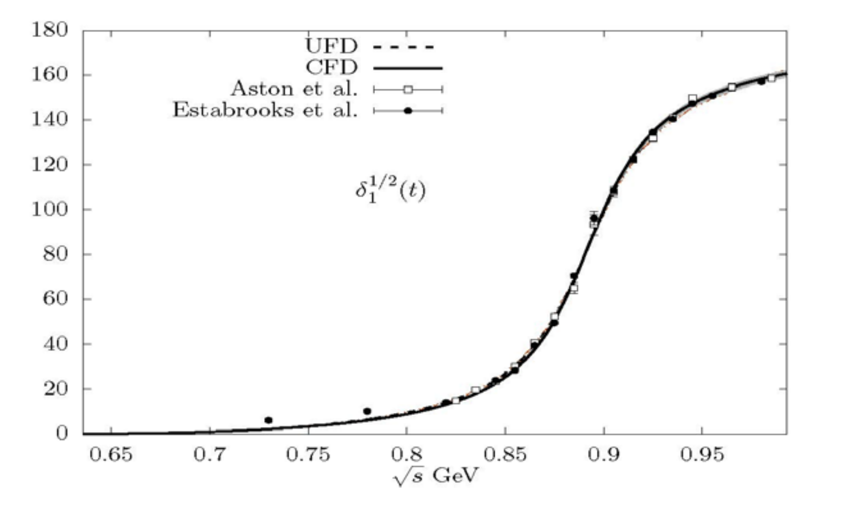
Preliminary!!

Constrained parameterizations suffer minor changes but still describe πK data fairly well. Here we compare the unconstrained fits (UFD) versus the constrained ones (CFD)









The “unphysical” rho peak in ππ→KK grows by 10% from UFD to CFD

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- As πK Checks: Large inconsistencies.
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV
- **Rigorous $\kappa/K_0^*(700)$ pole**

JRP, A.Rodas, in progress.
PRELIMINARY results
shown here

Dispersive pole analysis from constrained fit to data JRP, A. Rodas, in preparation

Preliminary!!

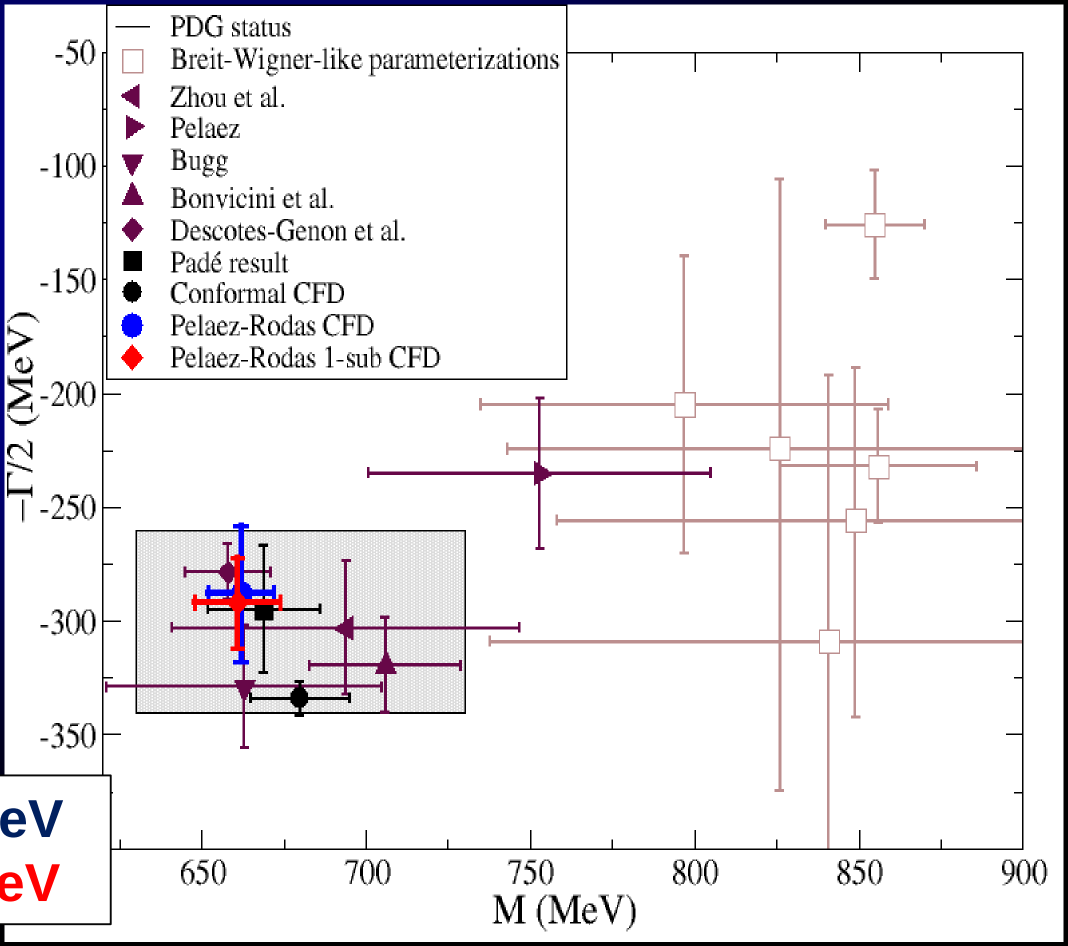
Recall Roy-Steiner SOLUTION from Paris group (658±13)-i(278.5±12) MeV

Decotes-Genon-Moussallam 2006

Now we have:

- Constrained **FIT TO DATA** (not solution but fit)
- Improved P-wave (consistent with data)
- Realistic ππ→KK uncertainties (none before)
- Improved Pomeron

- Constrained ππ→KK input with DR
- FDR up to 1.6 GeV
- Fixed-t Roy-Steiner Eqs.
- Hyperbolic Roy Steiner Eqs.
  both in real axis (not before)
  and complex plane
- Both one and no-subtraction
  for F- HDR (only the subtracted one before)



No sub: (662± 9)-i(288±31) MeV

1 sub: (661±13)-i(293±20) MeV

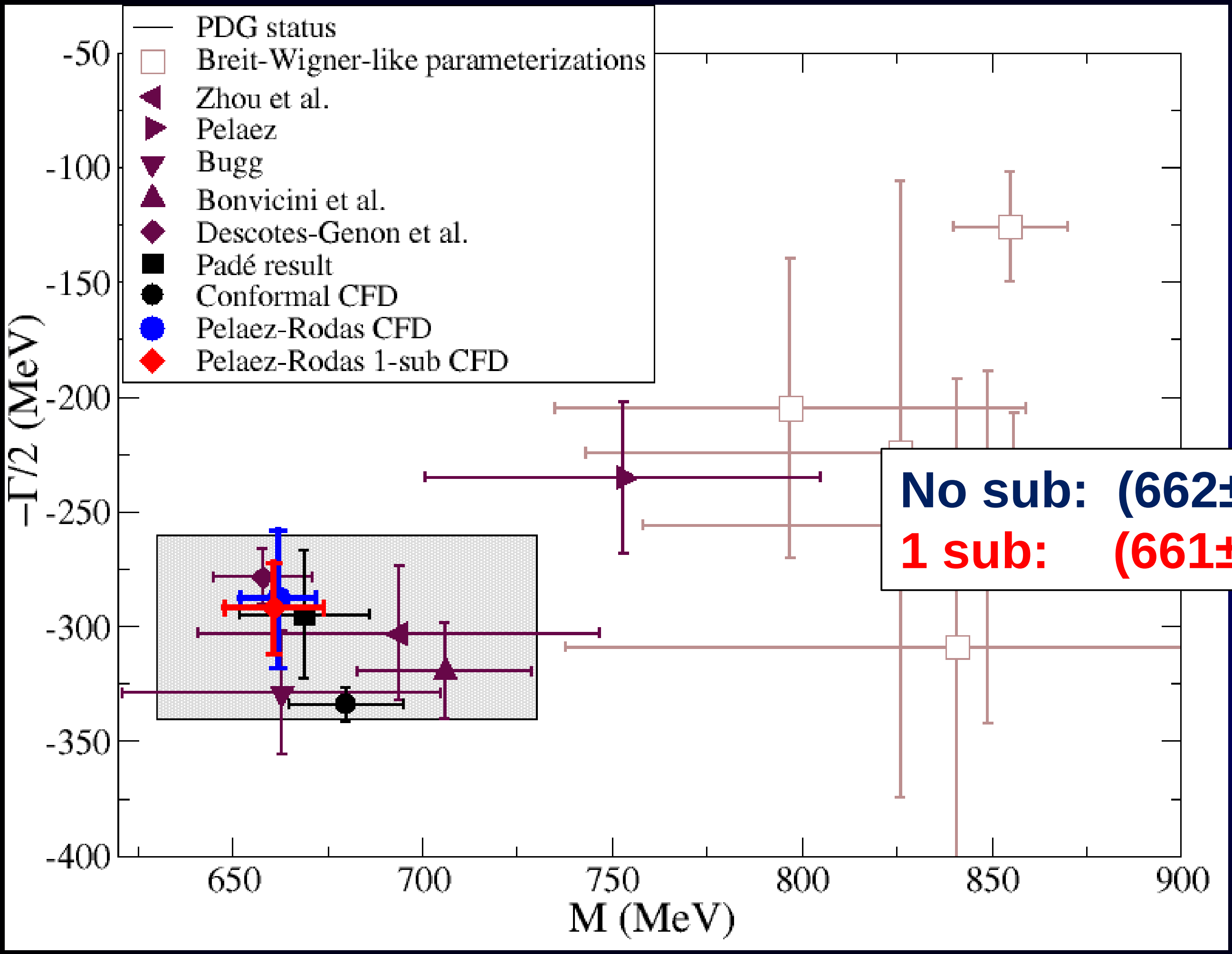
Summary

- The πK and $\pi\ \pi \rightarrow KK$ data do not satisfy well basic dispersive constraints
- Using dispersion relations as constraints we provide simple and consistent data parameterizations.
- Simple analytic methods of complex analysis can then reduce the model dependence in resonance parameter determinations.
- We are implementing partial-wave dispersion relations whose applicability range reaches the kappa pole. Our preliminary results confirm previous studies. We believe this resonance should be considered “well-established”, completing the nonet of lightest scalars.

Dispersive pole analysis from constrained fit to data JRP, A. Rodas, in preparation

Preliminary!!

When using the constrained fit to data both poles come out nicely compatible



No sub: (662± 9)-i(288±31) MeV
1 sub: (661±13)-i(293±20) MeV

Compatible with Paris group

Decotes-Genon-Moussallam 2006

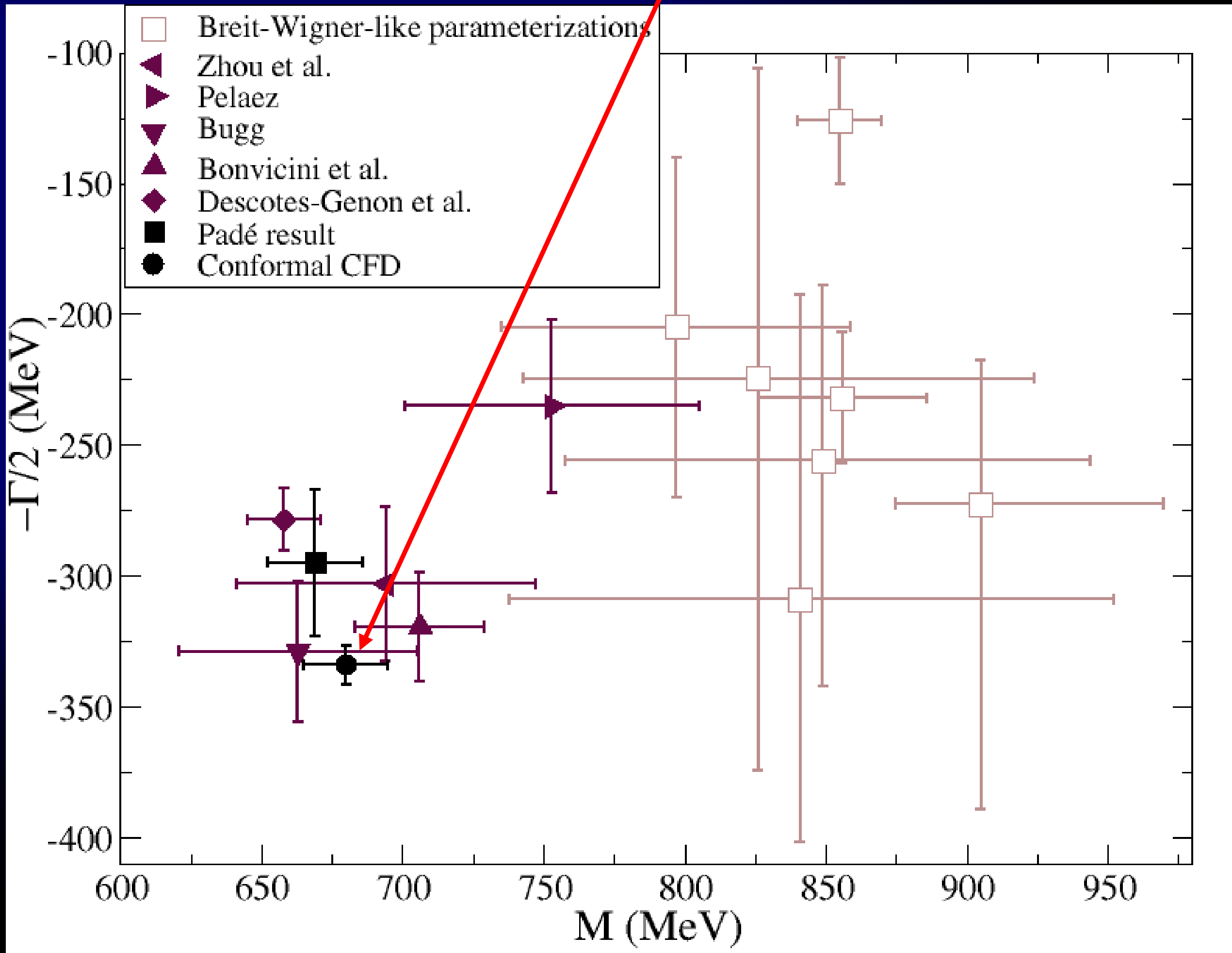
(658±13)-i(278.5±12) MeV

Kappa pole from CFD

1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

Fantastic analyticity properties, but not model independent

(680±15)-i(334±7.5) MeV



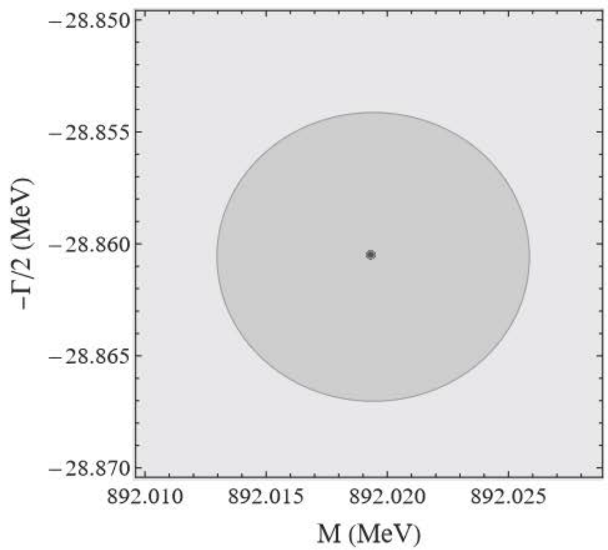
Strange resonance poles from CFD: Using Padé sequences

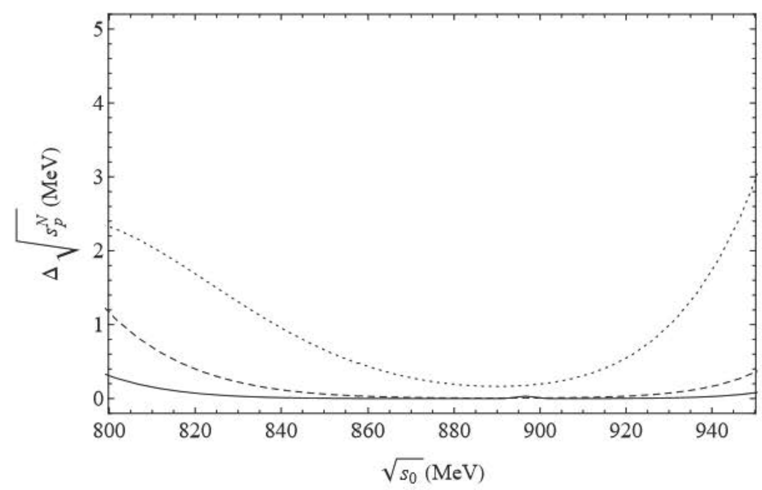
JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

Almost model independent: Does not assume any particular functional form (but local determination) **CAN BE USED FOR INELASTIC RESONANCES TOO**

Based on previous works by P.Masjuan, J.J. Sanz Cillero, I. Caprini, J.Ruiz de ELvira

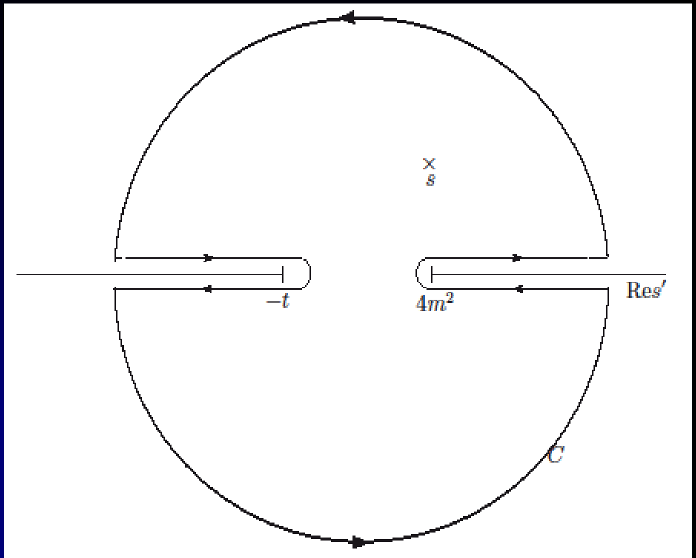
- For every fit we search the s_0 thats gives the minimum difference for the truncation of the sequence.
- We stop at a N ($N+1$ derivatives) where the systematic uncertainty is smaller than the statistical one (usually $N=4$ is enough).
- Run a montecarlo for every fit to calculate the parameters an errors of every resonance.





Why use dispersion relations?

CAUSALITY:
Amplitudes T(s,t) are ANALYTIC in complex s plane but for cuts for thresholds.
Crossing implies left cut from u-channel threshold



Cauchy Theorem determines T(s,t) at ANY s, from an INTEGRAL on the contour

If T->0 fast enough at high s, curved part vanishes

$$T(s,t,u) = \underbrace{\frac{1}{\pi}\int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\,T(s',t,u')}{s'-s}}_{\text{Right cut}} + \underbrace{\frac{1}{\pi}\int_{-\infty}^{-t} ds' \frac{\mathrm{Im}\,T(s',t,u')}{s'-s}}_{\text{Left cut}}$$

Otherwise, determined up to a polynomial (subtractions)
Left cut usually a problem

Good for:

1) Calculating T(s,t) where there is not data

2) Constraining data analysis

3) ONLY MODEL INDEPENDENT extrapolation to complex s-plane
without extra assumptions

Kappa pole from CFD

1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

Fantastic analyticity properties, but not model independent

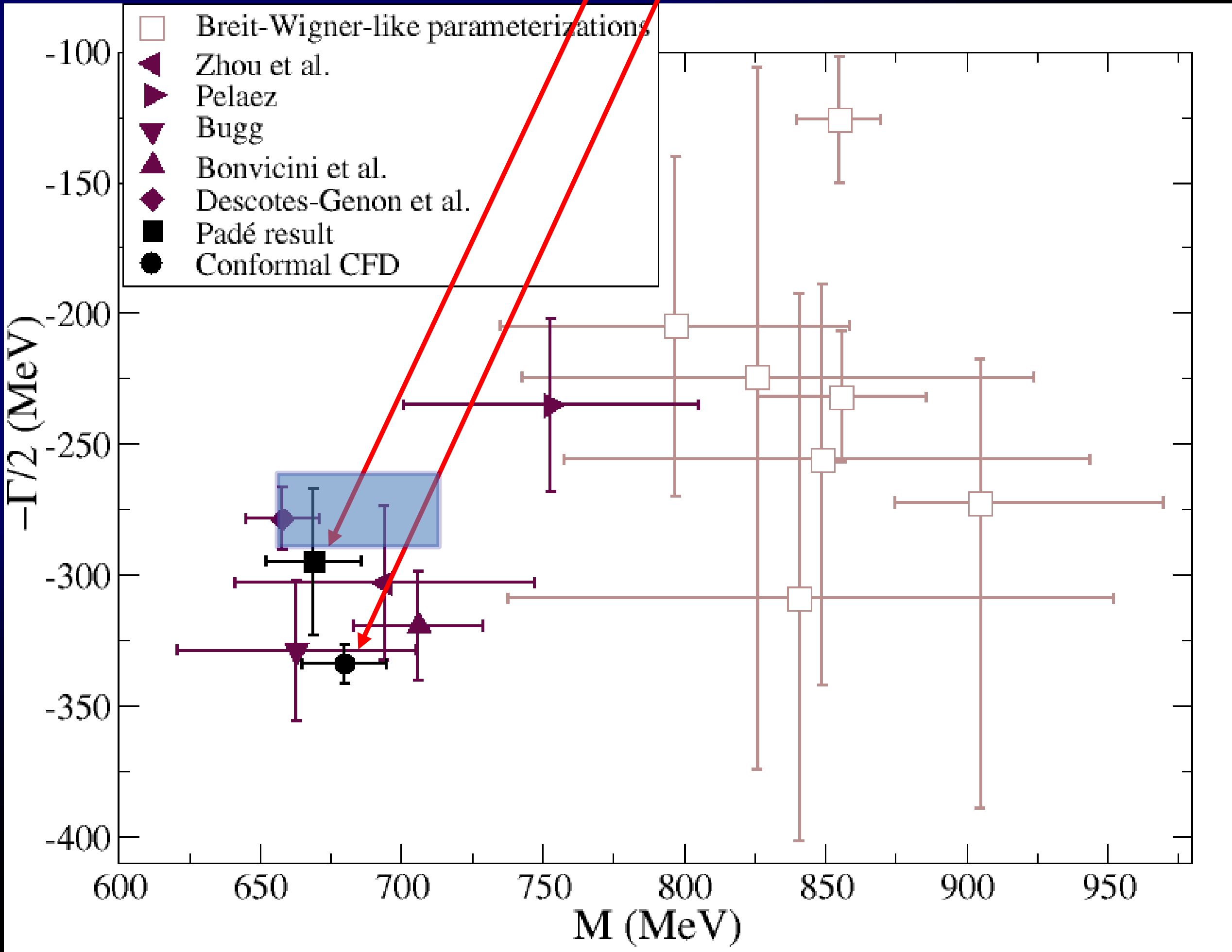
(680±15)-i(334±7.5) MeV

2) Using Padé Sequences…

JRP, A. Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017) 77:91

(670±18)-i(295± 28) MeV

Compare to PDG2017:
(682±29)-i(273±12) MeV



The resonance is NO LONGER the κ nor the $K_0{*}(800)$

But Still “Needs Confirmation” !

Best analysis so far: Roy-Steiner dispersion relations

Plenty of room for improvement on parameters

Our Pade sequences