Muon-electron scattering at NLO

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12^{th} International Workshop on e^+e^- collisions from Φ to Ψ

BINP and NSU, Novosibirsk, February 24 - March 1 2019

with M. Alacevich, M. Chiesa, G. Montagna, O. Nicrosini and F. Piccinini

Outline

- Short introduction
- NLO (QED & EWK) corrections to $\mu^\pm e^- \to \mu^\pm e^-$ (*i.e.* our paper*)
  - Details of the calculation
  - Phenomenology of NLO corrections
    - ✓ QED corrections (and splitting into gauge-invariant subsets)
    - ✓ EWK corrections
    - ✓ finite e-mass effects

*M. Alacevich *et al.*, JHEP 02 (2019) 155,
published on February 25

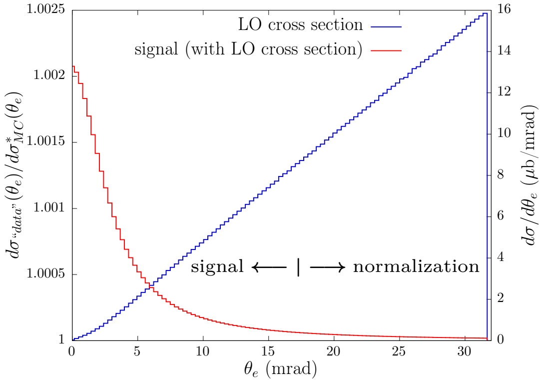
- Towards NNLO: "easy" deliverables at NNLO
- Conclusions and outlook

Intro

$$\text{Our signal} \equiv \frac{dN_{data}(O_i)}{dN_{MC}(O_i)|_{\Delta\alpha_{had}(t)=0}} \equiv \frac{dN_{data}(O_i)}{dN^*_{MC}(O_i)} =$$

$$= \frac{d\sigma_{data}(O_i)}{d\sigma^*_{MC}(O_i)} = \frac{dN_{data}(O_i)}{N^{norm}_{data}} \times \frac{\sigma^{norm}_{MC}}{d\sigma^*_{MC}(O_i)} \simeq$$

$$\simeq 1 + 2\left[\Delta\alpha_{\text{lep}}(O_i) + \Delta\alpha_{\text{had}}(O_i)\right] \qquad \text{(at LO)}$$



A first step, radiative corrections at NLO in QED

- The μe cross section and distributions must be known as precisely as possible
  $\mapsto$ radiative corrections (RCs) are mandatory

⋆ First step are QED $\mathcal{O}(\alpha)$ (i.e. QED NLO, next-to-leading order) RCs

The NLO cross section is split into two contributions,

$$\sigma_{NLO} = \sigma_{2\to 2} + \sigma_{2\to 3} = \sigma_{\mu e\to\mu e} + \sigma_{\mu e\to\mu e\gamma}$$

$\mapsto$ IR singularities are regularized with a vanishingly small photon mass $\boldsymbol{\lambda}$

$\mapsto$ $[2\to 2]/[2\to 3]$ phase space splitting at an arbitrarily small γ-energy cutoff $\boldsymbol{\omega_s}$

- $\mu e \to \mu e$

$$\sigma_{2\to 2} = \sigma_{LO} + \sigma_{NLO}^{virtual} = \frac{1}{F}\int d\Phi_2(|\mathcal{A}_{LO}|^2 + 2\Re[\mathcal{A}_{LO}^* \times \mathcal{A}_{NLO}^{virtual}(\boldsymbol{\lambda})])$$

- $\mu e \to \mu e\gamma$

$$\sigma_{2\to 3} = \frac{1}{F}\int_{\omega>\boldsymbol{\lambda}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2 = \frac{1}{F}\left(\int_{\boldsymbol{\lambda}<\omega<\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2 + \int_{\omega>\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2\right)$$

$$= \Delta_s(\boldsymbol{\lambda},\boldsymbol{\omega_s})\int d\sigma_{LO} + \frac{1}{F}\int_{\omega>\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2$$

- the integration over the 2/3-particles phase space is performed with MC techniques and fully-exclusive events are generated

NLO: method and cross-checks

- Calculation performed in the on-shell renormalization scheme
- **Full mass dependency kept everywhere**, fermions' helicities kept explicit
- Diagrams manipulated with the help of `FORM`, independently by at least two of us [perfect agreement]

  J. Vermaseren, `https://www.nikhef.nl/~form`
- 1-loop tensor coefficients and scalar 2-3-4 points functions evaluated with `LoopTools` and `Collier` libraries [perfect agreement]

  T. Hahn, `http://www.feynarts.de/looptools`

  A. Denner, S. Dittmaier, L. Hofer, `https://collier.hepforge.org`
- UV finiteness and λ independence verified with high numerical accuracy
- 3 body phase-space cross-checked with 3 independent implementations [perfect agreement]
- Comparisons with past/present independent results [all good]

  T. V. Kukhto, N. M. Shumeiko and S. I. Timoshin, J. Phys. G **13** (1987) 725

  D. Y. Bardin and L. Kalinovskaya, DESY-97-230, `hep-ph/9712310`

  N. Kaiser, J. Phys. G **37** (2010) 115005

  Fael, Passera
- Also NLO weak RCs calculated [negligible, see later]

Simulation setups & RCs

- 4 setups have been considered for $E_\mu^{\text{beam}} = 150$ GeV. Notice:

$$\sqrt{s} \simeq 0.4055 \text{ GeV} \qquad t^{min}_{ee,\mu\mu} = -\lambda(s, m_\mu^2, m_e^2)/s \simeq -0.143 \text{ GeV}^2$$

Setup 1:

- $E_e \geq 0.2$ GeV $\quad (\to t^{max}_{ee} \lesssim -2.04 \cdot 10^{-4}$ GeV$^2) \quad$ and $\theta_e, \theta_\mu \leq 100$ mrad

Setup 2:

- $E_e \geq 1$ GeV $\quad (\to t^{max}_{ee} \lesssim -1.02 \cdot 10^{-3}$ GeV$^2) \quad$ and $\theta_e, \theta_\mu \leq 100$ mrad

Setup 3:

- **Setup 1 + acoplanarity cut**, *i.e.* acoplanarity $\equiv |\pi - (\phi_e - \phi_\mu)| \leq 3.5$ mrad

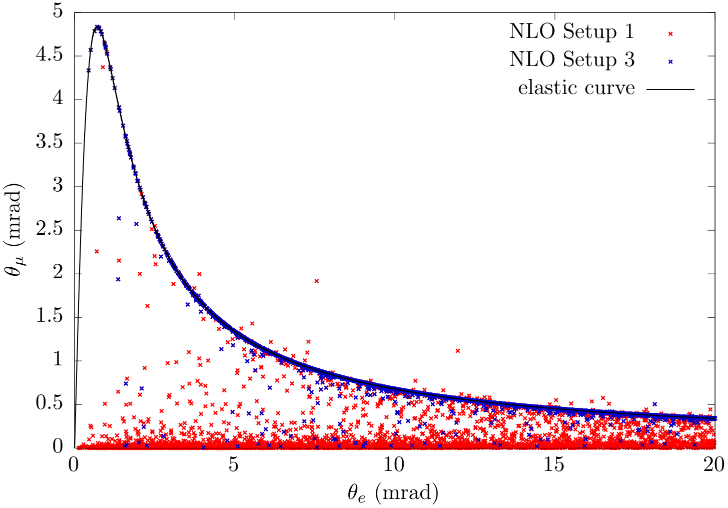
Setup 4:

- **Setup 2 + acoplanarity cut**

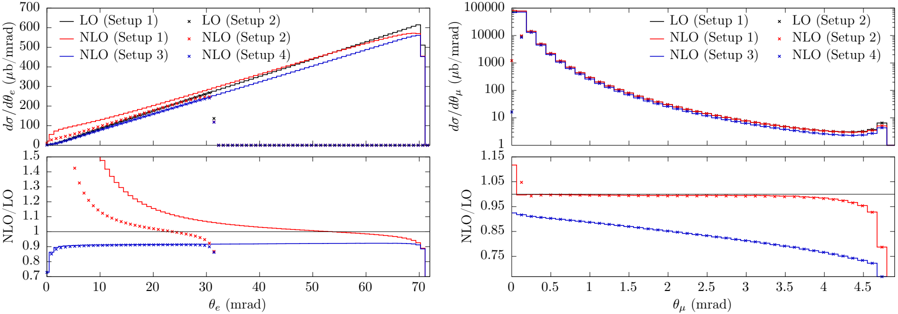
- both processes $\mu^\pm e^- \to \mu^\pm e^-$ considered
- full QED NLO, gauge-invariant subsets (e-, μ-line corrections, interference), $m_e \to 0$ limit, weak LO & NLO RCs, **any VP switched off**

[More realistic elasticity cuts are being explored together with experimental colleagues]

θ_e-θ_μ correlation **(in the lab. frame)**

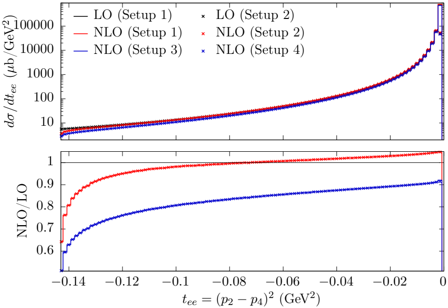


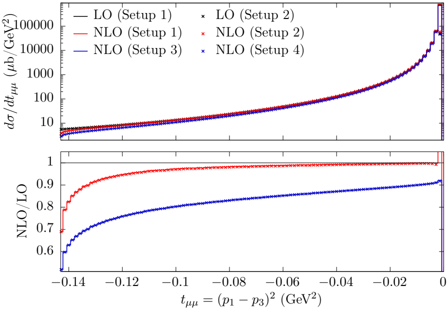
QED RCs on θ_e & θ_μ (incoming μ^+)



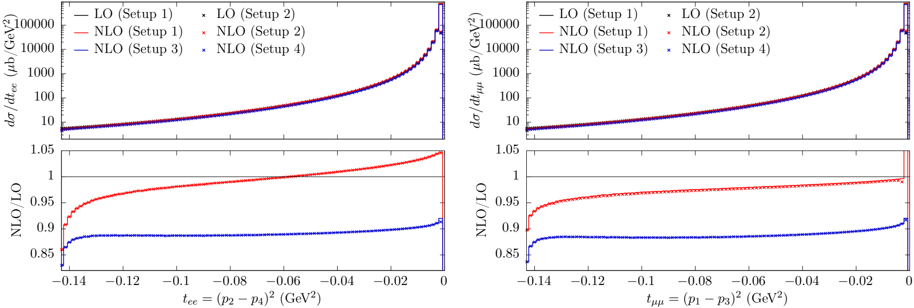
- Large RCs in Setup 1 & 3 induced by "hard" photon

QED RCs on t_{ee} & $t_{\mu\mu}$ (incoming μ^+)





QED RCs on t_{ee} & $t_{\mu\mu}$ (incoming μ^-)

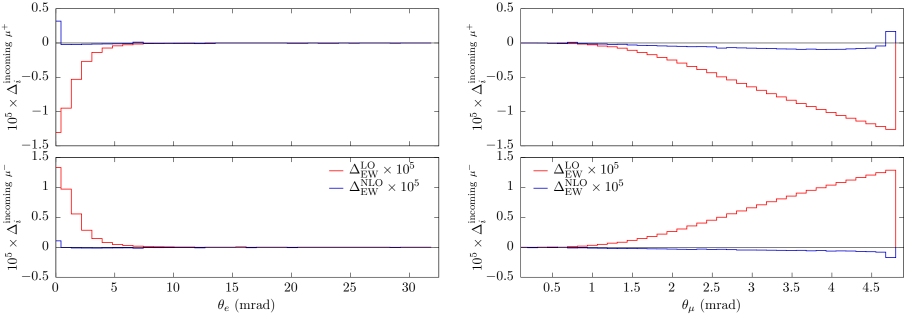


EWKology on θ_e & θ_μ

↦ Full EWK RCs calculated in the on-shell (complex mass) scheme with **RECOLA**

S. Actis *et al.*, JHEP 04:037, 2013

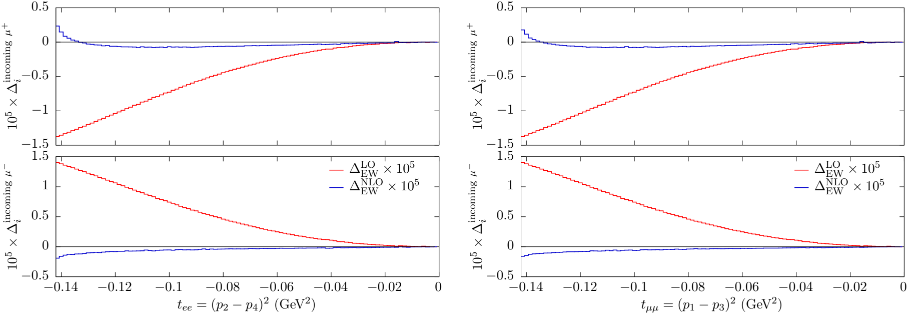
S. Actis *et al.*, CPC 214:140–173, 2017



$$\Delta_{\text{EW}}^{\text{LO}} = \frac{d\sigma_{\text{EW}}^{\text{LO}} - d\sigma_{\text{QED}}^{\text{LO}}}{d\sigma_{\text{QED}}^{\text{LO}}} \qquad \Delta_{\text{EW}}^{\text{NLO}} = \frac{\left(d\sigma_{\text{EW}}^{\text{NLO}} - d\sigma_{\text{EW}}^{\text{LO}}\right) - \left(d\sigma_{\text{QED}}^{\text{NLO}} - d\sigma_{\text{QED}}^{\text{LO}}\right)}{d\sigma_{\text{QED}}^{\text{NLO}}}$$

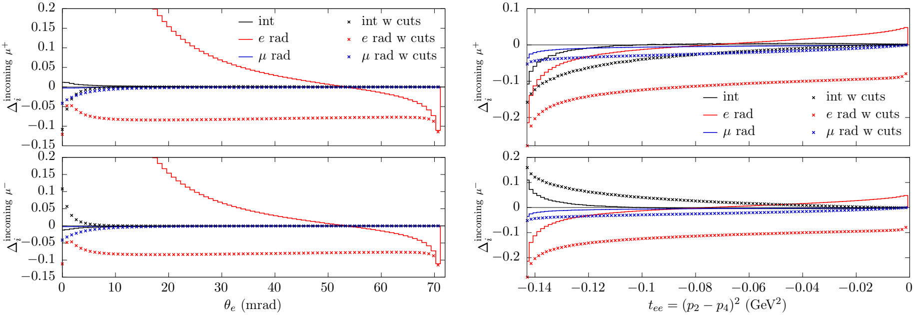
- $\Delta_{\text{EW}}^{\text{NLO}}$ measures the (gauge-invariant) purely weak RC, in QED NLO units

EWKology on t_{ee} & $t_{\mu\mu}$



- tree-level Z-exchange important at the 10^{-5} level
- purely weak RCs (in QED NLO units) at a few 10^{-6} level

Gauge-invariant subsets on θ_e and t_{ee} (Setup 1 & 3)



$$\Delta_i^{\text{incoming } \mu^\pm} = \frac{d\sigma_i^{\text{NLO}} - d\sigma^{\text{LO}}}{d\sigma^{\text{LO}}}$$

- "$e(\mu)$ rad": QED RCs only on electron (muon) current
- "int": full $-$ [e rad] $-$ [μ rad]

⋆ in general: $|e \text{ rad}| > |\text{int}| > |\mu \text{ rad}|$

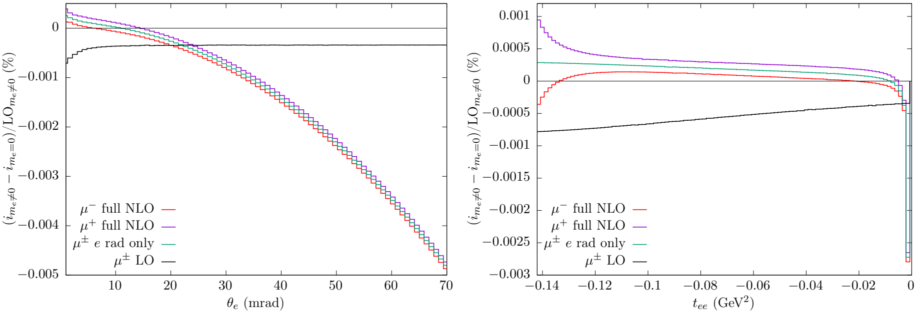
Electron mass effects ($m_e \to 0$ limit)

✓ Studied at NLO.
Can it give a grasp of finite m_e effect at NNLO?

1. Fully massive 4-momenta, phase space and flux kept.
[Otherwise the frame where e^- is at rest can't be defined]
2. $2 \to 2$ amplitudes expressed as functions of s and t.
3. Virtual amplitudes: fully reduced to scalar functions.
Everything $\propto \log\lambda$ is kept massive [**IR part**].
In the non-IR part, $u = 2m_\mu - s - t$ and everything $\propto m_e$ is neglected, *except* $\log m_e^2$.
4. Soft real: similarly, full m_e dependency in IR terms $\propto \log(\omega_s/\lambda)$, m_e neglected in the remainder, *except* $\log m_e^2$.
5. Real $(\omega \geq \omega_s)$: m_e kept everywhere.
Finite m_e corrections come from the interplay [phase-space integration]/[matrix elements], difficult to disentangle unambiguously.

- Following **1.-5.**, no spurious IR dependence is left.

Electron mass effects (limit $m_e \to 0$) at NLO

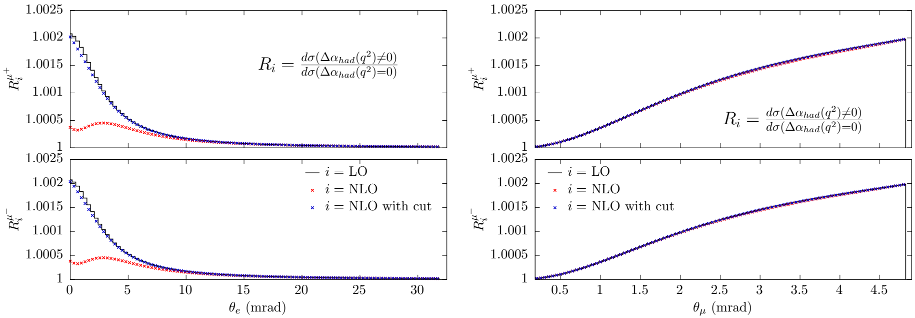


- with our definitions, finite m_e effects at NLO lie in the range of some 10^{-5}, dominated by e current corrections
- ***My*** guess: at NNLO finite e mass can be neglected

Signal sensitivity to RCs

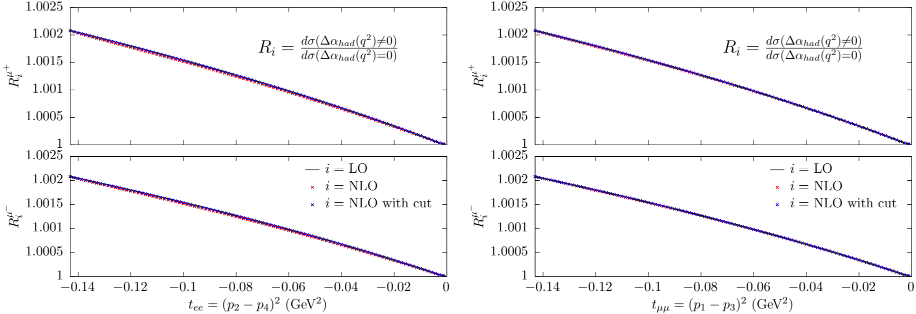
Our "signal" on observable $O \equiv \frac{d\sigma^{\text{best}}(O, \Delta\alpha_{\text{had}}(q^2) \neq 0)}{d\sigma^{\text{best}}(O, \Delta\alpha_{\text{had}}(q^2) = 0)}$

- Does it survive radiative corrections?



- Elasticity cuts mandatory to keep signal sensitivity on θ_e
- θ_μ is more "robust" under RCs (in particular "hard" photon radiation)

Signal sensitivity to RCs



“Quick” deliverables at NNLO (from Monte Carlo point of view)

⇝ An impressive amount of work is currently put in NNLO/resummation calculations

M. Fael and M. Passera, `arXiv:1901.03106`

M. Fael, JHEP 1902 (2019) 027

S. Di Vita *et al.*, JHEP 1809 (2018) 016

P. Mastrolia *et al.*, JHEP 1711 (2017) 198

2nd ThinkStart/WorkStop: Theory of μ-e scattering @ 10ppm, Zurich, February 4-7 '19

↦ QED NLO to $\mu^\pm e^- \to \mu^\pm e^- \gamma$
- ✓ (almost) straightforward

↦ $\mu^\pm e^- \to \mu^\pm e^- e^+ e^-, \to \mu^\pm e^- \mu^+ \mu^-$
- ✓ partially cancelled by NNLO fermionic RCs
- ✗ how the experiment will deal with these final states?

↦ $\mu^\pm e^- \to \mu^\pm e^- \pi^0 (\to \gamma\gamma), \to \mu^\pm e^- \pi^+ \pi^-$
- ✓ collaboration with Henryk Czyż

↦ Full virtual NNLO photonic RCs to e and μ currents separately?
- ✓ it can be the first step towards full fixed-order NNLO MC
- ✓ it can be the testing playground to implement matching with exponentiation at NNLO
  (*e.g.* along the lines of NLO matching with QED Parton Shower in `BabaYaga@NLO`)

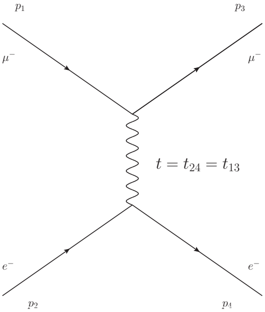
Conclusions & Outlook

- $\mu^\pm e^- \to \mu^\pm e^-$ under control at NLO in the SM and available into a MC generator
- MC easy to be extended to fixed order NNLO
  (once amplitudes are available, also partially or in sound approximation)
- Need to define an elasticity region, preserving sensitivity to $\Delta\alpha_{\text{had}}(t)$ on "golden" observables
- Full QED NNLO mandatory
- Leptonic and hadronic pairs need to be studied with realistic exp. criteria
- QED resummation/exponentiation needed
- Consistent matching with fixed order NNLO needs to be developed

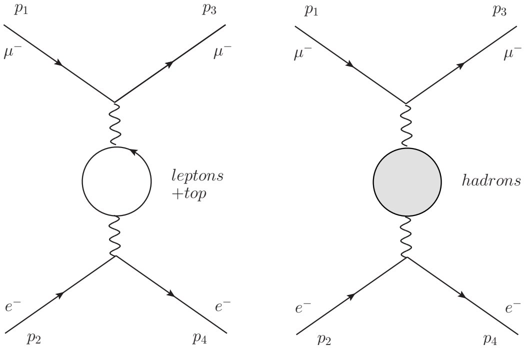
SPARES

LO and NLO vacuum polarization diagrams

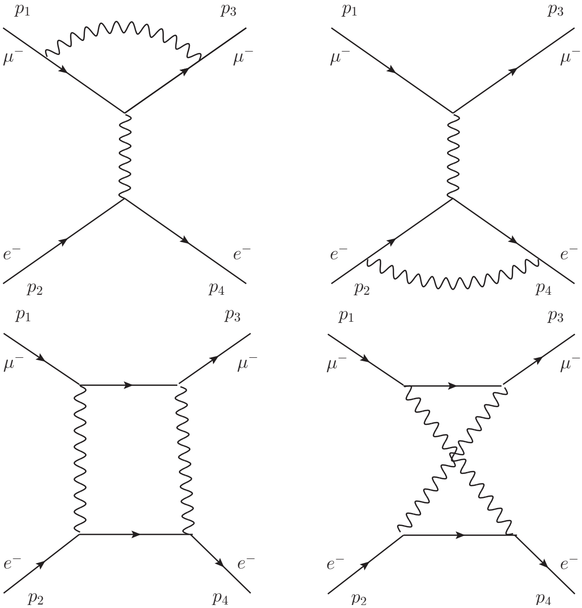
- $\mathcal{A}_{LO}$



- $\mathcal{A}_{NLO}^{virtual}$

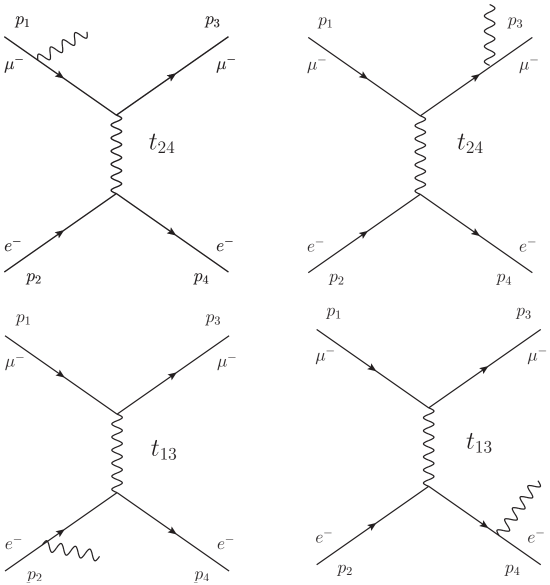


NLO virtual diagrams $\mathcal{A}_{NLO}^{virtual}$ (dependent on λ)

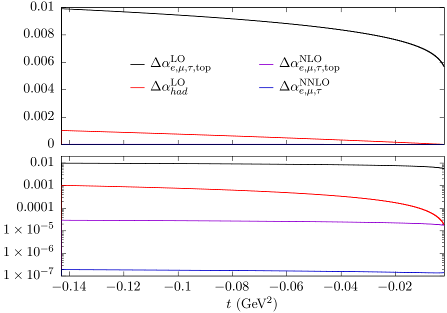


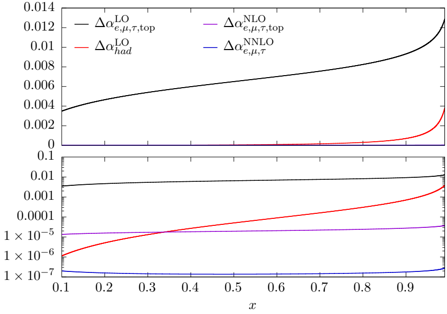
+ counterterms

NLO real diagrams $\mathcal{A}^{1\gamma}_{NLO}$

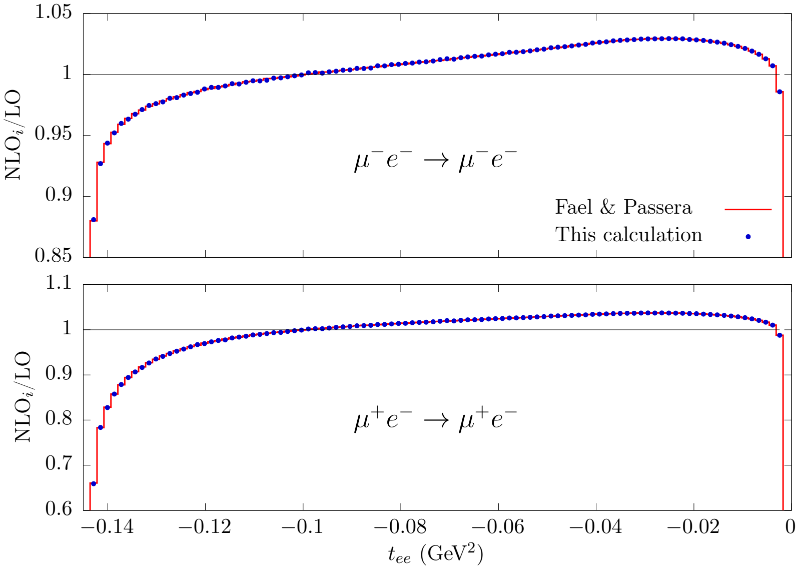


$\Delta\alpha_{\mathsf{lep}}(t)$ at higher orders





Tuned comparison with Fael & Passera



Matching NLO and PS (in BabaYaga@NLO)

As originally developed for $e^+e^- \to e^+e^-, \to \mu^+\mu^-, \to \gamma\gamma$ at flavour factories

Balossini *et al.*, Nucl. Phys. **B758** (2006) 227, CMCC *et al.*, Nucl. Phys. Proc. Suppl. **131** (2004) 48

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual (SV) corrections and hard-bremsstrahlung (H) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma^{\infty}_{PS} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$
- $d\sigma^{\alpha}_{PS} = [1 + C_{\alpha,PS}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_{1,PS}|^2 d\Phi_3 \equiv d\sigma^{SV}_{PS}(\varepsilon) + d\sigma^{H}_{PS}(\varepsilon)$
- $d\sigma^{\alpha}_{\rm NLO} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_{1}|^2 d\Phi_3 \equiv d\sigma^{SV}_{\rm NLO}(\varepsilon) + d\sigma^{H}_{\rm NLO}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,PS})$ $\qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,PS}|^2}{|\mathcal{M}_{1,PS}|^2}$

$$d\sigma^{\infty}_{\rm matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$$

$d\Phi_n$ is the exact phase space for n final-state particles

(2 fermions + an arbitrary number of photons)

Any approximation is confined into matrix elements

⤳ The same QED PS & NLO matching framework successfully applied also to Drell-Yan processes (HORACE) and $H \to 4\ell$ (Hto4l)

CMCC *et al.*, JHEP 0710 (2007) 109; CMCC *et al.*, JHEP 0612 (2006) 016; S. Boselli *et al.*, JHEP 1506 (2015)

Matching NLO and PS (in BabaYaga@NLO)

- F_{SV} and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of leading-logs
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\rm NLO}$
- Exponentiation of higher orders LL (PS) contributions is preserved
- The cross section is still fully differential in the momenta of the final state particles (F's correction factors are applied on an event-by-event basis)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV\ |\ H,i} \otimes$ [leading-logs]

  G. Montagna et al., **PLB** 385 (1996)
- The theoretical error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO) *not infrared*, singly collinear terms: naively and roughly (for photonic corrections)

$$\frac{1}{2!}\alpha^2 L \equiv \frac{1}{2!}\alpha^2 \log\frac{s}{m_e^2} \sim 3.5\times 10^{-4}$$

- **Need to generalize to NNLO matching!**
  Then, the error will be at the level of

$$\frac{1}{3!}\alpha^3 L^2 \sim 1.1\times 10^{-5}$$

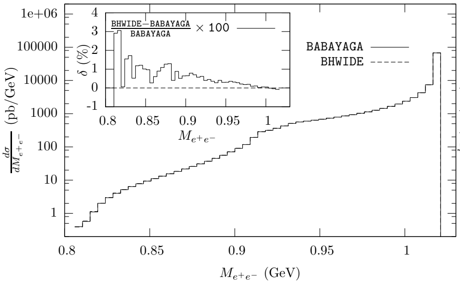
- Can we further improve with analytic resummation?

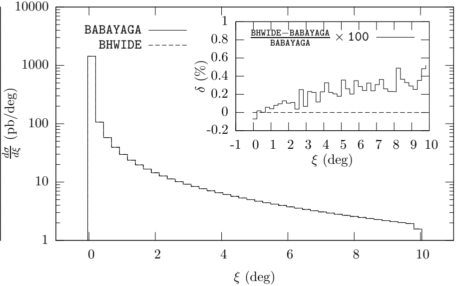
S. Actis et al. Eur. Phys. J. C **66** (2010) 585

"Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data"

- It is extremely important to compare independent calculations/implementations/codes, in order to
  - $\mapsto$ asses the technical precision, spot bugs (with the same th. ingredients)
  - $\mapsto$ estimate the theoretical error when including partial/incomplete higher-order corrections
- E.g. comparison `BabaYaga@NLO` vs. `Bhwide` at KLOE

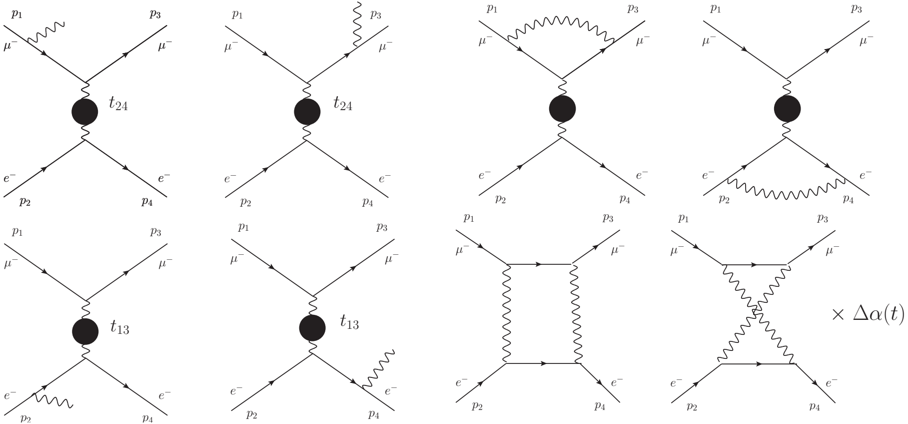
S. Jadach *et al.* PLB 390 (1997) 298





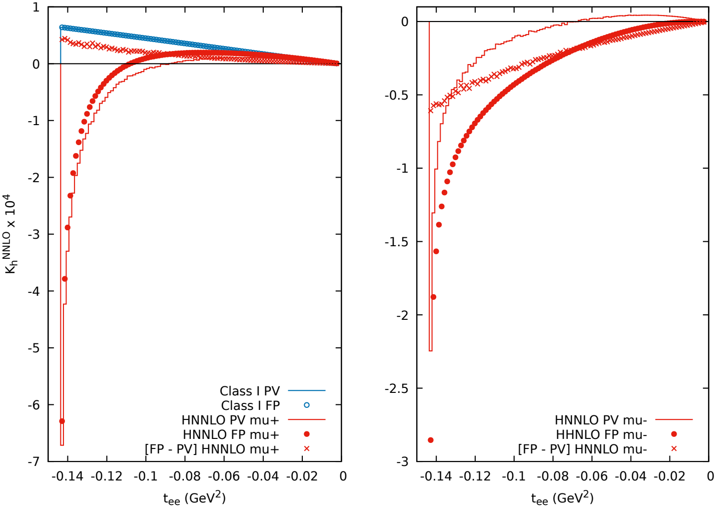
see also CMCC *et al.*, JHEP **1107** (2011) 126

- VP ($\Delta\alpha(q^2)$) can be inserted into QED NLO to approximate fermionic NNLO
  - $\rightsquigarrow$ exactly into $\mu^\pm e^- \to \mu^\pm e^- \gamma$ and into "non-loop γ" of vertex corrections
  - $\rightsquigarrow$ insertion into box diagrams can be approximated as $\mathcal{A}^{\text{box}} \cdot \mathcal{A}^0 \to \mathcal{A}^{\text{box}} \cdot \mathcal{A}^0 \times \Delta\alpha(t)$
  - $\rightsquigarrow$ VP insertion into "loop γ" at the vertex is missed



Approximate NNLO hadronic corrections

- E.g., isolating only corrections $\propto \Delta\alpha_{\text{had}}$ and comparing with Fig. 2 of M. Fael and M. Passera, **arXiv:1901.03106**

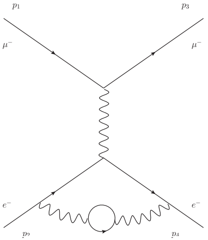


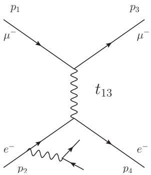
- The bulk of HNNLO corrections is catched (better for μ^+ than μ^-)

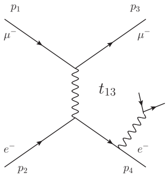
Example: first estimate of e^+e^- pair corrections (on e^- current)

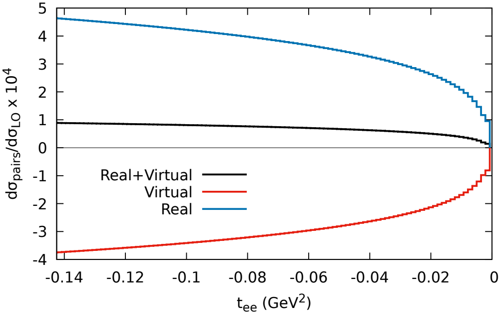
G. J. H. Burgers, Phys. Lett. **164B** (1985) 167, G. Montagna *et al.*, Nucl.Phys. B547 (1999) 39

A. Arbuzov *et al.*, Phys. Atom. Nucl. 60 (1997) 591, Yad. Fiz. 60N4 (1997) 673









$$L = \log \frac{-t}{m_e^2} \qquad \ell = \log \frac{2\Delta E}{\sqrt{s}}$$

$$\delta^{e^+e^-} = \overbrace{\left(\frac{\alpha}{\pi}\right)^2 \left(-\frac{1}{18}L^3 + \frac{19}{36}L^2 + \mathcal{O}(L)\right)}^{\text{virtual}} + \overbrace{\frac{1}{18}\left(\frac{\alpha}{\pi}\right)^2 \left((L-2\ell)^3 - 5\,(L-2\ell)^2 + \mathcal{O}(L-2\ell)\right)}^{\text{real}}$$