# Short-Period Undulators with Electrostatic Field


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**Abstract.** Due to strong sextupole corrections, the vertical dynamic aperture of a low-emittance storage ring is rather small. Therefore, in-vacuum short-period undulators can provide a sufficiently high field. In particular, a significant electrostatic field value can be obtained near electrodes. To eliminate the field emission, the high-field surfaces should be at a positive electric potential. In this paper, we consider the feasibility of an undulator using a comb of such electrodes. It is worth noting that a variable-period design may be rather simple for electrostatic undulators.


## INTRODUCTION

The electron energy of contemporary storage rings for high-brightness x-ray sources is defined by the desirable wavelength of radiation (see, e. g., review paper [1]). Using the formula for the critical wavelength of synchrotron radiation [2], one can find the minimum electron energy, which depends on the electromagnetic field on the particle trajectory. On the other hand, the radiation brightness can be increased by using a big number of sources (undulator poles). To maximize this number, they use the shortest possible undulator period.

Due to strong sextupole corrections, the vertical dynamic aperture of low-emittance storage ring is rather small. Therefore, in-vacuum short-period undulators can provide a sufficiently high field. In particular, a significant electrostatic field value can be obtained near electrodes. To eliminate the field emission, the high-field surfaces should be at a positive electric potential. In this paper, we consider the feasibility of an undulator using a comb of such electrodes. It is worth noting that a variable-period design [3 – 5] may be rather simple for electrostatic undulators.

## THE SHAPE OF A SINGLE ELECTRODE

Let us consider a conducting ellipsoid with an electric charge $Q$ described by the equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad . \tag{1}$$

The potential (see textbook [6]) is

$$\varphi=\frac{Q}{2}\int_{\xi}^{\infty}\frac{du}{\sqrt{\left(u+a^2\right)\left(u+b^2\right)\left(u+c^2\right)}} \quad , \tag{2}$$

where the elliptic coordinate $\xi$ can be found from the equation

$$\frac{x^2}{a^2+\xi}+\frac{y^2}{b^2+\xi}+\frac{z^2}{c^2+\xi}=1 \quad . \tag{3}$$

Then an on-axis field, e. g., $E_y$, is

$$E_y\left(0,y,0\right)=\frac{Q}{\sqrt{\left(y^2-b^2+a^2\right)\left(y^2-b^2+c^2\right)}}=\frac{CU}{\sqrt{\left(y^2-b^2+a^2\right)\left(y^2-b^2+c^2\right)}}\,, \tag{4}$$

where, according to Eq. (2),

$$\frac{1}{C}=\frac{1}{2}\int_0^\infty\frac{du}{\sqrt{\left(u+a^2\right)\left(u+b^2\right)\left(u+c^2\right)}}\quad , \tag{5}$$

and $U=\varphi\left(\xi=0\right)$ is the potential of the conducting ellipsoid. Let us maximize the field at some distance $y_1$ from the surface:

$$E_y\left(0,b+y_1,0\right)=\frac{2U}{\sqrt{\left(y_1^2+2by_1+a^2\right)\left(y_1^2+2by_1+c^2\right)}}\left[\int_0^\infty\frac{du}{\sqrt{\left(u+a^2\right)\left(u+b^2\right)\left(u+c^2\right)}}\right]^{-1}. \tag{6}$$

Due to symmetry of Eq. (6), the maximum takes place at $a = c$. Then

$$C=\begin{cases}\dfrac{\sqrt{b^2-a^2}}{\operatorname{acosh}\dfrac{b}{a}} & \text{for} \quad b>a\\[2ex] \dfrac{\sqrt{a^2-b^2}}{\operatorname{acos}\dfrac{b}{a}} & \text{for} \quad b<a\end{cases}\,, \tag{7}$$

and

$$E_y\left(0,b+y_1,0\right)=\frac{U/y_1}{a_1+\dfrac{1}{a_1}+2\dfrac{b}{a}}\left(\begin{array}{ll}\dfrac{\sqrt{\dfrac{b^2}{a^2}-1}}{\operatorname{acosh}\dfrac{b}{a}} & \text{for} \quad b>a\\[2ex] \dfrac{\sqrt{1-\dfrac{b^2}{a^2}}}{\operatorname{acos}\dfrac{b}{a}} & \text{for} \quad b<a\end{array}\right), \tag{8}$$

where $a_1=a/y_1$.

The maximum field

$$\left[E_y\left(0,b+y_1,0\right)\right]_{\max}=\frac{U}{\pi y_1} \tag{9}$$

takes place at $a = y_1$ and $b = 0$ (a disk of the radius $y_1$). For a sphere of the same radius, it is only slightly less: $U/\left(4y_1\right)$. Therefore, for simplicity, below we will consider such spherical electrodes. In particular,

$$E_y\left(0,y,z\right)=\frac{Uay}{\left(y^2+z^2\right)^{3/2}}\,. \tag{10}$$

## RADIATION FROM A SINGLE POLE

Let the electron velocity $v$ be parallel to the $z$ axis at the point with the coordinates (0, $2y_1$, 0). Then for $eU/(mc^2) << 1$, the spectral intensity of radiation along the $z$ axis is [2]

$$dE_\omega \approx \frac{2e^4\gamma^2}{\pi m^2 c^5}\left[\int_{-\infty}^{\infty} E_y(0,2y_1,z)\cos\frac{\omega z}{2\gamma^2 c}dz\right]^2 do\frac{d\omega}{2\pi} = \frac{2e^4\gamma^2 U^2}{\pi m^2 c^5}\left(\frac{\omega y_1}{\gamma^2 c}\right)^2 K_1^2\left(\omega\frac{y_1}{\gamma^2 c}\right) do\frac{d\omega}{2\pi}, \quad (11)$$

where $K_1$ is the first order modified Bessel function of the second kind. Equation (11) shows that the minimum wavelength of the spectrum is about $2\pi y_1/\gamma^2$.

## UNDULATOR ELECTRODES

The scheme of undulator electrodes is shown in Fig. 1.

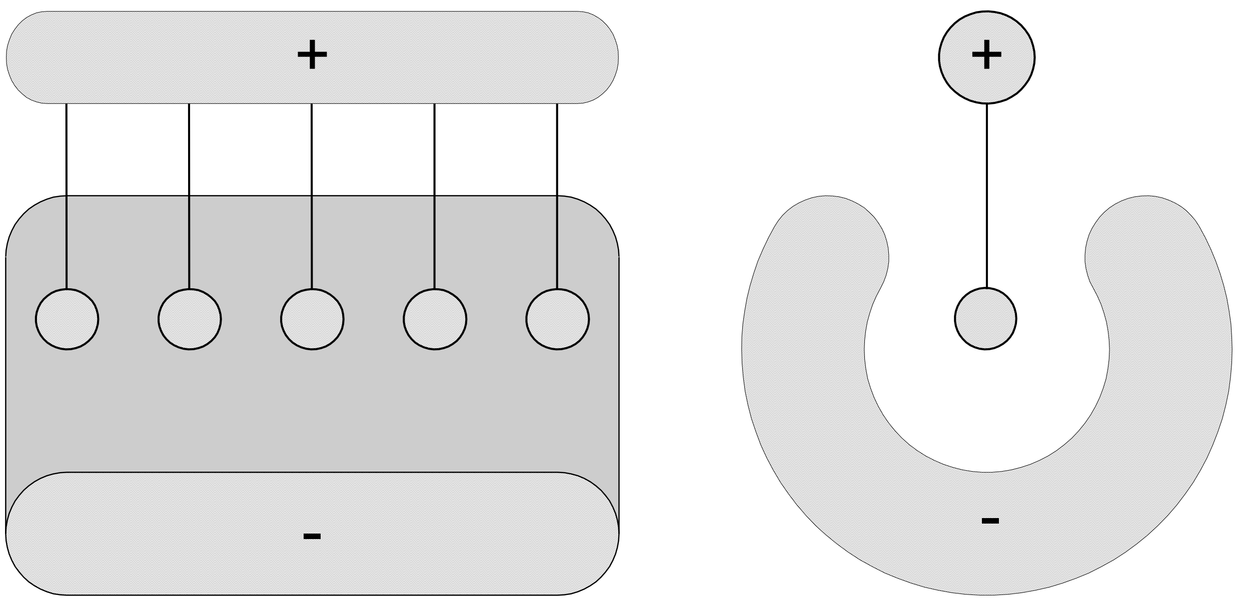


**FIGURE 1.** Scheme of undulator electrodes.

We have a comb of thin conducting rods with conducting balls at the ends. Graphite seems to be a good material for both wires and balls. For a rather long distance between neighbor balls, one can estimate the field using Eq. (10). For example, applying ± 300 kV to the electrodes, one obtains $U$ = 600 kV = 2 kG cm, and for $y_1$ = 1 mm one obtains a maximum field of 5 kG. The minimum period is about 3 mm. Mechanical variation of the period using a pantograph or other devices seems to be feasible (see [3-5]). The simplest way to compensate the average vertical deflection of electron passing through such undulator is the use of homogeneous horizontal magnetic field.

## UNDULATOR FIELD

For a rather long period $d >> a$, the undulator field is simply the sum of fields of $N = 2M + 1$ charged balls:

$$E_y(0,y,z)=\sum_{n=-M}^{n=M}\frac{Qy}{\left[y^2+(z-nd)^2\right]^{3/2}}. \tag{12}$$

The Fourier harmonics of this field component is

$$E_k=\int_{-\infty}^{\infty}E_y(0,y,z)\cos kz dz=2Q\frac{\sin(Nkd/2)}{\sin(kd/2)}kK_1(ky). \tag{13}$$

For first harmonics of the undulator field $k=k_u=2\pi/d$, Eq. (13) gives a simple result:

$$E_1=\int_{-\infty}^{\infty}E_y(0,y,z)\cos(k_u z)dz=2NQk_uK_1(k_u y). \tag{14}$$

The corresponding undulator parameter is

$$K=\frac{e}{2\pi mc^2}\frac{2E_1}{N}=\frac{2eQk_u}{\pi mc^2}K_1(k_u y). \tag{15}$$

At smaller periods, the balls will have not only a charge, but also a quadrupole moment, as the charge distribution will be squeezed along the *z*-direction. We will neglect this quadrupole correction. To find the charge of ball $Q=U\left(\sum_{j=1}^{N}C_{ij}^{-1}\right)^{-1}$, one should calculate the capacity matrix $C_{ij}$ of the chain of *N* balls. In the first approximation [6],

$$C_{ij}^{-1}\approx\frac{1}{a}\delta_{ij}+\frac{1-\delta_{ij}}{d|i-j|}\exp\left(-\frac{d}{w}|i-j|\right), \tag{16}$$

where the screening length *w* is about half of the inner radius of the negative electrode (see Fig. 1). Then

$$\sum_{j=1}^{N}C_{ij}^{-1}\approx\frac{1}{a}+\frac{2}{d}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{d}{w}n\right)=\frac{1}{a}-\frac{2}{d}\ln\left[1-\exp\left(-\frac{d}{w}\right)\right]. \tag{17}$$

Substituting Eq. (17) in Eq. (15), one can find the dependence of the *K* parameter on the undulator period:

$$K=\frac{2eU}{\pi mc^2}F(k_u y_1,a_1,w_1)\approx\frac{U}{0.8\text{MV}}F(k_u y_1,a_1,w_1), \tag{18}$$

where $w_1=w/y_1$ and

$$F(t,a_1,w_1)=\frac{K_1(t+ta_1)}{1/(ta_1)-\ln\left[1-\exp\left(-2\pi/(tw_1)\right)\right]/\pi}. \tag{19}$$

It is easy to show that $F(t,a_1,w_1)\le F(0,a_1,w_1)=a_1/(1+a_1)<1$. Then, according to Eq. (18), for reasonable voltages, which are limited, in particular, by high-voltage feedthroughs, $K<1$. Therefore, the variable-period design is desirable for this type of undulators.

In many cases of practical interest, the logarithmic correction of capacity in Eq. (17) can be neglected. Then Eq. (19) gives

$$F(k_u y_1,a/y_1,0)=\frac{a}{y}k_u yK_1(k_u y)\approx\frac{a}{y}e^{-k_u y}\sqrt{1+1.7k_u y}. \tag{20}$$

For $a=y_1$,

$$F(k_u y,1,0)=k_u y_1K_1(2k_u y_1)\approx\frac{1}{2}e^{-2k_u y_1}\sqrt{1+3.4k_u y_1}. \tag{21}$$

## CONCLUSION

In this paper, we considered the feasibility of electrostatic undulators. Their main advantage is the capability to operate at periods of a few millimeters. With such undulators, one can build high-brightness x-ray sources using lower (1.5 – 2 GeV) electron energies. The main limitation of the electrode voltage is caused by surface discharge at high-voltage vacuum feedthroughs. It can be increased with a dedicated feedthrough design and more exotic techniques like, e. g., in-vacuum Van de Graaff generators.

## REFERENCES


1. N. A. Vinokurov and E. B. Levichev, Phys. Usp. **58**, No. 9, 850–871 (2015).
2. L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, Oxford, 1980).
3. G. K. Shenoy, J.W. Lewellen, D. Shu and N.A Vinokurov, J. Synchrotron Radiat. **10**, 205–213 (2003).
4. N. A. Vinokurov, O. A. Shevchenko and V. G. Tcheskidov, Phys. Rev. Spec. Top. - Accel. Beams. **14**, 040701–1 - 040701–7 (2011).
5. J. Mun, *et al*., Phys. Rev. Spec. Top. - Accel. Beams. **17**, 080701–1 - 080701–6 (2014).
6. L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1963).