# Numerical modeling of the electrons confinement in the multicusp magnetic trap 

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#### Abstract

A charge-exchange target for neutralizing a negative ion beam with energies up to 10 MeV and higher requires the development of a highly efficient plasma trap which allows to form and confine plasma with a linear density up to $10^{17} \mathrm{~cm}^{-2}$ and higher. The magnetic systems in which the condition of magnetohydrodynamic stability of the plasma is satisfied are most interesting to obtain a high-density plasma. The electron confinement efficiency in a magnetic trap with a quasi-spherically symmetric multicusp magnetic field geometry with a "minimum B" at the center of the system, in which all cusps are point-type cusps is studied using numerical methods. The results of numerical experiments are compared with a collisionless model of particle motion in a trap.


## INTRODUCTION

To obtain beams of high-energy neutral hydrogen atoms, beams of accelerated negative ions must be passed through a neutralizing target.

For an energy range of $30-50 \mathrm{MeV}$ or lower (for example, at an energy of 30 MeV as the first stripping target for the implementation of multi-turn charge-exchange injection from the linear accelerator LU-30 to the proton synchrotron $\mathrm{U}-1.5$ ) the use of foil targets is associated with technological difficulties in manufacturing thin foils and operations of moving foils in the accelerator beam.

In addition, foil targets have a common disadvantage, caused by the destruction of targets at high beam currents, which reduces the stability of neutralization and leads to loss of accelerator operation time for replacement of targets.

There is not such disadvantage in gas and plasma charge-exchange targets, which have no limitation relative to the integral flow of ions through the target in the pulsed mode of operation.

The advantages of plasma charge-exchange targets are higher yield of neutral atoms in comparison with a gas target [1] as well as the possibility of reducing the influence of the working target on the vacuum conditions in the channel of the negative ion accelerator by confining the plasma ions and electrons by the magnetic field of the trap.

The generation and confinement of target plasma for this range of ion energies requires the development of highly efficient trap that allows obtaining plasma with a linear density of up to $10^{21} \mathrm{~m}^{-2}$ and higher, while simultaneously meeting the condition of minimizing the number of plasma particles flowing into the accelerator channel.

To obtain high-density plasma, magnetic systems in which the condition of magnetohydrodynamic stability of the plasma is fulfilled are most interesting. In addition, the trap should be characterized by a sufficiently high confinement time of charged particles, which should be comparable to the duration of the pulse of the ion beam from the accelerator.

Recently developed target plasma traps [2,3] use axisymmetric multicusp magnetic systems that represent a sequence of ring cusps of the magnetic field. The escape of plasma particles from them occurs through a set of annular magnetic field cusps and end holes, designed to pass an ion beam. The plasma flow through the end holes in this geometry leads to a significant additional load on the vacuum system of the negative ion accelerator.

In order to reduce the gas load, a quasi-spherically symmetric multicusp geometry of a magnetic trap in which all cusps are only point - type cusps [4], and the axis of the beam passing through the neutralizer does not coincide with any of the cusp axes can be used. This mutual arrangement of axes does not allow the electrons and plasma ions, leaving the trap mainly through the magnetic field cusps, to enter the accelerator transport line, and thus minimizes the impact of the charge-exchange target on the vacuum conditions in the accelerator. In this case, the magnetic field of the trap, which confines the target plasma with electron and ion energies of 10 eV scale, has a negligible effect on the trajectories of the ion beam with energies greater than 10 MeV .

Previously, it was shown experimentally that high-density plasma can be obtained in such traps, in paper [5] plasma with an electron density of up to $9 \times 10^{21} \mathrm{~m}^{-3}$ was obtained in a quasi-spherically symmetric multicusp trap with a system size of about 0.2 m and a pulse duration of about $10^{-5} \mathrm{~s}$.

To examine the possibility of using a quasi-spherically symmetric multicusp magnetic system as target plasma trap for a negative ion beam neutralizer, it is necessary to investigate the dependences of electron confinement efficiency on the magnitude of magnetic field induction and electron energy by numerical methods, and on the basis of these dependences to develop recommendations for choosing the operating parameters of the trap.

## THE GEOMETRY OF THE SYSTEM AND THE COLLISIONLESS MODEL CONFINEMENT OF ELECTRONS

In numerical modeling we consider a quasi-spherically symmetric multicusp magnetic field, created by current coils, located on the faces of the cube and switched on in such a way that the same poles of the magnetic field each of these coils are directed to the center of the device.

The number of cusps in such a system, equal to the sum of the number of vertices and the number of faces, for a cube is 14 . Six cusps lie on the axes passing through the center of the cube and the centers of its faces (the centers of the coils), eight cusps lie on the axes passing through the center of the cube and its vertices.

It is assumed that the axis of the beam of negative ions can pass through the middle of the opposite edges of the cube, and the deviation of ions in magnetic fields can be ignored.

The geometry of the main elements of the trap considered in the work is shown in Fig. 1.


FIGURE 1. The basic elements of a quasi-spherically symmetric multicusp magnetic trap.

In the paper [4], the confinement time of an electron $\tau_{c}$ in a trap with a quasi-spherically symmetric multicusp geometry is estimated by the value

$$
\begin{equation*}
\tau_{c}=\tau_{\text {transit }} \times M^{*}=\frac{2 R}{v} \times M^{*} \tag{1}
\end{equation*}
$$

were $\tau_{\text {transit }}=2 R / v$ - free transit time of an electron through the system, $R-$ radius of system, $M^{*}=B_{0} / B_{\text {min }}$ -
mirror ratio for a point cusp. Here $B_{0}$ is the maximum value of the magnetic field induction on the cusp axis, Bmin is the value of the magnetic field induction at the point, where the adiabatic invariant of motion for the electron begins to be conserved.

To find the value Bmin, it is assumed that at this point the Larmor radius of the electron is approximately equal to the scale of the change in the magnitude of the magnetic field induction

$$
\begin{equation*}
L \equiv B /|\nabla \mathrm{B}| \approx r_{L} \tag{2}
\end{equation*}
$$

where $r_{L}$ is the Larmor radius of the electron.
In the case of a quasi-spherically symmetric magnetic field, the magnetic field induction value can be represented as $B=B_{0}(r / R)^{n}$, where the index $n$ depends on the geometry of the system.

By equating $L=r_{L}=m v / q B$, we can find the value Bmin, starting from which the electron movement can be considered adiabatic

$$
\begin{equation*}
B_{\min }=\left(B_{0}\right)^{\frac{1}{n+1}}\left(\frac{n m v}{q}\right)^{\frac{n}{n+1}} R^{-\frac{n}{n+1}} \tag{3}
\end{equation*}
$$

and the value of the mirror ratio

$$
M^{*}=\frac{B_{0}}{B_{\min }}=\left(\frac{R}{n}\right)^{\frac{n}{n+1}} q^{\frac{n}{n+1}} B_{0}^{\frac{n}{n+1}}(2 m E)^{-\frac{n}{2(n+1)}}=\left(\frac{R}{n r_{L}}\right)^{\frac{n}{n+1}},
$$

here $q$ is the electron charge and $m v=(2 m E)^{1 / 2}$ is replaced .
The electron confinement time according to the formula (1) is equal

$$
\begin{equation*}
\tau_{c}=\tau_{\text {transit }} \times M^{*}=2^{\frac{1}{2(n+1)}} R^{\frac{2 n+1}{n+1}}(n)^{-\frac{n}{n+1}}\left(q B_{0}\right)^{\frac{n}{n+1}} m^{\frac{1}{2(n+1)}} E^{-\frac{2 n+1}{2(n+1)}} \tag{4}
\end{equation*}
$$

## NUMERICAL MODELING

To verify the dependences of the confinement time of sample particles in this trap on the magnetic field induction and electron energy, resulting from the relation (4) and mentioned above, numerical experiments on the confinement of electrons in a multicusp trap with a quasi-spherically symmetric magnetic field geometry were performed. Numerical experiments were performed using the COMSOL 5.2 software package [6], which allows simulating the 3-dimensional geometry of magnetic fields and electrons motion.

The magnetic field in the model system is created by six identical current coils located on the faces of a cube with edge length of 10 cm . The center of the magnetic system is located at a point with coordinates $(0,0,0)$. The centers of the inner ends of the coils have coordinates $(5,0,0),(-5,0,0),(0,5,0),(0,-5,0),(0,0,5),(0,0,-5) \mathrm{cm}$. The coils have an outer radius of 5 cm , an inner radius of 3 cm , and a length of 2 cm . The number of turns in each coil was 80 turns. The current value varied from 62.5 to 500 A . The magnetic field induction $B_{0}$ at these values varied from 0.0635 T to 0.508 T .

The distribution of magnetic field induction, calculated for the current value in coils 125 A in the $\mathrm{X}=0$ plane is shown in Fig. 2. The distribution of the magnetic induction value along one of the coordinate axes is shown in Fig.3.

To compare the results of numerical experiments with the theoretical dependence (formula 4), it was necessary to determine the index of power " $n$ " of the values of the magnetic field induction on the axis of the system. To do this, a power function was found that approximates the values of magnetic induction along one of the axes (for example, the Z axis).


Fig. 2. Distribution of magnetic field induction for the current value in coils 125 A ( 10 kA * turns) in the plane $\mathrm{X}=0$. Two current coils out of six are marked.


Fig. 3. Distribution of the magnetic induction value along the Z axis $(\mathrm{X}=0, \mathrm{Y}=0)$. For reference, the position of two current coils on the Z axis is indicated.

The values of magnetic induction were approximated by a power function for $Z$ values in the range $Z=0-4 \mathrm{~cm}$. As a result of this approximation, the value of the index $n \approx 5 / 2$ was obtained.

Accordingly, formula (4) takes the form

$$
\begin{equation*}
\tau_{c}=\tau_{\text {transit }} \times M^{*} \approx 1.4 \times 10^{-2} \times R^{12 / 7} \times E^{-6 / 7} \times B_{0}^{5 / 7} \tag{5}
\end{equation*}
$$

where $R$ is expressed in $\mathrm{m}, E$ in eV , and $B_{0}$ in T .
Electron emission was modeled from a disk-shaped surface with a radius of 1 cm , located inside a magnetic system with the coordinates of the disk center $(0,0,0) \mathrm{cm}$. Electrons were emitted from the disk surface into a hemisphere.

The particle counter registered electrons that reached the edges of a cube with an edge length of 14 cm centered at a point $(0,0,0)$, completely covering the magnetic system.

In each numerical experiment, 100 electrons were simulated. An example of calculating trajectories for the electron energy of 10 eV at the value $B_{0}=0.127 \mathrm{~T}$ (at $10 \mathrm{kA} \cdot$ turns) is shown in Fig. 4.

From this drawing, it can be seen that the electrons leave the system through cusps whose axes pass through the centers of the faces or vertices of the cube.

The electron confinement time $\tau_{c}$ was taken as the time, during which half of the initial number of emitted electrons flew out to the particle counter.

At the value $B_{0}=0.127 \mathrm{~T}$, numerical experiments were performed with the energy of emitted electrons $1.4 ; 3 ; 5$; 7; 10; 12; 20; 50 eV .

At an electron energy of 10 eV , numerical experiments at the values of currents in coils $62.5 \mathrm{~A}, 100 \mathrm{~A}, 125 \mathrm{~A}$, $187.5 \mathrm{~A}, 250 \mathrm{~A}, 312.5 \mathrm{~A}, 500 \mathrm{~A}$ were performed. At these values of currents, the value of induction $B_{0}$ was 0.0635 $\mathrm{T}, 0.102 \mathrm{~T}, 0.127 \mathrm{~T}, 0.19 \mathrm{~T}, 0.254 \mathrm{~T}, 0.317 \mathrm{~T}$, and 0.508 T , respectively.


Fig. 4. Trajectories of electrons with an energy of 10 eV at the value $B 0=0.127 \mathrm{~T}(10 \mathrm{kA}$ *turns $)$, calculated for the time interval $2.5 \cdot 10^{-6} \mathrm{~s}$.

## CORRECTION OF THE COLLISIONLESS MODEL CONFINEMENT OF ELECTRONS

As a result of simulations, a significant difference between the values of the confined time $\tau_{c}$ obtained in numerical experiments and calculated in accordance with the formula (5) was found.

To explain these discrepancies, it was assumed that the value $R$ (the size of the system) in the formula (1) should be replaced by the value $r_{r}$, where $r_{r}$ is the distance from the center to the point, where the particle is reflected from the magnetic field mirror.

The magnetic field induction at the turning point $B_{r}$ when the adiabatic motion condition is fulfilled, is determined by relation

$$
\frac{B_{r}\left(r_{r}\right)}{B_{\min }}=\frac{E}{E_{\perp}}
$$

where $B_{\text {min }}$ is the magnitude of magnetic field induction at the point at which begins to satisfy the condition of conservation of adiabatic invariant of motion for an electron (2), $E$ - full kinetic energy of the electron, $E \perp$ is the kinetic energy of the transverse motion of the electron at the point in which begins to satisfy the condition of conservation of adiabatic invariant of motion for the electron in accordance with the formula (2).

Using the found value Bmin from formula (3), the value $r_{r}$ is found and substituted for $R$ in formula (4) to calculate $\tau_{c}$. Thus, instead of formula (4), we get the value for $\tau_{\mathrm{c}}$

$$
\begin{aligned}
\tau_{c} & =\left(\frac{E}{E_{\perp}}\right)^{\frac{1}{n}} 2^{\frac{1}{(n+1)}} R^{\frac{2 n}{n+1}}(n)^{-\frac{n-1}{n+1}}\left(q B_{0}\right)^{\frac{n-1}{n+1}} m^{\frac{1}{(n+1)}} E^{-\frac{n}{n+1}}= \\
& \approx\left(\frac{E}{E_{\perp}}\right)^{2 / 5} \times 5 \times 10^{-4} \times R^{10 / 7} \times B_{0}^{3 / 7} \times E^{-5 / 7}
\end{aligned}
$$

At $B<B$ min (electron movement is free), we can consider $\left\langle v_{x}^{2}\right\rangle=\left\langle v_{y}^{2}\right\rangle=\left\langle v_{z}^{2}\right\rangle=1 / 3\left\langle v^{2}\right\rangle$ or $\left\langle v_{x}^{2}\right\rangle+\left\langle v_{y}^{2}\right\rangle=$ $2 / 3\left\langle v^{2}\right\rangle$. Then, taking $E_{\perp}=2 / 3 E$, we get

$$
\begin{equation*}
\tau_{c} \approx 6 \times 10^{-4} \times R^{10 / 7} \times B_{0}^{3 / 7} \times E^{-5 / 7} \tag{6}
\end{equation*}
$$



Fig.5. The dependences of the confinement time $\tau_{\mathrm{c}}$ on the electron energy (a), and magnetic field induction in cusp $B 0$, (b), calculated using the formula (5), calculated using the formula (6) and the points, obtained from numerical experiments.

Figure 5 shows the dependences of the confinement time $\tau_{c}$ on the electron energy and magnetic field induction in cusp $B_{0}$, calculated using the formula (5), calculated using the formula (6) and the points, obtained from numerical experiments.

As can be seen from the graphs, formula (6), which takes into account the dependence of the reflection point position on the electron energy, corresponds much better to the results of numerical experiments.

## DISCUSSION

The results of numerical experiments for the confinement time of electrons in a trap with a quasi-spherically symmetric multicusp geometry of the magnetic field allow us to conclude that the updated formula for the confinement time (6) corresponds fairly well to the results of numerical experiments.

In addition, the confinement time value obtained from formula (6) agrees well enough with the experimental result [5] under the assumption of the average energy of plasma electrons $E \approx 5 \mathrm{eV}$.

Based on this, formula (6) can be used for scaling parameters when designing a target plasma trap.
For example, to obtain the confinement time $\tau_{\mathrm{c}} \approx 30 \mu \mathrm{~s}$ (which corresponds to the duration of the negative ion beam for injection into the proton synchrotron $\mathrm{U}-1.5$ ), assuming an average plasma electron energy of about 5 eV , based on the formula (6), it is necessary to obtain a parameter $R^{10 / 7} B_{0}^{3 / 7}$ of about 0.16 .

The corresponding dependence the radius of the system on the magnetic field induction is shown in Fig. 6.


Fig. 6. The dependence of the radius $R$ (solid line) and the energy deposited to create the plasma $W$ (dotted line), on the magnetic field $B_{0}$ to ensure the confinement time $\tau_{\mathrm{c}} \approx 30 \mu \mathrm{~s}$ and the thickness of the target $10^{21} \mathrm{~m}^{-2}$.

To achieve the maximum yield of neutral atoms at a beam energy of 30 MeV , the thickness of the plasma target must be $n_{e} l=n_{e} 2 \sqrt{2} \mathrm{R} \approx 10^{21} \mathrm{~m}^{-2}$, i.e. the electron density is $n_{e} \approx 3.5 \cdot 10^{20} / R^{\mathrm{m}^{-2}}$.

Assuming the energy price of the ion of the created plasma $\omega$ is about 150 eV (which is close to the values obtained in the experiment [5]), the energy deposited to create the plasma should be the value

$$
W=\omega \times n_{e} \times 4 / 3 \pi R^{3} \approx 3,5 \cdot 10^{4} \times R^{2}[\mathrm{~J}]
$$

Figure 6 shows the dependences of the radius of the system $R$ and the energy deposited to create the plasma $W$, depending on the magnitude of the magnetic field in the cusp of the system while providing a confinement time of $\tau_{\mathrm{c}} \approx 30 \mu \mathrm{~s}$ and the thickness of the target $10^{21} \mathrm{~m}^{-2}$.

Based on these data, it seems optimal to choose the value of the magnetic field in the cusp in the range of 0.5 to 1 T , while the radius of the system lies in the range of 0.28 to 0.34 m , and the energy required to create plasma lies in the range of 2.6 to 4 kJ .

## CONCLUSION

The representation of electron motion in the collisionless approximation in the form of reflections from individual mirrors and free flights through the central region of the magnetic field with loss of adiabaticity, expressed as the ratio (6), taking into account the dependence of the reflection point position on the electron energy, is quite well consistent with the results of numerical modeling.

The values of the magnetic field induction, the system size, and the energy consumed, which are obtained in this approximation, are considered acceptable and can be used for development a plasma target of a beam neutralizer with an energy of 30 MeV and duration of $30 \mu \mathrm{~s}$ for the implementation of multi-turn charge-exchange injection from a linear accelerator LU-30 to a proton synchrotron U-1.5.

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