# ASPECTS OF LOW-ENERGY EFFECTIVE ACTION IN EXTENDED SUPERSYMMETRIC GAUGE THEORIES

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## Goals

- Construction of the background field methods and getting the manifestly supersymmetric and gauge invariant effective action for $6D, \mathcal{N} = (1,0)$ non-Abelian vector multiplet coupled to hypermultiplet
- Calculation of the one-loop off-shell divergences in vector multiplet and hypermultiplet sectors for arbitrary $\mathcal{N} = (1,0)$ gauge theory
- Analysis of divergences in the $\mathcal{N} = (1,1)$ SYM theory and prove that the one-loop off-shell divergence are completely absent

Based on:


I.L.B, E.A. Ivanov, B.M. Merzlikin, K.V. Stepanayantz, Phys.Lett., **B 763, 375, 2016;**

**I.L.B, E.A. Ivanov, B.M. Merzlikin, K.V. Stepanayantz, JHEP, 01 (2017) 128;**

**I.L.B, E.A. Ivanov, B.M. Merzlikin, K.V. Stepanayantz, Nucl. Phys., B 921, (2017) 127;**

**I.L.B, E.A. Ivanov, B.M. Merzlikin, K.V. Stepanayantz, Phys.Lett. B 778, 252, 2018;**

**I.L.B, E.A. Ivanov, B.M. Merzlikin, K.V. Stepanayantz, Symmetry, 11, 1, 2019.**

## Basic Motivations. Superstrings

Modern interest to study higher dimensional supersymmetric field theories is associated with superstring theory.

Specific feature of the superstring theory is existence of so called $D$ branes which are the $D+1$ dimensional surfaces in the ten-dimensional space-time. In the low-energy limit the $D$ brane is associated with $D+1$-dimensional supersymmetric gauge theory. Therefore, study of low-energy limit of superstring theory can be related to (extended) supersymmetric field theory in various dimensions.

## Basic Motivations. Study of quantum field models with large number of symmetries

- Explicit symmetries: gauge symmetry, global symmetries, supersymmetries.
- Quantization procedure with preservation of all explicit symmetries.
- Perturbation theory with preservation of all explicit symmetries.
- Hidden (on-shell) symmetries. Preservation of hidden symmetries.
- Divergences, renormalization and effective actions.
- Construction of the new extended supersymmetric invariants as the the quantum contributions to effective action

Some problems of higher dimensional supersymmetric gauge theories.

1. Problem of describing the quantum structure of six-dimensional supersymmetric gauge theories dimensionally reduced from superstrings (N. Seiberg, E. Witten, 1996; N. Seiberg, 1997).

2. Problem of field description of the interacting multiple $M5$-branes (see e.g. review J. Bagger, N. Lambert, S. Mikhu, C. Papageorgakis, Phys.Repts. 527 (2013) 1).

- Hypothetic $M$-theory is characterized by two extended objects: $M2$-brane and $M5$-brane in eleven dimensional space.
- The field description of interacting multiple $M2$-branes is given by Bagger-Lambert-Gustavsson (BGL) theory which is $3D$, $\mathcal{N}=8$ supersymmetric gauge theory.
- Lagrangian description of the interacting multiple $M5$-branes is not constructed so far.

3. Problem of miraculous cancelation of some on-shell divergences in higher dimensional maximally supersymmetric gauge theories (theories with 16 supercharges). All these theories are non-renormalizable by power counting.

- Field limit of superstring amplitude shows that $6D, \mathcal{N} = (1,1)$ SYM theory is on-shell finite at one-loop (M.B. Green, J.H. Schwarz, L. Brink, 1982).
- Analysis based on on-shell supersymmetries, gauge invariance and field redefinitions (P.S. Howe, K.S. Stelle, 1984, 2003; G. Bossard, P.S. Howe, K.S. Stelle, 2009).
- Direct one-loop and two-loop component calculations (mainly in bosonic sector and mainly on-shell) (E.S. Fradkin, A.A. Tseytlin, 1983; N. Marcus, A. Sagnotti, 1984, 1985.)
- Direct calculations of scattering amplitudes in $6D, \mathcal{N} = (1,1)$ theory up to five loops and in $D8, 10$ theories up to four loops (L.V. Bork, D.I. Kazakov, M.V. Kompaniets, D.M. Tolkachev, D.E. Vlasenko, 2015).

Results: On-shell divergences in maximally extended $6D$ SYM theory start at three loops. One-shell divergences in $8D$ and $10D$ SYM theories start at one loop.

## Some properties of $6D, \mathcal{N} = (1,1)$ SYM theory

Purpose: to show that the $\mathcal{N} = (1,1)$ SYM theory is off-shell finite at one-loop in spite of it is non-remormalizable on power counting

$6D, \mathcal{N} = (1,1)$ SYM theory possesses some properties close or analogous to $4D, \mathcal{N} = 4$ SYM theory.

- The $6D, \mathcal{N} = (1,1)$ SYM theory can be formulated in harmonic superspace as well as the $4D, \mathcal{N} = 4$ SYM theory
- The $6D, \mathcal{N} = (1,1)$ SYM theory possesses the manifest $\mathcal{N} = (1,0)$ supersymmetry and additional hidden $\mathcal{N} = (0,1)$ supersymmetry analogous to $4D, \mathcal{N} = 4$ SYM theory where there is the manifest $\mathcal{N} = 2$ supersymmetry and additional hidden $\mathcal{N} = 2$ supersymmetry
- The $6D, \mathcal{N} = (1,1)$ SYM theory is anomaly free as well as the $4D, \mathcal{N} = 4$ SYM theory and satisfies some non-renormaization theorems

# Superfield description of the supersymmetric theories

## Reminder of the notions

- Supersymmetric field models can be formulated in terms of conventional bosonic and fermionic fields. Component approach.
- In some cases the supersymmetric field models can be formulated in terms of superfields. A superfield depends on space-time coordinates $x$ and some number of anicommuting (Grassmann) coordinates $\theta$. The coefficients of expansion of superfield in anticommuting coordinates are the conventional bosonic and fermionic fields of supermultiplet.
- Advantage of component formulation: close relation with conventional field theory, convenient in classical field theory and to calculate the scattering amplitudes. Disadvantage: supersymmetry is not manifest.
- Advantage of superfield formulation: manifest supersymmetry, convenient in quantum field theory to study off-shell effects, simple proof of the non-renormalization theorems.
- Problem of unconstrained formulation. Harmonic superspace approach.

# Plan

- Review part
  1. $6D$ supersymmetry
  2. $6D, \mathcal{N} = (1,0)$ harmonic superspace
  3. $6D, \mathcal{N} = (1,0)$ hypermultiplet
  4. $6D, \mathcal{N} = (1,0)$ vector multiplet
  5. Action for vector multiplet coupled to hypermultiplet
  6. $\mathcal{N} = (1,1)$ SYM theory in terms of $\mathcal{N} = (1,0)$ harmonic supersfields
- Background field method
- Structure of one-loop counterterms
- Divergent part of one-loop effective action
- Summary

P.S. Howe, G. Sierra, P.K. Townsend, 1983.

$6D$ Minkowski space

- Coordinates $x^M$, $M = 0, 1, 2, 3, 4, 5$
- Metric $\eta_{MN} = diag(1, -1, -1, -1, -1, -1)$
- Proper Lorentz group $SO(1, 5)$

Two types of $6D$ Spinors

- Left $(1, 0)$ spinors $\psi_a$, $a = 1, 2, 3, 4$
- Right $(0, 1)$ spinors $\phi^a$, $a = 1, 2, 3, 4$

## Dirac matrices

- $8 \times 8$ Dirac matrices $\Gamma_M$,

$$\Gamma_M \Gamma_N + \Gamma_N \Gamma_M = 2\eta_{MN}$$

- Representation of the Dirac matrices

$$\Gamma_M = \begin{pmatrix} 0 & \tilde{\gamma}_M \\ -\gamma_M & 0 \end{pmatrix},$$

- Antisymmetric $4 \times 4$ Pauli-type matrices $\gamma_M$ and $\tilde{\gamma}_M$,

$$\gamma_M \tilde{\gamma}_N + \gamma_N \tilde{\gamma}_M = 2\eta_{MN}$$

$$(\tilde{\gamma}_M)^{ab} = \frac{1}{2} \epsilon^{abcd} (\gamma_M)_{cd}$$

- Spinor representation of the vectors, $V_{ab} = \frac{1}{2} (\gamma^M)_{ab} V_M$

## $6D$ superalgebra

- Two types of independent supercharges $Q^I_a, Q^a_J$, $I = 1, ..., 2m; J = 1, ..., 2n$
- Anticommutational relations for supercharges

$$\{Q^I_a, Q^K_b\} = 2\Omega^{IK} P_{ab}$$

$$\{Q^a_J, Q^b_L\} = 2\Omega_{JL} P^{ab}$$

Matrix $\Omega_{IK}$ belongs to $USp(2n)$ group (R-symmetry group), $\Omega_{IK}\Omega^{KJ} = \delta^J_I$

- $\mathcal{N} = (m, n)$ supersymmetry
- $\mathcal{N} = (1, 0)$ superspace
  Coordinates $z = (x^M, \theta^a_i), i = 1, 2$
- Basic spinor derivatives

$$D^i_a = \frac{\partial}{\partial\theta^a_i} - i\theta^{ib}\partial_{ab}, \quad \{D^i_a, D^j_b\} = -2i\Omega^{ij}\partial_{ab}$$

## Harmonic superspace

Basic references:

$4D$

A.Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky, E. Sokatchev, 1985.

A.Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic Superspace, 2001.

General purpose: to formulate $\mathcal{N}=2$ models in terms of unconstrained $\mathcal{N}=2$ superfields. General idea: to use the parameters $u^{\pm i}(i=1,2)$ (harmonics) related to $SU(2)$ automorphism group of the $\mathcal{N}=2$ superalgebra and parameterizing the 2-sphere,

$u^{+i}u_i^{-}=1$

It allows to introduce the $\mathcal{N}=2$ superfields with the same number of anticommuting coordinates as in case of the $\mathcal{N}=1$ supersymmetry. Prices for this are the extra bosonic variables, harmonics $u^{\pm i}$.

$6D$

P.S. Howe, K.S. Stelle, P.C. West, 1985.

B.M. Zupnik, 1986; 1999.

G. Bossard, E. Ivanov, A. Smilga, JHEP, 2015.

# $6D,(1,0)$ harmonic superspace

## $(1,0)$ harmonic superspace

- $USp(2) \sim SU(2), I = i$ The same harmonics $u^{\pm i}$ as in $4D, \mathcal{N} = 2$ supersymmetry
- Harmonic $6D,(1,0)$ superspace with coordinates $Z = (x^M, \theta^a_i, u^{\pm i})$
- Analytic basis $Z_{(an)} = (x^M_{(an)}, \theta^{\pm a}, u^\pm_i)$,
  $x^M_{(an)} = x^M + \frac{i}{2}\theta^{-a}(\gamma^M)_{ab}\theta^{+b}, \quad \theta^{\pm a} = u^\pm_i \theta^{ai}$
  The coordinates $\zeta = (x^M_{(an)}, \theta^{+a}, u^\pm_i)$ form a subspace closed under $(1,0)$ supersymmetry
- The harmonic derivatives

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} + i\theta^+ \not\partial \theta^+ + \theta^{+a}\frac{\partial}{\partial\theta^{-a}},$$

$$D^{--} = u^{-i}\frac{\partial}{\partial u^{+i}} + i\theta^- \not\partial \theta^- + \theta^{-a}\frac{\partial}{\partial\theta^{+a}},$$

$$D^0 = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} + \theta^{+a}\frac{\partial}{\partial\theta^{+a}} - \theta^{-a}\frac{\partial}{\partial\theta^{-a}}$$

- Spinor derivatives in the analytic basis

$$D^+_a = \frac{\partial}{\partial\theta^{-a}}, \quad D^-_a = -\frac{\partial}{\partial\theta^{+a}} - 2i\partial_{ab}\theta^{-b}, \quad \{D^+_a, D^-_b\} = 2i\partial_{ab}$$

## Hypermultiplet in conventional $6D$ superspace

- The $\mathcal{N}=(1,0)$ hypermultiplet is described in conventional $6D, \mathcal{N}=(1,0)$ superspace by the superfields $q^i(x,\theta)$ and their conjugate $\bar{q}_i(x,\theta)$, where $\bar{q}_i=(q^i)^+$ under the constraint

$$D_a^{(i}q^{j)}(x,\theta)=0$$

- On-shell component form of the hypermultiplet

$$q^i(z)=f^i(x)+\theta^{ai}\psi_a(x)$$

  where the scalar field $f^i(x)$ and the spinor field $\psi_a(x)$ satisfy the equations $\Box f^i=0, \partial^{ab}\psi_b=0$

- The on-shell $\mathcal{N}=(1,0)$ hypermultiplet in six dimensions has 2 bosonic+2 fermionic complex degrees of freedom.

Hypermultiplet in harmonic superspace: off-shell Lagrangian formulation

- Off-shell hypermultiplet is described by the analytic superfield $q^+_A(\zeta, u)$, $D^+_a q^+_A(\zeta, u) = 0$, satisfying the reality condition $\widetilde{(q^{+A})} \equiv q^+_A = \varepsilon_{AB} q^{+B}$. Pauli-Gürsey indices $A, B = 1, 2$
- Off-shell hypermultiplet harmonic superfield contains infinite set of auxiliary fields which vanish on-shell due to the equations of motion

$$D^{++} q^+(\zeta, u) = 0$$

- The equations of motion follow from the action

$$S_{HYPER} = -\frac{1}{2} \int d\zeta^{(-4)} du \; q^{+A} D^{++} q^+_A$$

Here $d\zeta^{(-4)} = d^6 x d^4 \theta^+$.

The $\mathcal{N}=(1,0)$ non-Abelian vector multiplet in $6D$ conventional superspace

- Gauge covariant derivatives

$$\nabla_{\mathcal{M}} = D_{\mathcal{M}} + \mathcal{A}_{\mathcal{M}}, \quad [\nabla_{\mathcal{M}}, \nabla_{\mathcal{N}}] = T_{\mathcal{M}\mathcal{N}}{}^{\mathcal{L}} \nabla_{\mathcal{L}} + F_{\mathcal{M}\mathcal{N}}$$

  with $D_{\mathcal{M}} = \{\partial_M, D^i_a\}$ being the flat covariant derivatives and $\mathcal{A}_M$ being the gauge connection taking the values in the Lie algebra of the gauge group.

- The constraints

$$F^{ij}_{ab} = 0, \quad \{\nabla^i_a, \nabla^j_b\} = -2i\varepsilon^{ij}\nabla_{ab}, \quad [\nabla^i_c, \nabla_{ab}] = -2i\varepsilon_{abcd}W^{id}$$

  Here $W^{ia}$ is the superfield strength obeying the Bianchi identities.

The constraints are solved in the framework of the harmonic superspace

The $\mathcal{N}=(1,0)$ non-Abelian vector multiplet in $6D$, $\mathcal{N}=(1,0)$ harmonic superspace

- Harmonic covariant derivative

$$\nabla^{++} = D^{++} + V^{++}$$

Connection $V^{++}$, takes the values in the Lie algebra of the gauge group, this is an unconstrained analytic potential of the $6D, \mathcal{N}=(1,0)$ SYM theory.

- On-shell contents: $V^{++} = \theta^{+a}\theta^{+b}A_{ab} + 2(\theta^{+})_a\lambda^{-a}$, $A_{ab}$ is a vector field, $\lambda^{-a} = \lambda^{ai}u_i^-, \lambda^{ai}$ is a spinor field.
- The superfield action of $6D, \mathcal{N}=(1,0)$ SYM theory is written in the form

$$S_{SYM} = \frac{1}{f^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{tr}\int d^{14}z du_1 \ldots du_n \frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)}$$

Here $f$ is the dimensional coupling constant ($[f]=-1$)

- Gauge transformations

$$V^{++\prime} = -ie^{i\lambda}D^{++}e^{-i\lambda} + e^{i\lambda}V^{++}e^{-i\lambda}, \qquad q^{+\prime} = e^{i\lambda}q^+$$

Theory of $\mathcal{N} = (1,0)$ non-Abelian vector multiplet coupled to hypermultiplet

- Action

$$S[V^{++}, q^{+}] = \frac{1}{f^2}\sum_{n=2}^{\infty}\frac{(-i)^n}{n}\mathrm{tr}\int d^{14}z\,du_1\ldots du_n\,\frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)}$$

$$-\int d\zeta^{-4}du\tilde{q}^{+}\nabla^{++}q^{+}$$

- Harmonic covariant derivative

$$\nabla^{++} = D^{++} + iV^{++}$$

- Equations of motion

$$\frac{1}{2f^2}F^{++} - i\tilde{q}^{+}q^{+} = 0\,, \qquad \nabla^{++}q^{+} = 0\,.$$

$$F^{++} = (D^{+})^4V^{--},\; D^{++}V^{--} - D^{--}V^{++} + i[V^{++}, V^{--}] = 0$$

# $\mathcal{N} = (1,1)$ SYM theory

$\mathcal{N} = (1,1)$ SYM theory can be formulated in terms of $\mathcal{N} = (1,0)$ harmonic superfields as the $\mathcal{N} = (1,0)$ vector multiplet coupled to hypermultiplet in adjoint representation. The theory is manifestly $\mathcal{N} = (1,0)$ supersymmetric and possesses the extra hidden $\mathcal{N} = (0,1)$ supersymmetry.

- Action

$$S[V^{++}, q^{+}] = S_{SYM}[V^{++}] + S_{HYPER}[q^{+}, V^{++}]$$

- The action is manifestly $\mathcal{N} = (1,0)$ supersymmetric.
- The action is invariant under the transformations of extra hidden $\mathcal{N} = (0,1)$ supersymmetry

$$\delta V^{++} = \epsilon^{+} q^{+}, \qquad \delta q^{+} = -(D^{+})^{4}(\epsilon^{-} V^{--})$$

where the transformation parameter $\epsilon^{\pm}_{A} = \epsilon_{aA}\theta^{\pm A}$.

# General scheme of calculations

- We start with harmonic superfield formulations of vector multiplet coupled to hypermultiplet.
- Effective action is formulated in the framework of the harmonic superfield background field method. It provides manifest $\mathcal{N} = (1, 0)$ supersymmetry and gauge invariance of effective action under the classical gauge transformations.
- Effective action can be calculated on the base of superfield proper-time technique. It provides preservation of manifest $\mathcal{N} = (1, 0)$ supersymmetry and manifest gauge invariance at all steps of calculations.
- The effective action can also be calculated perturbatively on the base of Feynman diagrams in superspace (supergraph technique).
- We study the model where the $\mathcal{N} = (1, 0)$ vector multiplet interacts with hypermultiplet in the arbitrary representation of the gauge group. Then, we assume in the final result for one-loop divergences, that this representation is adjoint what corresponds to $\mathcal{N} = (1, 1)$ SYM theory.

## Background field method

Aim: construction of gauge invariant effective action

Realization

- The superfields $V^{++}, q^{+}$ are splitting into the sum of the background superfields $V^{++}, Q^{+}$ and the quantum superfields $v^{++}, q^{+}$
$$V^{++} \to V^{++} + f v^{++}, \qquad q^{+} \to Q^{+} + q^{+}$$
- The action is expending in a power series in quantum fields. As a result, we obtain the initial action $S[V^{++}, q^{+}]$ as a functional $\tilde{S}[v^{++}, q^{+}; V^{++}, Q^{+}]$ of background superfields and quantum superfields.
- The gauge-fixing function are imposed only on quantum superfiled
$$\mathcal{F}^{(+4)}_{\tau} = D^{++} v^{++}_{\tau} = e^{-ib} (\nabla^{++} v^{++}) e^{ib} = e^{-ib} \mathcal{F}^{(+4)} e^{ib} \ ,$$
where $b(z)$ is a background-dependent gauge bridge superfield and $\tau$ means $\tau$-frame. In the non-Abelian gauge theory, the gauge-fixing function is background-dependent.
- Faddev-Popov procedure is used. Ones obtain the effective action $\Gamma[V^{++}, Q^{+}]$ which is gauge invariant under the classical gauge transformations. Background field construction in the case under consideration is analogous to one in $4D, \mathcal{N} = 2$ SYM theory (I.L.B, E.I. Buchbinder, S.M. Kuzenko, B.A. Ovrut, 1998).

- The effective action $\Gamma[V^{++}, Q^{+}]$ is written in terms of path integral

$$e^{i\Gamma[V^{++},Q^{+}]} = Det^{1/2}\,\widehat{\Box} \int \mathcal{D}v^{++}\,\mathcal{D}q^{+}\,\mathcal{D}\mathbf{b}\,\mathcal{D}\mathbf{c}\,\mathcal{D}\varphi\; e^{iS_{quant}[v^{++},q^{+},\mathbf{b},\mathbf{c},\varphi,V^{++},Q^{+}]}$$

- The quantum action $S_{quant}$ has the structure

$$S_{quant} = S[V^{++} + fv^{++}, Q^{+} + q^{+}] +$$

$$+ S_{GF}[v^{++}, V^{++}] + S_{FP}[\mathbf{b}, \mathbf{c}, v^{++}, V^{++}] + S_{NK}[\varphi, V^{++}].$$

- Gauge fixing term $S_{GF}[v^{++}, V^{++}]$, Faddeed-Popov ghost action $S_{FP}[\mathbf{b}, \mathbf{c}, v^{++}, V^{++}]$, Nelson-Kalosh ghost action $S_{NK}[\varphi, V^{++}]$
- Operator $\widehat{\Box}$

$$\widehat{\Box} = \eta^{MN}\nabla_M\nabla_N + W^{+a}\nabla^{-}_{a} + F^{++}\nabla^{--} - \frac{1}{2}(\nabla^{--}F^{++})$$

- All ghosts are the analytic superfields

One-loop approximation. Only quadratic in quantum fields and ghosts terms are taken into account in the path integral for effective action. It gives after some transformation the one-loop contribution $\Gamma^{(1)}[V^{++}, Q^{+}]$ to effective action in terms of formal functional determinants in analytic subspace of harmonic superspace

$$\Gamma^{(1)}[V^{++}, Q] = \frac{i}{2} Tr_{(2,2)} \ln[\delta^{(2,2)}\, \widehat{\square}^{AB} - 2f^2 Q^{+\,m}(T^A G_{(1,1)} T^B)_m{}^n Q^+_n] -$$

$$-\frac{i}{2} Tr_{(4,0)} \ln \widehat{\square} - iTr \ln(\nabla^{++})^2_{\mathrm{Adj}} + \frac{i}{2} Tr \ln(\nabla^{++})^2_{\mathrm{Adj}} + iTr \ln \nabla^{++}_{\mathrm{R}}$$

As usual, $TrlnO = lnDetO$, $Tr$ means the functional trace in analytic subspace and matrix trace.
$(T^A)_m{}^n$ are are generators of the representation for the hypermultiplet.
The $G_{(1,1)}$ is the Green function for the operator $\nabla^{++}$.
Index $A$ numerates the generators, $V^{++} = V^{++A} T^A$. Operator $\widehat{\square}$ acts on the components $V^{++A}$ as $(\widehat{\square} V^{++})^A = \widehat{\square}^{AB} V^{++B}$
$\mathrm{Adj}$ and $\mathrm{R}$ mean that the corresponding operators are taken in the adjoint representation and in the representation for hypermultiplet.

## Superfield Feynman diagrams (supergraphs)

- Perturbation theory can be given in terms of Feynman diagrams formulated in superspace
- Vector multiplet propagator

$$G^{(2,2)}(1|2) = -2\frac{(D_1^+)^4}{\Box_1}\delta^{14}(z_1 - z_2)\delta^{(-2,2)}(u_1, u_2)$$

- Hypermultiplet propagator

$$G^{(1,1)}(1|2) = \frac{(D_1^+)^4(D_2^+)^4}{\Box_1}\frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}$$

- Ghost propagators have the analogous structure
- Superspace delta-function

$$\delta^{14}(z_1 - z_2) = \delta^6(x_1 - x_2)\delta^8(\theta_1 - \theta_2)$$

- The vertices are taken from the superfield action as usual

## Power counting

Superficial degree of divergence $\omega$-total degree in momenta in loop integral.

- Consider the $L$ loop supergraph $G$ with $P$ propagators, $V$ vertices, $N_Q$ external hypermultiplet legs, and an arbitrary vector multiplet external legs.
- One can prove that due to structure of the propagators and the Grassmann delta-functions in the propagators, any supergraph for effective action can be written through the integrals over full $\mathcal{N} = (1, 0)$ superspace and contains only a single integral over $d^8\theta$ (non-renormalization theorem).
- Mass dimensions: $[x] = -1$, $[p] = 1$, $[\int d^6p] = 6$, $[\theta] = -\frac{1}{2}$, $[\int d^8\theta] = 4$, $[q^+] = 1$, $[V^{++}] = 0$.
- After summing all dimensions and using some identities, power counting gives $\omega(G) = 2L - N_Q - \frac{1}{2}N_D$
- $N_D$ is a number of spinor derivarives acting on external lines
- A number of space-time derivatives in the counterterms increases with $L$. The theory is multiplicatively non-renormalizable.
- One loop approximation $\omega_{1-loop}(G) = 2 - N_Q$
- The possible divergences correspond to $\omega_{1-loop} = 2$ and $\omega_{1-loop} = 0$

Calculations of $\omega$ are analogous to ones in $4D, \mathcal{N} = 2$ gauge theory (ILB, S.M. Kuzenko, B.A. Ovrut, 1998).

## Structure of one-loop counterterms

Possible candidate for one-loop divergences can be constructed on the based of dimensions, gauge invariance and $\mathcal{N}=(1,0)$ supersymmetry in the form (G. Bossard, E. Ivanov, A. Smilga, 2015)

$$\Gamma^{(1)}_{div} = \int d\zeta^{(-4)}\, du \Big[c_1(F^{++A})^2 + ic_2 F^{++A}(\widetilde{q}^+)^m (T^A)_m{}^n (q^+)_n + c_3\Big((\widetilde{q}^+)^m (q^+)_m\Big)^2$$

Where $c_1, c_2, c_3$ are arbitrary dimensionless real numbers.
We consider a possible form of one-loop counterterms on the base of superficial degree of divergences in quadratic approximation in fields and then restore a complete result of the base of gauge invariance.

- Let $N_Q=0$, $N_D=0$, so that $\omega$=2, and we use the dimensional regularization. The corresponding counterterm has to be quadratic in momenta and given by the full $\mathcal{N}=(1,0)$ superspace integral. The only admissible possibility is

$$\Gamma^{(1)}_1 \sim \int d^{14}z du V^{--}\Box V^{++}$$

  After some transformation it coincides with first term in $\Gamma^{(1)}_{div}$, with dimensionless divergent coefficient $c_1$. Being dimensionless, this coefficient must be proportional to $1/\varepsilon$ , where $\varepsilon = d-6$ is a regularization parameter.

- Let $N_Q = 2$, $N_D = 0$ so that $\omega = 0$ and we use the dimensional regularization. The corresponding counterterm has to be momentum independent and given by the full $\mathcal{N} = (1,0)$ superspace integral. The only admissible possibility is

$$\Gamma_2^{(1)} \sim \int d^{14}z du \widetilde{q}^{+} V^{--} q^{+}$$

  After some transformation it coincides with second term in $\Gamma^{(1)}_{div}$, with dimensionless divergent coefficient $c_2$. Being dimensionless, this coefficient must be proportional to $1/\varepsilon$ , where $\varepsilon = d - 6$ is a regularization parameter.
- Let $N_Q = 4$, $N_D = 0$ so that $\omega = -2$ and the corresponding supergraph is convergent. It means that $c_3 = 0$ in $\Gamma^{(1)}_{div}$. As a result, all one-loop $(q^+)^4$ possible contributions to effective action are finite.

# Manifestly covariant calculation

Calculating the one-loop divergences of superfield functional determinants is carried out in the framework of proper-time technique (superfield version of Schwinger-De Witt technique). Such technique allows us to preserve the manifest gauge invariance and manifest $\mathcal{N}=(1,0)$ supersymmetry at all steps of calculations.

General scheme of calculations

- Proper-time representation

$$TrlnO \sim Tr\int_0^\infty \frac{d(is)}{(is)^{1+\varepsilon}} e^{isO_1}\delta(1,2)|_{2=1}$$

- Here $s$ is the proper-time parameter and $\varepsilon$ is a parameter of dimensional regularization.
- Typically the $\delta(1,2)$ contains $\delta^8(\theta_1-\theta_2)$, which vanishes at $\theta_1=\theta_2$
- Typically the operator $O$ contains some number of spinor derivatives $D^+_a, D^-_a$ which act on the Grassmann delta-functions $\delta^8(\theta_1-\theta_2)$ and can kill them. Non-zero result will be only if all these $\delta$-functions are killed.
- Only these terms are taking into account which have the pole $\frac{1}{\varepsilon}$ after integration over proper-time.

Results of calculations

$$\Gamma^{(1)}_{div}[V^{++},Q^{+}] = \frac{C_2 - T(R)}{3(4\pi)^3\varepsilon} tr \int d\zeta^{(-4)} du (F^{++})^2 -$$

$$-\frac{2if^2}{(4\pi)^3\varepsilon} \int d\zeta^{(-4)} du \widetilde{Q}^{+m}(C_2\delta_m{}^n - C(R)_m{}^n)F^{++}Q^{+}{}_n.$$

- The quantities $C_2, T(R), C(R)$ are defined as follows

$$tr(T^A T^B) = T(R)\delta^{AB}$$

$$tr(T^A_{Adj}T^B_{Adj}) = f^{ACD}f^{BCD} = C_2\delta^{AB}$$

$$(T^A T^A)_m{}^n = C(R)_m{}^n.$$

- Results of calculations correspond to analysis done on the base of power counting. The coefficients $c_1, c_2$ are found. The coefficient $c_3 = 0$ as we expected.
- In $\mathcal{N} = (1,1)$ SYM theory, the hypermultiplet is in the same representation as the vector multiplet. Then $C_2 = T(R) = C(R)$. Then $\Gamma^{(1)}_{div}[V^{++},Q^{+}] = 0$!

## Summary

- The six-dimensional $\mathcal{N} = (1,0)$ supersymmetric theory of the non-Abelian vector multiplet coupled to hypermultiplet in the $6D$, $\mathcal{N} = (1,0)$ harmonic superspace was considered.
- Background field method in harmonic superspace was constructed .
- Manifestly supersymmetric and gauge invariant effective action, depending both on vector multiplet anf hypermultiplet superfields, was defined.
- Superficial degree of divergence is evaluated and structure of one-loop counterterms was studied.
- An efficient manifestly gauge invariant and $\mathcal{N} = (1,0)$ supersymmetric technique to calculate the one-loop effective action was developed. As an application of this technique, we found the one-loop divergences of the theory under consideration.
- The same one-loop divergences have been calculated independently with help of $\mathcal{N} = (1,0)$ supergraphs.
- It is proved that $\mathcal{N} = (1,1)$ SYM theory is one-loop off-shell finite. There is no need to use the equations of motion to prove this property.

# Open problems

- Two-loop off-shell finiteness?
- Low-energy effective action?
- Divergences in the higher derivative gauge theory?

THANK YOU VERY MUCH!