Gauge dependence of effective average action

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PML, I.L. Shapiro, JHEP 1306 (2013) 086;

arXiv:1212.2577 [hep-th].

PML, Phys. Lett. B791 (2019) 293;

arXiv:1805.02149 [hep-th]

PML, Phys. Lett. B803 (2020) 135314;

arXiv:1911.00194 [hep-th]

Contents

It is well-known fact that Green functions in gauge theories (and therefore the effective action being the generating functional of one-particle irreduciuble Green function or vertices) depend on gauges. On the other hand elements of S-matrix should be gauge independent. It means that gauge dependence of effective action should be a very special form.

The gauge dependence is a problem in quantum description of gauge theories beginning with famous papers by Jackiw

(R. Jackiw, Functional evaluation of the effective potential, Phys. Rev. D9 (1974) 1686)

and Nielsen

(N.K. Nielsen, On the gauge dependence of spontaneous symmetry breaking in gauge theories, Nucl. Phys. B101 (1975) 173)

where the gauge dependence of effective potential in Yang-Mills theories has been found.

For Yang-Mills theories in the framework of the Faddeev-Popov quantization method

(L.D. Faddeev, V.N. Popov, Feynman rules for the Yang-Mills field, Phys. Lett. B25 (1967) 29)

the gauge dependence problem have been found in our papers

(PML, I.V. Tyutin, On the structure of renormalization in gauge theories, Sov. J. Nucl. Phys. 34 (1981) 156; On the generating functional for the vertex functions in Yang-Mills theories, Sov. J. Nucl. Phys. 34 (1981) 474)

and for general gauge theories within the Batalin-Vilkovisky formalism

(I.A. Batalin, G.A. Vilkovisky, Gauge algebra and quantization, Phys. Lett. B102 (1981) 27)

in our paper

(B.L. Voronov, PML, I.V. Tyutin, Canonical transformations and gauge dependence in general gauge theories, Sov. J. Nucl. Phys. 36 (1982) 292)

respectively.

Over the past three decades, there has been an increased interest in the nonperturbative approach in Quantum Field Theory known as the functional renormalization group (FRG) proposed by Wetterich

(C. Wetterich, Average action and the renormalization group equation, Nucl.Phys. B352 (1991) 529).

The FRG approach has got further developments and numerous applications. There are many reviews devoted to detailed discussions of different aspects of the FRG approach and among them one can find

(J.M. Pawlowski, Aspects of the functional renormalization group, Ann.Phys. 322 (2007) 2831;
O.J. Rosten, Fundamentals of the exact renormalization group, Phys.Repots. 511 (2012) 177;
H.Gies, Introduction to the functional RG and applications to gauge theories, Notes Phys. 852 (2012) 287)

with qualitative references.

As a quantization procedure the FRG belongs to covariant quantization schemes which meets in the case of gauge theories with two principal problems: the unitarity of S-matrix and the gauge dependence of results obtained. Solution to the unitarity problem requires consideration of canonical formulation of a given theory on quantum level and use of the Kugo-Ojima method

(T. Kugo, I. Ojima, Local covariant operator formalism of non-abelian gauge theories and quark confinement problem, Prog.Theor.Phys.Suppl. 66 (1979) 1)

in construction of physical state space. Within the FRG the unitarity problem is not considered at all because main efforts are connected with finding solutions to the flow equation for the effective average action.

In turn the gauge dependence problem exists for the FRG approach as unsolved ones if one does not take into account the reformulation based on composite operators

(PML, I.L. Shapiro, On the functional renormalization group approach for Yang-Mills fields, JHEP 1306 (2013) 086)

where the problem was discussed from point of view the basic principles of the quantum field theory.

Later on the gauge dependence problem in the FRG was discussed in our papers several times for Yang-Mills and quantum gravity theories but the reaction from the FRG community was very weak and came down only to mention without any serious study

Situation with the gauge dependence in the FRG is very serious because without solving the problem a physical interpretation of results obtained is impossibly. It is main reason to return for discussions of the gauge dependence problem of effective average action in the FRG approach.

We start with the action $S_0[A]$ of fields A for given Yang-Mills theory. Generating functional of Green functions, $Z[J]$, can be constructed by the Faddeev-Popov rules in the form of functional integral

$$Z[J] = \int D\phi \ \exp\Big\{\frac{i}{\hbar}\big(S_{PF}[\phi] + J_i\phi^i\big)\Big\}.$$

where $\phi = \{\phi^i\} = (A, B, C, \bar{C})$ is full set of fields including the ghost C and antighost $\bar{C}$ Faddeev-Popov fields and auxiliary fields B (Nakanishi-Lautrop fields), $J = \{J_i\}$ are external sources to fields ϕ, $S_{PF}[\phi]$ is the Faddeev-Popov action

$$S_{PF}[\phi] = S_0[A] + \Psi[\phi]_{,i}R^i(\phi).$$

Here $\Psi[\phi]$ is gauge fixing functional (in the simplest case having the form $C\partial A$), , and notation $X_{,i} = \delta X/\delta\phi^i$ is used.

The Faddeev-Popov action $S_{FP}[\phi]$ obyes very important property of invariance under global supersymmetry - BRST (Becchi - Rouet - Stora- Tyutin) symmetry,

$$\delta_B S_{FP}[\phi] = 0, \quad \delta_B \phi^i = R^i(\phi)\mu, \quad \mu^2 = 0,$$

where $R^i(\phi)$ are generators of BRST transformations.

From definition it follows that the functional $Z[J]$ depends oh gauges To study the character of this dependence, let us consider an infinitesimal variation of gauge fixing functional $\Psi[\phi] \ \rightarrow \ \Psi[\phi] + \delta\Psi[\phi]$ in the functional integral for $Z[J]$. Then we obtain ($\partial_J = \delta/\delta J$)

$$\begin{aligned} \delta Z[J] &= \frac{i}{\hbar}\int D\phi \; \delta\Psi_{,i}[\phi]R^i(\phi) \; \exp\Big\{\frac{i}{\hbar}\big(S_{PF}[\phi] + J_i\phi^i\big)\Big\} = \\ &= \frac{i}{\hbar}\delta\Psi_{,i}[-i\hbar\partial_J]R^i(-i\hbar\partial_J)Z[J]. \end{aligned}$$

Gauge dependence in Yang-Mills theories

There exists an equivalent presentation of the variation for $Z[J]$ under variations of gauge conditions. Indeed, making use the change of integration variables in the functional integral for Z[J] with the choice $\Psi[\phi] + \delta\Psi[\phi]$ in the form of the BRST transformations,

$$\delta\phi^i = R^i(\phi)\mu[\phi],$$

taking into account that the corresponding Jacobian, J, is equal to

$$J = \exp\{-\mu[\phi]_{,i}R^i(\phi)\},$$

choosing the functional $\mu[\phi]$ in the form $\mu[\phi] = (i/\hbar)\delta\Psi[\phi]$, then we have

$$\begin{aligned} \delta Z[J] &= \frac{i}{\hbar}\int D\phi\; J_i R^i(\phi)\delta\Psi[\phi]\exp\Big\{\frac{i}{\hbar}\big(S_{PF}[\phi] + J_i\phi^i\big)\Big\} = \\ &= \frac{i}{\hbar}J_i R^i(-i\hbar\partial_J)\;\delta\Psi[-i\hbar\partial_J]\;Z[J]. \end{aligned}$$

Both relations are equivalent due to the evident equality

$$\int D\phi\, \partial_{\phi^j}\Big(\Psi[\phi]R^j(\phi)\exp\Big\{\frac{i}{\hbar}\big(S_{PF}[\phi]+J_i\phi^i\big)\Big\}\Big)=0,$$

where the following equations

$$S_{PF,i}[\phi]R^i(\phi)=0, \quad R^i{}_{,i}(\phi)=0, \quad R^i{}_{,j}(\phi)R^j(\phi)=0,$$

should be used.

In terms of the functional $W[J]=-i\hbar\ln Z[J]$ the above relations rewrite as

$$\delta W[J]=J_iR^i(\partial_J W-i\hbar\partial_J)\,\delta\Psi[\partial_J W-i\hbar\partial_J]\cdot 1,$$
$$\delta W[J]=\delta\Psi_{,i}[\partial_J W-i\hbar\partial_J]R^i(\partial_J W-i\hbar\partial_J)\cdot 1.$$

Introducing the effective action, $\Gamma = \Gamma[\Phi]$, through the Legendre transformation of $W[J]$,

$$\Gamma[\Phi] = W[J] - J_i\Phi^i, \quad \Phi^i = \frac{\delta W}{\delta J_i}, \quad \frac{\delta\Gamma}{\delta\Phi^i} = -J_i,$$

the gauge dependence of effective action is described by the equivalent relations

$$\delta\Gamma[\Phi] = -\frac{\delta\Gamma}{\delta\Phi^i}\, R^i(\hat{\Phi})\, \delta\Psi[\hat{\Phi}] \cdot 1,$$
$$\delta\Gamma[\Phi] = \delta\Psi_{,i}[\hat{\Phi}]\, R^i(\hat{\Phi}) \cdot 1,$$

where the notations

$$\hat{\Phi}^i = \Phi^i + i\hbar(\Gamma''^{-1})^{ij}\,\frac{\delta}{\delta\Phi^j}, \quad (\Gamma'')_{ij} = \frac{\delta^2\Gamma}{\delta\Phi_i\delta\Phi_j}, \quad \left(\Gamma''^{-1}\right)^{ik} \cdot \left(\Gamma''\right)_{kj} = \delta^i_j,$$

are used.

From the above presentation it follows the important statement that the effective action does not depend on the gauge conditions at the their extremals,

$$\delta\Gamma\Big|_{\partial_\Phi\Gamma=0} = 0,$$

making possible the physical interpretation of results obtained in the Faddeev-Popov -method for Yang-Mills theories.

There exists another description of gauge dependence of effective action for Yang-Mills theories: The effective action can be presented in the form of gauge independent functional in which all gauge dependence contains in their arguments.

Gauge dependence of effective average action

Main idea of the functional renormalization group is to modify the behavior of propagators in IR region with the help of a regulator action $S_k[\phi]$ being quadratic in fields. In the case of Yang-Mills theories it leads to action

$$S_{Wk}[\phi] = S_{FP}[\phi] + S_k[\phi], \quad S_k[\phi] = \frac{1}{2} R_{k|ij} \phi^j \phi^i.$$

Standard choice of regulators $R_{k|ij}$ is

$$R_{k|ij} = z_{ij} \frac{\Box \exp\{-\Box/k^2\}}{1 - \exp\{-\Box/k^2\}}, \quad \Box = \partial_\mu \partial^\mu,$$

with properties

$$\lim_{k \to 0} R_{k|ij} = 0.$$

The action $S_{Wk}[\phi]$ is not invariant under the BRST transformations

$$\delta_B S_{Wk}[\phi] = \delta_B S_k[\phi] = R_{k|ij} \phi^j R^i(\phi) \mu \neq 0.$$

The generating functional of Green functions has the form

$$Z_k[J] = \int D\phi \, \exp\left\{\frac{i}{\hbar}\left[S_{Wk}[\phi] + J_A\phi^A\right]\right\} = \exp\left\{\frac{i}{\hbar}W_k[J]\right\},$$

Variation of $\delta Z_k[J]$ under change of gauge condition can be presented as

$$\delta Z_k[J] = \frac{i}{\hbar}\big(J_i - i\hbar R_{k|ij}\partial_{J_j}\big)R^i(-i\hbar\partial_J)\;\delta\Psi[-i\hbar\partial_J]\;Z[J].$$

In terms of $W_k[J]$ we have

$$\delta W_k[J] = \big(J_i + R_{k|ij}(\partial_{J_j}W_k - i\hbar\partial_{J_j})\big)R^i(\partial_J W - i\hbar\partial_J)\;\delta\Psi[\partial_J W - i\hbar\partial_J]\cdot 1,$$

Gauge dependence of effective average action

Introducing the effective average action, $\Gamma_k = \Gamma_k[\Phi]$, being the main quantity in the FRG through the Legendre transformation of $W_k[J]$,

$$\Gamma_k[\Phi] = W_k[J] - J_i\Phi^i, \quad \Phi^i = \frac{\delta W_k}{\delta J_i}, \quad \frac{\delta \Gamma_k}{\delta \Phi^i} = -J_i,$$

the gauge dependence of effective average action is described as

$$\delta\Gamma_k[\Phi] = -\Big(\frac{\delta\Gamma_k}{\delta\Phi^i} - R_{k|ij}\hat{\Phi}^j\Big)\, R^i(\hat{\Phi})\, \delta\Psi[\hat{\Phi}]\cdot 1,$$

with the notations

$$\hat{\Phi}^i = \Phi^i + i\hbar(\Gamma_k^{''-1})^{ij}\,\frac{\delta}{\delta\Phi^j}, \quad (\Gamma_k^{''})_{ij} = \frac{\delta^2\Gamma_k}{\delta\Phi_i\delta\Phi_j}, \quad \big(\Gamma_k^{''-1}\big)^{il}\cdot\big(\Gamma_k^{''}\big)_{lj} = \delta^i_j.$$

The effective average action remains gauge dependent even on their extremals

$$\delta\Gamma_k[\Phi]\Big|_{\partial_\Phi\Gamma_k=0} \neq 0$$

making impossible physical interpretation of results obtained in the FRG.

Above analysis of gauge dependence of effective average action $\Gamma_k[\Phi]$ does not convince people from the FRG community because it is based on theorems in Quantum Field Theory formulated within standard perturbation approach while it is assumed that the flow equatuion for $\Gamma_k[\Phi]$ in the FRG is considered non-perturbatively.
We are going to study the gauge dependence of effective average action found as a solution to the flow equation. To do this we find first of all the partial derivative of $Z_k[J]$ with respect to IR cutoff parameter k. The result reads

$$\begin{aligned}\partial_k Z_k[J] &= \frac{i}{\hbar}\int D\phi\,\partial_k S_k[\phi]\exp\Big\{\frac{i}{\hbar}\big[S_{Wk}[\phi]+J_A\phi^A\big]\Big\}\\ &= \frac{i}{\hbar}\partial_k S_k[-i\hbar\partial_J]Z_k[J].\end{aligned}$$

In deriving this result, the existence of functional integral is only used.

In terms of generating functional of connected Green functions we have

$$\partial_k W_k[J] = \partial_k S_k[\partial_J W_k - i\hbar\partial_J] \cdot 1.$$

The basic equation (flow equation) of the FRG approach has the form

$$\partial_k \Gamma_k[\Phi] = \partial_k S_k[\hat{\Phi}] \cdot 1,$$

where $\hat{\Phi} = \{\hat{\Phi}^i\}$ is defined above. It is assumed that solutions to the flow equations present the effective average action $\Gamma_k[\Phi]$ beyond the usual perturbation calculations.

Gauge dependence of flow equation

Now, we analyze the gauge dependence problem of the flow equation . Note that up to now this problem has never been discussed in the literature. To do this we consider the variation of $\partial_k Z_k[J]$ under an infinitesimal change of gauge fixing functional, $\Psi[\phi] \rightarrow \Psi[\phi] + \delta\Psi[\phi]$. Taking into account that $\partial_k S_k[\phi]$ does not depend on gauge fixing procedure, we obtain

$$\delta\partial_k Z_k[J] = \left(\frac{i}{\hbar}\right)^2 \partial_k S_k[-i\hbar\partial_J]\delta\Psi_{,i}[-i\hbar\partial_J]R^i(-i\hbar\partial_J)Z_k[J].$$

In terms of the functional $W_k[J]$ we have

$$\begin{aligned} &\delta\partial_k W_k[J] = \\ &= \partial_k S_k[\partial_J W_k - i\hbar\partial_J]\delta\Psi_{,i}[\partial_J W_k - i\hbar\partial_J]R^i(\partial_J W_k - i\hbar\partial_J)\cdot 1. \end{aligned}$$

Finally, the gauge dependence of the flow equation is described by the equation

$$\delta\partial_k\Gamma_k[\Phi] = \partial_k S_k[\hat{\Phi}]\delta\Psi_{,i}[\hat{\Phi}]R^i(\hat{\Phi})\cdot 1.$$

Therefore, at any finite value of k the flow equation depends on gauges. The same conclusion is valid for the effective average action.
But what is about the case when $k \to 0$? Usual argument used by the FRG community to argue gauge independence is related to statement that due to the property

$$\lim_{k\to 0}\Gamma_k = \Gamma,$$

where Γ is the standard effective action constructed by the Faddeev-Popov rules, the gauge dependence of average effective action disappears at the fixed point. In our opinion this property is not sufficient to claim the gauge independence at the fixed point. The reason to think so is the flow equation which includes the differential operation with respect to the IR parameter k.

Indeed, let us present the effective average action in the form

$$\Gamma_k = \Gamma + kH_k,$$

where functional H_k obeys the property

$$\lim_{k\to 0} H_k = H_0 \neq 0.$$

Then we have the relations

$$\partial_k \lim_{k\to 0} \Gamma_k = 0, \quad \lim_{k\to 0} \partial_k \Gamma_k = H_0.$$

These two operation do not commute and the statement of gauge independence at the fixed point seems groundless within the FRG approach. Due to this reason it seems as very actual problem for the FRG community to fulfil calculations of the effective average action at the fixed point using, for example, a family of gauges with one parameter and choice of two different values of the parameter.

Discussions

The gauge dependence problem in the framework of FP-method and of FRG approach have been analyzed. It is known that the FP- quantizations is characterized by the BRST symmetry which governs gauge independence of S-matrix elements. In turn the BRST symmetry is broken in the FRG approach with all negative consequences for physical interpretation of results. But usual reaction of the FRG community with this respect is that they are only interested in the effective average action evaluated at the fixed point where the gauge independence is expected. One of the goals of this investigation was to study the gauge dependence of the effective average action as a solution of the flow equation. For the first time the equation describing the gauge dependence of flow equation has been explicitly derived. It was found the gauge dependence of flow equation at any finite value of the IR parameter k.

Discussions

As to the limit $k \to 0$ there is a strong motivation about the gauge dependence of effective average action at the fixed point. In this regard, an important task for the FRG community is to calculate the effective average action in a family of one-parameter gauges which corresponds to two different value of gauge parameter.

Thank you for attention !