New information on the strong isospin symmetry breaking in the reactions of the $a_0^0(980)$ and $f_0(980)$ resonance production

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Abstract. We discuss the isotopic symmetry breaking as a tool of studying the production mechanisms and nature of light scalar mesons. The anomalous isospin breaking effects can appear not only due to the $a_0^0(980)-f_0(980)$ mixing, but also for any mechanism of the production of the $K\bar{K}$ pairs with a definite isospin in the S wave.

1 Introduction

The forty years ago we discovered theoretically a threshold phenomenon known as the mixing of $a_0^0(980)$ and $f_0(980)$ resonances that breaks the isotopic invariance considerably, since the effect is $\sim \sqrt{2(M_{K^0}-M_{K^+})/M_{K^0}} \approx 0.13$ in the module of the amplitude, but not $\sim (M_{K^0}-M_{K^+})/M_{K^0} \approx 1/126$, i.e., by the order of magnitude greater than it could be expected from the naive considerations [1]. This effect appears as a narrow resonant structure with the width of about $2(M_{K^0}-M_{K^+}) \approx 8$ MeV between the K^+K^- and $K^0\bar{K}^0$ thresholds due to $a_0^0(980) \to K\bar{K} \to f_0(980)$ transition and vice versa. Since that time many new proposals were appeared, concerning both the searching $a_0^0(980)-f_0(980)$ mixing and estimating the effects related with this phenomenon [2–30]. Here only a short list of references on this subject is presented. More details may be found in the reviews [20, 28]. Nowadays this phenomenon is discovered experimentally and studied with the help of detectors VES in Protvino [6] and BESIII in Beijing [7, 9, 14, 27] in the reactions: $\pi^-N \to \pi^-f_1(1285)N \to \pi^-f_0(980)\pi^0N \to \pi^-\pi^+\pi^-\pi^0N$ [6], $J/\psi \to \phi f_0(980) \to \phi a_0(980) \to \phi\eta\pi^0$ [7, 27], $\chi_{c1}(1P) \to a_0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ [7, 27], $J/\psi \to \gamma\eta(1405) \to \gamma f_0(980)\pi^0 \to \gamma 3\pi$ [9], and $J/\psi \to \phi f_1(1285) \to \phi f_0(980)\pi^0 \to \phi 3\pi$ [14]. After these experiments, it has become clear [16, 18, 20, 28] that the similar isospin breaking effects can appear not only due to the $a_0^0(980)-f_0(980)$ mixing, but also for any mechanism of the production of the $K\bar{K}$ pairs with a definite isospin in the S wave: $X_{I=0} \to (K^+K^- + K^0\bar{K}^0) \to a_0^0(980) \to \eta\pi^0$ and $X_{I=1} \to (K^+K^- + K^0\bar{K}^0) \to f_0(980) \to \pi^+\pi^-$.[1] Thus a new tool to study the production mechanisms and the nature of light scalars is emerged.

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[1] Each such mechanism reproduces both the narrow resonant peak and sharp jump of the phase in the reaction amplitude between the K^+K^- and $K^0\bar{K}^0$ thresholds (see, e.g., Figs. 2 and 3 below).

2 $a_0^0(980) - f_0(980)$ mixing

The main contribution to the $a_0^0(980)$-$f_0(980)$ mixing amplitude caused by the diagrams in Fig. 1 has the form $\Pi_{a_0^0 f_0}(m) \approx \frac{g_{a_0 K^+K^-} g_{f_0 K^+K^-}}{16\pi}\, i\left(\rho_{K^+K^-}(m) - \rho_{K^0\bar{K}^0}(m)\right)$, where the invariant virtual mass of scalar resonances $m \geq 2m_{K^0}$ and $\rho_{K\bar{K}}(m) = \sqrt{1 - 4m_K^2/m^2}$; if $0 < m < 2m_K$, then $\rho_{K\bar{K}}(m) \to i|\rho_{K\bar{K}}(m)|$. In the region between the $K\bar{K}$ thresholds, which is the 8 MeV wide, the $a_0^0(980) - f_0(980)$ transition amplitude is $|\Pi_{a_0 f_0}(m)| \approx \frac{|g_{a_0 K^+K^-} g_{f_0 K^+K^-}|}{16\pi} \sqrt{2(m_{K^0} - m_{K^+})/m_{K^0}} \approx 0.127\, \frac{|g_{a_0 K^+K^-} g_{f_0 K^+K^-}|}{16\pi} \simeq 0.03\,\mathrm{GeV}^2 \approx m_K \sqrt{m_{K^0}^2 - m_{K^+}^2} \approx m_K^{3/2} \sqrt{m_d - m_u}$. Note that the $\rho^0 - \omega$ and $\pi^0 - \eta$ mixing amplitudes are an order of magnitude smaller: $|\Pi_{\rho^0\omega}| \approx |\Pi_{\pi^0\eta}| \approx 0.003\,\mathrm{GeV}^2 \approx (m_d - m_u) \times 1\mathrm{GeV}$. Fig. 2 illustrates the resonance-like behavior of the $a_0^0(980) - f_0(980)$ mixing amplitude $\Pi_{a_0^0 f_0}(m)$ as a function of m [3, 20]. Fig. 3 shows that the $a_0^0(980) - f_0(980)$ mixing cuts a narrow resonant structure from the $f_0(980)$ and $a_0^0(980)$ resonance distributions, having in the $\pi^+\pi^-$ and $\eta\pi^0$ channels, respectively, the normal widths of about 50-100 MeV. [2]

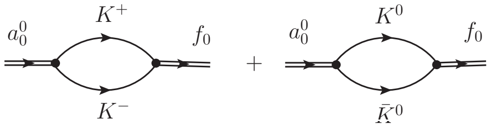

Figure 1. The $K\bar{K}$ loop mechanism of the $a_0^0(980) - f_0(980)$ mixing.

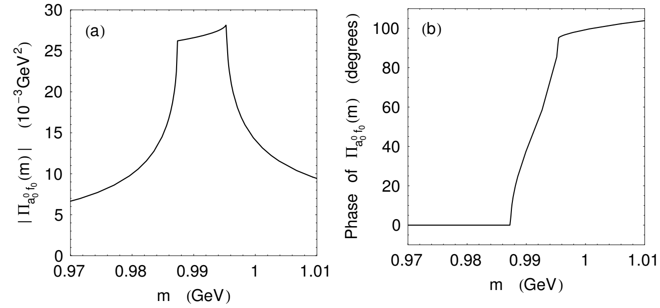

Figure 2. (a) An example of the modulus and (b) the phase of the $a_0^0(980) - f_0(980)$ mixing amplitude in the region of the K^+K^- and $K^0\bar{K}^0$ thresholds.

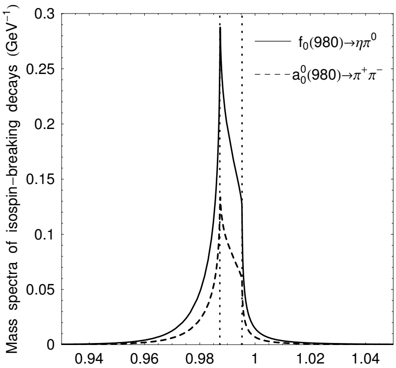

Figure 3. Mass spectra in the isospin-violating decays $f_0(980) \to \eta\pi^0$ and $a_0^0(980) \to \pi^+\pi^-$, caused by the $a_0^0(980) - f_0(980)$ mixing. They are the integrands in the equations: $BR(f_0(980) \to K\bar{K} \to a_0^0(980) \to \eta\pi^0) = \int \Gamma_{a_0^0\to\eta\pi^0}(m) \times \frac{2m^2}{\pi} \left| \frac{\Pi_{a_0^0 f_0}(m)}{D_{a_0^0}(m) D_{f_0}(m) - \Pi^2_{a_0^0 f_0}(m)} \right|^2 dm \approx 0.3\%$, $BR(a_0^0(980) \to K\bar{K} \to f_0(980) \to \pi^+\pi^-) = \int \Gamma_{f_0\to\pi\pi}(m) \times \frac{2m^2}{\pi} \left| \frac{\Pi_{a_0^0 f_0}(m)}{D_{a_0^0}(m) D_{f_0}(m) - \Pi^2_{a_0^0 f_0}(m)} \right|^2 dm \approx 0.14\%$, where $D_{a_0^0}(m)$ and $D_{f_0}(m)$ are the propagators of $a_0^0(980)$ and $f_0(980)$, respectively.

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[2] Here we have used the values of the coupling constants of the $f_0(980)$ with the $\pi\pi$ and $K\bar{K}$ channels and the $a_0^0(980)$ with the $K\bar{K}$ and $\eta\pi$ channels obtained in Ref. [18] from the BESIII data [7] for the central values of the $f_0(980) \to a_0^0(980)$ and $a_0^0(980) \to f_0(980)$ transition intensities measured in the reactions $J/\psi \to \phi f_0(980) \to \phi a_0(980) \to \phi\eta\pi^0$ and $\chi_{c1}(1P) \to a_0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$.

3 $a_0^0(980) - f_0(980)$ mixing in polarization phenomena

The phase jump of the $a_0^0(980)$-$f_0(980)$ mixing amplitude suggests the idea to study this mixing in polarization phenomena [3]. If a process amplitude with a spin configuration is dominated by the $a_0^0(980)$-$f_0(980)$ mixing then the spin asymmetry of the cross section jumps near the $K\bar{K}$ thresholds. An example is the reaction on a polarized proton target $\pi^- p_\uparrow \to (a_0^0(980) + f_0(980))n \to a_0^0(980)n \to \eta\pi^0 n$. The corresponding differential cross section has the form

$$d^3\sigma/dtdmd\psi = \left[|M_{++}|^2 + |M_{+-}|^2 + 2\Im(M_{++}M^*_{+-})P\cos\psi\right]/(2\pi),$$

and the spin asymmetry is $A(t,m) = 2\,\Im(M_{++}M^*_{+-})\,/\left[|M_{++}|^2 + |M_{+-}|^2\right]$.[3] Fig. 4 illustrates the strong asymmetry jump which is the direct manifestation of the $a_0^0(980)$-$f_0(980)$ mixing amplitude M^{π}_{+-} interfering with the isospin allowed amplitude $M^{\rho_2}_{++}$ in the π and ρ_2 Regge exchange model. These polarization phenomena are still in waiting for their investigators.

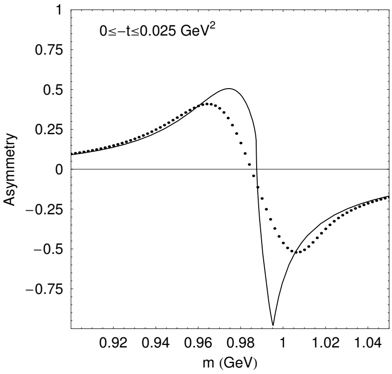

Figure 4. Manifestation of the $a_0^0(980) - f_0(980)$ mixing effect in the reaction $\pi^- p_\uparrow \to (a_0^0(980) + f_0(980))n \to a_0^0(980)n \to \eta\pi^0\, n$ on a polarized proton target at $P^{\pi^-}_{lab} = 18.3$ GeV. The solid (dotted) curve shows the spin asymmetry $A(0 \le -t \le 0.025\ \text{GeV}^2, m)$ as a function of the $\eta\pi^0$ invariant mass m (smoothed with 10 MeV mass resolution).

4 Observation of $a_0^0(980) - f_0(980)$ mixing

Recently, the BESIII Collaboration [27] has reported a new observation of $a_0^0(980) - f_0(980)$ mixing in the decays of $J/\psi \to \phi f_0(980) \to \phi a_0(980) \to \phi\eta\pi^0$ and $\chi_{c1}(1P) \to a_0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ (after the first studying of it in 2011 [7]). The signals of $f_0(980) \to a_0^0(980)$ and $a_0^0(980) \to f_0(980)$ mixing have been observed at levels of statistical significance of 7.4σ and 5.5σ, respectively. The corresponding branching fractions and mixing intensities ($\xi_{fa} \approx 1\%$, or $\approx 0.4\%$, and $\xi_{af} \approx 0.4\%$) have been measured; see for details [27]. Note that one of the most important feature of the $a_0^0(980) - f_0(980)$ mixing has been observed in this experiment. Namely, the width of the $f_0(980)$ signal in the $\eta\pi^0$ decay channel appears significantly narrower than the world average value of the $f_0(980) \to \pi\pi$ decay width. As for the coupling constants $g_{f_0K^+K^-}$ and $g_{a_0^0K^+K^-}$, their values estimated using these data are in agreement with many previous experimental results and also with the $q^2\bar{q}^2$ model for the $f_0(980)$ and $a_0^0(980)$ mesons.

5 Decay $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$

Very interesting situation takes place in the case of the $f_1(1285)$ resonance. According to the VES result [6], the isospin breaking decay of the $f_1(1285)$ into $f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ is so strong,

$$\frac{BR(f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{BR(f_1(1285) \to a_0^0(980)\pi^0 \to \eta\pi^0\pi^0)} = (2.5 \pm 0.9)\%,$$

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[3] Here M_{+-} and M_{++} are the s-channel helicity amplitude with and without nucleon helicity flip interfering in the polarized experiment, ψ is the angle between the normal to the reaction plain formed by the momenta of the π^- and $\eta\pi^0$ system, and the transverse (to the π^- beam axis) polarization of the the proton target, and P is a degree of this polarization.

that its description due to the transition mechanism $f_1(1285) \to a^0_0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ requires the "terrible" $a^0_0(980) - f_0(980)$ mixing and, as a result, the inconvenient values of the coupling constants of the scalar mesons with the pseudo-scalar ones in the many cases [18, 20]. In fact, the strong isospin breaking effect discovered in the decay $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ denotes a more general $K\bar{K}$ loop mechanism of the isospin symmetry breaking in this decay; see Fig. 5.

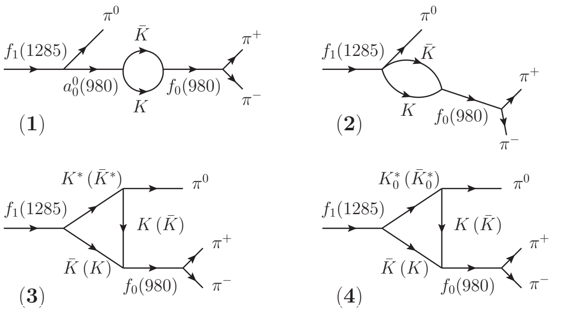

Figure 5. We have analyzed in detail four possible $K\bar{K}$ loop mechanisms [18] shown in this figure for the isospin breaking decay $f_1(1285) \to \pi^+\pi^-\pi^0$. We point out that existing data [6, 14] should be more precise, and they are difficult to explain by the single specific mechanism from those listed here.

Taking the decay $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ as an example, we also suggested the general approach to the description of the $K\bar{K}$ loop mechanism of the isotopic symmetry breaking (in the absence of logarithmic singularities in the amplitude) in the form of some consistency condition between two sets of the experimental data on $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ and $f_1(1285) \to K^+K^-\pi^0$ [18].

6 Decay $\eta(1405) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$

The BESIII Collaboration [9] investigated the isospin breaking decay $\eta(1405) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ and measured the branching ratio $BR(J/\psi \to \gamma\eta(1405) \to \gamma f_0(980)\pi^0 \to \gamma\pi^+\pi^-\pi^0) = (1.50 \pm 0.16) \times 10^{-5}$. In addition, the BESIII obtained the ratio

$$\frac{BR(\eta(1405) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{BR(\eta(1405) \to a^0_0(980)\pi^0 \to \eta\pi^0\pi^0)} = (17.9 \pm 4.2)\%\,,$$

that rules out practically the explanation of the discovered effect by means of the $a_0(980) - f_0(980)$ mixing. This large isospin breaking may be associated with manifestations of the anomalous Landau thresholds in the form of logarithmic triangle singularities, which are in the transition amplitude $\eta(1405) \to (K^*\bar{K} + \bar{K}^*K) \to (K^+K^- + K^0\bar{K}^0)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ [8, 10, 11, 16, 17, 25]; see Fig. 6.

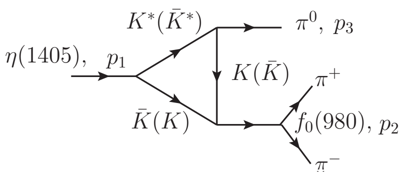

Figure 6. In the hypothetical case of the stable $K^* = K^*(892)$ meson the logarithmic singularity appears in the imaginary part of the amplitude of this triangle diagram in the $\eta(1405)$ meson region. However, its contribution can be correctly estimated only in view of the finite width of the K^* [16, 25].

Taking into account $\Gamma_{K^*\to K\pi} \approx 50$ MeV we showed that the calculated width of the decay $\eta(1405) \to (K^*\bar{K} + \bar{K}^*K) \to K\bar{K}\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ is about a factor of $6 - 8$ smaller than in the hypothetical case of the stable K^* ($\Gamma_{K^*\to K\pi} = 0$). Assuming the dominance of the decay $\eta(1405) \to (K^*\bar{K} + \bar{K}^*K) \to K\bar{K}\pi$ we also obtained $BR(J/\psi \to \gamma\eta(1405) \to \gamma f_0(980)\pi^0 \to \gamma\pi^+\pi^-\pi^0) \approx 1.12 \times 10^{-5}$ that agrees reasonably with experiment.

7 Isospin symmetry breaking in decays of D_s^+, D^0, and Υ' mesons, and in central diffractive $f_1(1285)$ and $a_0^0(980)$ production

Recently we showed that the decays $D_s^+ \to \eta\pi^0\pi^+$ [21], $D^0 \to K_S^0\pi^+\pi^-$, $D^0 \to K_S^0\eta\pi^0$ [22], and $\Upsilon(10860) \to \Upsilon(1S)\eta\pi^0$ [23] have potential for the $a_0^0(980) - f_0(980)$ mixing detection; see, as an example, Fig. 7 from [21].

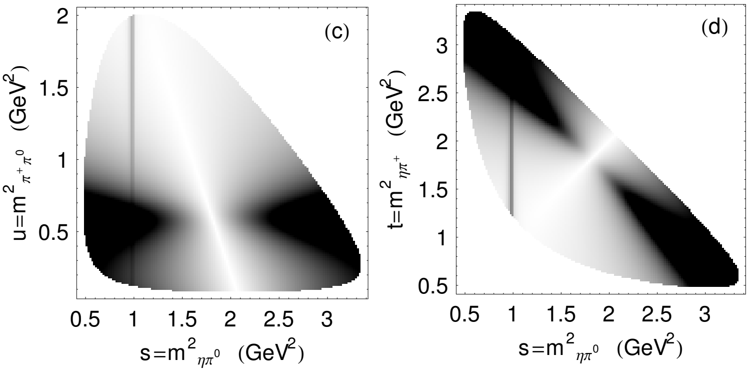

Figure 7. The $D_s^+ \to [f_0(980) \to K\bar{K} \to a_0^0(980)]\pi^+ \to \eta\pi^0\pi^+$ transition, caused by the $a_0^0(980) - f_0(980)$ mixing, manifests itself as the vertical bands in the Dalitz plot distributions for the $D_s^+ \to \eta\pi^0\pi^+$ events against the main mechanism $D_s^+ \to \eta\rho^+ \to \eta\pi^0\pi^+$ with $\eta\rho^+$ in the intermediate state.

We also draw attention to the central diffractive production processes $pp \to p(f_1(1285))p \to p(\pi^+\pi^-\pi^0)p$ and $pp \to p(K\bar{K})p \to p(a_0^0(980))p \to p(\eta\pi^0)p$ at the LHCb energies in which the anomalous breaking of the isotopic symmetry can be expected [26].

8 Last news and outlook

Very recently, there appeared a proposal to investigate the isospin breaking decay $J\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$, which can proceed in the main via the $a_0^0(980) - f_0(980)$ mixing [29]. This will be possible, since the BESIII will have 10^{10} J/ψ events by the end of 2019.

The latest proposal concerns of direct CP violation in multi-body B decays with the $a_0^0(980) - f_0(980)$ mixing [30]. Measuring these decays could provide a new way to verify the existence of the $a_0^0(980) - f_0(980)$ mixing and be helpful in clarifying the configuration nature of the light scalar mesons.

We also note that the mass differences for the charmed mesons D^+, D^0 and D^{*+}, D^{*0} are approximately the same as for the K^+ and K^0 mesons. Therefore, various dynamic effects of the strong isotopic symmetry breaking may also be expected in the charmonium family near the corresponding decay thresholds; see, for example, [31].

It is not improbable that in the super-b-factories it will be possible to search for the effects related to the B meson mass difference $m_{B^0} - m_{B^\pm} \approx 0.3$ MeV.

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