Theory of Multibunch Storage Ring with Transverse Feedback

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Abstract. Most contemporary storage rings operate in the multibunch mode. In this case, the transverse dynamics in the presence of feedback may be complicated. Indeed, generally, the amplified signal of beam position monitor kicks all circulating bunches. In this paper, the stability of such system with many degrees of freedom is considered. Damping times are estimated for the simplest cases.

INTRODUCTION

A typical transverse feedback system (see, e. g., [1]) of storage ring consists of a beam position monitor (BPM), signal processing electronics, and a kicker. In the simplest (and fastest) case, the signal from the BPM (which is proportional to the transverse displacement of bunch passed through the BPM) is amplified and transmitted to the transverse kicker. The main goal of transverse feedback system is to decrease the damping time of coherent betatron oscillations. Most contemporary storage rings operate in the multibunch mode. In this case, the transverse dynamics in the presence of feedback may be complicated. Indeed, generally, the amplified signal of BPM kicks all circulating bunches. In this paper, the stability of such system with many degrees of freedom is considered. Damping times are estimated for the simplest cases.

CHARACTERISTIC EQUATION

Let us consider a storage ring with a kicker fed by the amplified signal from a transverse BPM. The scheme of such machine is shown in Fig. 1.

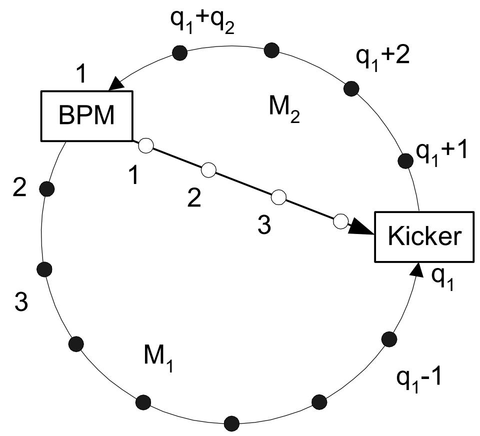

FIGURE 1. Storage ring with transverse feedback.

As in [2], we will consider the case of periodic bunches (the period T may be an integer of the radiofrequency period). To simplify equations, we assume the bunches to have the same charge, and the time of flight from the BPM to the kicker to be a multiple of T. Let q_1 bunches be travelling through the arc with a transport matrix M_1 from the BPM to the kicker, and q_2 bunches be flying from the kicker to the BPM (transport matrix M_2). It means that the times of flight through the first and the second arcs are q_1T and q_2T, respectively. We will suppose that the bunch length is smaller than the time resolution of the feedback electronics and consider the bunches as macroparticles. Therefore, the beam state is described by a column of $2(q_1 + q_2)$ components. To take into account the delay between a bunch pass through the BPM and the kick, we will present the kick $\Delta x' = \sum_{n=1}^{q_3} K_n x_1\left(-nT\right)$ as a linear response to last q_3 results of the coordinate measurements. Then the state of the whole system is described by a column of $2(q_1 + q_2) + q_3$ components, and the change of state in the time T is described by the corresponding square matrix **TMS**, which is the product of the following three matrices. The Matrix **S** describes pass of bunch 1 through the BPM, $X^{(f)}_{2q_1+2q_2+1} = X^{(i)}_1$, and the bunch with the number q_1 through the kicker, $X^{(f)}_{2q_1+2} = X^{(i)}_{2q_1+2} + \sum_{n=1}^{q_3} K_n X^{(i)}_{2q_1+2q_2+n}$, the lower right $q_3 \times q_3$ diagonal box describing the shift of pulses propagating from the BPM to the kicker:

$$\mathbf{S} = \begin{pmatrix}
E & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & E & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & E & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & E & 0 & 0 & 0 & \multicolumn{3}{c}{\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ K_1 & K_2 & K_3 & K_4 & K_5 & K_6 \end{matrix}} \\
0 & 0 & 0 & 0 & E & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & E & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & E & 0 & 0 & 0 \\
\begin{matrix} 1 & 0 \\ 0 & 0 \end{matrix} & 0 & 0 & 0 & 0 & 0 & 0 & \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix}
\end{pmatrix}, \qquad (1)$$

where E is the 2×2 unity matrix. To clarify notations, we have chosen $q_1 = 3$, $q_2 = 4$, and $q_3 = 6$ in Eqs. (1) - (3). The matrices

$$
\mathbf{M}=\begin{pmatrix}
M_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & E & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & E & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & M_2 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & E & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & E & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & E & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & E & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E
\end{pmatrix} \tag{2}
$$

and

$$
\mathbf{T}=\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & E & 0 & 0 & 0\\
E & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & E & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & E & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & E & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & E & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & E & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & E & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E
\end{pmatrix} \tag{3}
$$

describe propagation of bunches through the storage ring. The corresponding characteristic equation is

$$
\begin{aligned}
0 &= \left|TMS-\lambda E\right| = \left|MS-\lambda T^{-1}\right| = \\
&-\lambda^{q_3}\begin{vmatrix} M_1 & -\lambda^{q_1}E \\ -\lambda^{q_2}E & M_2 \end{vmatrix} + \sum_{n=1}^{q_3} K_n \lambda^{q_3-n} \begin{vmatrix} (M_1)_{12} & -\lambda^{q_1}E & 0 \\ (M_1)_{22} & & 0 \\ 0 & M_2 & (M_2)_{12} \\ -\lambda^{q_2} & & (M_2)_{22} \end{vmatrix} = \\
&-\lambda^{q_3}\left(1-2\lambda^{q_1+q_2}\cos\mu+\lambda^{2q_1+2q_2}\right)+\lambda^{q_1}\sum_{n=1}^{q_3}K_n\lambda^{q_3-n}\left[(M_1)_{12}+\lambda^{q_1+q_2}(M_2)_{12}\right]
\end{aligned} , \tag{4}
$$

or

$$
0=\cos\left[(q_1+q_2)\varphi\right]-\cos\mu-\frac{\sqrt{\beta_1\beta_2}}{2}\left[e^{-iq_2\varphi}\sin\psi+e^{iq_1\varphi}\sin(\mu-\psi)\right]\sum_{n=1}^{q_3}K_n e^{-in\varphi} , \tag{5}
$$

where $\lambda=\exp(i\varphi)$, $(M_1)_{12}=\sqrt{\beta_1\beta_2}\sin\psi$, $(M_2)_{12}=\sqrt{\beta_1\beta_2}\sin(\mu-\psi)$, β_1 and β_2 are the beta functions at the BPM and the kicker, respectively, and ψ is the betatron phase advance between these points.

SOLUTION TO THE CHARACTERISTIC EQUATION

It is worth noting that the feedback "optical strength" $|K_n|$ is limited by the corresponding kicker electronics and is typically small, $|K_n|\sqrt{\beta_1\beta_2} << 1$. Then the solution to the characteristic equation of the first order in this small parameter gives the complex shift of the betatron phase advance

$$\left(q_1+q_2\right)\varphi-\mu \approx -\frac{\sqrt{\beta_1\beta_2}}{2}e^{i\frac{q_1\mu}{q_1+q_2}-i\psi}\sum_{n=1}^{q_3}K_n e^{-i\frac{n\mu}{q_1+q_2}} . \tag{6}$$

The real part of this expression is the coherent betatron tune shift multiplied by 2π, and the imaginary part is the attenuation of the amplitude of coherent betatron oscillations per turn.

Since the response K_n decreases at large n, one can substitute $q_3 = \infty$ to Eqs. (5) and (6). In the theoretically simplest case of wide-band feedback, only one K_n is not zero. Equation (6) shows that if the bandwidth (which is about $2\pi n_{\max}/T$, where $n_{\max}$ is the number of the non-zero K_n) is less than the betatron frequency, the complex shift is reduced.

The simplest example is $K_n = K\delta_{n,q_1}$. Then each bunch is kicked by a field proportional to its measured coordinate, and Eq. (6) gives

$$\left(q_1+q_2\right)\varphi-\mu \approx -\frac{K\sqrt{\beta_1\beta_2}}{2}e^{-i\psi} . \tag{7}$$

It is also interesting to write down characteristic equation Eq. (4) for this case:

$$0 = 1-2\lambda^{q_1+q_2}\cos\mu+\lambda^{2q_1+2q_2}-K\left[\left(M_1\right)_{12}+\lambda^{q_1+q_2}\left(M_2\right)_{12}\right]. \tag{8}$$

Its exact solution is

$$\lambda^{q_1+q_2} = \cos\mu+\frac{1}{2}K\left(M_2\right)_{12}\pm i\sqrt{\sin^2\mu-K\left[\cos\mu\left(M_2\right)_{12}+\left(M_1\right)_{12}\right]-\left[\frac{1}{2}K\left(M_2\right)_{12}\right]^2} . \tag{9}$$

Equation (7) shows that at the optimal betatron phase advance $\psi = (n+1/2)\pi$, the real part of the tune shift is zero, and the amplitude damping per turn is $|K|\sqrt{\beta_1\beta_2}/2$. Moreover, at the betatron phase advance $\psi = 2n\pi$, the imaginary part of the tune shift is zero, and the real one is $-K\sqrt{\beta_1\beta_2}/2$, which at $\beta_1 = \beta_2$ is the betatron tune shift caused by a weak lens of optical strength $-K$ multiplied by 2π.

On the other hand, one can consider the narrow-band resonance feedback $K_n = K_0 e^{-\frac{n\omega T}{2Q}}\cos\left(n\omega T+\chi\right)$. Then

$$\left(q_1+q_2\right)\varphi-\mu \approx$$

$$-K_0 Q\frac{\sqrt{\beta_1\beta_2}}{2\omega T}e^{i\frac{q_1\mu}{q_1+q_2}-i\psi}\left[\frac{\frac{\omega T}{2Q}e^{i\chi}}{1-e^{-\frac{\omega T}{2Q}}\cos\left(\omega T-\frac{\mu}{q_1+q_2}\right)+ie^{-\frac{\omega T}{2Q}}\sin\left(\omega T-\frac{\mu}{q_1+q_2}\right)}+\frac{\frac{\omega T}{2Q}e^{-i\chi}}{1-e^{-\frac{\omega T}{2Q}}\cos\left(\omega T+\frac{\mu}{q_1+q_2}\right)-ie^{-\frac{\omega T}{2Q}}\sin\left(\omega T+\frac{\mu}{q_1+q_2}\right)}\right]. \tag{10}$$

At $\omega = \left| \frac{\mu}{(q_1+q_2)T} + m\frac{2\pi}{T} \right|$, m is an integer, and the maximum amplitude damping per turn is $|K_0| Q\sqrt{\beta_1\beta_2}/(2\omega T)$. This option is prospective for very fast damping systems. It may use a high-quality RF cavity.

It is easy to modify Eq. (6) for a single circulating bunch. In this case, K_n is not zero only for $n = q_1+m(q_1+q_2)$. Then

$$(q_1+q_2)\varphi - \mu \approx -\frac{\sqrt{\beta_1\beta_2}}{2} e^{-i\psi} \sum_{m=0}^{\infty} K_{q_1+m(q_1+q_2)} e^{-im\mu}. \tag{11}$$

In particular, for a single bunch and narrow-band resonance feedback, instead of Eq. (10) one has

$$(q_1+q_2)\varphi - \mu \approx -K_0 e^{-\frac{q_1}{2Q}\omega T} Q \frac{\sqrt{\beta_1\beta_2}}{2\omega T_r} e^{-i\psi} \left[\frac{\frac{\omega T_r}{2Q} e^{i\chi + iq_1\omega T}}{1 - e^{-\frac{\omega T_r}{2Q}}\cos(\omega T_r - \mu) + ie^{-\frac{\omega T_r}{2Q}}\sin(\omega T_r - \mu)} + \frac{\frac{\omega T_r}{2Q} e^{-i\chi - iq_1\omega T}}{1 - e^{-\frac{\omega T_r}{2Q}}\cos(\omega T_r + \mu) - ie^{-\frac{\omega T_r}{2Q}}\sin(\omega T_r + \mu)} \right], \tag{12}$$

where $T_r = (q_1+q_2)T$ is the revolution period.

CONCLUSION

In this paper, we considered the influence of transverse feedback system on the dynamics of multibunch beam in a storage ring. The damping time is frequently limited by the kicker strength. Nevertheless, the damping time can be decreased using resonance feedback.

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