

Semileptonic D^0 and D^+ decays as a probe of the $a_0(980)$ nature

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Abstract. The $D^+ \rightarrow d\bar{d}e^+\nu \rightarrow a_0^0(980)e^+\nu \rightarrow \pi^0\eta e^+\nu$ and $D^0 \rightarrow d\bar{u}e^+\nu \rightarrow a_0^-(980)e^+\nu \rightarrow \pi^-\eta e^+\nu$ decays (with the charge conjugated ones) are the direct probe of the constituent two-quark components in the $a_0^\pm(980)$ and $a_0^0(980)$ wave functions. Recent BESIII experiment is the first step in experimental study of these decays. We suggest adequate formulas for the data analysis and present a variant of $\eta\pi$ invariant mass distribution when $a_0(980)$ has no constituent two-quark component at all.

1 Introduction

The $a_0(980)$ and $f_0(980)$ mesons are well-established parts of the assumed light scalar meson nonet [1]. From the beginning, the $a_0(980)$ and $f_0(980)$ mesons became one of the central problems of nonperturbative QCD, as they are important for understanding the way chiral symmetry is realized in the low-energy region and, consequently, for understanding confinement. Many experimental and theoretical papers have been devoted to this subject.

There is much evidence that supports the four-quark model of light scalar mesons [2, 3].

Recently BES Collaboration measured the decays $D^0 \rightarrow d\bar{u}e^+\nu \rightarrow a_0^-(980)e^+\nu \rightarrow \pi^-\eta e^+\nu$ and $D^+ \rightarrow d\bar{d}e^+\nu \rightarrow a_0^0(980)e^+\nu \rightarrow \pi^0\eta e^+\nu$ for the first time.

It will be shown that in the scenario, based on the four-quark model, it is possible to describe the data on different reactions in agreement with the BESIII data [4], while $a_0(980)$ has no constituent two-quark component at all, that is, $g_{d\bar{u}a_0^-} = g_{d\bar{d}a_0^0} = 0$, $g_{a_0^0\gamma\gamma}^{(0)} = 0$. More precise data would allow to check this variant better.

2 The model

The amplitude of the $D^0 \rightarrow S$ (scalar) $e^+\nu$ decay reads [5]

$$M[D^0(p) \rightarrow S(p_1)W^+(q) \rightarrow S(p_1)e^+\nu] = \frac{G_F}{\sqrt{2}} V_{cd} A_\alpha L^\alpha, \quad (1)$$

$$\begin{aligned} A_\alpha &= f_+^S(q^2)(p + p_1)_\alpha, \\ L_\alpha &= \bar{\nu}\gamma_\alpha(1 + \gamma_5)e, \quad q = (p - p_1). \end{aligned} \quad (2)$$

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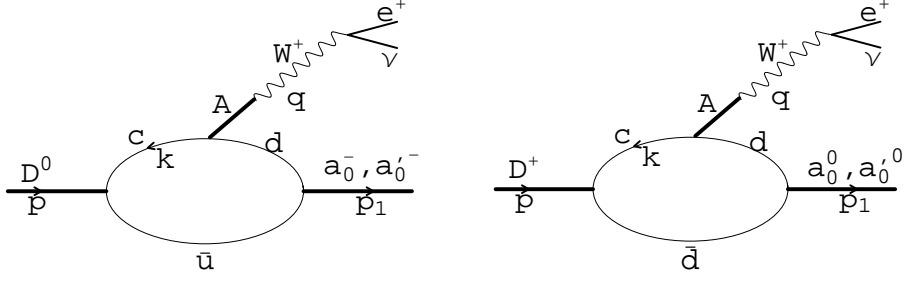


Figure 1. Model of the $D^0 \rightarrow (a_0^-, a_0'^-) e^+ \nu$ and $D^+ \rightarrow (a_0^0, a_0'^0) e^+ \nu$ decays.

$$f_+^S(q^2) = f_+^S(0) \frac{m_A^2}{m_A^2 - q^2} = f_+^S(0) f_A(q^2) = g_{D^0 c \bar{u}} F_S g_{d \bar{u} S} f_A(q^2), \quad (3)$$

where $A = D_1(2420)^\pm$, F_S - loop integral, assumed to be constant in the region of interest.

The decay rate into the stable S state is

$$\frac{d\Gamma(D^0 \rightarrow S e^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cd}|^2}{24\pi^3} p_1^3(q^2) |f_+^S(q^2)|^2, \quad (4)$$

$$p_1(q^2) = \frac{\sqrt{m_{D^0}^4 - 2m_{D^0}^2(q^2 + m_S^2) + (q^2 - m_S^2)^2}}{2m_{D^0}}. \quad (5)$$

The amplitude of the $D^0 \rightarrow d\bar{u} e^+ \nu \rightarrow [a_0^-(980) + a_0'^-] e^+ \nu \rightarrow \eta\pi^- e^+ \nu$ decay is

$$\begin{aligned} M(D^0 \rightarrow d\bar{u} e^+ \nu \rightarrow \eta\pi^- e^+ \nu) &= \frac{G_F}{\sqrt{2}} V_{cd} L^\alpha (p + p_1)_\alpha g_{D^0 c \bar{u}} f_A(q^2) \\ &\times \frac{1}{\Delta(m)} \left(F_{a_0^-} g_{d\bar{u} a_0^-} D_{a_0'^-}(m) g_{a_0 \eta \pi} + F_{a_0^-} g_{d\bar{u} a_0^-} \Pi_{a_0^- a_0'^-}(m) g_{a_0' \eta \pi} \right. \\ &\left. + F_{a_0'^-} g_{d\bar{u} a_0'^-} \Pi_{a_0'^- a_0^-}(m) g_{a_0 \eta \pi} + F_{a_0'^-} g_{d\bar{u} a_0'^-} D_{a_0^-}(m) g_{a_0' \eta \pi} \right), \quad (6) \end{aligned}$$

where m is the invariant mass of the $\eta\pi^-$ system, $\Delta(m) = D_{a_0'^-}(m) D_{a_0^-}(m) - \Pi_{a_0'^- a_0^-}(m) \Pi_{a_0^- a_0'^-}(m)$, $D_{a_0^-}(m)$ and $D_{a_0'^-}(m)$ are the inverted propagators of the a_0^- and $a_0'^-$ mesons, and $\Pi_{a_0^- a_0'^-}(m) = \Pi_{a_0'^- a_0^-}(m)$ is the nondiagonal element of the polarization operator, which mixes the a_0^- and $a_0'^-$ mesons [5].

Table 1. Properties of the resonances and the description quality

$m_{a_0^0}$, MeV	988.3	$m_{a_0'}$, MeV	1423.9	R , fm	6.3
$g_{a_0^0 K^+ K^-}$, GeV	4.06	$g_{a_0' K^+ K^-}$, GeV	4.19	λ	1
$g_{a_0 \eta \pi}$, GeV	3.99	$g_{a_0' \eta \pi}$, GeV	0.80	$\chi_{\gamma\gamma}^2 / 36$ points	13.8
$g_{a_0 \eta' \pi}$, GeV	-4.24	$g_{a_0' \eta' \pi}$, GeV	1.27	$\chi_{sp}^2 / 49$ points	65.5
$g_{a_0^0 \gamma \gamma}^{(0)}$	0	$g_{a_0' \gamma \gamma}^{(0)}$, 10^{-3} GeV $^{-1}$	-12.90	$\chi_{corr}^2 / 29$ points	28.4
$m_{a_0^+}$, MeV	997.6	$C_{a_0 a_0'}$, GeV 2	-0.163	$(\chi_{\gamma\gamma}^2 + \chi_{sp}^2 + \chi_{corr}^2) / \text{n.d.f.}$	107.8/99

The double differential rate of the $D^0 \rightarrow d\bar{u} e^+ \nu \rightarrow [a_0^-(980) + a_0'^-] e^+ \nu \rightarrow \eta \pi^- e^+ \nu$ decay is

$$\begin{aligned}
 \frac{d^2\Gamma(D^0 \rightarrow \eta \pi^- e^+ \nu)}{dq^2 dm} &= \\
 &= \frac{G_F^2 |V_{cd}|^2}{192 \pi^5} g_{D^0 c\bar{u}}^2 |f_A(q^2)|^2 p_1^3(q^2, m) \rho_{\eta\pi^-}(m) m \left| \frac{1}{\Delta(m)} \right|^2 \\
 &\times \left| F_{a_0^-} g_{d\bar{u}a_0^-} D_{a_0^-}(m) g_{a_0 \eta \pi} + F_{a_0'} g_{d\bar{u}a_0'} \Pi_{a_0^- a_0'}(m) g_{a_0' \eta \pi} \right. \\
 &\quad \left. + F_{a_0^-} g_{d\bar{u}a_0^-} \Pi_{a_0^- a_0^-}(m) g_{a_0 \eta \pi} + F_{a_0'} g_{d\bar{u}a_0'} D_{a_0'}(m) g_{a_0' \eta \pi} \right|^2, \tag{7}
 \end{aligned}$$

where $\rho_{\eta\pi^-}(m) = \sqrt{(1 - (m_\eta + m_{\pi^-})^2/m^2)(1 - (m_\eta - m_{\pi^-})^2/m^2)}$.

If a_0 does not contain two-quark state, then

$$\begin{aligned}
 \frac{d^2\Gamma(D^0 \rightarrow \eta \pi^- e^+ \nu)}{dq^2 dm} &= \frac{G_F^2 |V_{cd}|^2}{192 \pi^5} g_{D^0 c\bar{u}}^2 |f_A(q^2)|^2 p_1^3(q^2, m) \rho_{\eta\pi^-}(m) m \left| \frac{1}{\Delta(m)} \right|^2 \\
 &\times |F_{a_0'} g_{d\bar{u}a_0'}|^2 \left| \Pi_{a_0^- a_0'}(m) g_{a_0 \eta \pi} + D_{a_0'}(m) g_{a_0' \eta \pi} \right|^2, \tag{8}
 \end{aligned}$$

The $D^+ \rightarrow d\bar{d} e^+ \nu \rightarrow S e^+ \nu$ and $D^+ \rightarrow \eta \pi^0 e^+ \nu$ decays are described in the same way. It is enough to substitute $D^0 \rightarrow D^+$, $d\bar{u} \rightarrow d\bar{d}$, $a_0^- \rightarrow a_0^0$, $a_0'^- \rightarrow a_0'^0$, and $\pi^- \rightarrow \pi^0$. The coupling $g_{d\bar{d}a_0^0} = g_{d\bar{u}a_0^-} / \sqrt{2}$.

3 Results and perspectives

The results are shown in table 1 and figures 2, 3, 4, and 5. The data is described well, the $a_0(980)$ coupling constants agree with the four-quark model scenario, see [5].

In [10, 11] the program of studying light scalars in semileptonic D and B decays was suggested. Processes of interest are:

$$\begin{aligned}
 D_s^+ &\rightarrow s\bar{s} e^+ \nu \rightarrow [\sigma(600) + f_0(980)] e^+ \nu \rightarrow \pi^+ \pi^- e^+ \nu \\
 D^+ &\rightarrow d\bar{d} e^+ \nu \rightarrow [\sigma(600) + f_0(980)] e^+ \nu \rightarrow \pi^+ \pi^- e^+ \nu \\
 D^0 &\rightarrow d\bar{u} e^+ \nu \rightarrow a_0^- e^+ \nu \rightarrow \pi^- \eta e^+ \nu \\
 D^+ &\rightarrow d\bar{d} e^+ \nu \rightarrow a_0^0 e^+ \nu \rightarrow \pi^0 \eta e^+ \nu \\
 B^0 &\rightarrow d\bar{u} e^+ \nu \rightarrow a_0^- e^+ \nu \rightarrow \pi^- \eta e^+ \nu \\
 B^+ &\rightarrow u\bar{u} e^+ \nu \rightarrow a_0^0 e^+ \nu \rightarrow \pi^0 \eta e^+ \nu \\
 B^+ &\rightarrow u\bar{u} e^+ \nu \rightarrow [\sigma(600) + f_0(980)] e^+ \nu \rightarrow \pi^+ \pi^- e^+ \nu
 \end{aligned}$$

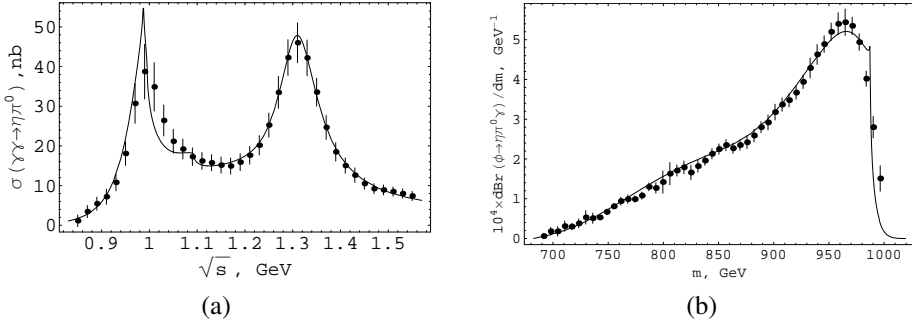


Figure 2. Results of our fit, see Tables I and II, on a) the Belle data on $\gamma\gamma \rightarrow \eta\pi^0$ cross-section [6], and b) the KLOE data on the $\phi \rightarrow \eta\pi^0\gamma$ decay [7], m is the invariant $\eta\pi^0$ mass.

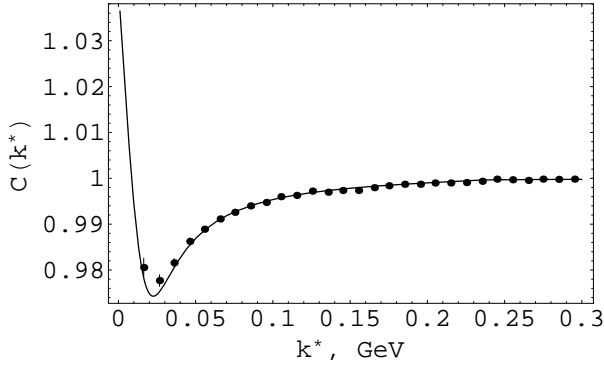


Figure 3. The $K_S^0 K^+$ correlation $C(k^*)$, see Ref. [8] and references therein. Solid line represents our fit, points are experimental data [9].

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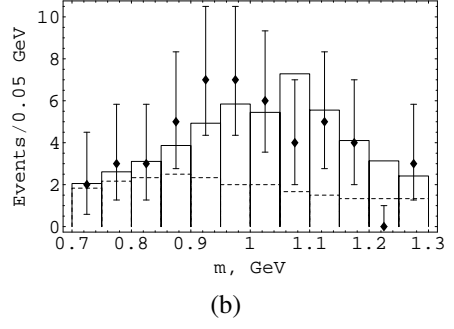
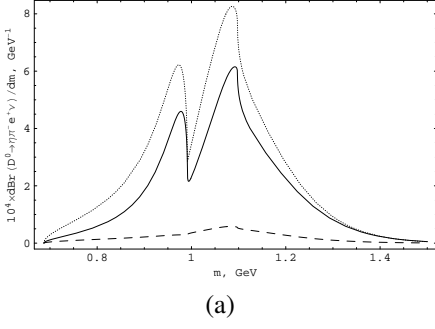


Figure 4. (a) The plot of $D^0 \rightarrow (a_0^-, a_0'^-) e^+ \nu \rightarrow \eta \pi^- e^+ \nu$ spectrum with parameters of our fit. The solid line is the total contribution, the dotted line is the term $\sim F_{a_0^-} g_{d\bar{u}a_0^-} \Pi_{a_0^- a_0^-}(m) g_{a_0 \eta \pi}$ contribution, and the dashed line is the term $\sim F_{a_0^-} g_{d\bar{u}a_0^-} D_{a_0^-}(m) g_{a_0 \eta \pi}$ contribution. (b) The data on the $D^0 \rightarrow (a_0^-, a_0'^-) e^+ \nu \rightarrow \eta \pi^- e^+ \nu$ decay and our fit. The solid histogram is the total contribution, and the dashed histogram represents the sum of backgrounds.

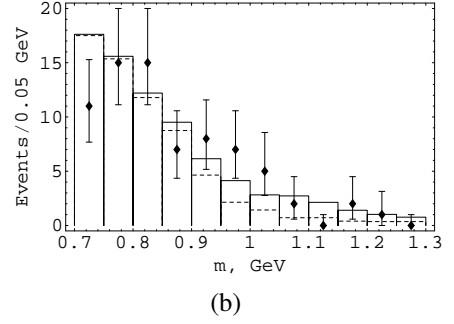
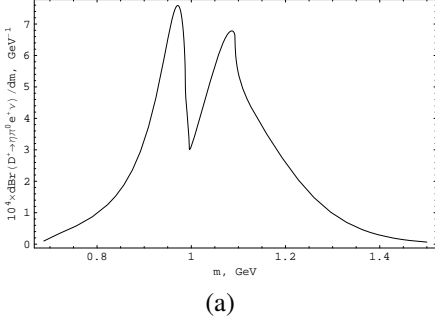


Figure 5. (a) The spectrum of the $D^+ \rightarrow (a_0^0, a_0'^0) e^+ \nu \rightarrow \eta \pi^0 e^+ \nu$ decay with parameters of our fit. (b) The spectrum of the $D^+ \rightarrow (a_0^0, a_0'^0) e^+ \nu \rightarrow \eta \pi^0 e^+ \nu$ decay. The solid histogram is the total contribution, and the dashed histogram represents the sum of backgrounds.