Invariant structures in extended Higgs sectors

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Why do we extend the Higgs sector?

The problem of the vacuum stability!

If the Higgs boson has mass $\backsim$ 125 ГэВ, else the Universe is in a metastable state [1, 2].

[1] Elias-Miro J. Higgs mass implications on the stability of the electroweak vacuum / J. Elias-Miro [et al.] // e-print: arXiv:1112.3022v1.

[2] Degrassi G. Higgs mass and vacuum stability in the Standard Model at NNLO / G. Degrassi [et al.] // e-print: arXiv:1205.6497v2.

Baryon asymmetry!

The Sakharov conditions [3]:

- the violation of baryon number;
- C-violation and CP-violation;
- the deviation from thermodynamic equilibrium.

[3] Sakharov A.D. Violation of CP invariance, C-asymmetry and baryon asymmetry of the Universe // Pisma v ZhETF, 1967, Vol. 5, Issue 1, pp. 32–35. [in Russian].

Why do we extend the Higgs sector?

The Supersymmetry!

SM: $\mathcal{L}_{Yukawa} = y^L_{\alpha\beta}\bar{L}_\alpha E_\beta H + y^D_{\alpha\beta}\bar{Q}_\alpha D_\beta H + y^U_{\alpha\beta}\bar{Q}_\alpha U_\beta \tilde{H}$, where $\tilde{H} = i\tau_2 H^\dagger$

SUSY-models:

$\Rightarrow \quad H \to H_1,\ \tilde{H} \to H_2,$

где $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$

[4] Gelfand, A. Yu., Likhtman E. P. Extension of the algebra of Poincare generators and violation of P invariance // JETP Letters.-1971.-T. 13.-vol.8.-P. 452-455.

[5] In Akulov.P. Volkov D. V. Goldstone fields with spin half // Teor. Mat. physics 1972. - Vol. 18. - P. 39-50.

[6] Wess J., Zumino B. Supergauge transformations in four dimensions. // Nucl. Phys. B. - 1974. - V.70. - P.39-49.

[7] Martin S., The Primer supersymmetry // e-print: hep-ph/9709356v6

Models with the extended Higgs sector

	superfield	SM field	spin	superpartner	spin
quarks/	$\hat{Q}$	$Q = \begin{pmatrix} U_\alpha \\ D_\alpha \end{pmatrix}_L$	$\frac{1}{2}$	$\tilde{Q} = \begin{pmatrix} \tilde{U}_\alpha \\ \tilde{D}_\alpha \end{pmatrix}_L$	0
squarks	$\hat{U}$	$U_{\alpha R}$	$\frac{1}{2}$	$\tilde{U}_{\alpha R}$	0
	$\hat{D}$	$D_{\alpha R}$	$\frac{1}{2}$	$\tilde{D}_{\alpha R}$	0
leptons/	$\hat{L}$	$L_{\alpha L}$	$\frac{1}{2}$	$\tilde{L}_{\alpha L}$	0
sleptons	$\hat{E}$	$E_{\alpha R}$	$\frac{1}{2}$	$\tilde{E}_{\alpha R}$	0
gauge	$\hat{G}$	G^a_μ	1	$\tilde{G}^a_\mu$	$\frac{1}{2}$
bosons /	$\hat{W}$	$W^\pm,\ W^0$	1	$\tilde{W}^\pm,\ \tilde{W}^0$	$\frac{1}{2}$
gaugino	$\hat{B}$	B^0	1	$\tilde{B}^0$	$\frac{1}{2}$
Higgs/	$\hat{H}_1$	$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$	0	$\tilde{H}_1 = \begin{pmatrix} \tilde{H}_1^0 \\ \tilde{H}_1^- \end{pmatrix}$	$\frac{1}{2}$
higgsino,	$\hat{H}_2$	$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$	0	$\tilde{H}_2 = \begin{pmatrix} \tilde{H}_2^+ \\ \tilde{H}_2^0 \end{pmatrix}$	$\frac{1}{2}$
S field/ singlino	$\hat{S}$	S	0	$\tilde{S}$	$\frac{1}{2}$

models	2HDM	MSSM	NMSSM
physical Higgs states	2 neutral CP-even 1 neutral CP-odd 2 charged	2 neutral CP-even 1 neutral CP-odd 2 charged	3 neutral CP-even 2 neutral CP-odd 2 charged

The CP-violation problem

[3] Kobayashi M., Maskawa T. CP Violation in the Renormalizable Theory of Weak Interaction // Prog. Theor. Phys.–1973.–V. 49.– P. 652-657.

Matrix CKM: $\hat{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$

$$= \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}\mathbf{e}^{\mathbf{i}\delta} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13}\mathbf{e}^{\mathbf{i}\delta} & C_{12}C_{23} - S_{12}S_{23}S_{13}\mathbf{e}^{\mathbf{i}\delta} & S_{23}C_{13} \\ S_{12}S_{23} - C_{12}C_{23}S_{13}\mathbf{e}^{\mathbf{i}\delta} & -S_{23}C_{12} - S_{12}C_{23}S_{13}\mathbf{e}^{\mathbf{i}\delta} & C_{23}C_{13} \end{pmatrix},$$

where $C_{ij} = \cos\Theta_{ij}, S_{ij} = \sin\Theta_{ij}, \delta-$complex phase.

[4] Gavela M.B. Standard model CP-violation and baryon asymmetry (II). Finite temperature / M.B. Gavela [et al.] // Nucl.Phys.B. - 1994. - V.430. - P.382-426. Matrix CKM is not enough to describe the baryon asymmetry!

Introduction

CP-violation in the lepton sector:

[5] Connecting Leptonic CP Violation, Lightest Neutrino Mass and Baryon Asymmetry Through Type II Seesaw // Int. J. Mod. Phys. A – 2015. – V.

[6] Petcov S.T. Leptonic CP violation and leptogenesis // Int. J. Mod. Phys.A – 2014.– V.30.–19P.

... etc.

CP-violation in the extended Higgs sector:

[7] Pilaftsis A., WagnerC.E.M. Higgs Bosons in the Minimal Supersyminetry Standard Model with Explicit CP Violation // Nucl. Phys. B. 1999. V. 553. P. 3-42.

[8] Choi S. K, Lee J.S. Decays of the MSSM Higgs Bosons with Explicit CP-Violation // Phys. Rev. D. 2000. V. Gl. P. 015003.

[9] Carena M. et al Higgs-Boson Pole Masses in the MSSM with Explicit CP Violation // NucL Phys. B. 2002. 625. P. 345-371.

The CP-violation in the extended Higgs sector:

[10] Akhmetzyanova E.N., Dolgopolov M.V., Dubinin M.N. Higgs Bosons in the Two-Doublet Model with CP Violation // Phys.Rev.D. V.71. N7. 2005. P.075008

[11] Akhmetzyanova E.N., Dolgopolov M.V., Dubinin M.N., Smirnov I.A., Shcherbakova E. S. Supersymmetric model with violation of CP invariance. 1. The decays of the Higgs boson $h \to gg$ и $h \to \gamma\gamma$ // Vestnik SamGU, 2003. N 2(28). C.122-136.

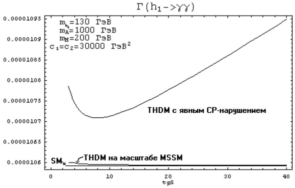

Fig.1. The dependence of the decay widths from $\tan\beta$ [11].

[12] Akhmetzyanova E. N., Dolgopolov M. V., Dubinin M. N. Supersymmetric model with violation of CP invariance. 3. The violation of CP invariance in the Higgs sector // Vestnik SamGU, 2003. N 4(30). P. 147-179.

... etc.

The two-doublet Higgs Potential for MSSM

$$U_{eff}(\phi_1,\phi_2) = -\frac{1}{2}\mu_1^2(\phi_1^\dagger\phi_1) - \frac{1}{2}\mu_2^2(\phi_2^\dagger\phi_2) - {\mu_{12}}^2(\phi_1^\dagger\phi_2) - ({\mu_{12}}^2)^*(\phi_2^\dagger\phi_1)$$

$$+\lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$

$$+\frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{{\lambda_5}^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + {\lambda_6}^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + {\lambda_7}^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$

Georgi H., Hadr. J. Phys. 1978, Lee T. D., Phys. Rev. D. 1973

Nilendra G. Deshpande, Ernest Ma, Phys.Rev. 1978

Dubinin M., Semenov A.,2004; Akhmetzyanova E.,Dolgopolov M., Dubinin M.,2005

$$\langle\phi_i\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_i \, with\, own\, spont\, CP\, phase \end{pmatrix} spont\, CP\, phase, \qquad (i=1,2).$$

$$v^2 = v_1^2 + v_2^2 = 246^2 \text{ GeV}^2, \qquad \tan\beta = \frac{v_2}{v_1}, \quad \mu_i^2(T),\ \lambda_i(T),\ v_{1,2}(T)$$

Highlight

The correlations in this report connect the parameter of the Higgs potential in an arbitrary and the Higgs bases. The considered U(2) transformations are general rotations between different bases of the Higgs isodoublets in TDM. The choice of basis is naturally arbitrary, but can be physically motivated by additional symmetries and symmetry violations, for example, CP invariance. It is preferable to express the observables of the physical Higgs sector in terms of invariants of the general U(2) transformation using unambiguously defined parameters.

In works concerning the physics of Higgs bosons, the specific basis in the space of isodoublets of complex scalar fields is chosen.
Higgs fields are considered near their vacuum expectations:

$$\langle \Phi_a^0 \rangle = \frac{v_a}{\sqrt{2}}, \qquad (a = 1, 2)$$

Parameter $\tan\beta = v_2/v_1$ plays an important role in phenomenology.
However, it cannot be unambiguously defined.
The meaning is lost upon transition to the Higgs basis in which the nontrivial vacuum expectation is in one of the doublets only.

Higgs basis

$$\langle \Phi_a^0 \rangle = \frac{v}{\sqrt{2}} \qquad\qquad \langle \Phi_b^0 \rangle = 0$$

is obtained from the general (arbitrary) basis by rotation in the space of scalar doublets:

$$\Phi_a = \Phi_1 c_\beta + e^{-i\theta}\Phi_2 s_\beta, \qquad\qquad \Phi_b = -\Phi_1 s_\beta + e^{i\theta}\Phi_2 c_\beta \tag{1}$$

$$\Phi_a = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \varphi_a^0 + iG^0) \end{pmatrix}, \qquad\qquad \Phi_b = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\varphi_b^0 + iA) \end{pmatrix} \tag{2}$$

Potential in the form of the renormalized and gauge invariant polynomial:

$$V = Y_{a\bar{b}}\Phi_{\bar{a}}^{\dagger}\Phi_{b} + \frac{1}{2}Z_{a\bar{b}c\bar{d}}\left(\Phi_{\bar{a}}^{\dagger}\Phi_{b}\right)\left(\Phi_{\bar{c}}^{\dagger}\Phi_{d}\right) \tag{3}$$

The potential in Higgs basis:

$$U(\Phi_a, \Phi_b) = Y_1(\Phi_a^\dagger \Phi_a) + Y_2(\Phi_b^\dagger \Phi_b) + \left[Y_3(\Phi_a^\dagger \Phi_b) + \text{H.c.}\right] + \tag{4}$$

$$+\frac{1}{2}Z_1(\Phi_a^\dagger \Phi_a)^2 + \frac{1}{2}Z_2(\Phi_b^\dagger \Phi_b)^2 + Z_3(\Phi_a^\dagger \Phi_a)(\Phi_b^\dagger \Phi_b) + Z_4(\Phi_a^\dagger \Phi_b)(\Phi_b^\dagger \Phi_a) + \tag{5}$$

$$+\left\{\frac{1}{2}Z_5(\Phi_a^\dagger \Phi_b)(\Phi_a^\dagger \Phi_b) + \left[Z_6(\Phi_a^\dagger \Phi_a) + Z_7(\Phi_b^\dagger \Phi_b)\right](\Phi_a^\dagger \Phi_b) + \text{H.c.}\right\}. \tag{6}$$

The form-invariants in an arbitrary basis:

$$
\begin{aligned}
&\mathrm{Y}_1 = -\mu_1^2 c_\beta^2 - \mu_2^2 s_\beta^2 - Re(\mu_{12}^2 e^{i\theta}) s_{2\beta}, \\
&\mathrm{Y}_2 = -\mu_1^2 s_\beta^2 - \mu_2^2 c_\beta^2 - Re(\mu_{12}^2 e^{i\theta}) s_{2\beta}, \\
&\mathrm{Y}_3 = \tfrac{1}{2}(\mu_1^2 - \mu_2^2) s_{2\beta} - Re(\mu_{12}^2 \mathbf{e} e^{i\theta}) c_{2\beta} - i Im(\mu_{12}^2 e^{i\theta}), \qquad (7) \\
&\text{and } \mathrm{Z}_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\big[c_\beta^2 Re(\lambda_6 e^{i\theta}) + s_\beta^2 Re(\lambda_7 e^{i\theta})\big], \\
&\mathrm{Z}_2 = \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\big[s_\beta^2 Re(\lambda_6 e^{i\theta}) + c_\beta^2 Re(\lambda_7 e^{i\theta})\big], \\
&\mathrm{Z}_3 = \tfrac{1}{4} s_{2\beta}^2 [\lambda_1 + \lambda_2 - 2\lambda_{345}] + \lambda_3 - s_{2\beta} c_{2\beta} Re[(\lambda_6 - \lambda_7) e^{i\theta}], \\
&\mathrm{Z}_4 = \tfrac{1}{4} s_{2\beta}^2 [\lambda_1 + \lambda_2 - 2\lambda_{345}] + \lambda_4 - s_{2\beta} c_{2\beta} Re[(\lambda_6 - \lambda_7) e^{i\theta}], \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{Z}_5 = \tfrac{1}{4}s_{2\beta}^2[\lambda_1 + \lambda_2 - 2\lambda_{345}] + Re(\lambda_5 e^{2i\theta}) + ic_{2\beta}Im(\lambda_5 e^{2i\theta}) - \\
&\text{-s}_{2\beta}c_{2\beta}Re[(\lambda_6 - \lambda_7)e^{i\theta}] - is_{2\beta}Im[(\lambda_6 - \lambda_7)e^{i\theta}], \\
&\mathrm{Z}_6 = \tfrac{1}{2}s_{2\beta}[\lambda_1 c_{2\beta}^2 - \lambda_2 s_{2\beta}^2 - \lambda_{345}c_{2\beta} - iIm(\lambda_5 e^{2i\theta})] + c_\beta c_{3\beta}Re(\lambda_6 e^{i\theta}) + \\
&+\mathrm{s}_\beta s_{3\beta}Re(\lambda_7 e^{i\theta}) + ic_\beta^2 Im(\lambda_6 e^{i\theta}) + is_\beta^2 Im(\lambda_7 e^{i\theta}), \\
&\mathrm{Z}_7 = \tfrac{1}{2}s_{2\beta}[\lambda_1 s_{2\beta}^2 - \lambda_2 c_{2\beta}^2 - \lambda_{345}c_{2\beta} - iIm(\lambda_5 e^{2i\theta})] + s_\beta s_{3\beta}Re(\lambda_6 e^{i\theta}) + \\
&+\mathrm{c}_\beta c_{3\beta}Re(\lambda_7 e^{i\theta}) + is_\beta^2 Im(\lambda_6 e^{i\theta}) + ic_\beta^2 Im(\lambda_7 e^{i\theta}), \qquad (7)
\end{aligned}
$$

The condition of extremum of scalar potential:

$$\hat{v}^*_{\bar{a}} \left[Y_{a\bar{b}} + \frac{1}{2} v^2 Z_{a\bar{b}c\bar{d}} \, \hat{v}^*_{\bar{c}} \hat{v}_d \right] = 0 \tag{7}$$

The corresponding conditions of minimum of the scalar potential are:

$$Y_1 = -\frac{1}{2} Z_1 v^2, \qquad Y_3 = -\frac{1}{2} Z_6 v^2 \tag{8}$$

G. C. Branco, L. Lavoura, and J. P. Silva, "CP Violation," in International Series of Monographs on Physics, No. 103 (Oxford Univ. Press, 1999); G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos, "CP-Odd Invariants in Models with Several Higgs Doublets," Phys. Lett. B 614, 187–194 (2005).

S. Davidson and H. E. Haber, "Basis-Independent Methods for the Two-Higgs-Doublet Model," Phys. Rev. D 72, 035004 (2005); J. F. Gunion, H. E. Haber, CP-violation in the General Two-Higgs-Doublet Model," hep-ph/0506227.

Summary

Connection of the parameters of the Higgs potential in an arbitrary and the Higgs bases is considered.
The considered $U(2)$ transformations are general rotations between different bases of the Higgs isodoublets in TDM. In particular, if expressions in terms of invariants (7) are substituted into the conditions of minimization, expressions for the masses of the bosons h, H, A and constants of self-interaction of scalars, the formulas in an arbitrary ("generic") basis should be obtained.
In MSSM, U(2) rotation in the space of isodoublets changes the form of Yukawa interaction and the Lagrangian for scalar quarks–Higgs bosons. The physical meaning of the general Lagrangian cannot and should not change.