Off-shell fermion polarization and t-quark production

Alexander Kaloshin[1,2,*] and *Vladimir* Lomov[3,1,**]

[1]Irkutsk State University, Irkutsk, Russia
[2]Joint Institute for Nuclear Research, Dubna, Russia
[3]Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia

Abstract. Standard calculation of the polarization of final electron for pure initial state may be reformulated as a problem of looking for the complete polarization axis of produced state. It gives method for calculation of polarization applicable for both final and intermediate state fermions. We discuss modification of the energy and spin projectors in theory with parity violation. The obtained projectors are used to give the most accurate parametrization of t-quark resonance curve and simultaneously for its off-shell polarization.

1 Introduction

A concept of polarization of fermion in final state is widely used in particle physics and may be found in textbook [1]. As for polarization of particle in intermediate state, this notion is not so generally accepted but is used in some cases. We can mention the account of polarization in the method of equivalent photons, see, e.g. [2]. Another example – experimental and theoretical activity concerning of polarization of t-quark produced in hadron collisions, see reviews [3, 4]. Note that in the last case a naive definition is used in analogy with a final state particle.

If to say about off-shell fermion density matrix, first of all one should introduce some off-shell energy projectors. There are different variants, for example, in the method of quasi-real electrons [5, 6] the energy poles are accompanied by rather artificial not-orthogonal energy projectors. Note that in this approach polarization of intermediate electrons is not discussed. Another problem is related with spin projectors in fermion density matrix. In theory with γ^5 the standard spin projectors do not commute with dressed propagator and should be modified.

We start our consideration from the problem of search for a complete polarization axis for scattered electron. One can show that the found 4-axis coincides with polarization of final fermion $s^{(f)}$ calculated by the standard method for pure initial state. Since the proposed method is used an amplitude but not its square, it can be applied also for intermediate state.

To give exact meaning for fermion polarization in an intermediate state, we suggest to use the covariant form of propagator (10), which is in fact is a particular case of the spectral representation and can be generalized with account of interactions.

*e-mail: kaloshin@physdep.isu.ru
**e-mail: v.p.lomov@icc.ru

2 Polarization of final fermion

Let electron line starts from the initial state (with momentum $p_{1\mu}$ and polarization vector $s_{1\mu}$) and comes to final state with vectors $p_{2\mu}$ and $s_{2\mu}$ (polarization selected by a detector). Corresponding amplitude is

$$\mathscr{M} = \bar{u}_2(p_2, s_2)\Gamma u_1(p_1, s_1), \tag{1}$$

where $u_1(p, s)$ are bispinors and matrix Γ characterizes the scattering process.

To find the final electron polarization one needs to calculate square of amplitude

$$|\mathscr{M}|^2 = \mathrm{Sp}\,(u_2\bar{u}_2\Gamma u_1\bar{u}_1\tilde{\Gamma}) = A + B_\mu s_2^\mu = A\Big(1 + \frac{B_\mu}{A} s_2^\mu\Big), \quad \tilde{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0. \tag{2}$$

Here are shown terms dependent on detector polarization s_2 and independent on it. Since $(s_2 p_2) = 0$, only transverse part of vector $B_\mu^\perp$ remains in (2).

The matrix element square (2) is in fact projection of the scattered electron density matrix (its spin part is defined by the vector $s_\mu^{(f)}$) onto the detector density matrix ρ'. Thus comparison of (2) with

$$\mathrm{Sp}\,(\rho'\rho) = \mathrm{Sp}\,\Big(\frac{m + \hat{p}_2}{2m} \cdot \frac{1 + \gamma^5 \hat{s}_2}{2} \cdot \frac{1 + \gamma^5 \hat{s}^{(f)}}{2}\Big) = \frac{1}{2}\Big(1 - (s_2 s^{(f)})\Big) \tag{3}$$

gives the final electron polarization $s_\mu^{(f)} = -B_\mu^\perp / A$.

Besides this standard approach, there exists another method to calculate polarization of the final electron. After scattering, described by the amplitude (1), we obtain a new state

$$u(p_1, s_1) \to \Lambda^+(p_2)\Gamma u(p_1, s_1) = \Lambda_2^+\Gamma\Lambda^+(p_1)\Sigma_1 u(p_1, s_1), \quad \Lambda^+(p) = \frac{1}{2}(1 + \frac{\hat{p}}{m}). \tag{4}$$

Let us consider the problem of looking for complete polarization axis z_μ of bispinor of scattered electron

$$\frac{1 + \gamma^5 \hat{z}}{2} \cdot \Lambda^+(p_2)\Gamma u_1 = \Lambda^+(p_2)\Gamma u_1, \quad (z n_2) = 0. \tag{5}$$

We know in advance that this problem has a solution.

One can expect that two problems are equivalent: calculation of the scattered fermion polarization (2) (for pure initial state) and looking for complete polarization axis (5) of bispinor. This fact may be verified, the proof needs some portion of algebra and may be found in [7]. The feature of the problem (5) is the use of amplitude instead of its square and it allows to apply it also for intermediate fermion.

3 Spectral representation of fermion propagator

The problem of looking for complete polarization axis (5) can be applied for fermion in an intermediate state, if to use a spectral representation of propagator.

The start point is the eigenvalue problem for inverse propagator $S(p)$. Due to γ-matrix basis, one can solve a matrix problem, i.e. to look for eigenvalues λ_i and eigenprojectors Π_i

$$S\Pi_i = \lambda_i \Pi_i, \qquad \Pi_i \Pi_k = \delta_{ik}\Pi_i. \tag{6}$$

Having solved this problem, we can construct the spectral representation of $S(p)$

$$S(p) = \lambda_1 \Pi_1 + \lambda_2 \Pi_2. \tag{7}$$

If the system of projectors is complete, then propagator $G(p) = S^{-1}(p)$ is

$$G(p) = \frac{1}{\lambda_1}\Pi_1 + \frac{1}{\lambda_2}\Pi_2, \tag{8}$$

i.e. zeroes of eigenvalues λ_i define the poles of propagator. Let us consider how the propagator spectral representation is shown up in particular cases.

In case of bare propagator Π_i are in fact the known off-shell projector operators $\Lambda^{\pm}_W$

$$\Lambda^{\pm}_W = \frac{1}{2}\Big(1 \pm \frac{\hat{p}}{W}\Big), \quad p^2 = W^2, \tag{9}$$

where W is center-of-mass energy. As a result the bare propagator

$$G_0(p) = \frac{1}{\hat{p} - m_0} = \frac{1}{W - m_0}\Lambda^{+}_W + \frac{1}{-W - m_0}\Lambda^{-}_W, \quad \lambda_{1,2} = \pm W - m_0 \tag{10}$$

looks as a sum of poles with positive and negative energies in covariant form.

For dressed propagator

$$S(p) = \hat{p} - m_0 - \Sigma(p), \qquad \Sigma(p) = A(p^2) + \hat{p}B(p^2) + \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2) \tag{11}$$

spectral representation looks differently depending on interaction.

- If theory does not have γ^5 in a vertex, $\Sigma(p)$ contains unit matrix and $\hat{p}$. In this case eigenprojectors coincide with the such for bare propagator (9) and the propagator spectral representation has form

$$G(p) = \frac{1}{\hat{p} - m_0 - \Sigma(p)} = \frac{1}{W - m_0 - \Sigma^{+}(W)}\Lambda^{+}_W + \frac{1}{-W - m_0 - \Sigma^{-}(W)}\Lambda^{-}_W, \tag{12}$$

  where $\Sigma^{\pm}(W) = A(W^2) \pm WB(W^2)$.
- In theory with γ^5 the self-energy contribution also has γ^5 terms and in this case the eigenprojectors take a more complicated form [8]:

$$\begin{gathered}\Pi_{1,2}(p) = \frac{1}{2}(1 \pm \hat{n}\tau), \quad \hat{n} = \frac{\hat{p}}{W}, \\ \tau = \frac{1}{R}\Big(1 - B - \gamma^5 D - \hat{n}\gamma^5 \frac{C}{W}\Big), \quad R = \sqrt{(1-B)^2 - D^2 + C^2/W^2},\end{gathered} \tag{13}$$

  and eigenvalues are $\lambda_{1,2}(W) = -m_0 - A(W^2) \pm WR(W^2)$.

An essential aspect related with the completeness of eigenprojectors system is the existence of spin projectors commuting with propagator. But the standard spin projectors

$$\Sigma_0(s) = \frac{1 + \gamma^5 \hat{s}}{2}, \quad s^2 = -1, \quad (sp) = 0, \tag{14}$$

cease to commute with propagator in the presence of γ^5 in a vertex.

Nevertheless, there exist [9] the generalized spin projectors with desired properties

$$\Sigma(s) = \frac{1}{2}(1 + \gamma^5 \hat{s}\tau), \tag{15}$$

where matrix τ is defined in (13). One can see that without interaction ($B = C = D = 0$) or in theory with parity conservation ($C = D = 0$) the projectors $\Pi_i(p)$ and $\Sigma(s)$ return to the standard form.

Furthermore, it can be seen that "under observation" of the energy eigenprojector $\Pi_i(p)$ the spin projector (15) is significantly simplified

$$\Pi_i(p)\Sigma(s) = \Pi_i(p)\,\frac{1}{2}(1+\gamma^5\hat{s}\hat{n}), \quad n^\mu = p^\mu/W. \tag{16}$$

The problem of looking for complete polarization axis of bispinor (5) is naturally generalized for the case of virtual fermion. Using free propagator (10) one needs only to change projector Λ_2^+ in (5) to one of off-shell projectors (9). Now (5) is turned into that:

$$\frac{1}{2}(1+\gamma^5\hat{z}^\pm)\cdot\Lambda_W^\pm(p)\Gamma\Lambda_1^+\Sigma_1 u_1 = \Lambda_W^\pm(p)\Gamma\Lambda_1^+\Sigma_1 u_1, \quad (z^\pm p) = 0, \tag{17}$$

and such problem also has solution: there exists a vector $z_\mu^\pm$, $(z^\pm)^2 = -1$.

The aforesaid is also true for dressed propagator in theory with γ^5 as well. In this case we have problem where the dressed energy and spin projectors (13), (15) are involved:

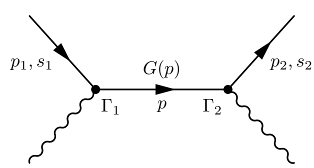
$$\Sigma(z^\pm)\cdot\Pi^\pm(p)\Gamma\Lambda_1^+\Sigma_1 u_1 = \Pi^\pm(p)\Gamma\Lambda_1^+\Sigma_1 u_1, \quad (z^\pm p) = 0 \tag{18}$$

and this problem also has a solution, see details in [9].

4 Polarization in intermediate state and t-quark propagator

The spectral representation of propagator, where the orthogonal off-shell projectors are arisen allows to give an accurate definition of fermion polarization in an intermediate state.

Consider some process with intermediate fermion is born in s-channel. Here the external boson lines correspond either to some on-mass-shell particles or to external field.



The corresponding amplitude looks as

$$\mathscr{M} = \bar{u}_2(p_2, s_2)\Gamma_1 G(p)\Gamma_2 u_1(p_1, s_1). \tag{19}$$

For the case of bare propagator or theory without γ^5 the fermion propagator in the intermediate state has form

$$G(p) = \frac{1}{\lambda_1}\Lambda_W^+ + \frac{1}{\lambda_2}\Lambda_W^-, \tag{20}$$

where $\Lambda_W^\pm$ are off-shell energy projectors (9).

If to recall the problem of looking for complete polarization axis involving $\Lambda_W^\pm$ (17), the propagator in the amplitude (19) can be rewritten as following

$$G = \frac{1}{\lambda_1}\Lambda_W^+\Sigma_0(z^+) + \frac{1}{\lambda_2}\Lambda_W^-\Sigma_0(z^-). \tag{21}$$

This gives a natural definition for polarization of fermion in an intermediate state. Polarization vectors $z^\pm$ are different for poles with positive and negative energies. It is not so evident that spin density matrices in (21) are pure ones: $(z^\pm)^2 = -1$, however, it follows from the problem of looking for axis (17).

Let us take a look now on the case of the theory with γ^5 in vertex. In this case dressed fermion propagator in intermediate state may be represented as

$$G(p) = \frac{1}{\lambda_1}\Pi_1(p) + \frac{1}{\lambda_2}\Pi_2(p), \tag{22}$$

where $\Pi_{1,2}(p)$ are the energy projectors (13). Using the problem (18) one can see that the dressed propagator inside the diagram acquires spin projectors

$$G \to \tilde{G} = \frac{1}{\lambda_1}\Pi_1(p)\frac{1+\gamma^5\hat{z}^+\hat{n}}{2} + \frac{1}{\lambda_2}\Pi_2(p)\frac{1+\gamma^5\hat{z}^-\hat{n}}{2}, \quad (z^\pm)^2 = -1. \tag{23}$$

Recall that the dressed energy projectors $\Pi_i(p)$ presented here contain self-energy contributions and should be renormalized.

In Standard Model dressed inverse propagator of t-quark looks like

$$S = \hat{p} - m - \Sigma(p), \quad \Sigma(p) = \hat{p}B(p^2) + \hat{p}\gamma^5 D(p^2), \quad D = -B.$$

At first step let us consider the case of stable fermion. We will use the On-Mass-Shell (OMS) scheme, where the loop contributions are subtracted at mass point.

As a result, the renormalized dressed inverse propagator takes the form

$$S = \hat{p} - m - \Sigma^r(p), \quad \Sigma^r(p) = A^r(p^2) + \hat{p}B^r(p^2) + \hat{p}\gamma^5 D^r(p^2)$$

and renormalized mass operator $\Sigma^r(p)$ has zero of second order at $\hat{p} = m$.

Having the renormalized propagator we can calculate the eigenprojectors

$$\begin{gathered} \Pi^r_{1,2}(p) = \frac{1}{2}(1 \pm \hat{n}\tau), \quad \hat{n} = \hat{p}/W \\ \tau = (1 - B^r - \gamma^5 D^r)/R, \qquad R = \sqrt{(1-B^r)^2 - (D^r)^2}, \end{gathered} \tag{24}$$

and eigenvalues: $\lambda^r_{1,2}(W) = -m - A^r \pm WR(W^2)$. Here

$$A^r = m\varkappa, \qquad B^r = B(W^2) - B(m^2) - \varkappa, \qquad D^r = D(W^2) - D(m^2), \tag{25}$$

where $\varkappa = 2m^2 B'(m^2)$, see details in [8].

It leads to simple properties of eigenvalues and eigenprojectors

$$\lambda^r_1(m) = 0, \quad (\lambda^r_1)'(m) = 1, \quad \Pi_1(W = m) = \frac{1}{2}(1 + \frac{\hat{p}}{m}) \tag{26}$$

and similarly for λ_2, Π_2 at the negative energy pole $W = -m$.

If loop contributions have imaginary part at $W = m$, one can use the generalized OMS scheme [10]. In this case in above formulae we should subtract only real part of a loop.

As a result we have an approximate form of eigenvalue in vicinity of $W = m$

$$\lambda_1 = W - m + i\frac{\Gamma(W)}{2} + O(g^2(W-m)) + O(g^4)$$

and eigenprogectors

$$\Pi_{1,2} = \frac{1}{2}(1 \pm \hat{n}\tau), \qquad \tau(W^2) = 1 + i\frac{\Gamma(W)}{W}\gamma^5 + O(g^2(W^2 - m^2)) + O(g^4), \tag{27}$$

where $\Gamma(W) = W\,\mathrm{Im}\,B(W^2)$.

As for the generalized spin projectors, they may be written in density matrix in two equivalent forms

$$\Sigma(s) = \frac{1}{2}\left(1+\gamma^5\hat{s}\hat{\tau}\right) \quad \text{or} \quad \frac{1}{2}\left(1+\gamma^5\hat{s}\hat{n}\right).$$

As a result we come to approximate expression for t-quark propagator in vicinity $W = m$

$$G(p) \approx \frac{1}{W - m + i\Gamma(W)/2} \cdot \Pi_1(p)\cdot\Sigma(z^+). \tag{28}$$

Here modified energy and spin projectors are

$$\Pi_1(p) = \frac{1}{2}\left[1+\hat{n}(1+i\frac{\Gamma(W)}{W}\gamma^5)\right], \quad \Sigma(z^+) = \frac{1}{2}\left[1+\gamma^5\hat{z}^+(1+i\frac{\Gamma(W)}{W}\gamma^5)\right]$$

and off-shell polarization of positive energy term is defined by vector $z^+_\mu(W)$, see (17), (23).

4.1 Conclusions

We suggest to reformulate calculation of polarization of final electron as a problem of looking for the complete polarization axis of a produced state. As a result, one can use the same approach for fermion in intermediate state, if to write propagator in form of spectral representation.

The use of this form of propagator gives natural definition for polarization of fermion in an intermediate state. Another advantages of the representation (10) are covariant form and orthogonality of system of eigenprojectors.

Most interesting is the case of fermion resonance in theory with P-parity violation. Corresponding the energy and spin projection operators are modified in theory with γ^5 – we found their form. The obtained projectors are used to give the most accurate parametrization of t-quark resonance curve including for its off-shell polarization.

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