The decay $J/\psi \to \gamma X(J^P) \to \gamma\phi\phi$: Dynamical analysis of the $X(J^P) \to \phi\phi$ resonance contributions.

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Abstract. The dynamics of the $J^{PC} = 0^{-+}, 0^{++}$, and 2^{++} resonance contributions to the decay $J/\psi \to \gamma X(J^{PC}) \to \gamma\phi\phi$ is analysed using the data obtained by BESIII collaboration. The effective coupling constants parameterising invariant amplitudes of the transitions $J/\psi \to \gamma X(J^{PC})$ and $X(J^{PC}) \to \phi\phi$ and masses of $X(J^{PC})$ found from the fits are used to plot the $\phi\phi$ mass spectrum of each J^{PC} component.

1 Introduction

The interest in the decay $J/\psi \to \gamma\phi\phi$ [1] is related with the possible existence of the exotic glueball state decaying into the $\phi\phi$ pair [2]. The spin-parity quantum numbers of the resonance states decaying into $\phi\phi$ are reported to be $J^P = 0^+, 0^-$, and 2^+ [1, 3]. The partial wave analysis of the $\phi\phi$ system was performed in Ref. [1] based on the model with the coherent sum of the Breit-Wigner amplitudes with the constant widths. Following Ref. [1], the chain $J/\psi \to \gamma X \to \gamma\phi\phi$ is assumed here to give the dominant contribution to the amplitude. However, the dynamics of the decay chain $J/\psi \to \gamma X(J^P)$, $X(J^P) \to \phi\phi$ is relatively simple only in case of the $X(0^-)$ resonance admitting the only p wave contribution in both above vertices. In general, one should include the different spin-orbital momentum structures for different spin-parities of the $X(J^P)$ resonances in the $\phi\phi$ system, especially in case of the tensor contribution $J^{PC} = 2^{++}$. So it is reasonable to reanalyze the data of Ref. [1] in the model with the energy-dependent partial $\phi\phi$ width in order to extract the magnitudes of the effective coupling constants parameterizing the effective invariant amplitudes of the above transitions. The results of such analysis are presented below.

2 Amplitudes for specific $X(J^P)$ resonance

(a) $J^P = 0^-$. The effective amplitudes for the processes $J/\psi \to \gamma X(0^-)$ and $X(0^-) \to \phi\phi$ and their three-dimensional form in the respective rest frame systems are chosen as follows:

$$\begin{aligned} M_{J/\psi\to\gamma X(0^-)} &= g_{J/\psi\gamma X(0^-)}\epsilon_{\mu\nu\lambda\sigma}Q_\mu\epsilon_\nu k_\lambda e_\sigma = g_{J/\psi\gamma X(0^-)}m_{J/\psi}|\boldsymbol{k}|(\boldsymbol{n}\cdot[\boldsymbol{\xi}\times\boldsymbol{e}]), \\ M_{X(0^-)\to\phi\phi} &= g_{X(0^-)\to\phi\phi}\epsilon_{\mu\nu\lambda\sigma}k_{1\mu}\epsilon_{1\nu}k_{2\lambda}\epsilon_{2\sigma} = g_{X(0^-)\phi\phi}m_{12}|\boldsymbol{k}_1^*|(\boldsymbol{n}_1\cdot[\boldsymbol{\xi}_1\times\boldsymbol{\xi}_2]), \end{aligned} \tag{1}$$

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so that the amplitude of interest is

$$M_{J/\psi\to\gamma X(0^-)\to\gamma\phi\phi} = A^{(0^-)} m_{J/\psi} m_{12} |\boldsymbol{k}||\boldsymbol{k}_1^*| (\boldsymbol{\xi}[\boldsymbol{n}\times\boldsymbol{e}])(\boldsymbol{n}_1[\boldsymbol{\xi}_1\times\boldsymbol{\xi}_2]). \tag{2}$$

Here and in what follows $Q_\mu = (m_{J/\psi}, 0)$, $|\boldsymbol{k}_1^*| = (m_{12}^2/4 - m_\phi^2)^{1/2}$, $|\boldsymbol{k}| = (m_{J/\psi}^2 - m_{12}^2)/2m_{J/\psi}$; $m_{12} \equiv m_{\phi\phi}$ is the invariant mass of the $\phi\phi$ system; $\boldsymbol{\xi}$, $\boldsymbol{\xi}_{1,2}$ stand for the polarization vectors of the J/ψ and both ϕ mesons in their respective rest frames; $\boldsymbol{n}$ ($\boldsymbol{n}_1$) is, respectively, the unit vector in the direction of the photon in the J/ψ rest frame (one of the ϕ mesons in the X resonance rest frame); $\boldsymbol{e}$ is vector of polarization of the photon. Three pseudoscalar resonances $X_{1,2,3}(0^-) = \eta(2225), \eta(2100), \eta(2500)$ are included. Since the single p wave is present in both $J/\psi \to \gamma X_i(0^-)$ and $X_i(0^-) \to \phi\phi$ transition amplitudes, we consider two possible models for $A^{0^-)}$. The model A admits the mixing between resonances in the form [4]

$$A^{(0^-)} = G_{J/\psi\gamma X(0^-)} \begin{pmatrix} D_1 & -\Pi_{12} & -\Pi_{13} \\ -\Pi_{12} & D_2 & -\Pi_{23} \\ -\Pi_{13} & -\Pi_{23} & D_3 \end{pmatrix}^{-1} \begin{pmatrix} g_{X_1\phi\phi} \\ g_{X_2\phi\phi} \\ g_{X_3\phi\phi} \end{pmatrix}, \tag{3}$$

$G_{J/\psi\gamma X(0^-)} = \begin{pmatrix} g_{J/\psi\gamma X_1(0^-)} & g_{J/\psi\gamma X_2(0^-)} & g_{J/\psi\gamma X_3(0^-)} \end{pmatrix}$. Hereafter,

$$D_{X_i(J^P)}(m_{12}^2) = m_{X_i(J^P)}^2 - m_{12}^2 - i m_{X_i(J^P)}\Gamma'_{X_i} - i m_{12}\Gamma_{X_i(J^P)\to\phi\phi} \tag{4}$$

is the inverse propagator of the resonance $X_i(J^P)$ where the energy dependence of the $\phi\phi$ mode is taken into account while the constant Γ'_{X_i} approximately takes into account other possible modes. In the case of $J^P = 0^-$ the partial width and polarization operator Π_{ij} responsible for the mixing look as follows:

$$\begin{aligned} \Gamma_{X_i(0^-)\to\phi\phi} &= g^2_{X_i(0^-)\to\phi\phi}|\boldsymbol{k}_1^*|^3/8\pi, \\ \Pi_{ij} &= \mathrm{Re}\Pi_{ij} + i m_{12} g_{X_i(0^-)\phi\phi} g_{X_j(0^-)\phi\phi}|\boldsymbol{k}_1^*|^3/8\pi, \end{aligned} \tag{5}$$

where $\mathrm{Im}\Pi_{ij}$ is fixed by the unitarity relation while $\mathrm{Re}\Pi_{ij}$ are assumed to be some constants, $a_{12} \equiv \mathrm{Re}\Pi_{12}$, $a_{13} \equiv \mathrm{Re}\Pi_{13}$, and $a_{23} \equiv \mathrm{Re}\Pi_{23}$ to be determined from the fit. In the model B the mixing is neglected, $\Pi_{ij} \equiv 0$. The expression for the $J^{PC} = 0^{-+}$ resonance component of the $\phi\phi$ mass spectrum spectrum is

$$\frac{dN^{(0^-)}}{dm_{12}} = \frac{\mathcal{N}}{(2\pi)^3 \times 6} \left|A^{(0^-)}\right|^2 m_{12}^2 |\boldsymbol{k}|^3 |\boldsymbol{k}_1^*|^3, \tag{6}$$

where $\mathcal{N}$ is unknown overall normalization factor.

(b) $J^P = 0^+$. One has

$$\begin{aligned} M_{J/\psi\to\gamma X(0^+)} &= -g^{(1)}_{J/\psi\gamma X(0^+)}(\epsilon e) = g^{(1)}_{J/\psi\gamma X(0^+)}(\boldsymbol{\xi}\boldsymbol{e}), \\ M_{X(0^+)\to\phi\phi} &= -g^{(1)}_{X(0^+)\phi\phi}(\epsilon_1\epsilon_2) - g^{(2)}_{X(0^+)\phi\phi}(\epsilon_1 k_2)(\epsilon_2 k_1) = \\ & \quad f^{(0^+)}_{00}(\boldsymbol{\xi}_1\boldsymbol{\xi}_2) + f^{(0^+)}_{22}(\boldsymbol{\xi}_1\boldsymbol{n}_1)(\boldsymbol{\xi}_2\boldsymbol{n}_1), \\ M_{J/\psi\to\gamma X(0^+)\to\gamma\phi\phi} &= \frac{g^{(1)}_{J/\psi\gamma X(0^+)}(\boldsymbol{\xi}\boldsymbol{e})}{D_{X(0^+)}(m_{12}^2)}\left[f^{(0^+)}_{00}\delta_{ab} + f^{(0^+)}_{22} n_{1a}n_{1b}\right]\xi_{1a}\xi_{1b}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} f^{(0^+)}_{00} &= g^{(1)}_{X(0^+)\phi\phi}, \\ f^{(0^+)}_{22} &= \left[2g^{(1)}_{X(0^+)\phi\phi} + g^{(2)}_{X(0^+)\phi\phi} m_{12}^2\right]\frac{\boldsymbol{k}_1^{*2}}{m_\phi^2}. \end{aligned} \tag{8}$$

The $X(0^+) \to \phi\phi$ decay width is

$$\Gamma_{X(0^+)\to\phi\phi} = \frac{|\boldsymbol{k}_1^*|}{16\pi m_{12}^2}\left(2\left|f_{00}^{(0^+)}\right|^2 + \left|f_{00}^{(0^+)} + f_{22}^{(0^+)}\right|^2\right). \tag{9}$$

The $J^{PC} = 0^{++}$ resonance component of the spectrum which uses the single resonance contribution is given by the expression

$$\begin{aligned}\frac{dN^{(0^+)}}{dm_{12}} &= \mathcal{N}\frac{2m_{12}^2\Gamma_{J/\psi\to\gamma X(0^+)}\Gamma_{X(0^+)\to\phi\phi}}{\pi|D_{X(0^+)}|^2},\\ \Gamma_{J/\psi\to\gamma X(0^+)} &= |g^{(1)}_{J/\psi\gamma X(0^+)}|^2|\boldsymbol{k}|/12\pi m_{J/\psi}^2.\end{aligned} \tag{10}$$

(c) $J^P = 2^+$. The necessary amplitudes look as follows.

$$\begin{aligned}M_{J/\psi\to\gamma X(2^+)} &= -\left[g^{(1)}_{J/\psi\gamma X(2^+)}(\epsilon e)Q_\mu Q_\nu + g^{(2)}_{J/\psi\gamma X(2^+)}(\epsilon k)e_\mu k_\nu + g^{(3)}_{J/\psi\gamma X(2^+)}\epsilon_\mu e_\nu\right]T_{\mu\nu} =\\ &\left[g_{02}(\boldsymbol{\xi}\cdot\boldsymbol{e})n_in_j + g_{12}(\boldsymbol{\xi}\cdot\boldsymbol{n})e_in_j + g_{20}\xi_ie_j\right]t_{ij},\end{aligned} \tag{11}$$

where $T_{\mu\nu}$ (t_{ij}) is the polarization tensor of the $X(2^+)$ resonance (its three-dimensional counterpart in its rest frame) and

$$\begin{aligned}g_{02} &= \frac{m_{J/\psi}^2\boldsymbol{k}^2}{m_{12}^2}g^{(1)}_{J/\psi\gamma X(2^+)},\\ g_{12} &= \frac{\boldsymbol{k}^2}{m_{12}}\left[g^{(2)}_{J/\psi\gamma X(2^+)}q_0 + \frac{g^{(3)}_{J/\psi\gamma X(2^+)}}{q_0+m_{12}}\right],\\ g_{20} &= g^{(3)}_{J/\psi\gamma X(2^+)};\end{aligned} \tag{12}$$

$q_0 = (m_{J/\psi}^2 + m_{12}^2)/2m_{J/\psi}$. The $X(2^+) \to \phi\phi$ decay amplitude is

$$\begin{aligned}M_{X(2^+)\to\phi\phi} &= \Big\{g^{(1)}_{X(2^+)\phi\phi}\epsilon_{1\mu}\epsilon_{2\nu} + k_{1\mu}k_{2\nu}\left[g^{(2)}_{X(2^+)\phi\phi}(\epsilon_1\epsilon_2) + g^{(3)}_{X(2^+)\phi\phi}(\epsilon_1k_2)(\epsilon_2k_1)\right] +\\ &g^{(4)}_{X(2^+)\phi\phi}\left[\epsilon_{1\mu}k_{2\nu}(\epsilon_2k_1) + \epsilon_{2\mu}k_{1\nu}(\epsilon_1k_2)\right]\Big\} = \Big\{f_{20}^{(2^+)}\xi_{1i}\xi_{2j}+\\ &f_{02}^{(2^+)}(\boldsymbol{\xi}_1\cdot\boldsymbol{\xi}_2)n_{1i}n_{1j} + f_{22}^{(2^+)}\left[(\boldsymbol{\xi}_1\cdot\boldsymbol{n}_1)\xi_{2i} + (\boldsymbol{\xi}_2\cdot\boldsymbol{n}_1)\xi_{1i}\right]n_{1j}+\\ &f_{24}^{(2^+)}(\boldsymbol{\xi}_1\cdot\boldsymbol{n}_1)(\boldsymbol{\xi}_2\cdot\boldsymbol{n}_1)n_{1i}n_{1j}\Big\}t_{ij},\end{aligned} \tag{13}$$

where

$$\begin{aligned}f_{20}^{(2^+)} &= g^{(1)}_{X(2^+)\phi\phi},\\ f_{02}^{(2^+)} &= g^{(2)}_{X(2^+)\phi\phi}\boldsymbol{k}_1^{*2},\\ f_{22}^{(2^+)} &= \frac{\boldsymbol{k}_1^{*2}}{m_\phi}\left[\frac{g^{(1)}_{X(2^+)\phi\phi}}{k_{10}^*+m_\phi} + m_{12}g^{(4)}_{X(2^+)\phi\phi}\right],\\ f_{24}^{(2^+)} &= \frac{\boldsymbol{k}_1^{*4}}{m_\phi^2}\left[\frac{g^{(1)}_{X(2^+)\phi\phi}}{(k_{10}^*+m_\phi)^2} + 2g^{(2)}_{X(2^+)\phi\phi} + m_{12}^2g^{(3)}_{X(2^+)\phi\phi} + \frac{2m_{12}g^{(4)}_{X(2^+)\phi\phi}}{k_{10}^*+m_\phi}\right];\end{aligned} \tag{14}$$

$k_{10}^* = m_{12}/2$. The resulting amplitude $M^{(2^+)} \equiv M_{J/\psi\to\gamma X(2^+)\to\gamma\phi\phi}$ looks like

$$\begin{aligned}M^{(2^+)} &= \left\{g_{02}(\boldsymbol{\xi}\cdot\boldsymbol{e})n_in_j + g_{20}\xi_ie_j + g_{12}(\boldsymbol{\xi}\cdot\boldsymbol{n})e_in_j\right\}\Big[f_{20}^{(2^+)}\delta_{ka}\delta_{lb} + f_{02}^{(2^+)}\delta_{ab}n_{1k}n_{1l}+\\ &f_{22}^{(2^+)}(n_{1a}\delta_{kb} + n_{1b}\delta_{ka})n_{1l} + f_{24}^{(2^+)}n_{1a}n_{1b}n_{1k}n_{1l}\Big]\frac{\Pi_{ij,kl}}{D_{X(2^+)}(m_{12}^2)}\times\xi_{1a}\xi_{2b},\end{aligned} \tag{15}$$

where $\Pi_{ij,kl} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})/2 - \delta_{ij}\delta_{kl}/3$. The radiative width is

$$\Gamma_{J/\psi\to\gamma X(2^+)} = \frac{|\boldsymbol{k}|}{72\pi m^2_{J/\psi}}\left[4g^2_{02} + 7g^2_{20} - 4g_{02}g_{20} + 3(g_{12} + g_{20})^2\right]. \tag{16}$$

The expression for $\Gamma_{X(2^+)\to\phi\phi}$ can be found in Ref. [5]. Note that nine free parameters are required to fit the single 2^+ contribution. Taking into account three 2^+ contributions, as is made in Ref. [1], requires 27 free parameters. So, having in mind a limited statistics of the data [1], we will try to describe the 2^+ component with the single resonance using the expression

$$\frac{dN^{(2^+)}}{dm_{12}} = \mathcal{N}\frac{2m^2_{12}\Gamma_{J/\psi\to\gamma X(2^+)} \times \Gamma_{X(2^+)\to\phi\phi}}{\pi|D_{X(2^+)}|^2}. \tag{17}$$

Using Eqs. (2), (7), and (15) for the amplitudes $M_{J/\psi\to\gamma X(J^P)\to\gamma\phi\phi}$ one can prove that different J^P contributions to the $\phi\phi$ mass spectrum do not interfere [5].

3 Results

Let us present the results of the analysis. The magnitudes of the coupling constants necessary for plotting all the following J^P contributions to the $\phi\phi$ mass spectrum can be found in Ref. [5].

(a) $J^P = 0^-$. The results of fitting this component are presented in Figs. 1 and 2. One can see that three 0^- resonances in the models A and B interfere differently but the resulting spectra coincide.

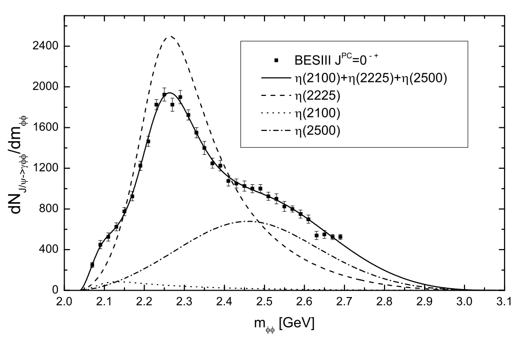

Figure 1. Fitting the 0^- component in the model A.

(b) $J^P = 0^+$. The fitting with Eq. (10) results in a poor fit with $\chi^2/n_{\rm d.o.f} = 37.8/28 = 1.4$. A better fit is obtained when adding the second 0^+ resonance. To this end, we neglect the mixing of the $X_1(0^+)$ and $X_2(0^+)$ analogously to the model B in the $J^P = 0^-$ case and use the expression

$$\frac{dN^{(0^+)}}{dm_{12}} = \frac{\mathcal{N}|\boldsymbol{k}||\boldsymbol{k}^*_1|}{(2\pi)^3 \times 12m^2_{J/\psi}}\left(2|A_0|^2 + |A_0 + A_2|^2\right), \tag{18}$$

where $\mathcal{N}$ is the same unknown overall normalization factor as in Eq. (6) and $A_0 = \sum_{i=1,2} g^{(1)}_{J/\psi\gamma X(0^+)} f^{(0^+)}_{00,i}/D_{X_i(0^+)}$, $A_2 = \sum_{i=1,2} g^{(1)}_{J/\psi\gamma X(0^+)} f^{(0^+)}_{22,i}/D_{X_i(0^+)}$. Corresponding curves are shown in Fig. 3.

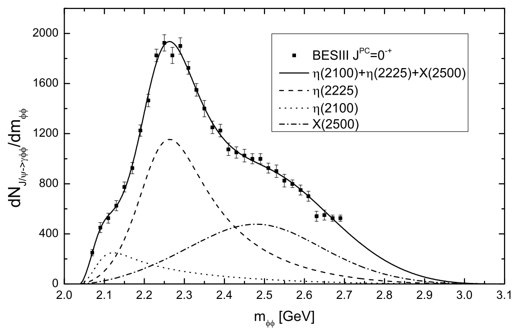

Figure 2. Fitting the 0^- component in the model B.

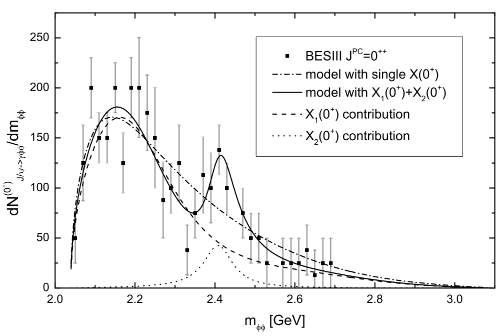

Figure 3. Fitting the 0^+ component in the models with one or two scalar resonances.

(c) $J^P = 2^+$. Surprisingly, but a rather good fit is obtained with the single tensor resonance; the corresponding curve is shown in Fig. 4.

4 Discussion and conclusion

The correct comparison of the above results with Ref. [1] and [3] requires the evaluation of the effective resonance peak positions and widths. A rough estimate can be obtained upon neglecting the resonance peak distortion due to the effects of the phase space volume. This can be made with help of Figs. 1, 2, 3, and 4 by evaluating the width at the half of height of the resonance peaks. In the case of the pseudoscalar resonances (in both models A and B) one finds the peak positions $m_{X_1(0^-)} \equiv m_{\eta(2250)} \approx 2260$ MeV, $m_{X_2(0^-)} \equiv m_{\eta(2100)} \approx 2120$ MeV, and $m_{X_3(0^-)} \equiv m_{\eta(2500)} \approx 2480$ MeV while the effective widths are $\Gamma_{X_1(0^-)} \equiv \Gamma_{\eta(2250)} \approx 220$ MeV, $\Gamma_{X_2(0^-)} \equiv \Gamma_{\eta(2100)} \approx 210$ MeV, and $\Gamma_{X_3(0^-)} \equiv \Gamma_{\eta(2500)} \approx 400$ MeV. Within one or two magnitudes of the experimental uncertainty they agree with the values given in Ref. [1]. When fitting the scalar resonance contribution, the first one designated here as $X_1(0^+)$, has the effective peak characteristics which, within the experimental accuracy, agree with those of the resonance $f_0(2100)$ observed in Ref. [1]. The second one, $X_2(0^+)$, included here to achieve the better description of the data, is new. However, taking into account rather large experimental

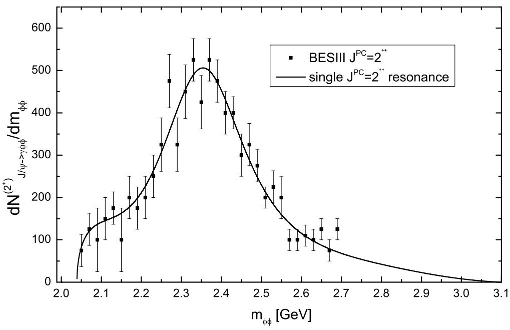

Figure 4. Fitting the 2^{+} component.

error bars in this sector, see Fig. 3, the latter conclusion should be treated as preliminary. The data with improved statistics could resolve the issue. The effective characteristics of the tensor resonance obtained here agree with those of $f_2(2340)$ cited in Ref. [3]. Note that the analysis of the tensor component of the $\phi\phi$ mass spectrum in Ref. [1] requires three tensor resonances.

To conclude, the dynamical analysis of the resonance contributions to the $J/\psi \to \gamma X \to \gamma\phi\phi$ decay amplitude is performed based on the effective amplitudes of the transitions $J/\psi \to \gamma X$ and $X \to \phi\phi$. The X-resonances with the quantum numbers $J^{PC} = 0^{-+}$, 0^{++}, and 2^{++} are taken into account to describe the $\phi\phi$ mass spectrum in the reaction $e^+e^- \to J/\psi \to \gamma X(J^{PC}) \to \gamma\phi\phi$ studied by BESIII collaboration [1]. Two models, with and without mixing of three $X(0^{-+})$ resonances were shown to give satisfactory description of the data, hence one cannot distinguish them with the present accuracy of the data. The scalar component of the $\phi\phi$ spectrum is better described in the model with two scalar resonances. The tensor component requires only one resonance, because the non-trivial behaviour shown in Fig. 4 at the left shoulder of the resonance peak is due to the dependence on the $\phi\phi$ invariant mass of the contributions with given spin and orbital angular momentum in the $X(2^{++}) \to \phi\phi$ vertex. Masses and effective coupling constants parametrizing invariant amplitudes are extracted from the fits and used for evaluation of branching fractions $B_{X(J^{PC})\to\phi\phi}$, relative branching fractions $B_{J/\psi\to\gamma X(J^{PC})\to\gamma\phi\phi}$, and for obtaining the photon angular distributions [5]. The consistency of the fits is supported by the evaluation of the incoherent sum of the 0^{-+}, 0^{++}, and 2^{++} resonance contributions to the $\phi\phi$ mass spectrum of the reaction $e^+e^- \to J/\psi \to \gamma\phi\phi$ and of the angular distribution of the final photons [5]. Their calculated magnitudes are shown [5] to agree with the data Ref. [1].

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