Dispersive analysis of the $\kappa/K_0^*(700)$ meson and other light strange resonances.

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Abstract. We briefly review our recent works where we use dispersion relations to constrain fits to data on $\pi K \to \pi K$ and $\pi\pi \to K\bar{K}$ providing a simple but consistent description of these processes. Then, simple analytic methods allow to extract parameters of poles associated to light strange resonances without assuming a particular model. We also present preliminary results on a model-independent determination of the controversial κ or $K_0^*(700)$ resonance parameters, by using those constrained parameterizations as input for partial-wave hyperbolic dispersion relations that allow to perform a rigorous analytic continuation to determine its associated pole.

1 Introduction

$\pi K \to \pi K$ scattering provides most of the information on strange resonances below 2 GeV. Unfortunately this process cannot be observed directly but has to be extracted as a sub-process within $\pi N \to \pi K N'$. This leads to large systematic uncertainties and to conflicting data sets. Moreover, the determination of the parameters of these resonances is very often affected by large model dependencies, due to the use of too simple models. The most extreme case is the κ meson, nowadays called $K_0^*(700)$, which is the lightest strange resonance, and whose very existence has been controversial over almost four decades. As a matter of fact, according to the last Review of Particle Physics [1], the $\kappa/K_0^*(700)$ still "Needs Confirmation". It should nevertheless be noted that, for several decades now, as soon as models describe the low-energy data, respect unitarity and have some basic analyticity properties, a κ pole is found around 600 to 900 MeV [2].

Mathematically, resonances are defined rigorously through their associated pole lying in the second Riemann sheet of the complex plane of any amplitude in which that resonance appears. The mass and width of the resonance is then related to the pole position by $\sqrt{s_{pole}} \simeq M_R - i\Gamma_R/2$ and its residue is related to the coupling to the elastic channel. Thus, in case the resonance is narrow its pole lies close to the physical axis (i.e. the real axis above the energy threshold). In cases where the resonance is also isolated from other analytic structures (other poles, thresholds, cuts etc...), it appears in experiment as a characteristic peak. In such cases even simple models describing the data in the vicinity of the peak, like the usual Breit-Wigner

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parameterization, may provide fair approximations to the pole position and residue. However, when resonances are wide, or overlap with other resonances, or have threshold cuts and other analytic or dynamical structures nearby, simple models are inadequate to determine the pole parameters or even the very existence of the resonant state.

Nevertheless by taking rigorously into account the analytic structure of the singularities that appear in amplitudes one can overcome all the above caveats. In particular, using Cauchy's Integral Formula, these singularities translate into integral relations known as dispersion relations. In this workshop we reviewed our recent analyses using such dispersion relations to constrain the of $\pi K \to \pi K$ and $\pi\pi \to K\bar{K}$ data fits. The resulting parameterizations can later be used as input to determine in a model-independent way the pole parameters of strange resonances.

2 Poles determined from Dispersive and Analytic methods

Thus, over a series of works, we have followed our goal of determining the existence of strange resonances as well as extracting their parameters from the existing data avoiding or minimizing model dependencies. Hence, in [3] we provided first simple unconstrained fits to $\pi K \to \pi K$ data on S, P, D, F partial waves up to 1.8 GeV, taking particular care on the estimation of systematic uncertainties. We then showed that such simple parameterizations lead to inconsistencies with Forward Dispersion Relations (FDR). Our main result was then to provide a set of partial-wave parameterizations, called Constrained Fits to Data (CFD), which is consistent with a complete set of FDRs up to 1.6 GeV, i.e., one FDR for the $s \leftrightarrow u$ symmetric F^+ amplitude and another one for the antisymmetric F^-, where s, t, u are the usual Mandelstamm variables.

Unfortunately, an analytic continuation to the complex plane for partial waves cannot be obtained from FDRs. Nonetheless, we can use of a powerful method [4] based on the convergence on the complex plane of series of Padé approximants constructed from information on the physical axis. Remarkably, this method does not require a choice of parameterization for the resonance pole, hence avoiding such a model dependence. In particular, the residue is not necessarily related to the pole position in a Breit-Wigner-like form, as it has been assumed in most analyses of strange resonances below 2 GeV. Obviously, the main caveat is that we are not able to calculate an infinite sequence of Padés, but it has to be truncated. In practice, the pole can only be determined with a few Padés, not an infinite number of them, and that effect yields a sizable contribution to the systematic uncertainty. Another relevant feature of this technique is that it can be applied not only in the elastic region but, more importantly, also in the inelastic region. Hence, our "Final result"+ poles in [5] obtained by applying this Padé method to our πK CFD fits in the inelastic region are shown in Fig.1. Hollow symbols stand for Breit-Wigner-like parameterizations whereas solid symbols represent T-matrix poles (references can be found in [1]). The large dispersion in older values is mostly due to model-dependence, which we have avoided with the Padé sequence technique.

Concerning the $\kappa/K_0^*(700)$, we illustrate in Fig.2, that its pole lies very deep in the complex plane in comparison with the $K^*(892)$ pole, for which a Breit-Wigner parameterization may provide a decent description. Actually, the κ pole is as close to its nominal mass region in the real axis as it is to the πK threshold, the Adler zero or the circular and left cuts. As a consequence, the use of Breit-Wigner-like parameterizations, which rely on the narrow resonance approximation, is not justified. Such Breit-Wigner poles are represented in Fig.3 by hollow symbols. More rigorous T-matrix pole determinations are obtained using analytic or dispersive methods (solid symbols) that may also include chiral symmetry constraints (Adler zeros at least or some matching with Chiral Pertubation Theory). In particular, we show the best dispersive determination by "Descotes-Genon et al." [6]. This is a very sound analysis
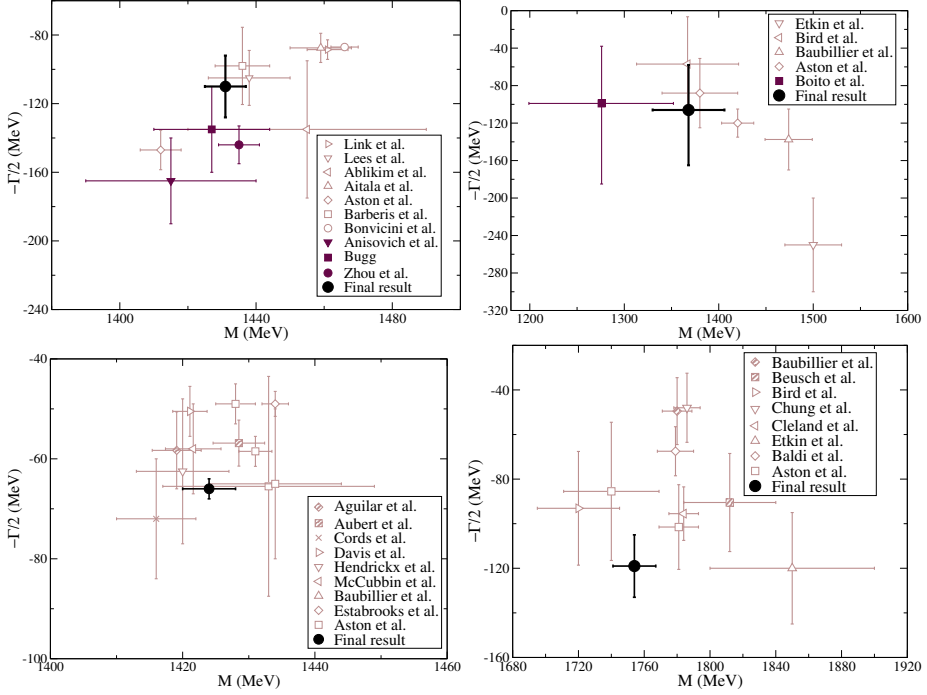
Figure 1. Pole positions of the $K_0^*(1430)$ (top-left), $K_1^*(1410)$ (top-right), $K_2^*(1430)$ (bottom-left), $K_3^*(1780)$ (bottom-right) extracted from data fits constrained with Forward Dispersion Relations and using sequences of Padé approximants for the analytic continuation to the complex plane. Also shown are the poles listed in the RPP (see [1] for references). The figures and our "final result" come from [5].

that makes use of partial-wave hyperbolic dispersion relations for the analytic continuation to the complex plane of a numerical solution (without fitting elastic data) of Roy-Steiner equations. In spite of this rigorous result, the κ, still called $K_0^*(800)$ in 2016, still "Needs confirmation" in the RPP.

It is worth noting that in the elastic regime, just by naively extrapolating to the complex plane our CFD description, which has been constructed matching simple parameterizations in different energy regions, we find a pole roughly consistent with the RPP estimate. In Fig.3 we have denoted this result by 'Conformal CF". However, such a simple determination relies on a specific parameterization and is therefore model dependent.

Remarkably, applying the Padé method explained above to our constrained fits to data in [3] we have also shown in [5] that there is a pole for the controversial $\kappa/K_0^*(700)$. It is shown as the "Padé resul" in Fig.3. Note that this result, with its very reduced model dependence, is in very good agreement with the dispersive prediction in [6] triggered the κ change of denomination at the 2018 RPP revision: from $K_0^*(800)$ to its present name $K_0^*(700)$. Still, even with this additional piece of evidence, the 2018 RPP [1] considers that this state "Needs Confirmation".

As a side remark, using our CFD or Padé pole position in [7] as the only input for a dispersive representation of the Regge trajectory, we showed that the $\kappa/K_0^*(700)$ resulting slope comes out non-linear with respect to the mass squared and is much smaller than that of ordinary mesons. This is a strong model-independent support for the non-ordinary nature of the $\kappa/K_0^*(700)$ meson and thus of the whole light scalar meson nonet. In other words, it is not predominantly made of just a quark and an antiquark as ordinary mesons in the
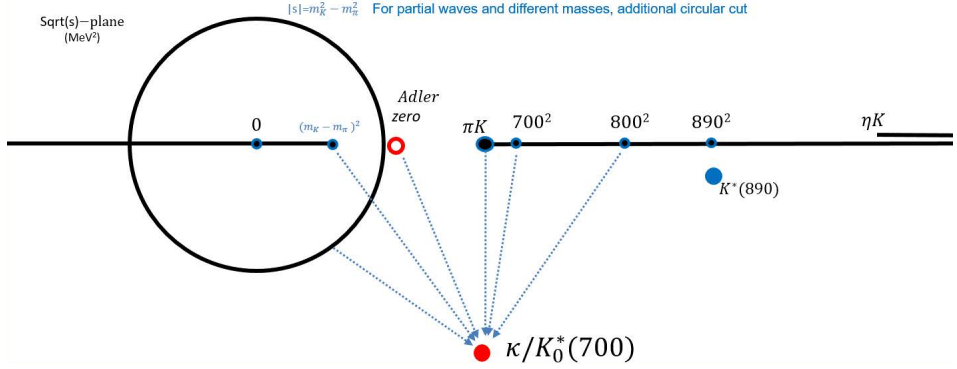
Figure 2. The analytic structure of πK partial waves has physical cuts on the real axis extending from each new threshold to infinity, a circular cut and a left cut due to crossed channels. The presence of an Adler zero in the subthreshold region is a requirement of chiral symmetry breaking. For typical Breit-Wigner-like resonances like the $K^*(892)$, their associated pole in the lower half of the complex plane (in blue) lies close to the real axis very near its nominal mass. In contrast, the $\kappa/K_0^*(700)$ pole (in red) lies very deep in the complex plane. Moreover, its pole is as close to its nominal mass region in the real axis as it is to the πK threshold, the Adler zero, the circular and left cuts. For a precise determination of the $\kappa/K_0^*(700)$ pole a careful calculation of such contributions is thus required.

quark model sense. This is not just a matter of being a wide resonance, but of the pole residue (i.e. its coupling to πK) being related to the mass and width in a different way than for ordinary resonances (those generically described with simple Breit-Wigner equations or slightly modified variants).

Therefore, with the aim of providing the confirmation requiered for the $\kappa/K_0^*(700)$, two of us are presently completing [9] a data analysis using partial-wave hyperbolic and fixed-t dispersion relations up to $\simeq 1$ GeV as constraints. In this way we take into account correctly all analytic structures in πK partial waves, as shown in Fig.2. This kind of dispersion relations, with crossing built in, are called Roy-Steiner equations.

Let us remark that in our ongoing analysis we are using data in the elastic region of the S, P, D, F waves. This is in contrast to what was done by the authors of [6], who found solutions for S and P waves only, but without input from data on those very same waves in the elastic region. In addition, note that we use the dispersion relations as constraints on fits, not to obtain solutions, as did in [6]. On top of that, we are constraining partial waves with both hyperbolic and fixed-t dispersion relations, contrary to [6], where the hyperbolic dispersion relations used as input a solution of the fixed-t ones.

Since they are needed for the $\pi K \to \pi K$ analysis, we obtained first constrained fits to data for $\pi\pi \to K\bar{K}$ scattering [8]. Actually, we showed that some of the existing $\pi\pi \to K\bar{K}$ data, as well as simple unconstrained fits to those data, fail to satisfy hyperbolic dispersion relations. Nevertheless we were able to provide Constrained Fits to $\pi\pi \to K\bar{K}$ Data (CFD), describing data while being consistent with the dispersion relations up to 1.47 GeV. Note that our CFD parameterizations can describe data up to 2 GeV, but, as shown in [8], 1.47 GeV is the maximum applicability region of hyperbolic dispersion relations.

In principle, with the required $\pi\pi \to K\bar{K}$ input we are now in a position to use partial-wave hyperbolic dispersion relations to rigorously continue analytically the experimental information on the real axis and obtain the $\kappa/K_0^*(700)$ pole. However, let us emphasize that it is important to use a description of data that is already consistent with dispersion relations in the real axis, otherwise the use of a rigorous analytic continuation does not guarantee that
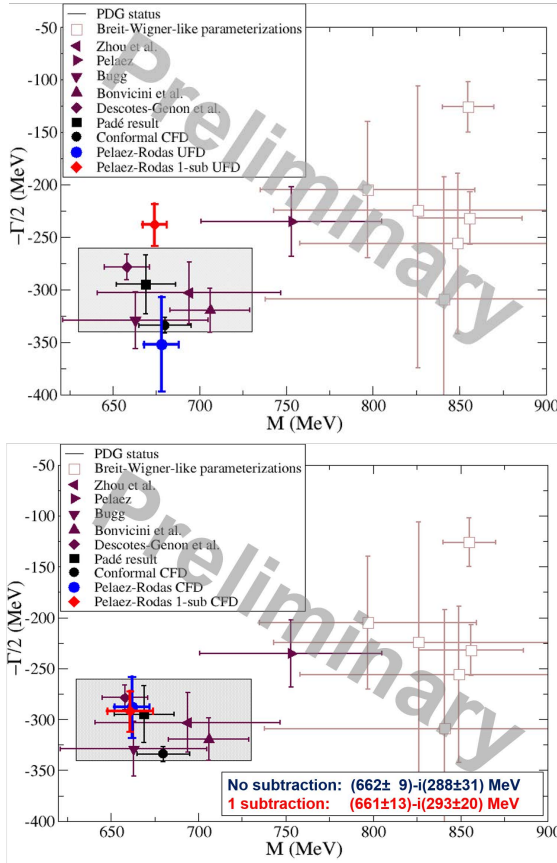
Figure 3. Preliminary results for the $\kappa/K_0^*(700)$ pole from a Roy-Steiner analysis. For comparison, we show the T-matrix poles listed in the RPP [1] (see references therein) and the grey rectangle stands for the uncertainty presently estimated there [1]. In the top panel we show how, even with the same UFD input, the red and blue poles, respectively obtained using one or no subtractions for F^- yield incompatible values. In contrast, we show in the lower panel that, when using our simple CFD parameterizations, the two red and blue poles come out consistently, determining precisely the $\kappa/K_0^*(700)$ pole position at the red and blue numerical values quoted in the inset rectangle, which are also preliminary.

the obtained pole is accurate. This is illustrated in the top panel of Fig.3, where we have used as input for the dispersion relations our unconstrained fits to πK data, which describe data but do not satisfy well the Roy-Steiner representation in the real axis. In such case, the pole obtained from the once-subtracted relations for F^- (in red) is incompatible with the pole obtained with the unsubtracted dispersion relation for F^- (in blue). And both come from the same data! If on top of that one were to use a simple model for a naive analytic continuation, the result would be even less reliable!! *To determine the $\kappa/K_0^*(700)$ pole it is therefore important to have both an input that satisfies the dispersive representation on the real axis and a dispersive integral to perform the analytic continuation to the complex plane.*

Therefore, in order to ensure that our work provides the rigorous confirmation needed to finally settle the $\kappa/K_0^*(700)$ discussion, we are now imposing as constraints partial-wave hyperbolic dispersion relations on data fits, besides the FDRs up to 1.6 GeV and partial-

wave dispersion relations obtained from fixed-t dispersion relations. Moreover, we have constrained the fits and determine the pole existence and parameters with both one or no subtractions for the antisymmetric amplitude (only the subtracted one was used before). Furthermore, we have included other improvements in our calculation. In particular, our isospin 1/2 P-wave describes the existing data, our $\pi\pi \to K\bar{K}$ input has associated uncertainties and satisfies the Roy-Steiner representation and we have improved the Pomeron determination to be consistent with factorization of kaon-nucleon data.

With all those improvements, our preliminary results for the $\kappa/K_0^*(700)$ pole are shown in the lower panel of Fig.3. Remarkably, the poles obtained either with one or no-subtractions (red and blue, respectively) lie on top of each other. Not only they are very compatible among themselves but also with our Padé result and with the previous Roy-Steiner prediction. Therefore, we think our preliminary results are on the way of providing the confirmation demanded by the RPP to finally settle the existence and parameters of the $\kappa/K_0^*(700)$ resonance and complete the members of the controversial light scalar nonet.

Acknowledgements

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