# Measurement of $\gamma^*\omega\pi^0$ form factor using SND detector at VEPP-2000

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# Introduction

**Motivation**

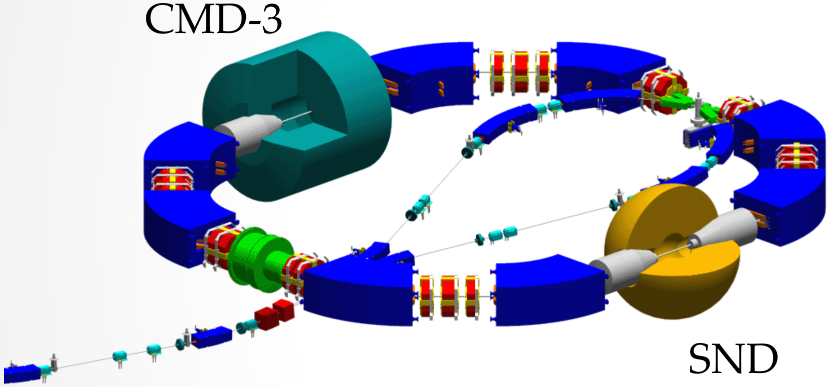
- $\omega\pi^0$ mechanism contribution dominates in $\pi^+\pi^-\pi^0\pi^0$ cross section in energy range 1 – 1.5 GeV and gives some contribution to hadronic vacuum polarization and anomalous magnetic momentum of muon
- $\gamma^*\omega\pi^0$ form factor is used for studying excited resonances, in particular, for measuring parameters of ρ’.
- In decay $\omega \to \mu^+\mu^-\pi^0$, significant discrepancy of $\gamma^*\omega\pi^0$ format factor was found in respect to VMD model predictions (NA60 experiment).

**Tasks**

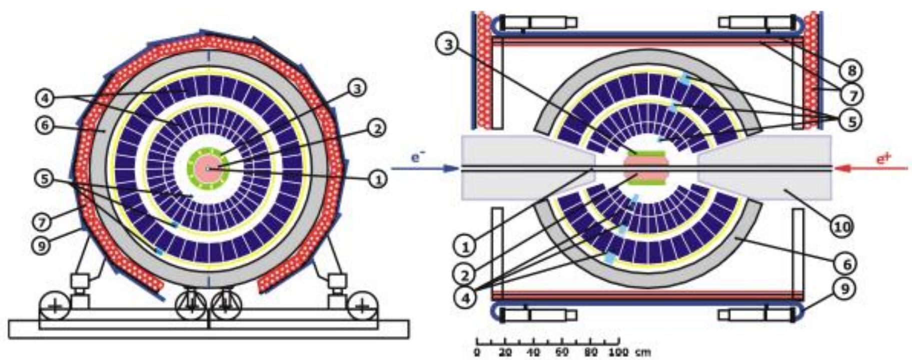
- Measuring $e^+e^- \to \omega\pi^0 \to \pi^+\pi^-\pi^0\pi^0$ cross section
- Determining efficiency corrections and systematic uncertainties
- Obtaining $\gamma^*\omega\pi^0$ from the measured cross section
- Fitting the form factor in VMD model

# Experiment SND at VEPP-2000 collider

## VEPP-2000



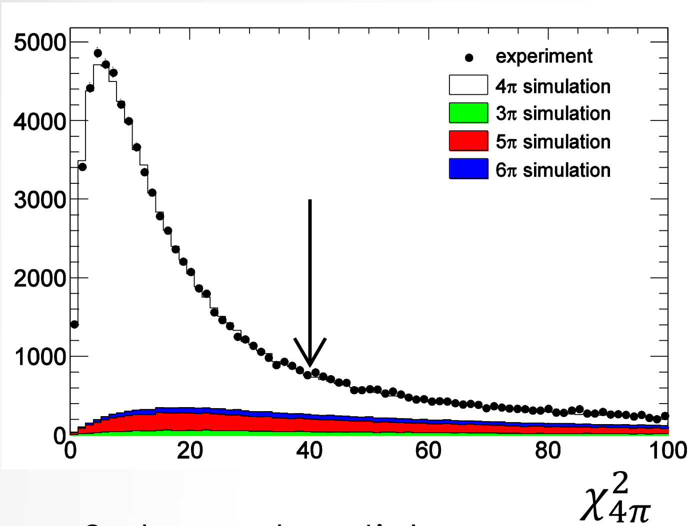

## Spherical Neutral Detector




1 – beam pipe, 2 – tracking system, 3 – aerogel cherenkov counter, 4 – NaI(Tl) crystals, 5 – phototriodes, 6 – iron muon absorber, 7–9 – muon detector, 10 – focusing solenoids.

| Experiment | C.M. Energy | Number of points | Luminosity |
|---|---|---|---|
| MHAD2011 | 1.05 – 2.00 GeV | 40 | 22 $pb^{-1}$ |
| MHAD2012 | 1.28 – 1.98 GeV | 16 | 13 $pb^{-1}$ |

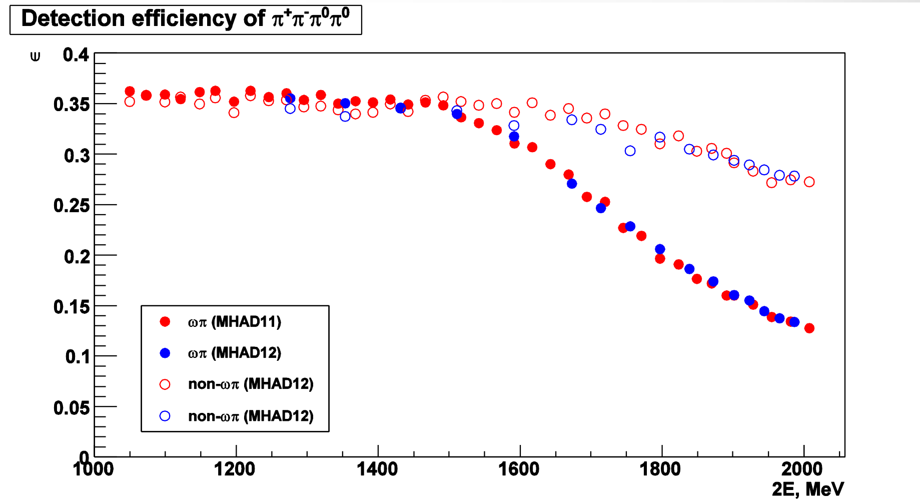
# Selection criteria and detection efficiency




- ≥ 2 charged particles
- ≥ 4 photons
- $|d_0| < 1$ cm
- $|z_0| < 15$ cm

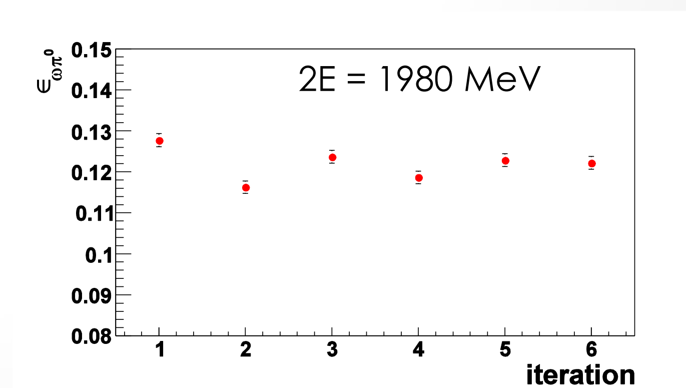
Kinematic reconstruction in $4\pi$ hypothesis:

- $70 < M_{\pi 0} < 200$ MeV
- $\chi^2_{4\pi} < 40$




Detection efficiency for $\omega\pi^0$ and other mechanisms of $4\pi$

Detection efficiency accounts for radiative corrections and is calculated using iterative method. Efficiency depending on the number of iterations:

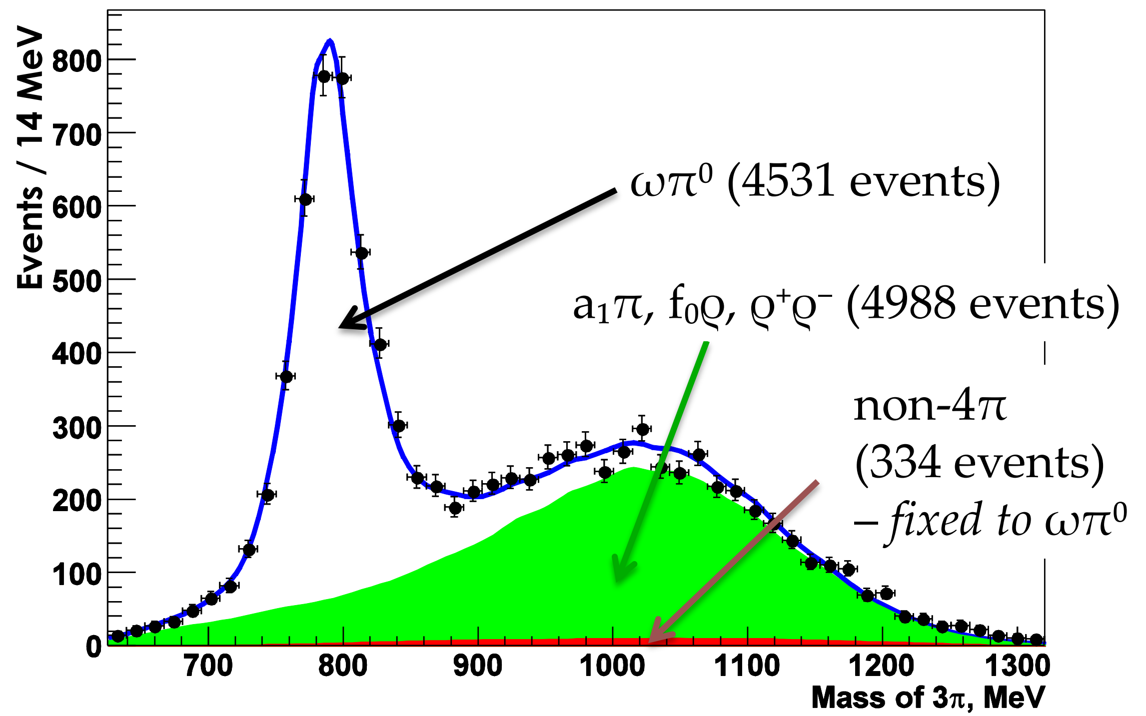


# Determination of $\omega\pi^0$ contribution

**RooFit**:
- Ubinned fit
- PDF by Kernel Estimation

Contribution of mechanism interference to the cross section:
0.5 – 7%
(upper estimate using MC)

Point 2E = 1520 MeV, Selection: $\chi^2_{4\pi} < 40$



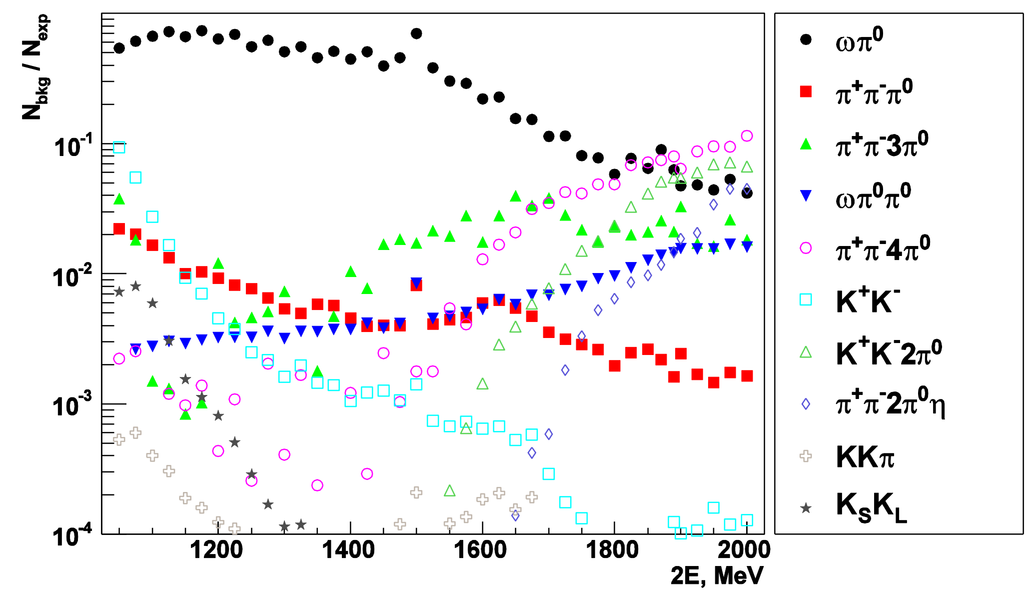

To obtain $\omega\pi^0$ contribution, the distribution of the $\pi^+\pi^-\pi^0$ invariant mass is fitted by signal – background model.
Signal and background distribution shapes are obtained from MC.

# Background processes

In each energy point for each background process:

$$N_{bkg}(E_i) = \sigma_{bkg} \cdot IL \cdot \varepsilon_{bkg}$$

Contributions of different background processes to events selected by $\chi^2_{4\pi} < 40$

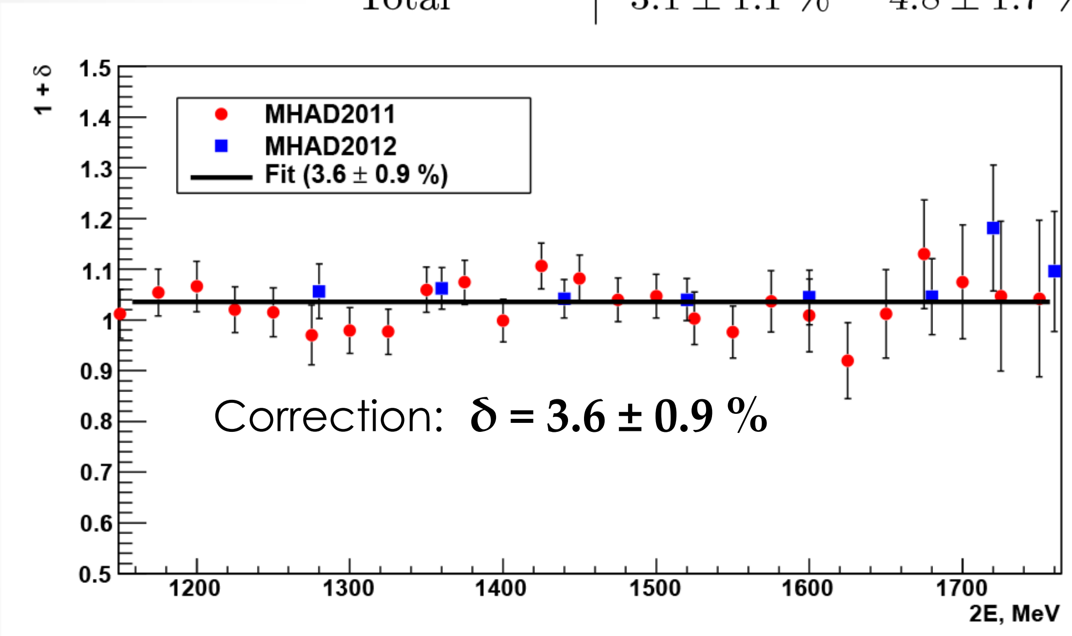



Contribution to systematic uncertainty

| 2E, МэВ | 1050 – 1600 | 1600 – 2000 |
|---|---|---|
| $\pi^+\pi^-3\pi^0$ | 0.0 – 1.8 % | 0.5 – 4.3 % |
| $\pi^+\pi^-4\pi^0$ | 0.0 – 0.4 % | 0.5 – 7.0 % |
| $\omega\pi^0\pi^0$ | 0.0 – 0.1 % | 0.1 – 1.5 % |
| $K^+K^-2\pi^0$ | 0.0 – 0.1 % | 0.1 – 1.4 % |
| other | 0.0 – 0.8 % | 0.2 – 1.9 % |

# Detection efficiency corrections

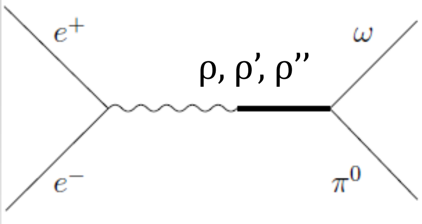
| | MHAD2011 | MHAD2012 | Mean |
|---|---|---|---|
| Lost track | $0.8 \pm 0.4$ % | $0.8 \pm 0.4$ % | $0.8 \pm 0.4$ % |
| Lost photon | $1.1 \pm 0.5$ % | $2.2 \pm 0.8$ % | $1.4 \pm 0.4$ % |
| $\chi^2$ | $1.1 \pm 0.9$ % | $1.8 \pm 1.4$ % | $1.3 \pm 0.7$ % |
| Total | $3.1 \pm 1.1$ % | $4.8 \pm 1.7$ % | $3.6 \pm 0.9$ % |




$$1 + \delta = \frac{\left(1 + \frac{N_{4\pi,\,lost}}{N_{4\pi}}\right)_{DATA}}{\left(1 + \frac{N_{4\pi,\,lost}}{N_{4\pi}}\right)_{MC}}$$

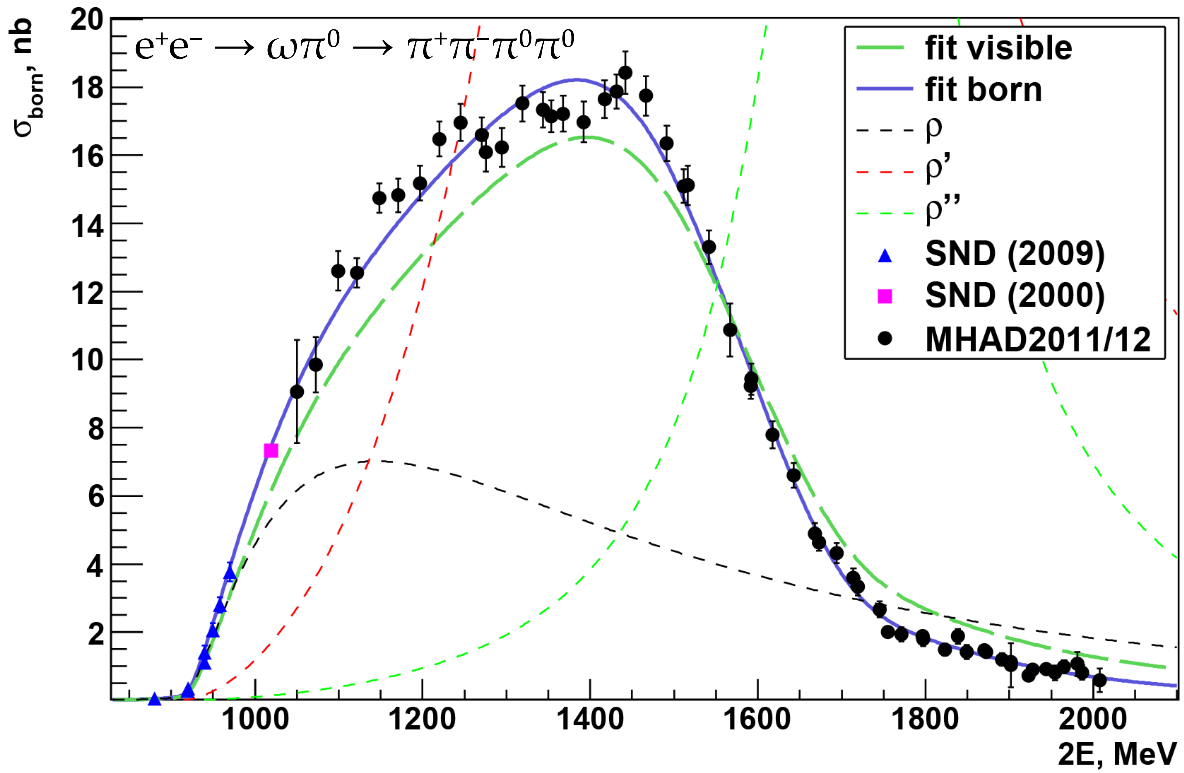
- We has used kinematic reconstruction recovering a lost track or a lost photon.
- Selection $\chi^2_{4\pi} < 40$ is extened to $\chi^2_{4\pi} < 100$.

# $e^+e^- \to \omega\pi^0$ cross section




| Model | VDM(3,PDG*) | VDM(3,PDG) |
|---|---|---|
| $A_\rho$ | $3.42 \pm 0.04$ | $3.10^{+0.08}_{-0.12}$ |
| $A_{\rho'}$ | $1.24 \pm 0.01$ | $1.28 \pm 0.87$ |
| $M_{\rho'}$, GeV | 1.465 * | $1.526 \pm 0.013$ * |
| $\Gamma_{\rho'}$, GeV | 0.400 * | $0.516^{+0.031}_{-0.020}$ * |
| $\phi_{\rho'}$, $^o$ | $235.2^{+1.1}_{-1.2}$ | $205.5^{+6.2}_{-36.4}$ |
| $A_{\rho''}$ | $0.36 \pm 0.00$ | $0.40 \pm 0.60$ |
| $M_{\rho''}$, GeV | 1.720 * | $1.722^{+0.016}_{-0.015}$ * |
| $\Gamma_{\rho''}$, GeV | 0.250 * | $0.268^{+0.049}_{-0.050}$ * |
| $\phi_{\rho''}$, $^o$ | $105.2^{+1.8}_{-1.9}$ | $44.0^{+17.8}_{-52.7}$ |
| $\chi^2_{\text{total}}$ | 134.3 / 61 | 94.9 / 61 |
| $P_{\text{total}}$ | $1.96 \cdot 10^{-7}$ | $3.56 \cdot 10^{-3}$ |
| $\chi^2_{\text{MHAD11/12}}$ | 113.8 / 56 | 74.6 / 56 |
| $\Gamma(\omega \to \pi^0\gamma)$ | 1.05 MeV | 0.72 MeV |
| $\chi^2_{\text{param}}$ | – | 9.8 / 4 |

* - fixed parameters




ISR correction:

$$\sigma_{vis}(E) = \int_0^{\varepsilon} F(x,E)\ \sigma_{born}\left(E\sqrt{1-x}\right) dx,$$

$$\sigma_{Born}(E) = \frac{4\pi\alpha^2}{E^3} \underbrace{|F(E)|^2}_{\text{Form factor}} \overbrace{P_f(E)}^{\text{phase space}}$$

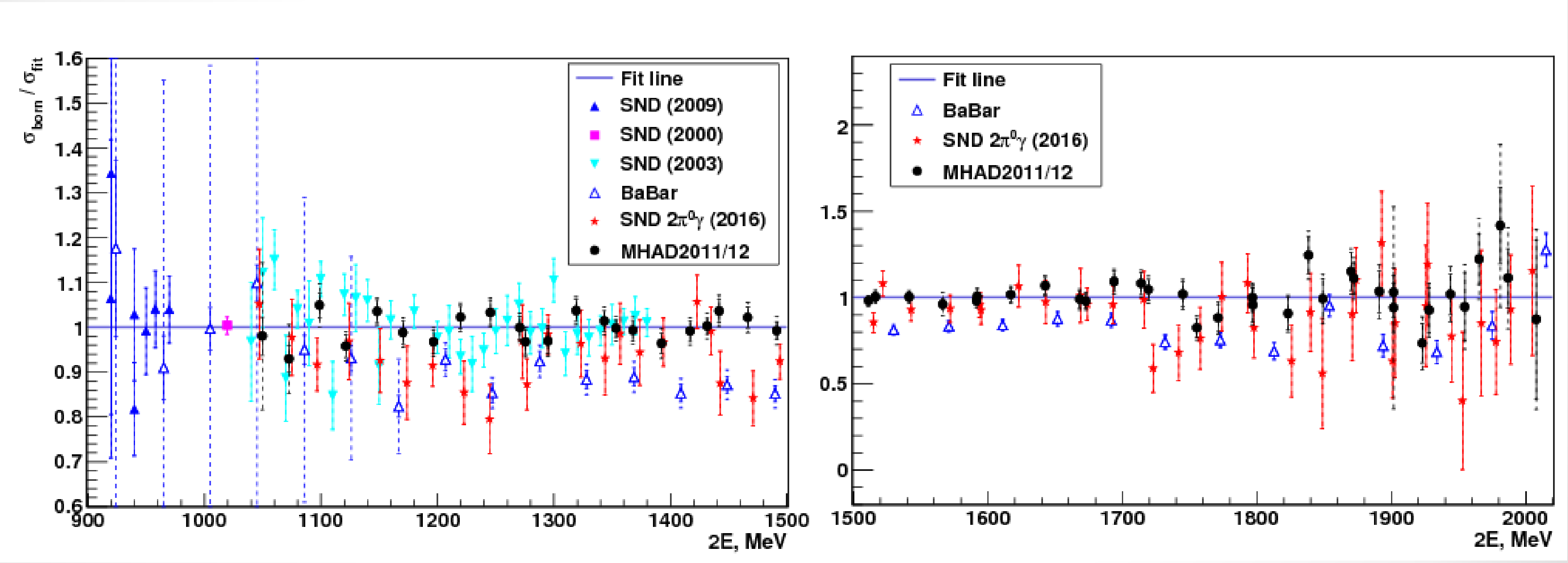
- Vector Meson Dominance (VMD) model: ρ, ρ'(1450), ρ''(1700)

$$F(E) = \sqrt{A_0}\hat{D}_\rho(E, m_\rho, \Gamma_\rho) + \sqrt{A_1}\hat{D}_\rho(E, m_{\rho1}, \Gamma_{\rho1})e^{i\phi1} + \sqrt{A_2}\hat{D}_\rho(E, m_{\rho2}, \Gamma_{\rho2})e^{i\phi2}$$

# Comparison of $e^+e^- \to \omega\pi^0$ cross sections

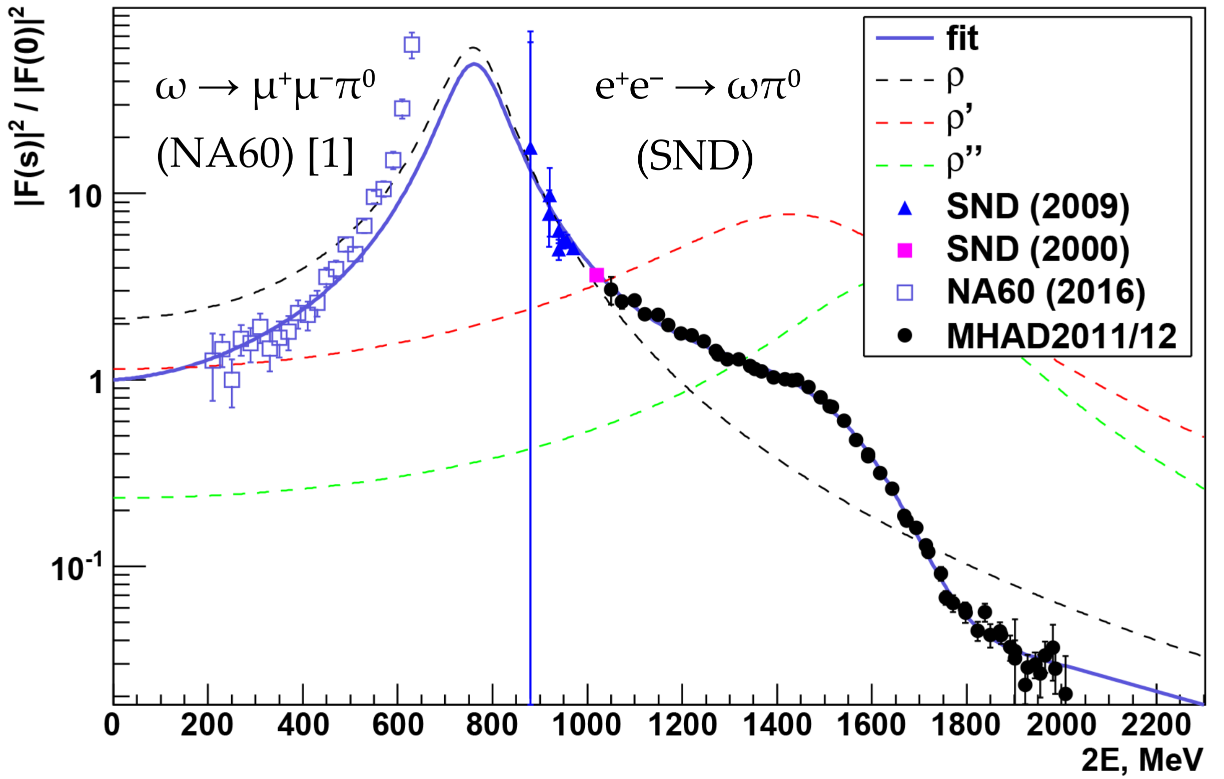
The measured $e^+e^- \to \omega\pi^0$ cross section has following uncertainties:

| 2E, MeV | 1050 – 1600 | 1600 – 2000 |
|---|---|---|
| systematical | 2.6 – 4.0 % | 4.4 – 14.0 % |
| statistical | 2.6 – 3.8 % | 4.0 – 21.7 % |



# Joint description of the form factor in $e^+e^- \rightarrow \omega\pi^0$ and $\omega \rightarrow \mu^+\mu^-\pi^0$ processes

| Model | VDM(3,PDG) | VDM(Sneid,PDG)[2] |
|---|---|---|
| $A_\rho$ | $3.42^{+0.09}_{-0.08}$ | $2.41 \pm 0.02$ |
| $A_{\rho'}$ | $2.50^{+0.42}_{-0.44}$ | $-0.24 \pm 0.01$ |
| $M_{\rho'}$, GeV | $1.486^{+0.026}_{-0.028}$ * | $1.330^{+0.027}_{-0.026}$ * |
| $\Gamma_{\rho'}$, GeV | $0.583^{+0.041}_{-0.044}$ * | $0.826^{+0.042}_{-0.041}$ * |
| $\phi_{\rho'}$, $^o$ | $231.6^{+4.0}_{-4.5}$ | $-132.9 \pm 7.5$ |
| $A_{\rho''}$ | $1.13^{+0.29}_{-0.26}$ | $-0.15 \pm 0.01$ |
| $M_{\rho''}$, GeV | $1.680^{+0.023}_{-0.021}$ * | $1.721^{+0.018}_{-0.017}$ * |
| $\Gamma_{\rho''}$, GeV | $0.431^{+0.052}_{-0.056}$ * | $0.541^{+0.044}_{-0.039}$ * |
| $\phi_{\rho''}$, $^o$ | $74.1^{+6.5}_{-6.9}$ | $420.4^{+7.8}_{-7.4}$ |
| $\chi^2_{\text{total}}$ | 270.8 / 84 | 379.0 / 84 |
| $P_{\text{total}}$ | $1.69 \cdot 10^{-21}$ | $4.53 \cdot 10^{-39}$ |
| $\chi^2_{\text{MHAD11/12}}$ | 77.7 / 56 | 116.8 / 56 |
| $\chi^2_{\text{NA60}}$ | 153.1 / 22 | 163.3 / 22 |
| $\chi^2_{\text{param}}$ | 17.3 / 4 | 88.4 / 4 |





[1] R. Arnaldi et al. (NA60), Phys. Lett. B757 437–444 (2016),

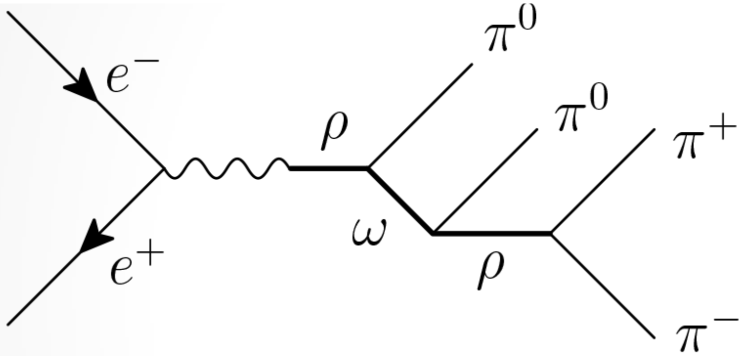
[2] S. P. Schneider, B. Kubis, and F. Niecknig, Phys. Rev. D 86, 054013 (2012).

# Conclusion

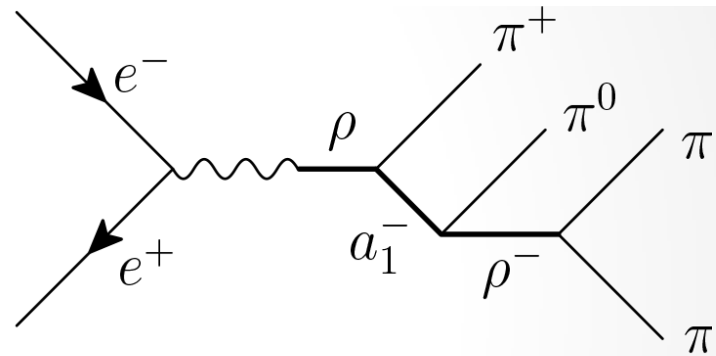
- The $e^+e^- \to \omega\pi^0 \to \pi^+\pi^-\pi^0\pi^0$ cross section is measured with high precision and is in agreement with previous measurement but has better accuracy.
- $\gamma^*\omega\pi^0$ form factor is obtained and fitted by VMD model taking into account ρ(770), ρ(1450) and ρ(1700) in several variants of VMD model.
- It is found that VDM is not capable to jointly describe $e^+e^- \to \omega\pi^0$ and $\omega \to \mu^+\mu^-\pi^0$ processes, even if ρ' and ρ'' are taken into account
- Parameters of ρ' and ρ'' mesons differ in different channels.

# Intermediate states of $e^+ e^- \to \pi^+ \pi^- \pi^0 \pi^0$ for $1 < \sqrt{s} < 2$ GeV

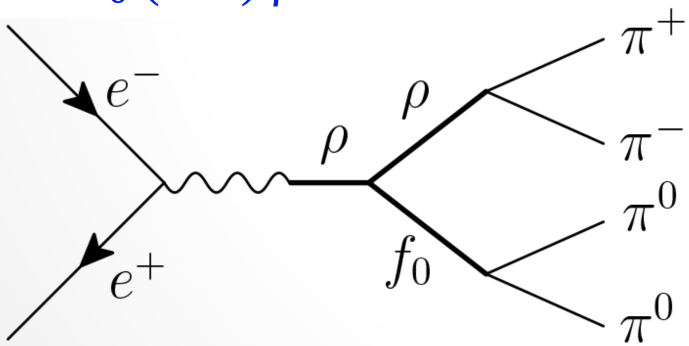
$e^+ e^- \to \omega\,(782)\,\pi^0 \to \pi^+ \pi^- \pi^0 \pi^0$



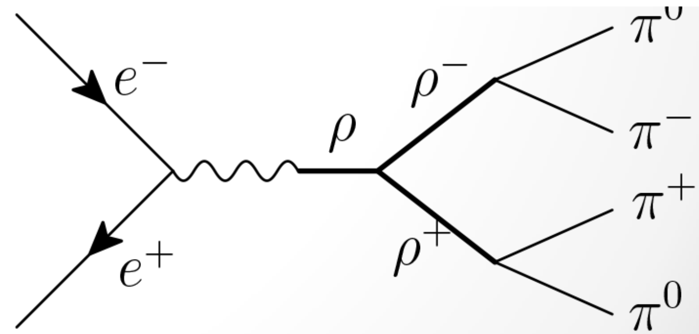

$e^+ e^- \to a_1\,(1260)\,\pi \to \pi^+ \pi^- \pi^0 \pi^0$




$e^+ e^- \to f_0\,(980)\,\rho \to \pi^+ \pi^- \pi^0 \pi^0$




$e^+ e^- \to \rho^+ \rho^- \to \pi^+ \pi^- \pi^0 \pi^0$



# Systematic uncertainty

| $2E$, MeV | 1000 – 1200 | 1200 – 1400 | 1400 – 1600 | 1600 – 1800 | 1800 – 2000 |
|---|---|---|---|---|---|
| Statistical (MHAD11) | 2.6 – 4.5 % | 2.1 – 2.7 % | 2.1 – 3.8 % | 4.0 – 10.8 % | 9.8 – 54.9 % |
| Statistical (MHAD12) | | 2.0 – 3.0 % | 1.8 – 2.9 % | 4.0 – 7.0 % | 8.6 – 16.1 % |
| Luminosity | 2.0 – 2.0 % | 2.0 – 2.0 % | 2.0 – 2.0 % | 2.0 – 2.0 % | 2.0 – 2.0 % |
| Eff. correction | 1.7 – 1.7 % | 1.7 – 1.7 % | 1.7 – 1.7 % | 1.7 – 1.7 % | 1.7 – 1.7 % |
| Background | 0.1 – 0.9 % | 0.1 – 0.4 % | 0.3 – 1.8 % | 2.1 – 4.9 % | 2.8 – 8.1 % |
| Interference | 0.0 – 0.3 % | 0.4 – 1.1 % | 1.3 – 2.5 % | 2.7 – 4.5 % | 4.8 – 7.0 % |
| Rad. correction | 0.1 – 0.1 % | 0.1 – 0.2 % | 0.2 – 0.5 % | 0.5 – 1.9 % | 1.9 – 8.7 % |
| Systematical | 2.6 – 2.8 % | 2.6 – 2.9 % | 2.9 – 4.0 % | 4.4 – 7.4 % | 6.6 – 14.0 % |

# VMD model

$$F_{\gamma\omega\pi}(s) = \sum_{i=0} \frac{g_{\rho(i)\omega\pi} M^2_{\rho(i)} e^{i\varphi_{\rho(i)}}}{M^2_{\rho(i)} - s - i\sqrt{s}\Gamma_{\rho(i)}(s)},$$

$$\Gamma_{\rho(0)}(s) = \Gamma_\rho \frac{M^2_\rho}{s} \left(\frac{s - 4m^2_\pi}{M^2_\rho - 4m^2_\pi}\right)^{3/2}$$

Kozhevnikov [1]:

$$C_{\rho\omega\pi}(E) = \frac{1 + (Rm_\rho)^2}{1 + (RE)^2},$$

$$\Gamma_\rho(E) \to C_{\rho\omega\pi}(E)\Gamma_\rho(E)$$


[1] Динамические эффекты в редких и многочастичных распадах векторных мезонов. / А. А. Кожевников. // Диссертация на соискание ученой степени доктора физ.-мат. наук. Новосибирск. 2004.


Schneider [2]:

$$F(s) = \frac{M^2_\rho + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M^2_\rho - s - iM_\rho\Gamma_\rho(s)} \exp\left(-\frac{sA_\pi(s)}{96\pi^2 F^2_\pi}\right) -$$

$$-\frac{\gamma s e^{i\phi_1}}{M^2_{\rho'} - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left(-\frac{s\Gamma_{\rho'} A_\pi(s)}{\pi M^3_{\rho'}\sigma^3_\pi(M^2_{\rho'})}\right) -$$

$$-\frac{\delta s e^{i\phi_2}}{M^2_{\rho''} - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left(-\frac{s\Gamma_{\rho''} A_\pi(s)}{\pi M^3_{\rho''}\sigma^3_\pi(M^2_{\rho''})}\right),$$

$$A_\pi(s) = \ln\frac{M^2_\pi}{M^2_\rho} + \frac{8M^2_\pi}{s} - \frac{5}{3} + \sigma^3_\pi(s)\ln\frac{1 + \sigma_\pi(s)}{1 - \sigma_\pi(s)},$$

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F^2_\pi}\sigma^3_\pi(s),$$

$$\sigma_\pi(s) = \sqrt{1 - \frac{4M^2_\pi}{s}},$$

$$\Gamma_{\rho'}(s) = \frac{M_{\rho'}}{\sqrt{s}} \left(\frac{s - 4M^2_\pi}{M^2_{\rho'} - 4M^2_\pi}\right)^{3/2} \Gamma_{\rho'}.$$


[2] S. P. Schneider, B. Kubis, and F. Niecknig, Phys. Rev. D 86, 054013 (2012).

# Radiative correction (ISR)

$$\sigma_{\text{vis}}(E) = \int\limits_0^{\epsilon} \sigma_{\text{born}}(E\sqrt{1-x})F(x,E)\,\mathrm{d}x,$$

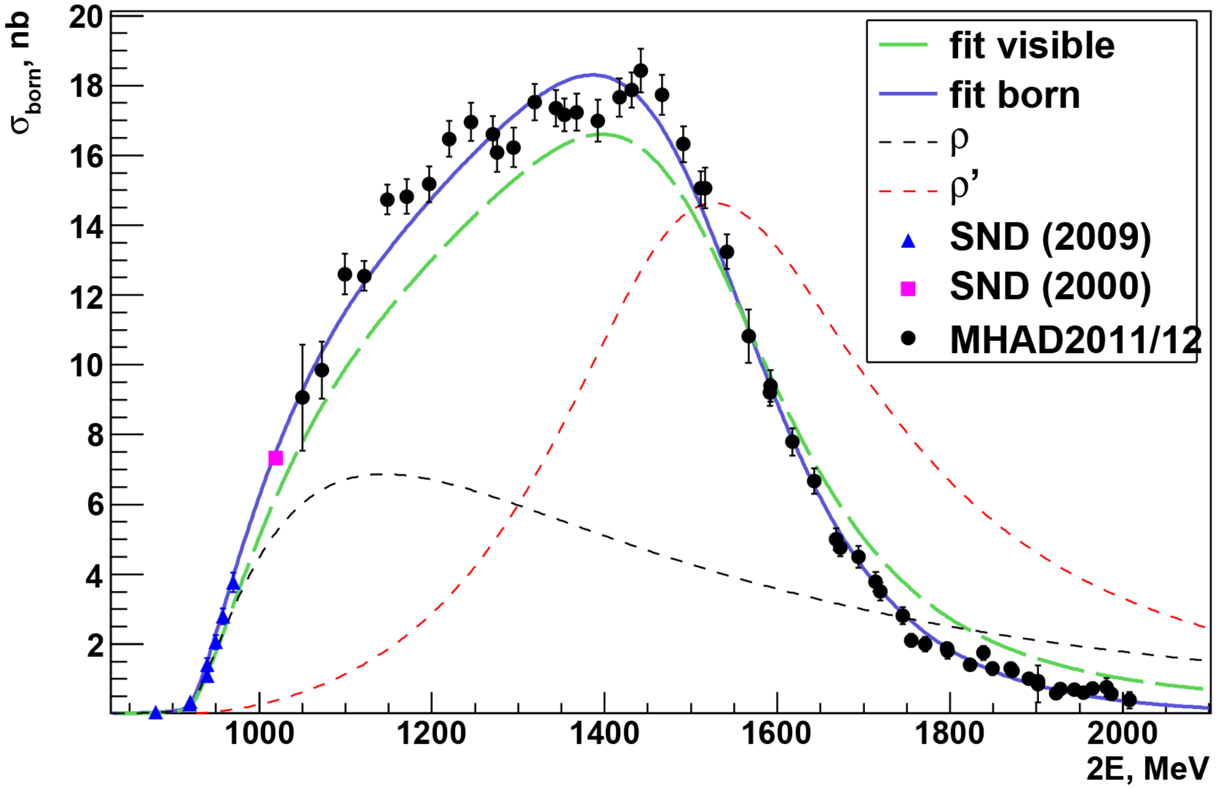
$$F(x,E) = \beta x^{\beta-1}\left[1+\frac{\alpha}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)+\frac{3\beta}{4}-\frac{\beta^2(E)}{24}\left(\frac{L}{3}+2\pi^2-\frac{37}{4}\right)\right]-$$
$$-\beta\left(1-\frac{x}{2}\right)+\frac{\beta^2}{8}\left[4(2-x)\ln\frac{1}{x}+\frac{1}{x}(1+3(1-x)^2)\ln\frac{1}{1-x}-6+x\right]+$$
$$+\left(\frac{\alpha}{\pi}\right)^2\left\{\frac{1}{6x}\left(x-\frac{2m}{E}\right)^{\beta}\left(2\ln\frac{Ex}{m}-\frac{5}{3}\right)^2\left[2-2x+x^2+\frac{\beta}{3}\left(2\ln\frac{Ex}{m}-\frac{5}{3}\right)\right]+\right.$$
$$\left.+\frac{L^2}{2}\left[\frac{2}{3}\frac{1-(1-x)^3}{1-x}-(2-x)\ln\frac{1}{1-x}+\frac{x}{2}\right]\right\}\Theta\left(x-\frac{2m}{E}\right),$$

$$L = 2\ln\frac{2E}{m_e}, \qquad \beta = \frac{2\alpha}{\pi}(L-1)$$

[*] Э. А. Кураев, В. С. Фадин. Вычисление радиационных поправок к сечению однофотонной аннигиляции с помощью структурных функций. Препринт ИЯФ 84-44.

# Approximation without ρ”(1700)

| Model | VDM(2) | VDM(2b) |
|---|---|---|
| $A_\rho$ | $3.17^{+0.05}_{-0.06}$ | $3.06 \pm 0.03$ |
| $A_{\rho'}$ | $0.68^{+0.01}_{-0.02}$ | $0.60 \pm 0.01$ |
| $M_{\rho'}$, GeV | $1.515 \pm 0.005$ | $1.517 \pm 0.005$ |
| $\Gamma_{\rho'}$, GeV | $0.444 \pm 0.017$ | $0.470 \pm 0.008$ |
| $\phi_{\rho'}$, $^o$ | $197.2^{+4.1}_{-6.1}$ | 180.0 * |
| $A_{\rho''}$ | | |
| $M_{\rho''}$, GeV | | |
| $\Gamma_{\rho''}$, GeV | | |
| $\phi_{\rho''}$, $^o$ | | |
| $\chi^2_{\text{total}}$ | 104.8 / 61 | 107.7 / 62 |
| $P_{\text{total}}$ | $4.17 \cdot 10^{-4}$ | $2.90 \cdot 10^{-4}$ |
| $\chi^2_{\text{MHAD11/12}}$ | 87.9 / 56 | 95.0 / 56 |
| $\Gamma(\omega \to \pi^0\gamma)$ | 0.85 MeV | 0.81 MeV |
| $\chi^2_{\text{param}}$ | | |




$$F(E) = \sqrt{A_0}\hat{D}_\rho(E, m_\rho, \Gamma_\rho) + \sqrt{A_1}\hat{D}_\rho(E, m_{\rho1}, \Gamma_{\rho1})e^{i\phi1}$$