Methods of parameterization of amplitudes and extraction of resonances, D-decay amplitudes

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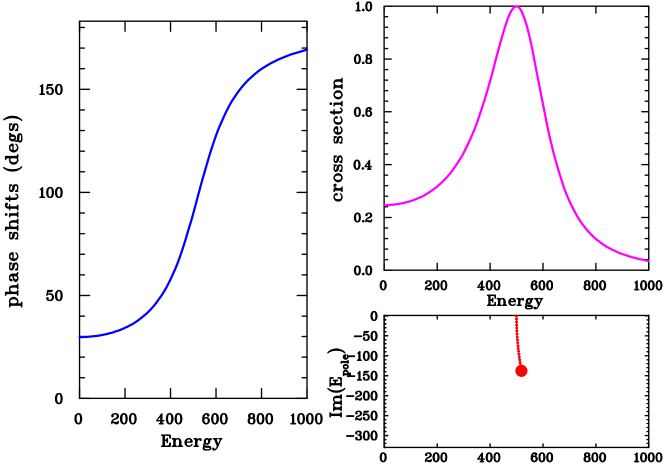
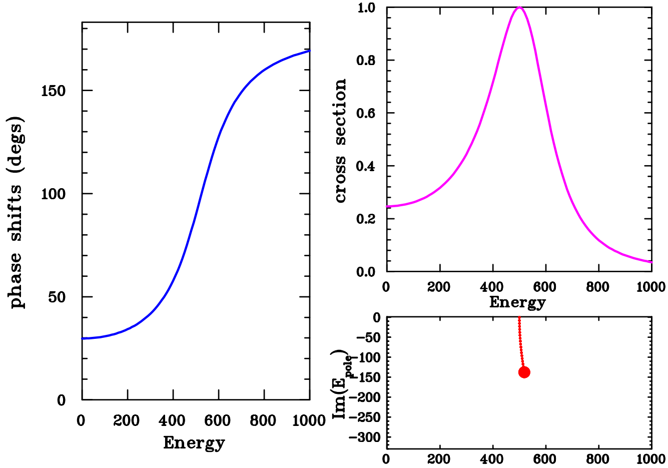
Resonances



phase shifts (degs)
0
50
100
150
0
200
400
600
800
1000
Energy
cross section
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000
Energy
Im(E_pole)
0
−50
−100
−150
−200
−250
−300
0
200
400
600
800
1000

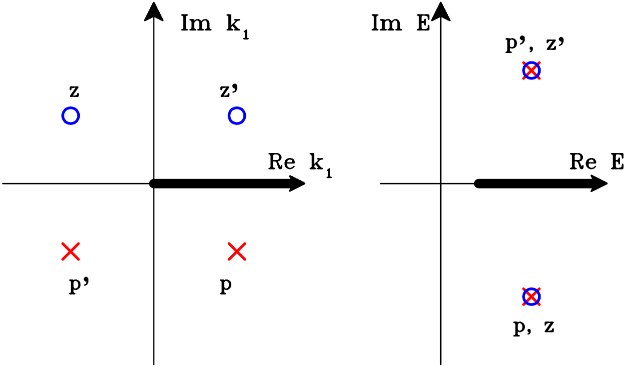
Resonances: Breit-Wigner (BW) approximation:

$$Ampl = \frac{C}{M_{BW} - E - i\Gamma_{BW}/2}, \; \delta = ArcTan\left(\frac{\Gamma/2}{M_{BW} - E}\right), \; \sigma = \frac{C^2}{(M_{BW} - E)^2 + \Gamma^2/4}$$



Complex momentum and energy space frame

$$E = 2\sqrt{(\pm k)^2 + m^2}$$



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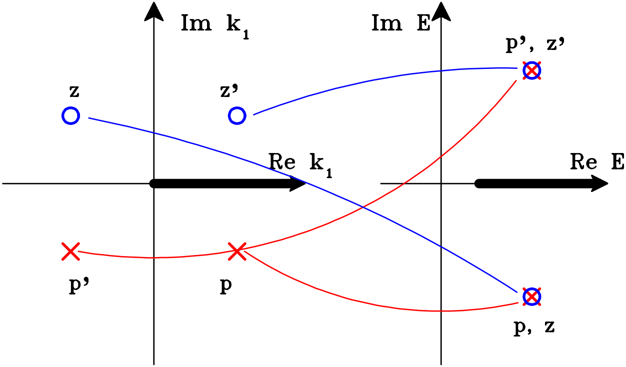

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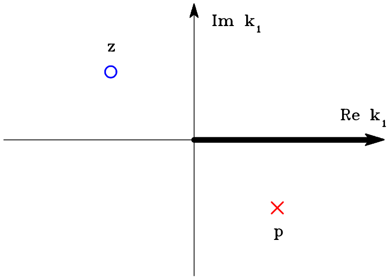

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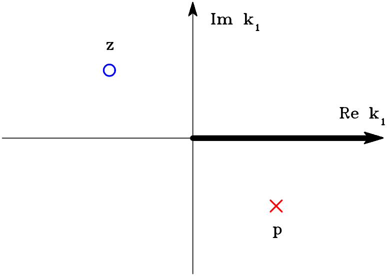

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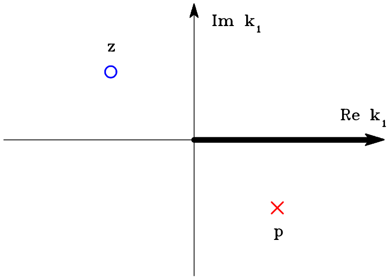

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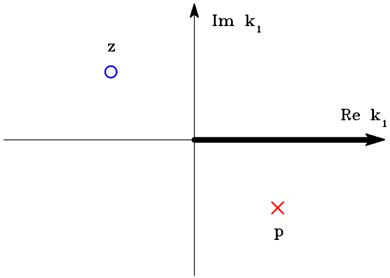

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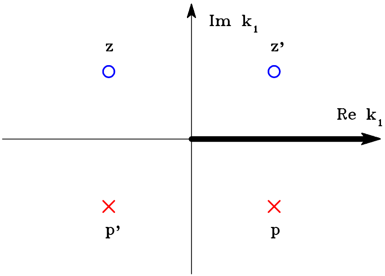

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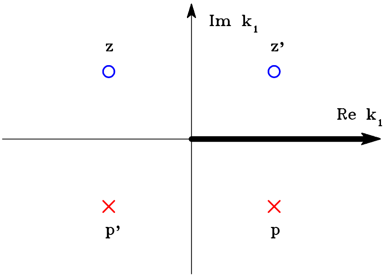
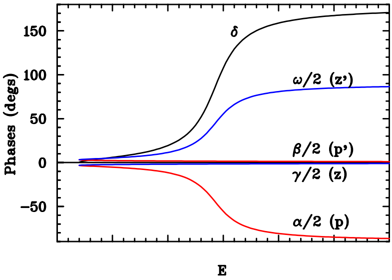
- then $|S(k)| = 1$
- and $\delta = (-\alpha - \beta + \gamma + \omega)/2$

$$angle = ArcTan(\frac{-Imk_j}{k - Rek_j})$$





Unitarity of Breit Wigner approximation

- $BW(E) = \frac{\Gamma/2k}{M_{BW} - E - i\Gamma/2}$
- $\sigma_\ell(E) = 4\pi(2\ell+1)|BW(E)|^2 = \frac{\pi}{k^2}(2\ell+1)\frac{\Gamma^2}{(M_{BW}-E)^2+\Gamma^2/4}$
- $BW(E) = \frac{S_{BW}(E)-1}{2ik}$
- $S_{BW}(E) = \frac{M_{BW}-E+i\Gamma/2}{M_{BW}-E-i\Gamma/2}$
- Defining $E_j = M_{BW} - i\Gamma/2$ we get
- $S_{BW}(E) = \frac{E-E_j^*}{E-E_j}$ and
- because $E = \sqrt{(\pm k)^2/4 + m^2}$ there are two poles and two zeroes and
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Limit of $S_{BW}(E)$ and $S(k)$ at the threshold

Let's check δ and $\sigma \sim |\frac{S-1}{2ik}|^2$ at $E \longrightarrow 2m$

then:

- $S(k) \longrightarrow \frac{-k_j * k_j}{-k_j * k_j}$ and $\delta(k) \longrightarrow 0$
  also $\sigma(k) \longrightarrow 0$

- $S_{BW}(E) \longrightarrow \frac{2m - E_j^*}{2m - E_j}$ and $\delta(E) \longrightarrow ArcTan(\frac{\Gamma/2}{M_{BW} - 2m}) \neq 0$
  also $\sigma(E) \neq 0$

Pole and mass of a resonance

- Let's have good BW fit to the data $\rightarrow$ mass $E = M_{BW}$ at $\delta = 90^o$
  - Let's fit amplitude $A_{S_{notU}}$ to the same data. $A_{S_{notU}}$ has a single pole at $k_j = a - ib$ then $\delta = ArcTan(\frac{-b}{k-a}) + ArcTan(\frac{b}{-k-a})$ and $M_{BW} \neq 2\sqrt{a^2 + m^2}$, additionally $|S| \neq 1$
  - Let's fit amplitude A_{S_U} to the same data. A_{S_U} has a two symmetric poles at $k_j = a - ib$ and $k = -a - ib$ then $\delta = ArcTan(\frac{2bk}{k^2-a^2-b^2}) + ArcTan(\frac{-2bk}{-k^2-a^2-b^2})$ and again $M_{BW} \neq 2\sqrt{a^2 + m^2}$, but now $|S| = 1$

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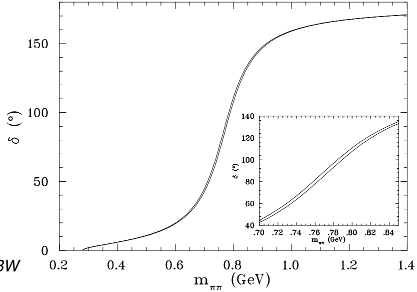
Let's check it for $\rho(770)$:
$M_{BW} = 775.26 \pm 0.25$ MeV (PDG'2016),
$\Gamma = 149.1 \pm 0.8$ MeV (PDG'2016),
$2\sqrt{a^2 + m^2} < M_{BW}$ by ≈ 9 MeV !!!

Left (upper) line:
A_{BW} fitted to the data

Right (lower) line:
A_S fitted to the data with $2\sqrt{a^2 + m^2} = M_{BW}$



First three remarks

(for one channel and one resonance only!)

1. Breit Wigner approximation is unitary

2. Breit Wigner approximation works well only for single resonances and not far from their maximum

3. Positions of poles and Breit Wigner masses are not the same!

bigger $\Gamma \Rightarrow$ bigger difference between $Re(E_{pole})$ and M_{BW}

More resonances (but still one channel)

Adding resonances (for simplicity two resonances, both with $S = e^{2i\delta}$):

- Isobar model: adding amplitudes (even unitary ones) violates unitarity:
  $T_{1,2} = T_1 + T_2 = \frac{S_1 - 1}{2ik} + \frac{S_1 - 1}{2ik} \to S_1 + S_2 = e^{2i\delta_1} + e^{2i\delta_2}$
  of course $|S_1 + S_2| \neq 1$,
- Product of S matrices: $|S_1 S_2| = 1$ in elastic case and $|S_1 S_2| < 1$ in inelastic case ($S = \eta e^{2i\delta}$)
  For example $S_{1,2} = \frac{(-k-k_1)(-k+k_1^*)(-k-k_2)(-k+k_2^*)}{(k-k_1)(k+k_1^*)(k-k_2)(k+k_2^*)}$
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- Sum of K matrices: $S = 1 + 2iT = (1 + iK)/(1 - iK)$ does noit violate unitarity, for example $T_{1,2} = \frac{1}{k}\frac{K_1 + K_2}{1 - iK_1 - iK_2}$

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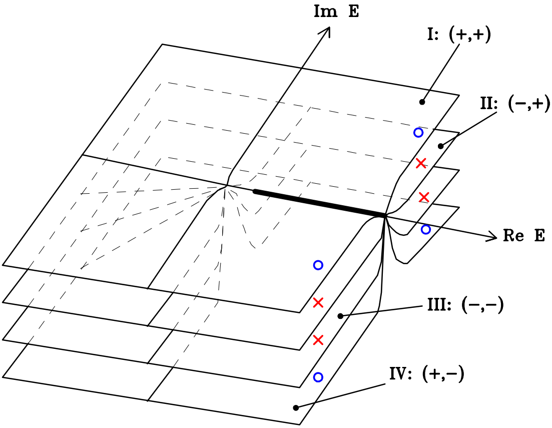

Multiplication and displacement of S matrix poles

- Multiplication:

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  1 pole $\longrightarrow$ 2^{n-1} poles due to $(\pm k)^2$ ambiguity and

- Displacement:

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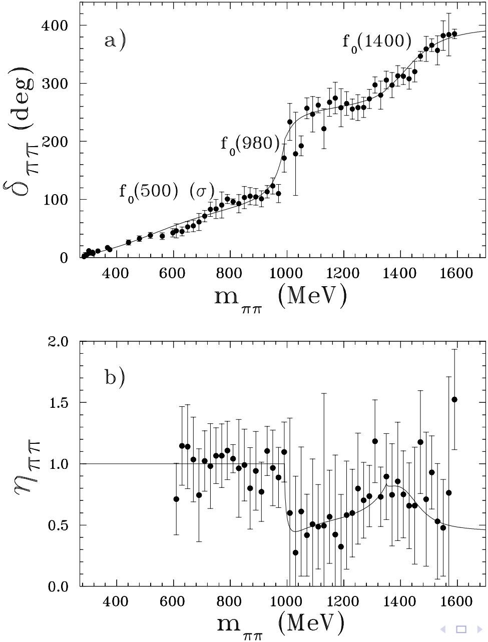
  $S = \frac{D_1(-k_1)D_2(k_2)+C(-k_1,k_2)}{D_1(k_1)D_2(k_2)+C(k_1,k_2)}$ in coupled case

$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} = \begin{pmatrix} \frac{D(-k_1,k_2)}{D(k_1,k_2)} & S_{12} \\ S_{21} & \frac{D(k_1,-k_2)}{D(k_1,k_2)} \end{pmatrix}$$

where $S_{12}^2 = S_{21}^2 = S_{11}S_{22} - \frac{D(-k_1,-k_2)}{D(k_1,k_2)}$

Example for two channels: $JI = S0$ wave



Example for two channels: $JI = S0$ wave

Pole	ReE_{pole} MeV	ImE_{pole} MeV	R. sheet
1	639.6	-323.9	$(-,-)$: *III*
1'	511.4	-230.6	$(-,+)$: *II*
2	982.0	-36.9	$(-,+)$: *II*
2'	432.4	-8.4	$(-,-)$: *III*
3	1431.7	-79.3	$(-,-)$: *III*
3'	1394.9	-120.6	$(-,+)$: *II*

Example for two channels: $JI = S0$ wave

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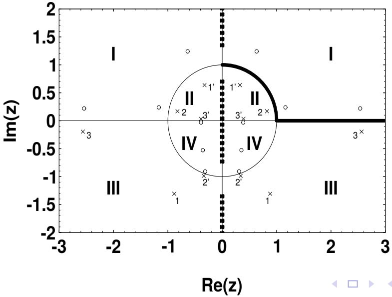
3	1431.7	-79.3	$(-,-)$: *III*
3’	1394.9	-120.6	$(-,+)$: *II*

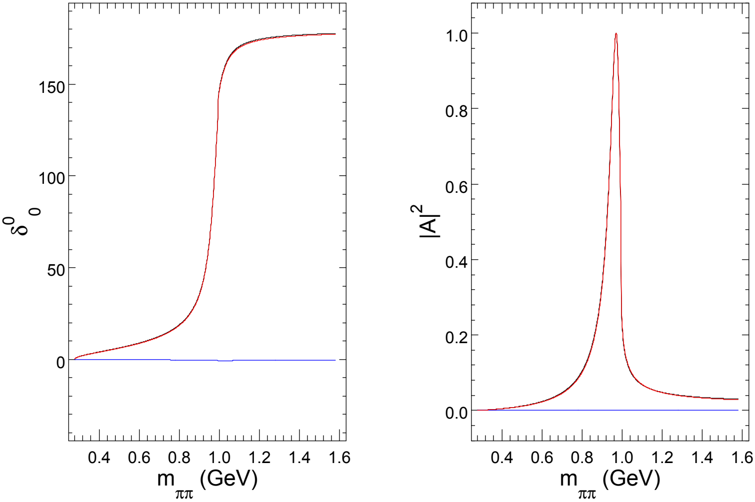
$$z = \frac{k_1 + k_2}{\sqrt{m_K^2 - m_\pi^2}}$$

Rysunek 16: Położenie biegunów (krzyże) i zer (kółka) elementu macierzowego S_{11} macierzy rozpraszania dla dopasowania do zestawu D_{CKM} A. Gruba linia ciągła oznacza obszar fizyczny rozpraszania w kanałach sprzężonych $\pi\pi$ i $K\overline{K}$. Grubą linią przerywaną przedstawione jest położenie cięć funkcji Josta. Cienką linią zaznaczony jest okrąg $|z| = 1$. Numeracja poszczególnych płatów i biegunów została wyjaśniona w tekście.



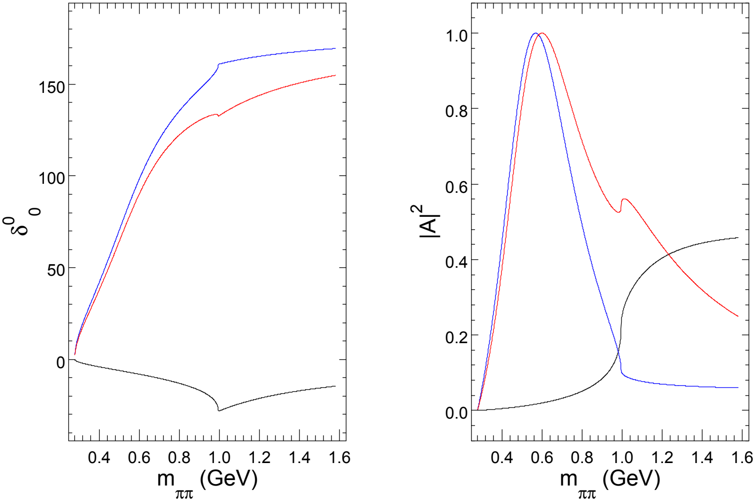
For two channels: TWO POLES 2 and 2' ($f_0(980)$)

2: 982.0-i*36.9 2': 432.4-i*8.4 both 2 and 2'



For two channels: TWO POLES 1 and 1' ($f_0(500)$)

1: 639.6-i*323.9 1': 511.4-i*230.6 both 1 and 1'

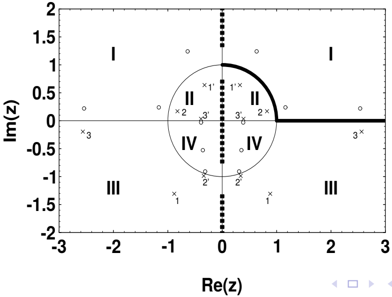


Example for two channels: $J0 = S0$ wave

Pole	ReE_{pole} MeV	ImE_{pole} MeV	R. sheet
1	639.6	-323.9	$(-,-)$: *III*
1'	511.4	-230.6	$(-,+)$: *II*
2	982.0	-36.9	$(-,+)$: *II*
2'	432.4	-8.4	$(-,-)$: *III*
3	1431.7	-79.3	$(-,-)$: *III*
3'	1394.9	-120.6	$(-,+)$: *II*

$$z = \frac{k_1 + k_2}{\sqrt{m_K^2 - m_\pi^2}}$$

Rysunek 16: Położenie biegunów (krzyże) i zer (kółka) elementu macierzowego S_{11} macierzy rozpraszania dla dopasowania do zestawu D_{CKM} A. Gruba linia ciągła oznacza obszar fizyczny rozpraszania w kanałach sprzężonych $\pi\pi$ i $K\overline{K}$. Grubą linią przerywaną przedstawione jest położenie cięć funkcji Josta. Cienką linią zaznaczony jest okrąg $|z| = 1$. Numeracja poszczególnych płatów i biegunów została wyjaśniona w tekście.



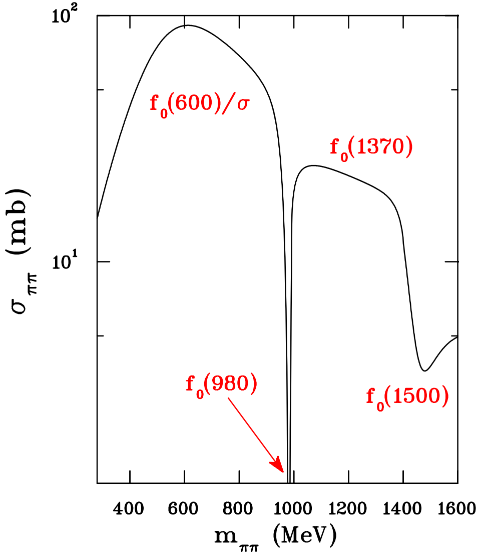
More than 2 channels

- more poles and more Riemann sheets (2^n)
- no similar "z" variable

2^n Riemann sheets for n channels

channel	$C=0$		$C=1$		sign	sheet
	ReE	ImE	ReE	ImE	Imk_π, Imk_K, Imk_3	
$\pi\pi$	658	-607	564	-279	$-,-,-$	VI
			518	-261	$-,+,+$	II
			211	0	$-,+,-$	VII
			532	-315	$-,-,+$	III
			235	0	$+,+,-$	VIII
$\pi\pi$	1346	-275	1405	-74	$-,-,-$	VI
			1445	-116	$-,+,+$	II
			1424	-94	$-,+,-$	VII
			1456	-47	$-,-,+$	III
$K\bar{K}$	881	-498	170	0	$+,-,-$	V
			159	0	$-,-,-$	VI
			418	-10	$-,-,+$	III
			1038	-204	$-,+,-$	VII
			988	-31	$-,+,+$	II
$\sigma\sigma$	118	-2227	4741	-4688	$-,-,-$	VI
			3687	-2875	$-,+,-$	VII
			3626	-3456	$+,-,-$	V
			3533	-579	$+,+,-$	VIII

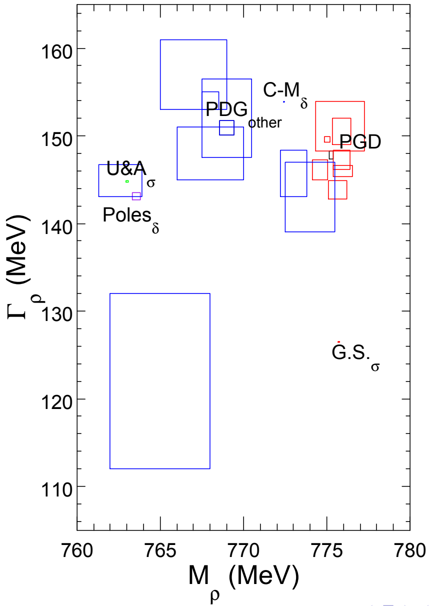
Puzzling ($J I$) $S0$ wave $\pi\pi$ cross section



2^n Riemann sheets for n channels

channel	$C = 0$		$C = 1$		sign	sheet	
	ReE	*ImE*	*ReE*	*ImE*	Imk_π, Imk_K, Imk_3		
$\pi\pi$	658	-607	564	-279	$-,-,-$	VI	
			518	-261	$-,+,+$	II	$\leftarrow f_0(500)$
			211	0	$-,+,-$	VII	
			532	-315	$-,-,+$	III	
			235	0	$+,+,-$	VIII	
$\pi\pi$	1346	-275	1405	-74	$-,-,-$	VI	$\leftarrow f_0(1370)?$
			1445	-116	$-,+,+$	II	
			1424	-94	$-,+,-$	VII	
			1456	-47	$-,-,+$	III	$\leftarrow f_0(1500)?$
$K\bar{K}$	881	-498	170	0	$+,-,-$	V	
			159	0	$-,-,-$	VI	
			418	-10	$-,-,+$	III	
			1038	-204	$-,+,-$	VII	
			988	-31	$-,+,+$	II	$\leftarrow f_0(980)$
$\sigma\sigma$	118	-2227	4741	-4688	$-,-,-$	VI	
			3687	-2875	$-,+,-$	VII	
			3626	-3456	$+,-,-$	V	
			3533	-579	$+,+,-$	VIII	

$\rho(770)$



C-M δ
PDG other
PGD
U&A σ
Poles δ
G.S. σ
160
150
140
130
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110
760
765
770
775
780
Γ ρ (MeV)
M ρ (MeV)

Gounaris-Sakurai pion EM FF and $\rho(770)$

Gounaris-Sakurai pion electromagnetic form factor at the elastic region.

What are the correct $\rho(770)$ meson mass and width values?
Erik Bartoŝ, Stanislav Dubniĉka, Andrej Liptaj Anna Zuzana Dubniĉkova and RK
PRD 96, 113004 (2017)

$$\sigma_{tot}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2(0)}{3s}\beta_\pi^3(s)\left|F_\pi^{EM,I=1}(s) + Re^{i\phi}\frac{m_\omega^2}{m_\omega^2 - s - im_\omega\Gamma_\omega}\right|^2$$

where pion "velocity" $\beta_\pi(s) = \sqrt{\frac{s-4m_\pi^2}{s}}$, R - amplitude for $\rho - \omega$ interference (free parameter), phase $\phi = ArcTan\frac{m_\rho\Gamma_\rho}{m_\rho^2 - m_\omega^2}$ is the $\delta_1^1 = \delta_\rho$ fixed at $s = m_\omega^2$

Fit to data for $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$
- M. Ablikin et al. (BESIII Collaboration), Phys. Lett. B 753, 629 (2016).
- J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 86, 032013 (2012).

Gounaris-Sakurai pion electromagnetic form factor

G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. 21, 244 (1968)

Assumption: $\frac{q^3}{\sqrt{s}} Cotg\delta_1^1(s) = a + bq^2 + h(s)q^2$ where $h(s) = \frac{2q}{\pi\sqrt{s}} Log\left(\frac{\sqrt{s}+2q}{2m_\pi}\right)$

Then $F_\pi^{GS}(s) = \frac{\sqrt{s}}{q^3} \frac{1}{Cotg\delta_1^1(s) - i}$ - no direct dependence on $\rho(770)$ parameters, however taking into account two conditions:

- $Cotg\delta_1^1(s)\Big|_{s=m_\rho^2} = 0$ and
- $F_\pi^{BW}(s) = \frac{m_\rho^2}{m_\rho^2 - s - i m_\rho \Gamma_\rho} \to \delta_1^1(s) = ArcTan \frac{m_\rho \Gamma_\rho}{m_\rho^2 - s} \to \frac{d\delta_1^1(s)}{ds}\Big|_{s=m_\rho^2} = \frac{1}{m_\rho \Gamma_\rho}$

$$a = \frac{4q_\rho^5}{m_\rho^2 \Gamma_\rho} + 4q_\rho^4 h'(m_\rho^2)$$

$$b = -\frac{4q_\rho^3}{m_\rho^2 \Gamma_\rho} - 4q_\rho^4 h'(m_\rho^2) - h(m_\rho^2)$$

$$F_\pi^{GS}(s) = \frac{m_\rho^2 + m_\rho \Gamma_\rho \left[\frac{3m_\pi^2}{\pi q_\rho^2} log\left(\frac{m_\rho + 2mq_\rho}{2m_\pi}\right) + \frac{m_\rho}{2\pi q_\rho} - \frac{m_\pi^2 m_\rho}{\pi q_\rho^3}\right]}{(m_\rho^2 - s) + \Gamma_\rho \frac{m_\rho^2}{q_\rho^3}\left[q^2\left(h(s) - h(m_\rho^2)\right) + q_\rho^2 h'(m_\rho^2)(m_\rho^2 - s)\right] - i m_\rho \Gamma_\rho \left(\frac{q}{q_\rho}\right)^3 \frac{m_\rho}{\sqrt{s}}}$$

Fit to unified BESIII-BABAR data at the elastic region using G-S model

- $\chi^2 = 40.6$ pdf
- $m_\rho = (775.73 \pm 0.10)$ MeV
- $\Gamma_\rho = (126.51 \pm 0.13)$ MeV
- PDG'2017:

  $m_\rho = 775.26 \pm 0.25$ MeV (from $e^+e^- \to \pi^+\pi^-$)
  $m_\rho = 769.0 \pm 0.9$ MeV (other)

  $\Gamma_\rho = 147.8 \pm 0.9$ MeV (from $e^+e^- \to \pi^+\pi^-$)
  $\Gamma_\rho = 150.9 \pm 1.7$ MeV (other)

in Ref. [6].

Now, the $\rho^0(770)$ meson parameters will be determined by an application of the U&A model of the pion EM FF to an optimal description of the same data on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process.

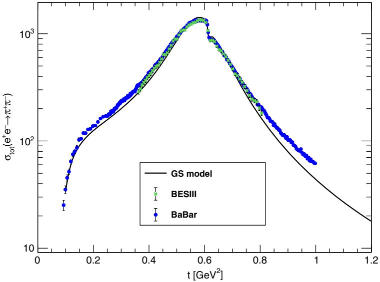

FIG. 2. Optimal description of the unified BESIII-*BABAR* data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ at the elastic region by the pion EM FF G.-S. model.

Fit to unified BESIII-BABAR data at the elastic region using U&A model

- $F_{\pi}^{VDM}(s) = \frac{m_{\rho}^2 f_{\rho\pi\pi}/f_{\rho}}{m_{\rho}^2 - s}$
- $F_{\pi}^{VDM}(q) = \frac{(i-q_{\rho})(i+q_{\rho})}{(q-q_{\rho})(q+q_{\rho})} f_{\rho\pi\pi}/f_{\rho}$
- $F_{\pi}^{VDM}(q) = \frac{(q-q_Z)(i-q_P)(i-q_{\rho})(i+q_{\rho})}{(q-q_P)(i-q_Z)(q-q_{\rho})(q+q_{\rho})} f_{\rho\pi\pi}/f_{\rho}$
- $\chi^2 = 1.54$ pdf
- $m_{\rho} = (763.03 \pm 0.14)$ MeV
- $\Gamma_{\rho} = (144.8 \pm 0.23)$ MeV

The optimal description of the recent data [1,2] on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process at the region of the ρ meson resonance by (21) (see Fig. 3), achieved with $\chi^2/ndf = 1.5443$, gives the ρ meson mass and width

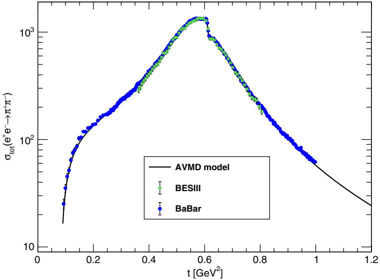

FIG. 3. Optimal description of the unified BESIII-*BABAR* data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ at the elastic region by the pion EM FF U&A model.

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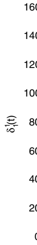

FIG.
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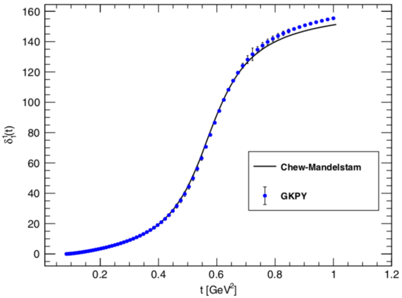
Fit to δ_1^1 data using Chew-Mandelstam type effective-range formula

- $\frac{q^3}{\sqrt{s}} Cotg\delta_1^1(s) = a + bq^2 + h(s)q^2$
  where $h(s) = \frac{2q}{\pi\sqrt{s}} Log\left(\frac{\sqrt{s}+2q}{2m_\pi}\right)$
- $a = 0.2860 \pm 0.0011$ MeV^2, $b = -2.7025 \pm 0.0089$
- $\chi^2 = 2.45$ pdf
- $m_\rho = (772.42 \pm 0.03)$ MeV
- $\Gamma_\rho = (153.85 \pm 0.11)$ MeV



Fit to δ_1^1 data

- $F_\pi^{EM,I=1}(s) =$

$$P_n(s) exp\Big[\frac{s}{\pi}\int_4^\infty \frac{\delta_1^1(s')}{s'(s'-s)}ds'\Big]$$

- $\delta_1^1(q) = ArcTan\frac{A_3 q^3 + A_5 q^5 + \dots}{1 + A_2 q^2 + A_4 q^4 + \dots}$
- $\equiv Log\frac{(q-q_2)(q-q_3)(q-q_4)(q-q_5)}{(q-q_2^*)(q-q_3^*)(q-q_4^*)(q-q_5^*)}$
- $\chi^2 = 0.024$ pdf
- $m_\rho = (763.56 \pm 0.51)$ MeV
- $\Gamma_\rho = (143.09 \pm 0.82)$ MeV

$$\delta_1^1(q) = arctg\frac{A_3 q^3 + A_5 q^5 + \dots}{1 + A_2 q^2 + A_4 q^4 + \dots}, \qquad (26)$$

where $A_1 \equiv 0$ in order to secure the threshold behavior of $\delta_1^1(q)$. An optimal description of the GKPY phase shift $\delta_1^1(q)$ data is achieved (see Fig. 5) with $\chi^2/ndf = 0.0244$ and four nonzero coefficients A_2, A_3, A_4, and A_5.

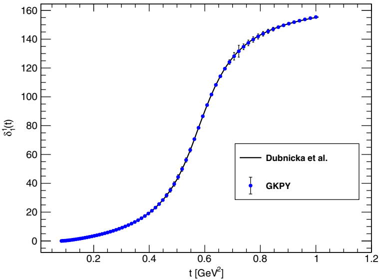
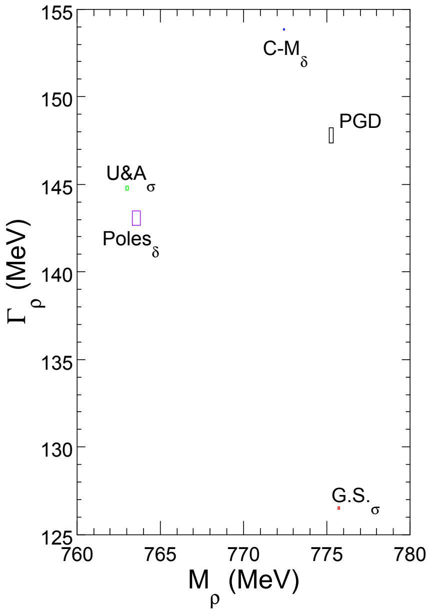


FIG. 5. Optimal description of the most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with model-independent parametrization (26).



C-M$_\delta$
PGD
U&A$_\sigma$
Poles$_\delta$
G.S.$_\sigma$
Γ_ρ (MeV)
M_ρ (MeV)
155
150
145
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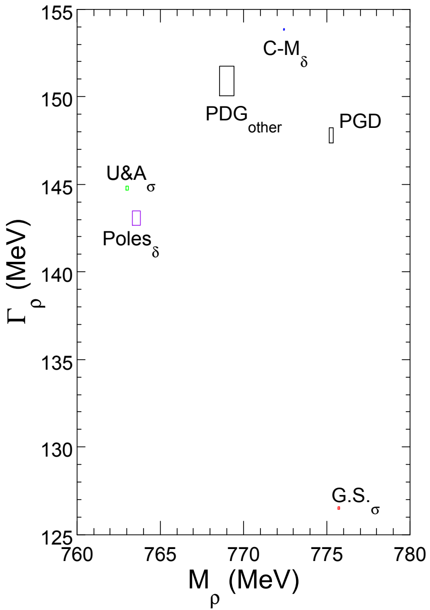

155
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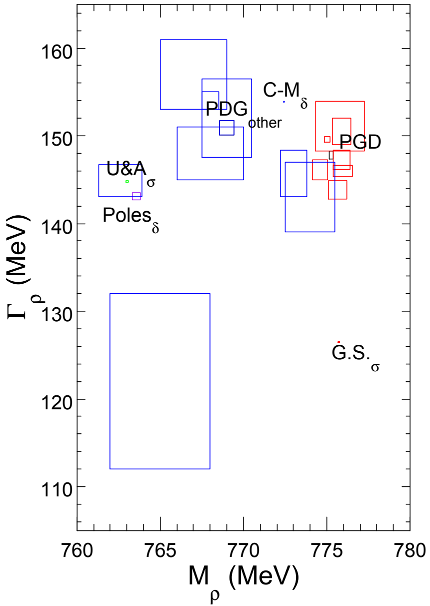
Γ_ρ (MeV)
M_ρ (MeV)
C-M_δ
PDG_{other}
PGD
U&A_σ
$Poles_\delta$
G.S.$_\sigma$



C-M$_\delta$
PDG$_{other}$
PGD
U&A$_\sigma$
Poles$_\delta$
G.S.$_\sigma$
160
150
140
130
120
110
760
765
770
775
780
Γ_ρ (MeV)
M_ρ (MeV)

Generalisation of Gounaris-Sakurai model to excited ρ mesons $\rho(1450)$ and $\rho(1700)$

- $F_\pi = \frac{1}{1+\beta+\gamma}\left[F^{GS}_{\rho(770)}(s)\left(1+\delta\frac{s}{m^2_\omega}BW_\omega(s)\right)+\beta F^{GS}_{\rho(1450)}(s)+\gamma F^{GS}_{\rho(1700)}(s)\right]$
- $\chi^2 = 0.98$ pdf

Generalisation of U&A model to excited ρ mesons

- $F_\pi = \frac{\Pi(q-q_i)}{\Pi(q+q_i^*)}$
- $\chi^2 = 1.84$ pdf

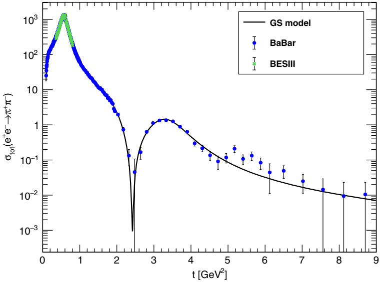


FIG. 6. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{\rm tot}(e^+e^-\to\pi^+\pi^-)$ by the generalized pion EM FF G.-S. model.

determined by the original pion EM FF G.-S. model (13) to be valid only at the elastic region.

A totally different situation is in a generalization of the U&A pion EM FF model. Here, the contribution of all three vector mesons is at an equal level. Only now, the effective inelastic threshold, which is left as a free parameter of the model, has to be taken into account explicitly. Therefore, instead of the q variable, the W variable

TABL
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Param

m_ρ
$m_{\rho'}$
$m_{\rho''}$
Γ_ρ
$\Gamma_{\rho'}$
$\Gamma_{\rho''}$
χ^2/nd

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Parameter	PDG MeV	G.S. MeV	U&A MeV
m_ρ	775.26 ± 0.25	774.81 ± 0.01	763.88 ± 0.04
$m_{\rho'}$	1465.00 ± 25.00	1497.70 ± 1.07	1326.35 ± 3.46
$m_{\rho''}$	1720.00 ± 20.00	1848.40 ± 0.09	1770.54 ± 5.49
Γ_ρ	149.10 ± 0.80	149.22 ± 0.01	144.28 ± 0.01
$\Gamma_{\rho'}$	400.00 ± 60.00	442.15 ± 0.54	324.13 ± 12.01
$\Gamma_{\rho''}$	250.00 ± 100.00	322.48 ± 0.69	268.98 ± 11.40
χ^2 pdf		0.98	1.84
		14 param.	11 param.

WHAT ARE THE CORRECT $\rho^0(770)$ MESON MASS …

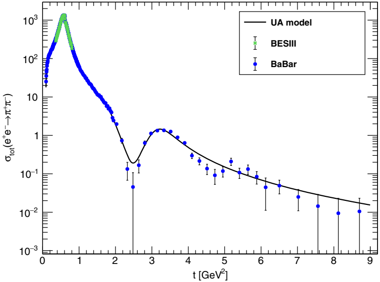

FIG. 7. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by the generalized pion

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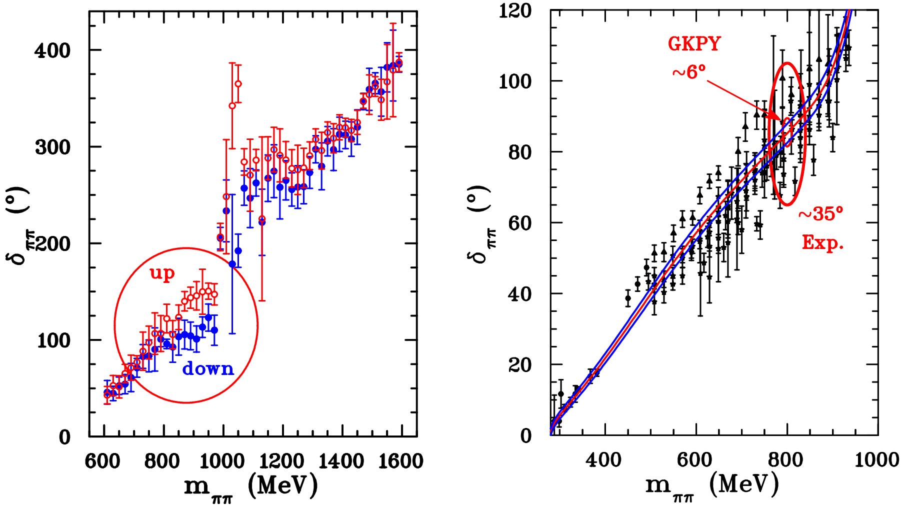
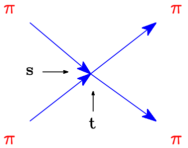
Experimental data for the $\pi\pi$ in the S0 wave (JI)

In PWA (CERN-Munich group'74) $A(s,t) \sim Cos(\theta_S - \theta_P)$



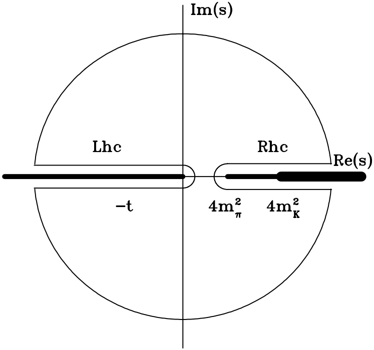
Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory $\longleftrightarrow$ experiment

crossing symmetry:



$\rightarrow \vec{T}_s(s,t) = \hat{C}_{st}\vec{T}_t(t,s)$

$\vec{T}(s,t)$ + crossing symmetry $\rightarrow$ dispersion relations for $\underline{4m_\pi^2 < s < \sim (1150\ \text{MeV})^2}$

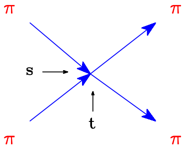


Once subtracted DR:

$$
\begin{aligned}
\text{Re}\,\vec{F}(s,t) \;=\;& \text{Re}\,\vec{F}(s_0,t) + \frac{s-s_0}{\pi} \\
&\times \left[\int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}\,\vec{F}(s',t)}{(s'-s_0)(s'-s)} \right. \\
&+ \left. \int_{-t}^{-\infty} ds' \frac{\text{Im}\,\vec{F}(s',t)}{(s'-s_0)(s'-s)} \right]
\end{aligned}
$$

Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory $\longleftrightarrow$ experiment

crossing symmetry:



$\rightarrow \vec{\boldsymbol{T}}_{\boldsymbol{s}}(s,t) = \hat{\boldsymbol{C}}_{st}\vec{\boldsymbol{T}}_{\boldsymbol{t}}(t,s)$

$\vec{\boldsymbol{T}}(\boldsymbol{s},\boldsymbol{t})$ + crossing symmetry $\rightarrow$ dispersion relations for $4m_\pi^2 < s < \sim (1150\ \mathrm{MeV})^2$

Once subtracted dispersion relations ("GKPY" for the S and P waves):

$$\mathrm{Re}\ t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C_{st}^{II'} a_0^{I'} + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s,s')\ \mathrm{Im}\ t_{\ell'}^{I'(IN)}(s')$$

$a_0^{I'}$ - subtraction constant $= \vec{\boldsymbol{T}}_{\boldsymbol{s}}(s = 4m_\pi^2, t = 0)$ - scattering lengths from only S wave

due to $\mathrm{Re}\ t_\ell^I(k) = k^{2\ell}(a_\ell^I + b_\ell^I k^2 + O(k^4))$ $\quad$ $\mathrm{Re}\ t_\ell^{I(OUT)}(s) - \mathrm{Re}\ t_\ell^{I(IN)}(s) \rightarrow 0$

GKPY equations and $\pi\pi$ amplitudes

partial waves: ***Jl***

experiment

F1 D2

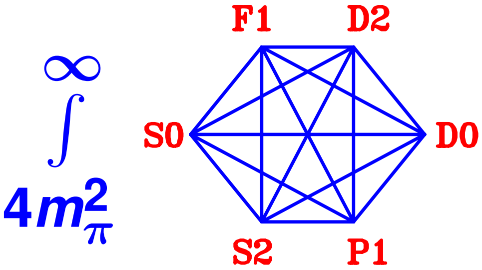
S0 D0

S2 P1

GKPY equations and $\pi\pi$ amplitudes

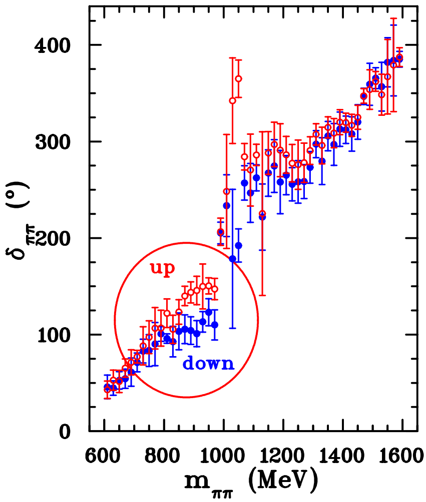
partial waves: *Jl*

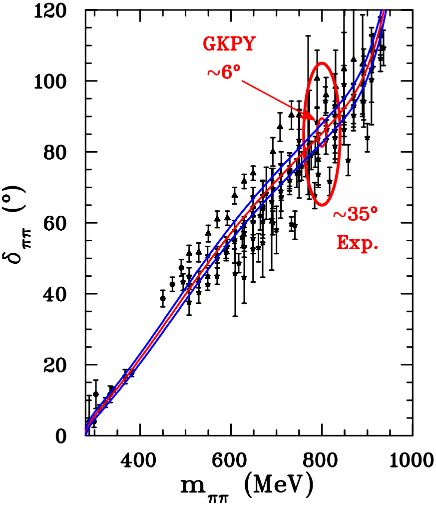
experiment + theory (GKPY)



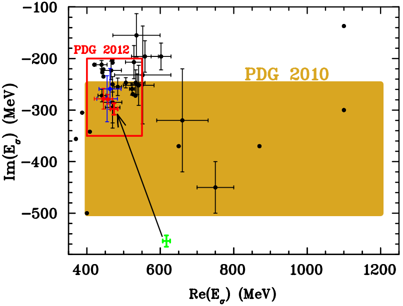
Experimental data for the $\pi\pi$ in the S0 wave (JI)

In PWA (CERN-Munich group'74) $A(s,t) \sim Cos(\theta_S - \theta_P)$





what forces GKPY eqs to pull up-left the sigma pole?



Two things: trigonometry and crossing symmetry algebra lead to narrower and lighter σ.

Modified $\pi\pi$ amplitude with σ pole PRD 90, 116005 (2014) P. Bydzovský, 1, R. Kamiński, V. Nazari

Nothing more and nothing instead of it is needed.

Resonance is near the threshold

1976 S. M. Flatté analyses the $\pi\eta$ and the $K\bar{K}$ coupled channel systems

$$A_i \sim \frac{M_R\sqrt{\Gamma_0\Gamma_i}}{M_R^2 - E^2 - iM_R(\Gamma_1 + \Gamma_2)}, \quad i = 1, 2.$$

$\Gamma_i = g_i k_i$ and $\Gamma_0 = g_1 q$ with $q = k_1(2m_K)$. So THREE free parameters: M_R, g_1, g_2.

One channel case:

$$T_{22} = \frac{\sin\delta_2}{k_2} e^{i\delta_2} \equiv \frac{1}{k_2\cot\delta_2 - ik_2},$$

$$k_2\cot\delta_2 \approx \frac{1}{a} + \frac{1}{2} r\, k_2^2 \longrightarrow T_{22} = \frac{1}{\frac{1}{a} - i\, k_2 + \frac{1}{2}\, r\, k_2^2}$$

where a is the scattering length and r is the effective range (both real).
Two channel case: A and R are complex so FOUR free parameters

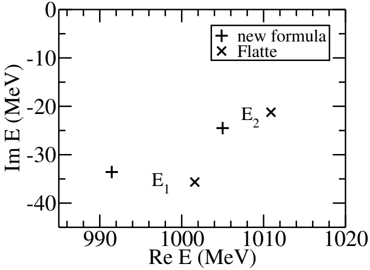
$$T_{22} = \frac{1}{2ik_2}(\eta e^{2i\delta_2} - 1) \longrightarrow T_{22} = \frac{1}{\frac{1}{A} - i\, k_2 + \frac{1}{2}\, R\, k_2^2}.$$

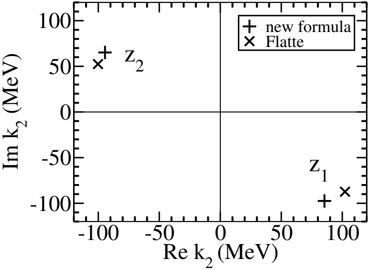
$$A = -i\left(\frac{1}{z_1} + \frac{1}{z_2}\right), \qquad R = \frac{2i}{z_1 + z_2}.$$

where z_1 and z_2 are zeroes of the S_{22} matrix element and are related with resonance.
Flatte approach: $ImR = 0$ so $Rez_1 = -Rez_2$

For $a_0(980)$

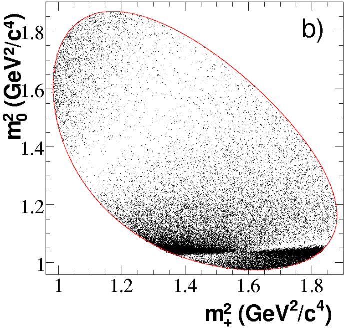
L. Leśniak, AIP Conf.Proc. 1030 (2008) 238



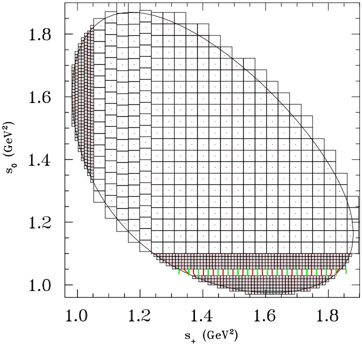


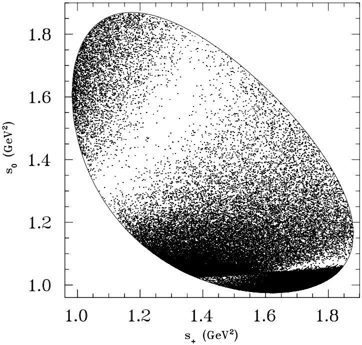
$D^0 \to K_S^0 K^+ K^-$ decays

BaBar: PRD 78, 034023 (2008) $\longrightarrow$



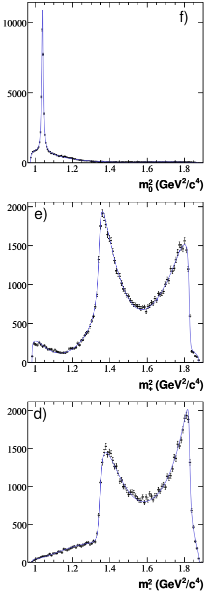
Our model *(temporary results)*:



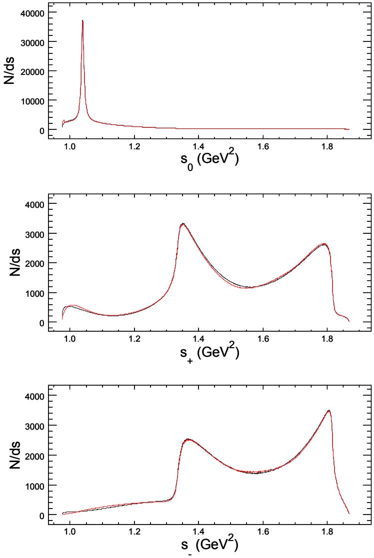


$D^0 \to K_S^0 K^+ K^-$ decays

PRD 78, 034023 (2008) (BABAR)



Our model *(temporary results)*



$D^0 \to K_S^0 K^+ K^-$ decays

BABAR analysis:

isobar model with 5 resonances: $a_0(980)$, $a_0(1450)$, $f_0(1370)$, $\phi(1020)$ and $f_2(1270)$ + their charge "twins": $A_{tot}(\mathbf{m}) = \sum_r a_r e^{i\Phi_r} A_r(\mathbf{m}) + a_{NR} e^{i\Phi_{NR}}$

Single-wave unitary analysis (our - DKLL model):

Quasi-two-body factorization: $A_{tot}(m) = S_0(m) + S_+(m) + S_-(m) + P_0(m) + D_0(m)$ with unitary $S_0(m), S_+(m), S_-(m), P_0(m), D_0(m)$ components

PRD 78, 034023 (2008) (BABAR):

Component	a_r	ϕ_r (deg)	Fraction (%)
$K_S^0 a_0(980)^0$	1	0	55.8
$K_S^0 \phi(1020)$	0.227 ± 0.005	-56.2 ± 1.0	44.9
$K_S^0 f_0(1370)$	0.04 ± 0.06	-2 ± 80	0.1
$K_S^0 f_2(1270)$	0.261 ± 0.020	-9 ± 6	0.3
$K_S^0 a_0(1450)^0$	0.65 ± 0.09	-95 ± 10	12.6
$K^- a_0(980)^+$	0.562 ± 0.015	179 ± 3	16.0
$K^- a_0(1450)^+$	0.84 ± 0.04	97 ± 4	21.8
$K^+ a_0(980)^-$	0.118 ± 0.015	138 ± 7	0.7

Sum of fractions, χ^2 and free parameters:

BABAR: 152.2%, $\chi^2 = 1.1$pdf, 14 free parameters
Our model: $\approx 130\%$, $\chi^2 \approx 1.2$pdf, 16 free parameters

Conclusions and what to do?

- Let's start analysis using S-matrix approach,
- always determine what definition of "resonance" we use,
- in case of resonances close to the threshold one has to specify what pole we use (on what Riemann sheet),
- Flatte approximation can be insufficient,
- for amplitudes with more than two channel one needs precise analysis of all poles especially when they are near the threshold,
- unitary amplitudes in wide energy range are quite simple and work well, please use them!