



R measurement at VEPP-4M collider

Korneliy Todyshev

on behalf of the KEDR collaboration

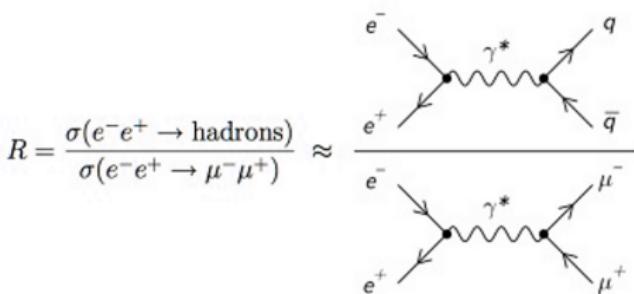
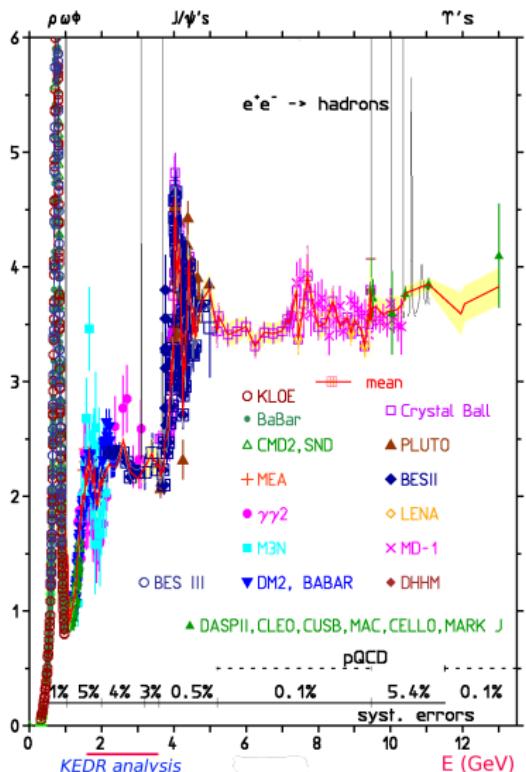
18 December 2019

BINP-IHEP Seminar



$R(s)$ measurement. Motivation.

"Priore loco" (In first)



In first approximation:

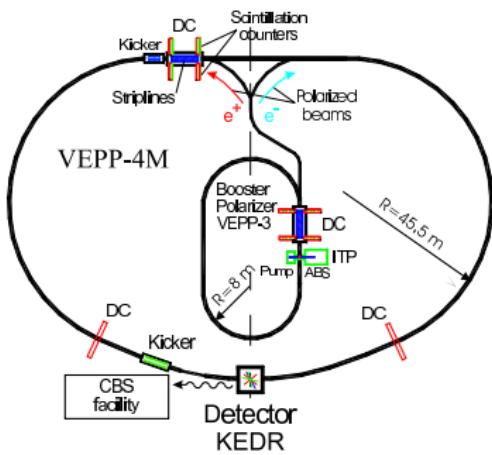
$$R(s) \simeq 3 \sum e_q^2$$

$R(s)$ is used to determine:

- $\alpha(M_Z^2)$
- $(g_\mu - 2)/2$
- $\alpha_s(s)$
- heavy quark masses

F. Jegerlehner arXiv:1511.0447

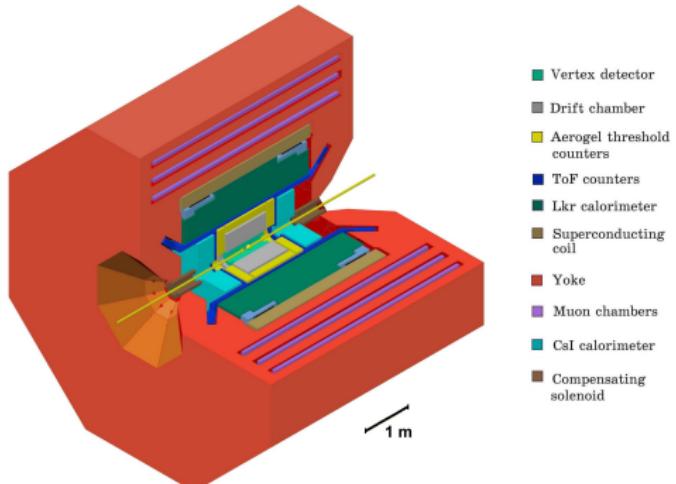
"Actum atque tractatum" (done and discussed)



| | |
|-------------------|---|
| Beam energy | $1 \div 5 \text{ GeV}$ |
| Number of bunches | 2×2 |
| Luminosity | 1.8 GeV $1.5 \times 10^{30} \text{ cm}^{-2} \text{ s}^{-1}$ |

Energy measurement:

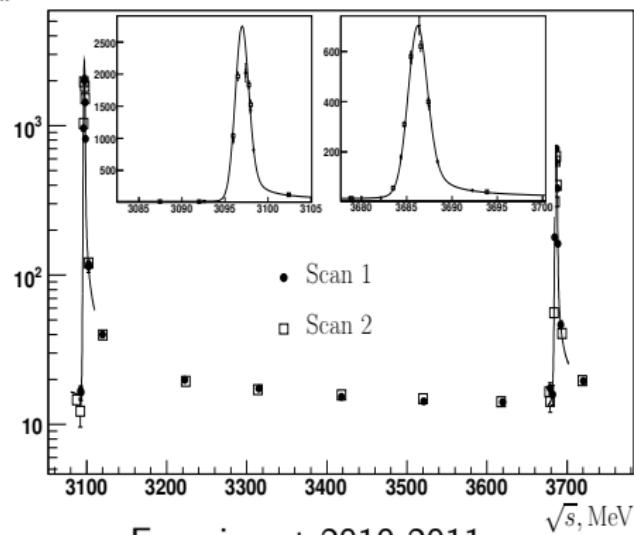
- Resonant depolarization method:
Instant measurement accuracy $\sim 1 \text{ keV}$
Energy interpolation accuracy $10 \div 30 \text{ keV}$
- Compton backscattering method $\sim 100 \text{ keV}$



"Consideratio naturae" (contemplation of nature)

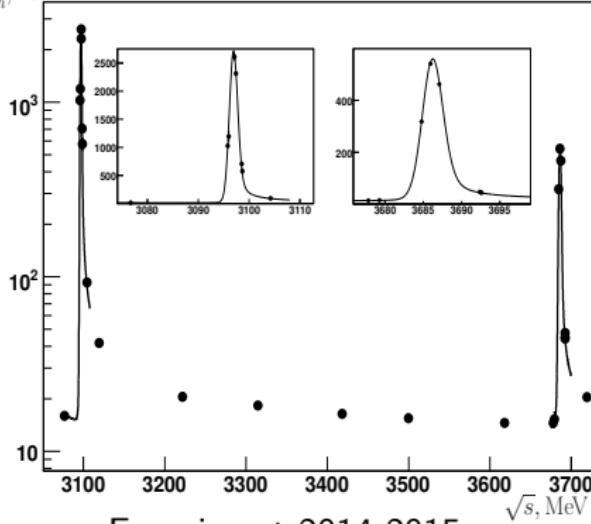
The observed multihadron cross section as a function of the c.m. energy

σ_{mh}^{obs} , nb



Experiment 2010-2011

σ_{mh}^{obs} , nb



Experiment 2014-2015

- The c.m. energy range between 3.076 and 3.72 GeV studied
- An integrated luminosity of 2.7 pb^{-1} collected at 9 energies 3.077, 3.120, 3.223, 3.315, 3.418, 3.500, 3.521, 3.618, 3.719 GeV
- $\sim (2 - 6) \times 10^3$ m.h. events per point, $\sim 38 \times 10^3$ in total



"Modus operandi" (procedure;method of operating)

The way that we are measuring R :

$$R = \frac{\sigma_{obs}(s) - \sum \varepsilon_{\psi}^{tail}(s)\sigma_{\psi}^{tail}(s) - \sum \varepsilon_{bg}^i(s)\sigma_{bg}^i(s)}{\varepsilon(s)(1 + \delta(s))\sigma_{\mu\mu}^0}$$

with $\sigma_{obs}(s) = \frac{N_{mh} - N_{res.bg.}}{\int \mathcal{L} dt}$. where N_{mh} represent all events pass hadronic selection criteria, $N_{res.bg.}$ – residual machine background

$\sum \varepsilon_{\psi}^{tail}(s)\sigma_{\psi}^{tail}(s)$ is contribution from J/ψ and $\psi(2S)$ resonances

$\sum \varepsilon_{bg}^i(s)\sigma_{bg}^i(s)$ is contribution from physical processes: $e^+e^- \rightarrow l^+l^-$, $\gamma\gamma$ -processes.

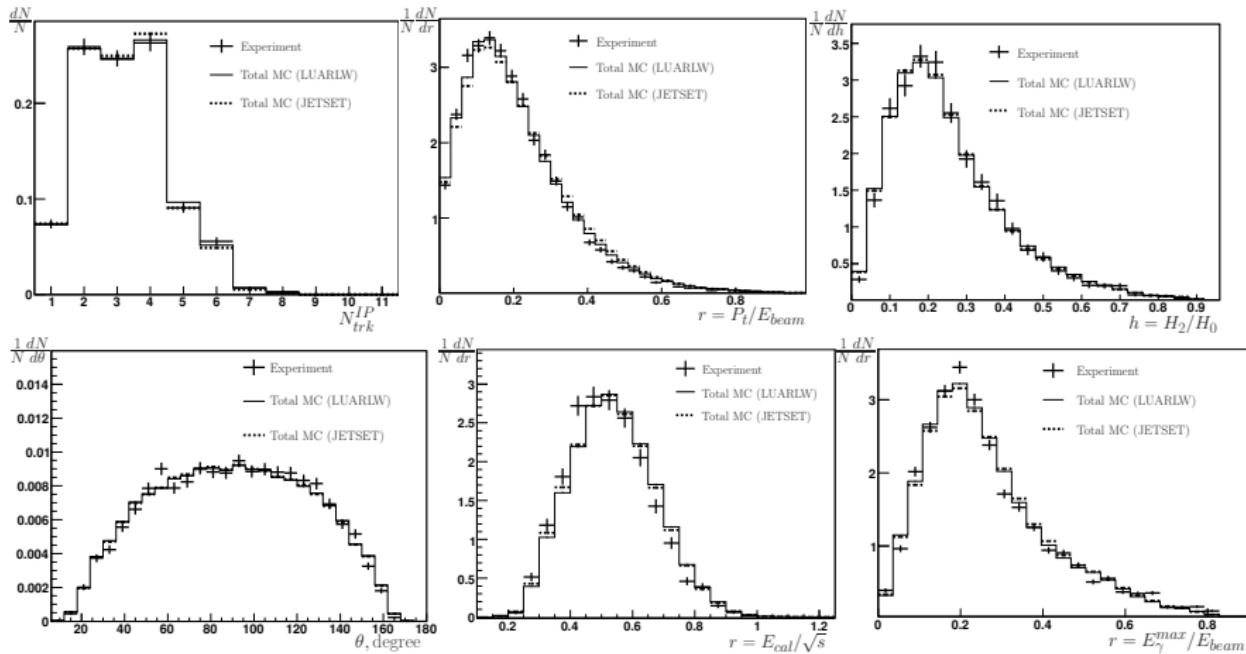
$\varepsilon(s)$ – multihadron efficiency.

$$1 + \delta(s) = \int dx \frac{1}{1-x} \frac{\mathcal{F}(s,x)}{|1 - \tilde{\Pi}(s(1-x))|^2} \frac{\tilde{R}(s(1-x))\varepsilon(s(1-x))}{R(s)\varepsilon(s)}$$

$\mathcal{F}(s,x)$ – radiative correction kernel ([E.A.Kuraev, V.S.Fadin Sov.J.Nucl.Phys.41\(466-472\)1985](#))

Here $\tilde{\Pi}$ and \tilde{R} does not includes J/ψ and $\psi(2S)$ resonances. To determine the contributions of the J/ψ and $\psi(2S)$ without external data, the additional data samples of about 0.4 pb^{-1} (2010-2011) and 0.34 pb^{-1} (2014-2015) were collected in the vicinity of peak regions.

"Punctum saliens" (The most important thing)



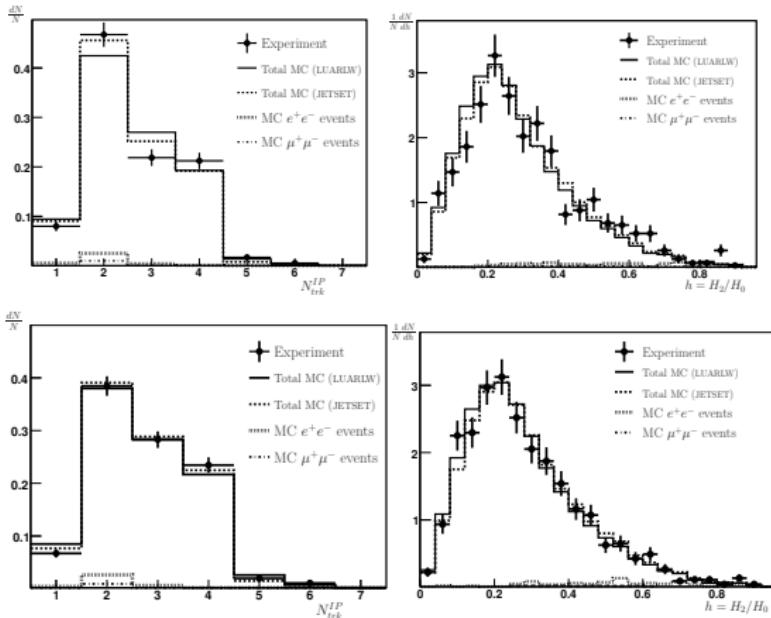
Properties of hadronic events produced in the uds continuum at 3.119 GeV (2014-2015).

Here N is the number of events, N_{IP}^{trk} is the number of tracks originated from IP, P_t is a transverse momentum of the track, H_2 and H_0 are Fox-Wolfram moments, θ is a polar angle of the track, E_{cal} is energy deposited in the calorimeter, E_γ^{\max} is energy of the most energetic photon.

"Fac simile" (to make alike; reproduction exacte)

- An integrated luminosity 0.66 pb^{-1} collected at 13 equidistant points with a step $\sim 0.1 \text{ GeV}$: $1.841, 1.937 \dots 3.048 \text{ GeV}$
- $\sim 10^3$ hadronic events per point, 14.8×10^3 events in total
- Simulation of the uds continuum based on the LUARLW generator, tuned JETSET alternatively used at 6 points for a cross-check.

Experimental distribution and two variants of MC simulation based on LUARLW and tuned JETSET are plotted ($\sqrt{s} = 1.94 \text{ GeV}$ and $\sqrt{s} = 2.14 \text{ GeV}$).





Systematic uncertainties

*"Satius est supervacua discere quam nihil"
(Better to learn more than necessary than nothing at all)*

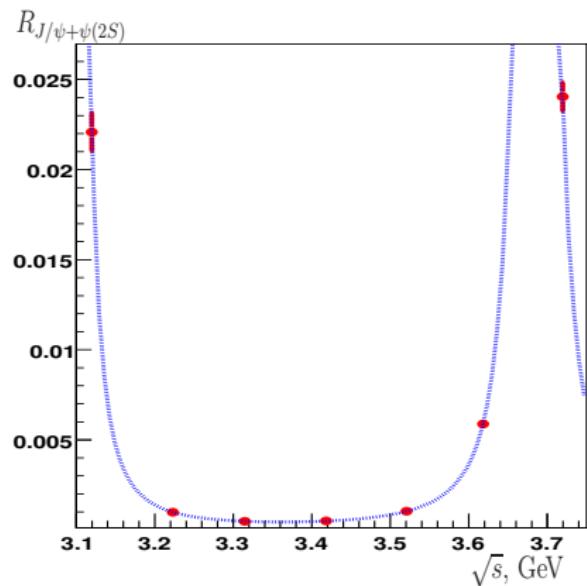
Seneca

| Source | Syst. uncertainty, % | | | |
|--------------------------|----------------------|---|----------------|----------------|
| | Scan. 2010 | Scan 1 and 2 2011 | Scan 2014-2015 | Corr. |
| Luminosity | 1.2 | 1.1 | 0.9 | 0.4 |
| Rad. corr. | $0.5 \div 2.0$ | $0.4 \div 0.6$ | $0.5 \div 0.8$ | $0.2 \div 0.4$ |
| Sim. uds continuum | $1.2 \div 6.6$ | $1.3 \div 2.0$ | 1.1 | 0.9 |
| Track reconstruction | 0.5 | 0.5 | 0.4 | – |
| J/ψ | – | $0.1 \div 2.7$ | $0.1 \div 1.8$ | – |
| $\psi(2S)$ (at 3.72 GeV) | – | 1.4 | 1.1 | – |
| I^+I^- | $0.3 \div 0.6$ | $0.1 \div 0.2$ | $0.3 \div 0.4$ | $0.1 \div 0.2$ |
| e^+e^-X | 0.2 | $0.1 \div 0.2$ | 0.1 | 0.1 |
| Trigger | 0.3 | 0.2 | 0.2 | 0.2 |
| Nuclear int. | 0.4 | 0.2 | 0.2 | 0.2 |
| Beam background | $0.4 \div 0.9$ | $0.5 \div 1.1$ | $0.4 \div 0.8$ | – |
| Selection criteria | 0.7 | 0.6 | 0.6 | – |
| Square sum. | $2.1 \div 7.1$ | $2.1 \div 3.6$ (corr. $1.8 \div 2.5$) | $1.9 \div 2.7$ | 1.1 |

Results

| \sqrt{s} , GeV | $R_{uds}(s)\{R(s)\}$ | $\frac{\delta R}{R} \left(\frac{\delta R_{syst}}{R} \right), \%$ |
|------------------|--------------------------------------|---|
| 1.841 | $2.226 \pm 0.139 \pm 0.158$ | $9.5(7.1)$ |
| 1.937 | $2.141 \pm 0.081 \pm 0.073$ | $5.1(3.4)$ |
| 2.037 | $2.238 \pm 0.068 \pm 0.072$ | $4.4(3.2)$ |
| 2.134 | $2.275 \pm 0.072 \pm 0.055$ | $4.0(2.4)$ |
| 2.239 | $2.208 \pm 0.069 \pm 0.053$ | $3.9(2.4)$ |
| 2.340 | $2.194 \pm 0.064 \pm 0.048$ | $3.7(2.2)$ |
| 2.444 | $2.175 \pm 0.067 \pm 0.048$ | $3.8(2.2)$ |
| 2.543 | $2.222 \pm 0.070 \pm 0.047$ | $3.8(2.1)$ |
| 2.645 | $2.220 \pm 0.069 \pm 0.049$ | $3.8(2.2)$ |
| 2.745 | $2.269 \pm 0.065 \pm 0.050$ | $3.6(2.2)$ |
| 2.850 | $2.223 \pm 0.065 \pm 0.047$ | $3.6(2.1)$ |
| 2.949 | $2.234 \pm 0.064 \pm 0.051$ | $3.7(2.3)$ |
| 3.048 | $2.278 \pm 0.075 \pm 0.048$ | $3.9(2.3)$ |
| 3.077 | $2.188 \pm 0.056 \pm 0.042$ | $3.2(2.1)$ |
| 3.120 | $2.212\{2.235\} \pm 0.042 \pm 0.049$ | $2.9(2.2)$ |
| 3.223 | $2.194\{2.195\} \pm 0.040 \pm 0.035$ | $2.4(1.6)$ |
| 3.315 | $2.219\{2.219\} \pm 0.035 \pm 0.035$ | $2.2(1.6)$ |
| 3.418 | $2.185\{2.185\} \pm 0.032 \pm 0.035$ | $2.2(1.6)$ |
| 3.500 | $2.224\{2.224\} \pm 0.054 \pm 0.040$ | $3.0(1.8)$ |
| 3.521 | $2.200\{2.201\} \pm 0.050 \pm 0.044$ | $3.0(2.0)$ |
| 3.618 | $2.212\{2.218\} \pm 0.038 \pm 0.035$ | $2.3(1.6)$ |
| 3.720 | $2.204\{2.228\} \pm 0.039 \pm 0.042$ | $2.6(1.9)$ |

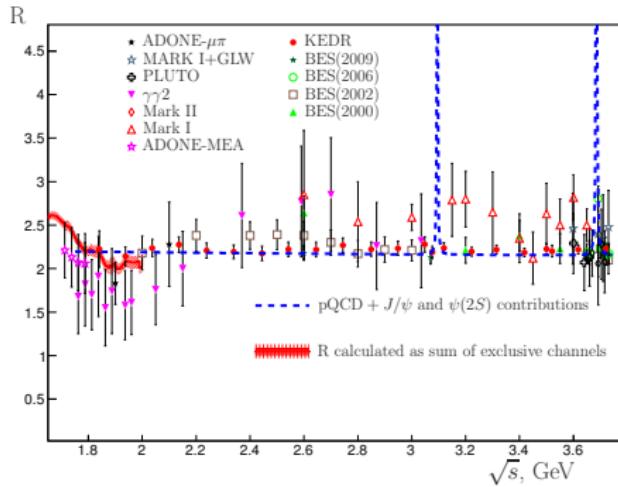
"Ut supra" (as (described) above)



Using J/ψ and $\psi(2S)$ parameters, we obtain

$$R_{uds}(s) + R_{J/\psi + \psi(2S)} \implies R(s)$$

"De omnibus dubitandum"(All is to be doubted)
René Descartes



The quantity R versus the c.m. energy and the sum of the prediction of perturbative QCD and a contribution of narrow resonances.

In the c.m.energy range 3.08-3.72 GeV the weighted average

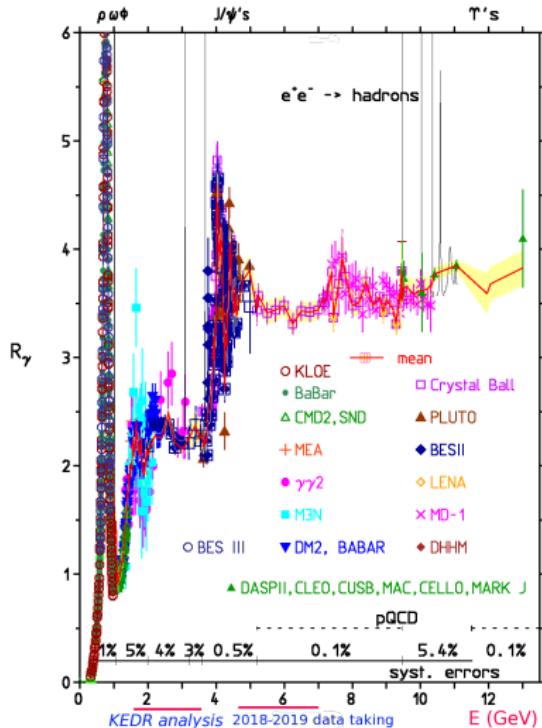
$\bar{R}_{uds} = 2.204 \pm 0.014 \pm 0.026$ is approximately one sigma higher than that theoretically expected, $R_{uds}^{pQCD} = 2.16 \pm 0.01$ calculated according to the pQCD In the lower c.m.energy range 1.84-3.05 GeV the weighted average is $2.225 \pm 0.020 \pm 0.047$ (the pQCD prediction of 2.18 ± 0.02).



"Omne futurum incertum"(Every future thing is uncertain)

R measurement in the energy range
4.56-6.96 GeV.

- We plan to measure R value in the energy range 4.6-7.0 GeV.
- First scan finished in 2018. An integrated luminosity $\sim 4 \text{ pb}^{-1}$ collected at 8 equidistant points with a step $\sim 0.3 \text{ GeV}$ from 4.71 to 6.81 GeV
- Second scan is in progress. An integrated luminosity $\sim 4 \text{ pb}^{-1}$ collected at 7 equidistant points with a step $\sim 0.3 \text{ GeV}$ from 4.6 to 6.4 GeV



F. Jegerlehner arXiv:1511.0447



Summary

"Jucundi acti labores"(past labors are pleasant)
Cicero

- KEDR measured the R values at 22 center-of-mass energies between 1.84 and 3.72 GeV.
In the energy range between 1.84 and 3.05 GeV the achieved accuracy is about or better than **3.9%** at most of the energy points with a systematic uncertainty less than **2.4%**.
For the energies above J/ψ resonance the total error is about or better than **2.6%** and a systematic uncertainty of about **1.9%**.

V. V. Anashin et al., Phys.Lett. B 770C, 174 (2017).[arXiv:1610.02827]

V. V. Anashin et al., Phys.Lett. B 753, 533-541 (2016).[arXiv:1510.02667]

V. V. Anashin et al., Phys.Lett. B 788, 42-51 (2019).[arXiv:1805.06235]

- Experiment in the energy range from 4.6 to 7 GeV is in progress.

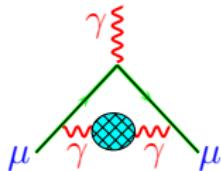
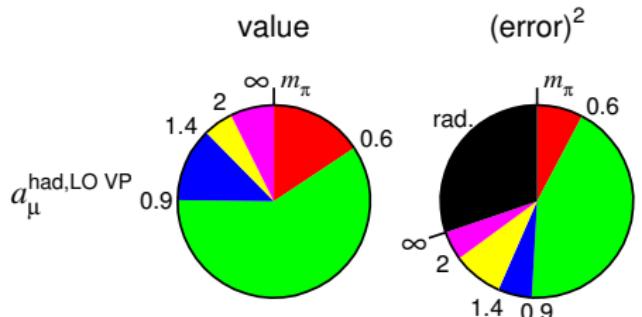
"Gratia gratiam parit"(thanks begets thanks)

Thank you for your time and
attention

BACKUP SLIDES

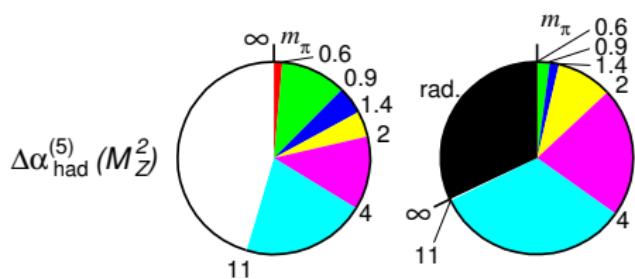
R contribution in a_μ and $\alpha(M_Z^2)$

$$a_\mu^{exp} = (g_\mu - 2)/2$$



$$a_\mu^{\text{LO VP}} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^\infty \frac{K(s)R(s)}{s^2} ds$$

Low energy contributions dominate



$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$$

$$\Delta\alpha = \sum_f \text{---} \gamma \text{---} \circlearrowleft \text{---} \gamma \text{---} = \Delta\alpha_{\text{lept}}(s) + \Delta\alpha_{\text{had}}(s)$$

$$\Delta\alpha^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{m_\pi^2}^\infty \frac{R(s)ds}{s(s - M_Z^2 - i\epsilon)}$$

K.Hagiwara et al. arxiv:1105.3149



An application of the $R(s)$

"Natura appetit perfectum, ita est lex"

(Nature desires perfection, so also does the law)

Correlated uncertainties of R_{uds} in %

| Source | Uncertainty in % | |
|--------------------------|------------------|--------------------------|
| | Data 2010 | Data 2010 / 2011,2014 |
| Luminosity | | |
| Cross section calc. | 0.5 | 0.4 |
| Calorimeter response | 0.7 | - |
| Calorimeter alignment | 0.2 | 0.2 |
| Rad. correction | | |
| \bar{n} approx. | 0.3 | 0.1 |
| $\delta R_{uds}(s)$ | 0.2 | 0.2 |
| $\delta \epsilon(s)$ | 0.3 | 0.2 |
| Continuum simulation | 1.2 | $0.4 \div 0.8$ |
| Track reconstr. | 0.5 | 0.4 |
| e^+e^-X contribution | 0.2 | 0.1 |
| j/ψ contribution | 0.3 | 0.2 |
| Trigger efficiency | 0.3 | 0.2 |
| Nuclear interaction | 0.4 | 0.2 |
| Sum in quadrature | 1.8 | $0.8 \div 1.1$ |

$$R_{uds}(s) \simeq 2 \times \left(1 + \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2} \times \left(\frac{365}{24} - 9\zeta_3 - \frac{11}{4} \right) \right)$$

where ζ is the Euler-Riemann zeta function,

$$\begin{aligned} \alpha_s(s) = & \frac{1}{b_0 t} \left(1 - \frac{b_1 l}{b_0^2 t} + \frac{b_1(l^2 - l - 1) + b_0 b_2}{b_0^4 t^2} \right. \\ & \left. + \frac{b_1^3(-2l^3 + 5l^2 + 4l - 1) - 6b_0 b_2 b_1 l + b_0^2 b_3^2}{2b_0^6 t^3} \right) \end{aligned}$$

with $t = \ln \frac{s}{\Lambda^2}$, $l = \ln t$ parametrized in terms of the QCD scale parameter Λ and coefficients b_0, b_1, b_3 (can be found in PDG). To determine Λ , we minimise the χ^2 function

$$\chi^2 = \sum_i \sum_j \left(R_{uds}^{\text{meas}}(s_i) - R_{uds}^{\text{calc}}(s_i) \right) C_{ij}^{-1} \left(R_{uds}^{\text{meas}}(s_j) - R_{uds}^{\text{calc}}(s_j) \right),$$

The obtained value of $\Lambda = 0.361^{+0.155}_{-0.174}$ GeV corresponds to $\alpha_s(m_\tau) = 0.332^{+0.100}_{-0.092}$. If the next order of pQCD is included in the expansion of R_{uds} , the fitting results are as follows: $\Lambda = 0.437^{+0.210}_{-0.215}$ GeV and $\alpha_s(m_\tau) = 0.378^{+0.173}_{-0.120}$.

$\alpha_s(m_\tau)$ determined from our $R(s)$ results is consistent with obtained in semileptonic τ decays ($\alpha_s(m_\tau) = 0.331 \pm 0.013$)

$\sigma^{e^+ e^- \rightarrow \text{hadrons}}$ and $\sigma^{e^+ e^- \rightarrow e^+ e^-}$ nearby a narrow resonance

In the soft photon approximation analytical expression for the annihilation cross section nearby a narrow resonance.

Ya.I. Azimov et al. JETP Lett. 21 (1975) 172. With up-today modifications one has

$$\sigma^{e^+ e^- \rightarrow \text{hadr}}(s) = \sigma_{\text{continuum}}^{e^+ e^- \rightarrow \text{hadr}} + \frac{12\pi}{s} (1 + \delta_{sf}) \left[\frac{\Gamma_{ee} \tilde{\Gamma}_h}{\Gamma M} \text{Im } f(s) - \frac{2\alpha \sqrt{R \Gamma_{ee} \tilde{\Gamma}_h}}{3\sqrt{s}} \lambda \text{Re } \frac{f^*(s)}{1 - \Pi_0} \right],$$

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega} \right)^{ee \rightarrow ee} &= \left(\frac{d\sigma}{d\Omega} \right)_{\text{QED}}^{ee \rightarrow ee} + \frac{1}{s} (1 + \delta_{sf}) \left\{ \frac{9}{4} \frac{\Gamma_{ee}^2}{\Gamma M} (1 + \cos^2 \theta) \text{Im } f - \right. \\ &\quad \left. \frac{3\alpha}{2} \frac{\Gamma_{ee}}{M} \left[(1 + \cos^2 \theta) \text{Re } \frac{f^*}{1 - \Pi_0(s)} - \frac{(1 + \cos \theta)^2}{(1 - \cos \theta)} \text{Re } \frac{f^*}{1 - \Pi_0(t)} \right] \right\}, \end{aligned}$$

Recently it was verified in the work X. Y. Zhou, Y. D. Wang and L. G. Xia, Chin. Phys. C 41 (2017) no.8, 083001

$$\delta = \frac{3}{4}\beta + \frac{\alpha}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right) + \beta^2 \left(\frac{37}{96} - \frac{\pi^2}{12} - \frac{L}{72} \right), \quad L = \ln \left(s/m_e^2 \right), \quad \beta = \frac{2\alpha}{\pi} (L - 1),$$

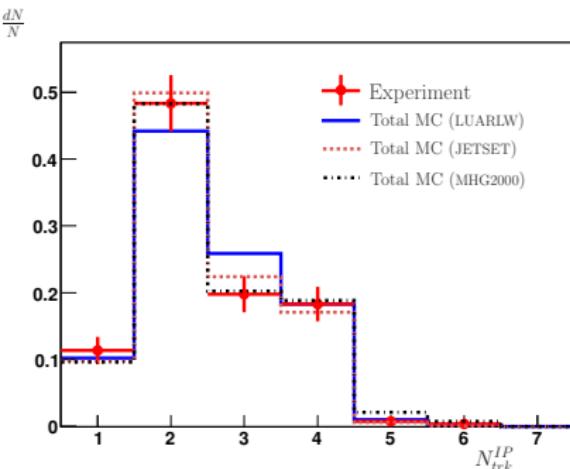
$$f(s) = \frac{\pi\beta}{\sin \pi\beta} \left(\frac{s}{M^2 - s - iM\Gamma} \right)^{1-\beta}$$

Γ_{ee} , Γ , M – 'dressed' parameters including corrections to the vacuum polarization,
 $\Gamma_{ee} = \Gamma_{ee}^{(0)}/|1 - \Pi_0|^2$, λ -parameter controls the resonance–continuum interference, $\tilde{\Gamma}_h \neq \Gamma_h$

Numerical convolution with the collision energy distribution is used to fit resonance.

Detection efficiency uncertainty in the energy range $\sqrt{s} = 1.84 \div 3.05$ GeV

- Used two essentially different MC generators (LUARLW and tuned JETSET)
- We validated our estimate of the systematic uncertainty related to simulation of the uds continuum using an unfolding method (Chinese Physics C Vol. 37, No. 6 (2013) 063001).
- The estimate at the most problematic energy point 1.84 GeV was additionally verified using the exclusive generator MHG2000.



Detection efficiency uncertainties obtained by different methods

| Energy, MeV | $\delta\epsilon/\epsilon$ | | |
|----------------------|---------------------------|-------------------------------|-------------------|
| | LUARLW | Unfolding JETSET method | LUARLW MHG2000 |
| 1841.0 | 6.6% | 3.6% | 3.8% |
| 1937.0 \div 2135.7 | 2.5% | 1.9% | - |
| 2135.7 \div 3048.1 | 1.2% | 0.5% | - |

Unfolding method

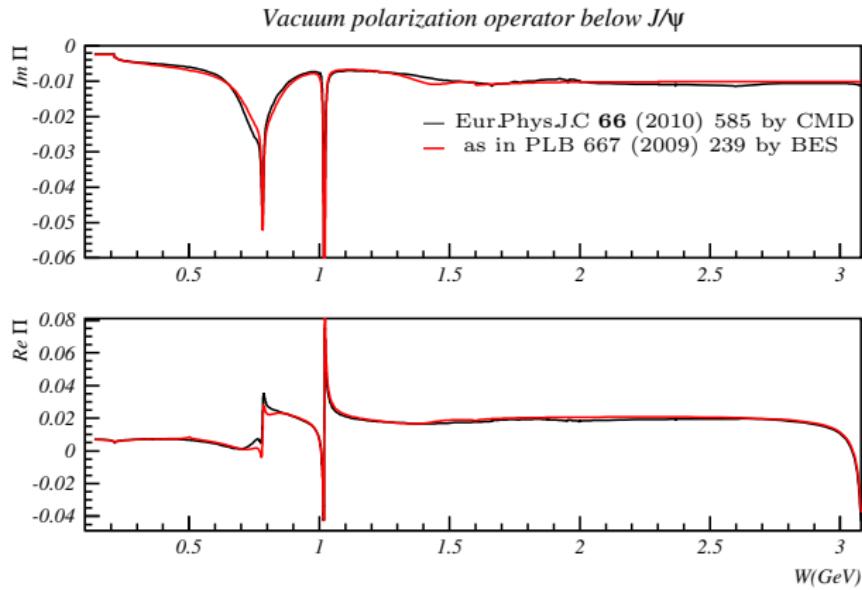
- An efficiency matrix ϵ_{ij} describes the efficiency of an event generated with j charged tracks to be reconstructed with i charged tracks.
- The distribution of the number of observed charged track events in data, N_i^{obs} , is known. The true multiplicity distribution in data can be estimated from the observed multiplicity distribution in data and the efficiency matrix by minimizing the χ^2 .
-

$$\chi^2 = \sum_{i=1}^{i=8} \frac{N_i^{obs} - \sum_{j=1}^{j=8} \epsilon_{ij} \times N_j}{N_i^{obs}}$$

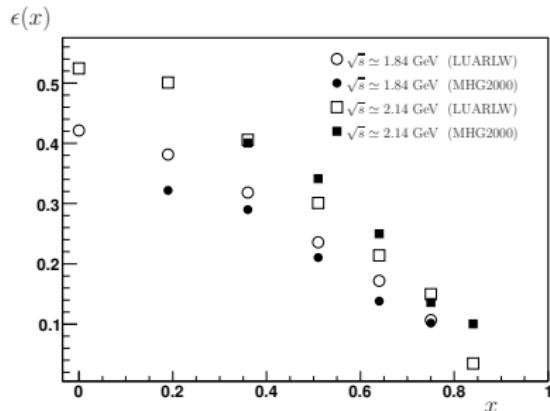
where the N_j ($j = 0, 2, 4, 6, 8$) describe the true multiplicity distribution in data and are taken as floating parameters in the fit.

- The total «true» number of events in data can be obtained by summing all fitted N_j .

$\Pi(s)$ calculation



Radiation correction calculation in the energy range 1.84 – 3.05 GeV



Detection efficiency vs variable x at 1.84 and 2.14 GeV.

$$\mathbf{1} + \delta(s) = \int \frac{dx}{\mathbf{1} - x} \frac{\mathcal{F}(s, x)}{|\mathbf{1} - \Pi((\mathbf{1} - x)s)|^2} \frac{R((\mathbf{1} - x)s)\varepsilon((\mathbf{1} - x)s)}{R(s)\varepsilon(s)}$$

$$R(s) = -\frac{3}{\alpha} \operatorname{Im} \Pi_{\text{hadr}}(s)$$

Vacuum polarization according to
CMD-2 data compilation:
Eur. Phys. J. C66 (2010) 585

Radiative correction factor $1 + \delta$

| \sqrt{s} , MeV | $1 + \delta$ | \sqrt{s} , MeV | $1 + \delta$ |
|------------------|---------------------|------------------|---------------------|
| 1841.0 | 1.0423 ± 0.0208 | 2542.6 | 1.0739 ± 0.0054 |
| 1937.0 | 1.0429 ± 0.0156 | 2644.8 | 1.0796 ± 0.0054 |
| 2037.3 | 1.0515 ± 0.0126 | 2744.6 | 1.0809 ± 0.0054 |
| 2135.7 | 1.0634 ± 0.0106 | 2849.7 | 1.0823 ± 0.0054 |
| 2239.2 | 1.0645 ± 0.0096 | 2948.9 | 1.0774 ± 0.0054 |
| 2339.5 | 1.0664 ± 0.0075 | 3048.1 | 1.0584 ± 0.0053 |
| 2444.1 | 1.0684 ± 0.0064 | | |

Selection criteria

Selection criteria for hadronic events which were used by AND.

| Variable | Allowed range | |
|--|---|---|
| | 3.12-3.72 GeV (2010-2011) | 1.84 - 3.05 GeV |
| $N_{\text{track}}^{\text{IP}}$ | ≥ 1 | ≥ 1 |
| E_{obs} | $> 1.6 \text{ GeV}$ | $> 1.4 \text{ GeV} \left(> 1.3 \text{ GeV if } E_{\text{beam}} < 1.05 \text{ GeV} \right)$ |
| $E_{\gamma}^{\max}/E_{\text{beam}}$ | < 0.8 | < 0.8 |
| $E_{\text{obs}} - E_{\gamma}^{\max}$ | | $> 1.2 \text{ GeV} \left(> 1.1 \text{ GeV if } E_{\text{beam}} < 1.05 \text{ GeV} \right)$ |
| E_{cal} | $> 0.75 \text{ GeV}$ | $> 0.55 \text{ GeV}$ |
| H_2/H_0 | < 0.85 | < 0.9 |
| $ P_z^{\text{miss}}/E_{\text{obs}} $ | < 0.6 | < 0.7 |
| $E_{\text{LKr}}/E_{\text{cal}}^{\text{tot}}$ | > 0.15 | > 0.15 |
| $ Z_{\text{vertex}} $ | $< 20.0 \text{ cm}$ | $< 15.0 \text{ cm}$ |
| | $N_{\text{particles}} \geq 4 \text{ or } \tilde{N}_{\text{track}}^{\text{IP}} \geq 2$ | $N_{\text{particles}} \geq 3 \text{ or } \tilde{N}_{\text{track}}^{\text{IP}} \geq 2$ |

The correlation matrix for systematic uncertainties of the R value obtained in the KEDR experiments

Point Correlation Matrix

| | | | | | | | | | | | | | | | | | | | | | | |
|----|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | 1 | 0.139 | 0.143 | 0.193 | 0.192 | 0.212 | 0.212 | 0.216 | 0.207 | 0.211 | 0.216 | 0.201 | 0.222 | 0.096 | 0.046 | 0.096 | 0.105 | 0.110 | 0.098 | 0.089 | 0.114 | 0.071 |
| 2 | 1 | 0.309 | 0.418 | 0.408 | 0.445 | 0.437 | 0.466 | 0.446 | 0.457 | 0.467 | 0.434 | 0.480 | 0.200 | 0.097 | 0.201 | 0.225 | 0.229 | 0.212 | 0.189 | 0.244 | 0.151 | |
| 3 | 1 | 0.423 | 0.425 | 0.470 | 0.470 | 0.480 | 0.460 | 0.463 | 0.480 | 0.442 | 0.486 | 0.212 | 0.101 | 0.212 | 0.232 | 0.243 | 0.218 | 0.198 | 0.253 | 0.158 | | |
| 4 | 1 | 0.575 | 0.635 | 0.635 | 0.649 | 0.622 | 0.610 | 0.649 | 0.598 | 0.637 | 0.287 | 0.137 | 0.286 | 0.314 | 0.329 | 0.295 | 0.268 | 0.342 | 0.213 | | | |
| 5 | 1 | 0.621 | 0.621 | 0.642 | 0.615 | 0.629 | 0.643 | 0.598 | 0.661 | 0.280 | 0.134 | 0.280 | 0.310 | 0.322 | 0.293 | 0.262 | 0.336 | 0.208 | | | | |
| 6 | 1 | 0.677 | 0.709 | 0.679 | 0.695 | 0.710 | 0.661 | 0.730 | 0.306 | 0.148 | 0.305 | 0.342 | 0.351 | 0.323 | 0.287 | 0.371 | 0.229 | | | | | |
| 7 | 1 | 0.709 | 0.679 | 0.695 | 0.710 | 0.661 | 0.730 | 0.304 | 0.148 | 0.305 | 0.342 | 0.348 | 0.323 | 0.287 | 0.371 | 0.229 | | | | | | |
| 8 | 1 | 0.695 | 0.710 | 0.725 | 0.675 | 0.745 | 0.320 | 0.153 | 0.320 | 0.351 | 0.368 | 0.330 | 0.299 | 0.382 | 0.238 | | | | | | | |
| 9 | 1 | 0.681 | 0.695 | 0.647 | 0.715 | 0.307 | 0.146 | 0.306 | 0.336 | 0.352 | 0.316 | 0.287 | 0.366 | 0.228 | | | | | | | | |
| 10 | 1 | 0.710 | 0.654 | 0.701 | 0.314 | 0.150 | 0.313 | 0.344 | 0.360 | 0.323 | 0.293 | 0.374 | 0.233 | | | | | | | | | |
| 11 | 1 | 0.675 | 0.745 | 0.321 | 0.153 | 0.320 | 0.351 | 0.368 | 0.330 | 0.300 | 0.382 | 0.238 | | | | | | | | | | |
| 12 | 1 | 0.687 | 0.298 | 0.142 | 0.298 | 0.327 | 0.342 | 0.307 | 0.279 | 0.356 | 0.222 | | | | | | | | | | | |
| 13 | 1 | 0.330 | 0.157 | 0.329 | 0.361 | 0.378 | 0.339 | 0.308 | 0.393 | 0.245 | | | | | | | | | | | | |
| 14 | 1 | 0.288 | 0.396 | 0.405 | 0.394 | 0.356 | 0.317 | 0.403 | 0.333 | | | | | | | | | | | | | |
| 15 | 1 | 0.345 | 0.347 | 0.345 | 0.305 | 0.275 | 0.345 | 0.288 | | | | | | | | | | | | | | |
| 16 | 1 | 0.486 | 0.475 | 0.427 | 0.380 | 0.483 | 0.400 | | | | | | | | | | | | | | | |
| 17 | 1 | 0.486 | 0.427 | 0.387 | 0.486 | 0.405 | | | | | | | | | | | | | | | | |
| 18 | 1 | 0.427 | 0.380 | 0.483 | 0.400 | | | | | | | | | | | | | | | | | |
| 19 | 1 | 0.340 | 0.427 | 0.356 | | | | | | | | | | | | | | | | | | |
| 20 | 1 | 0.384 | 0.318 | | | | | | | | | | | | | | | | | | | |
| 21 | 1 | 0.403 | | | | | | | | | | | | | | | | | | | | |
| 22 | | | | | | | | | | | | | | | | | | | | | | |