Caustics in non-linear Compton scattering

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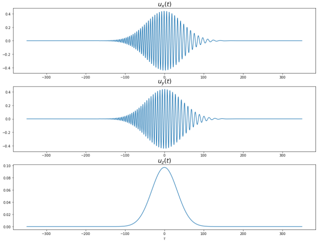
Mathematical model

A short pulse with a parametric envelope and chirp.

$$u_x \sim E\,(\tau, \alpha)\, sin(\Phi(\tau, \alpha) + \phi_x)$$

$$u_y \sim E\,(\tau, \alpha)\, sin(\Phi(\tau, \alpha) + \phi_y)$$

$$u_z \sim u_x^2 + u_y^2$$



We are interested in trying to optimize parameters and scattering angles for narrow-band radiated spectrum.

$$\frac{d^2 I}{d\omega d\Omega} \sim \left| \int_{-\infty}^{+\infty} \mathbf{n} \times [\mathbf{n} \times \mathbf{u}] \exp\left(i\omega(\tau + z - \mathbf{nr})\right) d\tau \right|^2 \qquad (1)$$

Possible approaches to optimization

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3. Analysis of stationary phases in the integral (1).
   - Analytic but asymptotic.
   - Only applicable for special cases of envelope and phase.
   - Saves many computational resources.

Stationary phase method

For integrals of type

$$\int g(x)e^{ikf(x)}dx$$

most of its value is contained at the critical points of $f(x)$ when $k \to \infty$ when $g(x)$ support is compact. The rest of oscillations cancel out.

Linearly-chirped pulse with **short** Gaussian envelope makes integral (1) exactly of that type.
By finding $\nabla f(x)$ we are able to compute the integral efficiently for given values of frequency, angles and chirp parameter.

Ray surfaces and caustics

The set of critical points $\nabla f(x) = 0$ can be represented as a surface in cylinder coordinates

$$\omega = \frac{1 - \frac{\chi\tau}{\sigma^2(1+\chi)}}{1 + u_z(1 - \cos\theta)},$$

Projecting the surface along the variable of integration is equivalent to computing the integral.

Caustic patterns emerge as bright lines $\Rightarrow$ bright and narrow radiated spectrum.

Numerical confirmation

For linearly chirped pulses the problem can be reduced from 3 to 1 parameter optimization with very efficient computations.

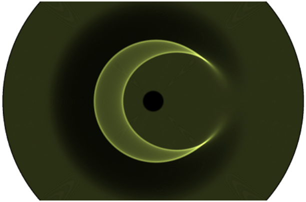


Figure: Ray surface projection with visible caustics

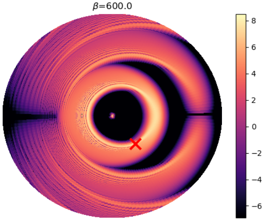
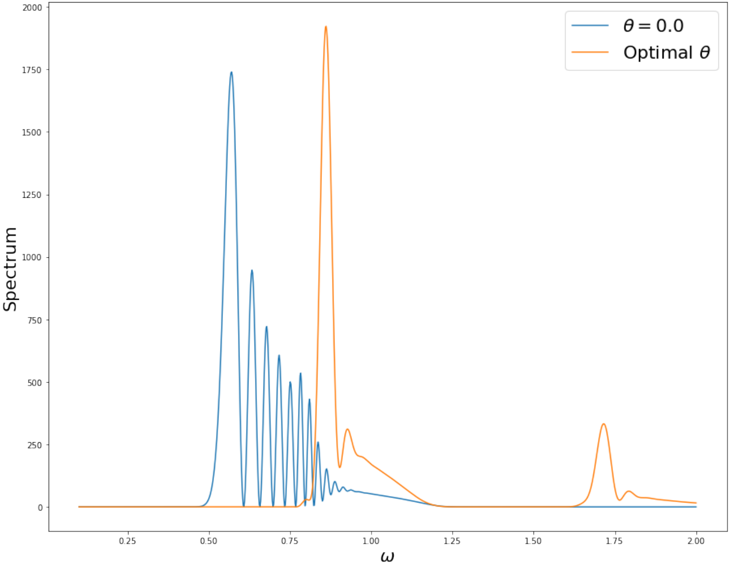


Figure: Numerical simulation. The red cross indicates the maximum of spectra, corresponding to the caustic.

Numerical confirmation

Optimized spectrum is much more concentrated and radiates brighter.



Conclusion

- We showed how stationary phase method and emergent caustics can be utilized to optimize the Compton scattered spectrum.
- The method can be extended to a range of settings as long as the set of critical points can be found.