An extension of BDK event generator

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Abstract. An extension of the well known $ee \to 4l$ event generator of F. A. Berends, P. H. Daverveldt, and R. Kleiss is presented. This extension accounts for production of narrow resonances, as well as the initial state radiation. In addition, it allows one to simulate the production of $l^+l^-\pi^+\pi^-$ with the lepton pair and the pion pair in 1^{--} states.

1 Introduction

BDK is the popular nickname for $ee \to 4l$ generator of F. A. Berends, P. H. Daverveldt, and R. Kleiss [1]. The authors named their generator DIAG36 according to number of QED diagrams accounted in $e^+e^- \to e^+e^- + e^+e^-$ simulation. Our work on its extension is inspired by needs of some analyses in KEDR [2] and Belle [3].

2 BDK generator

The generator is suitable in the energy region of $C-$ and $B-$ factories where electroweak corrections can be neglected. Electroweak processes are accounted in NEXTCALIBUR generator by F.A. Berends *et al.* [4].

DIAG36 includes four subgenerators for the four types of diagrams: multiperipheral, bremsstrahlung, annihilation and conversion. Generated events are reweighted according to the square of the exact lowest-order QED matrix element so that the interference effects are accounted.

The best experimental tests of generator for $e^+e^- \to e^+e^- + e^+e^-$ and $e^+e^- \to e^+e^- + \mu^+\mu^-$ has been performed by L3 experiment at LEP [5], the accuracy of about 2% has been reached.

3 Identity effect in $e^+e^- \to e^+e^- + e^+e^-$ process

The BDK generator takes into account all diagrams corresponding to permutation of identical particle momenta. It is known that in the total cross section the identity effect is negative and is suppressed by the factor $ln(s/m_e^2)$ [6]. However, BDK generator predicts that identity becomes important when the cut on the invariant mass is applied. This statement is illustrated by Figure 1. Worth note that identity effect is not considered in some popular two-photon event generators.

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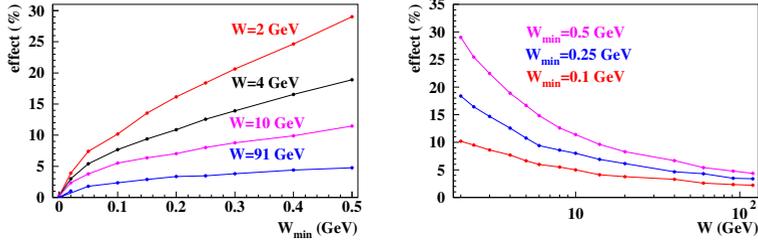
Figure 1. The dependencies of the identity effect on the cut W_{min} on invariant mass for a few energies (left) and on the energy W at the given cut (right).

4 Alternative matrix element expression

The matrix element expressions in the BDK extension have been obtained by P.A. Krachkov and A.I. Milstein. Using these expressions one can

- check the identity effect accounting
- simulate $e^+e^- \to l^+l^-\pi^+\pi^-$ for the pairs in 1^{--} states
- perform calculations, using C++, with the multiple precision, which is important for simulation of $e^+e^- \to e^+e^- + e^+e^-$ at high energies.

The two sets of matrix elements for leptonic processes give identical numerical results despite to very different approaches to calculations.

Among the $e^+e^- \to l^+l^-\pi^+\pi^-$ diagrams only those are considered which are not suppressed with the pion QED form-factor at the collision energy. They can not be treated accurately, however, they can be neglected at collision energy above 3 GeV. The code for the form-factor calculation was taken from the MCGPJ generator [7].

5 Accounting of narrow resonances

The narrow resonances ϕ, J/ψ, $\psi(2S)$, $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ were accounted via the vacuum polarization operator. In the original BDK codes and in the alternative matrix element calculations the photon q^2 was replaced with $(1 - \sum \Pi_R)\,q^2$, where the resonance contributions in the vacuum polarization is given by

$$\Pi_R(s) = \frac{3\Gamma_{ee}^{(0)}}{\alpha}\,\frac{s}{M_0}\,\frac{1}{s - M_0^2 + iM_0\Gamma_0}\,, \tag{1}$$

where $\Gamma_{ee}^{(0)}$ is the electron partial width, Γ_0 is the total resonance width and M_0 is its mass. The naked parameters employed differ from those in PDG tables. For discussion see [8]

The cross section of $e^+e^- \to \mu^+\mu^-\pi^+\pi^-$ process as a function of the muon and pion pairs invariant masses are shown in Figure 2. The resonance-continuum interference is clearly seen. The total cross section of $2\mu\,2\pi$ and 4μ productions at 10 GeV accounting for ISR are about 550 and 380 fb, respectively. The characteristic $\Upsilon(1S)$ cross section is 0.15 fb.

6 Accounting of initial state radiation

For the two-photon processes, the initial state radiation (ISR) is important if the multiperipheral graphs do not dominate. For the process $e^+e^- \to \mu^+\mu^-\mu^+\mu^-$ at 10 GeV the ISR correction to the total cross section is about 10%.
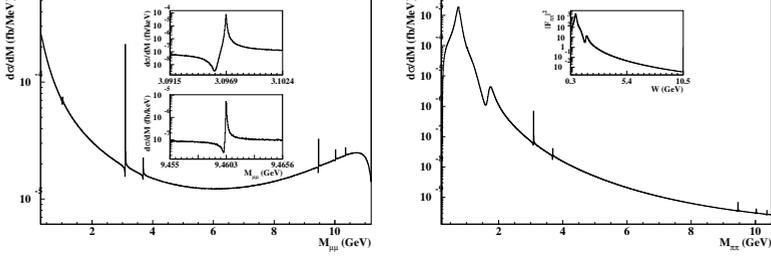
Figure 2. The cross section of $e^+e^- \to \mu^+\mu^-\pi^+\pi^-$ process as a function of the muon pair invariant mass (left) the pion pair invariant mass (right). The square of the pion form-factor is shown for comparison in the insertion on right.

The ISR is accounted using the result of E.A. Kuraev and V.S. Fadin [9] for the cross section $\sigma(s)$,

$$\sigma(s) = \int dx \, \sigma_0((1-x)s) \, \mathcal{F}(s, x), \tag{2}$$

where s is the c.m. energy squared, $\mathcal{F}(s, x)$ is the radiative correction function, and $\sigma_0(s)$ is the cross section without account of radiation. To calculate it, the preparatory run of the generator is required. The cross sections of $e^+e^- \to \mu^+\mu^-\mu^+\mu^-$ reaches the maximum of about 950 fb at the energy of 2.5 GeV approximately. For the $e^+e^- \to \mu^+\mu^-\pi^+\pi^-$ processes correspondent numbers are 2500 fb and 2 GeV.

Eq. (2) is used to generate the amount of radiated energy in the event and to determine the collision energy at which the event should be generated. The produced particles are boosted to the laboratory frame under the assumption that only one photon is radiated along or opposite to the initial electron.
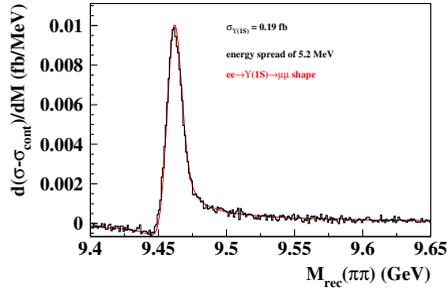
Figure 3. $\Upsilon(1S)$ signal in the recoil mass of the pion pair in the two-photons process $e^+e^- \to \pi^+\pi^-\mu^+\mu^-$ at the c.m. energy of 10.52 GeV. Normalized shape of the single photon $e^+e^- \to \Upsilon(1S)$ signal as a function of e^+e^- energy is show in red.

Accounting of ISR becomes essential when the recoil mass method is employed in the data analysis. In this method it is assumed that the energy and the momentum of the produced system equal to those of colliding beams. The machine energy spread and ISR violate this assumption, however, the recoil mass analysis is useful in many cases.

Figure 3 illustrates the applications of this method to studying of $e^+e^- \to \Upsilon(1S)\pi^+\pi^-$ process with the dimuon decay of $\Upsilon(1S)$ at Belle conditions. The two-photon production $e^+e^- \to \pi^+\pi^-\mu^+\mu^-$ gives the peaking background to the single photon process. Because

of the large mass of $\Upsilon(1S)$ the shape of recoil mass signal is rather similar to that of the $e^+e^- \to \Upsilon(1S)(\gamma)$ cross section.

7 Comparison with data

The prediction of the BDK extension are compared with the experimental data at the c.m. energy of 10.52 GeV in Figure 4. The distribution in the pion invariant mass were obtained for the muon pair mass in the range 8.7-9.4 GeV.
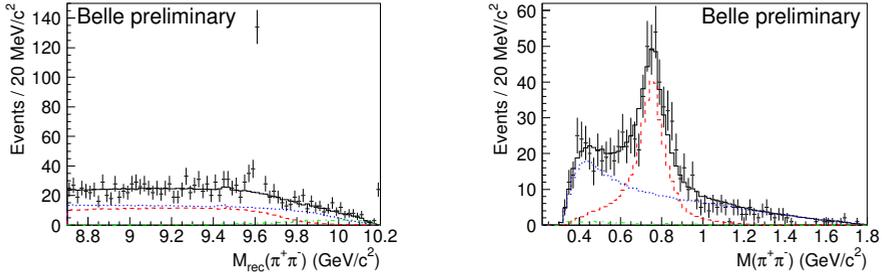
Figure 4. The recoil mass for pion pairs (left), their invariant mass (right). Points with error bar represent preliminary Belle data [3], red lines show simulated contributions of $e^+e^- \to 2\pi2\mu$, green and blue lines are for contributions of misidentified muon and electron pairs, respectively. The black lines show the sum. The peaks at 9.6 and 10.2 GeV are reflections of the transition between narrow Υ states, which was not simulated.

Conclusion

We presented the extension of BDK generator for simulation of $e^+e^- \to 4l$ processes which allows for accounting of the production of narrow 1^{--} resonances and the initial state radiation. The simulation of $l^+l^-\pi^+\pi^-$ with lepton and pion pairs in 1^{--} states is added. It is accurate enough for e^+e^- energy above 3 GeV. The generation of one event $2\pi2\mu$ at the energy of 11 GeV takes about 2 seconds at 3.4 GHz CPU. There is a good agreement between data obtained at Belle and results of this BDK extension. We are planing to increase generator performance, account for nonresonant vacuum polarization, enhance accuracy of ISR simulation and account for the beam polarization.

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