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Analytical model of a stationary electric double layer under laser irradiation of a target

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22, October, 2026

Stationary Electric Double Layer (DL): Introduction

Quasi-neutral expansion

A.V. Gurevich …, JETP, 49, 647 (1965).

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Charge-separation models

P. Mora, Phys.Rev.Lett., 90, 185002 (2003).

E.A. Govras…, JETPLett., 98, 78 (2013).

Coulomb explosion

A.E.Kaplan, Phys.Rev.Lett., 91, 143401 (2003).

V.Yu.Bychenkov, JETPLett., 94, 101 (2011).

Stationary Double Layer (DL)

J.E.Crow, … J.Pl.Phys., 14, 65 (1975). | J.Schreiber, … PRL, 97, 045005 (2006). | M.Passoni, … Laser&Part.Beams, 22, 163 (2004).

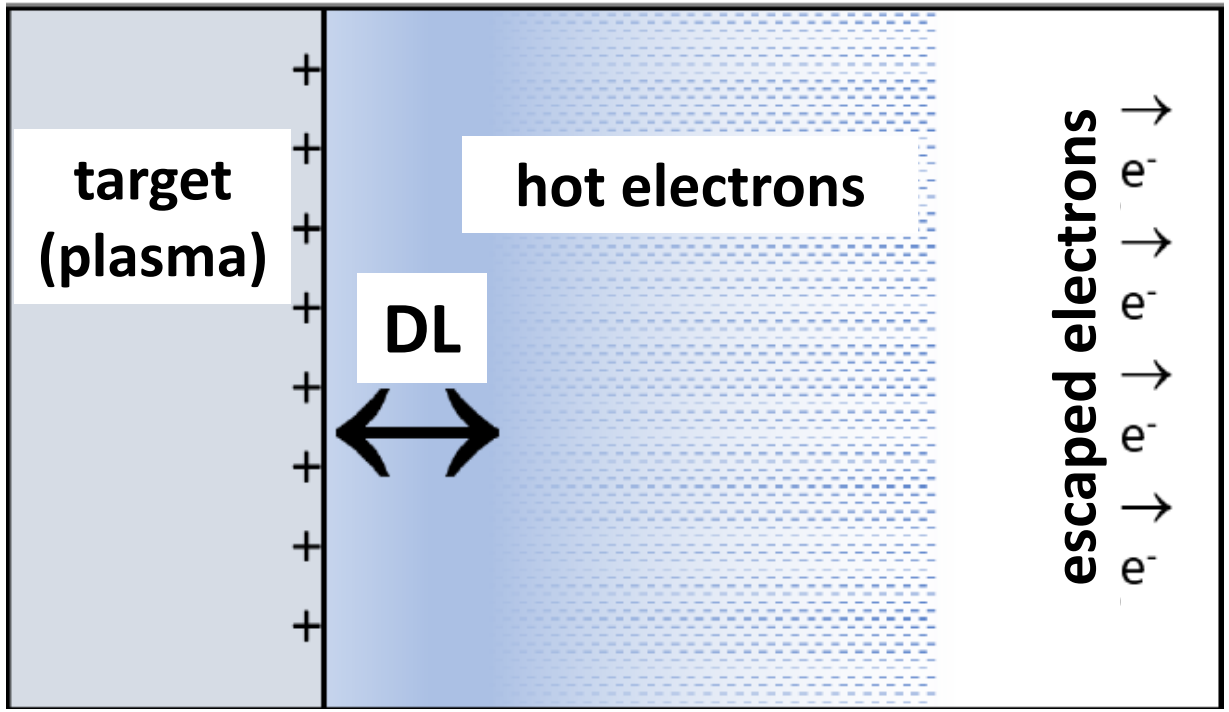
M.Lontano, … Phys.Plasmas, 13, 042102 (2006). | A.Macchi, *A Superintense Laser-Plasma Interaction Theory Primer,* (2013).

V.Mazhukin, … Mathematica Montisnigri, XLIV, 110 (2019). **Dependence upon laser-plasma parameters?**

Key Points for Discussion of DL Structure

1. Theoretical model for the nonlinear steady-state electric DL at the target-vacuum interface.
2. Model is described by an integrable system, enabling an analytical solution for n_e and $\mathcal{E}$ within DL.
3. DL formed by laser interaction with a dense target is characterized in terms of three control parameters:
   1) the total charge of the laser-heated ("hot") electrons,
   2) the magnitude of the charge lost by the target, and
   3) the hot-electron temperature.
4. For significant charge losses the maximum energy of ions accelerated within the double layer via the TNSA exhibits a non-trivial dependence on the laser pulse duration and focusing sharpness.

DL Theoretical Model



$Q_e > 0$ -- total charge of the laser-heated electrons;
$\Delta Q_e > 0$ -- total charge loss by the target

$\Sigma_e = Q_e/S$; $\sigma_e = \Delta Q_e/S$, $S \sim$ (10-100) μm^2;

Typical laser-plasma parameters

$\omega_{pe}^{-1} \ll \tau \lesssim \omega_{pi}^{-1}$; $T_e \sim (1-10)$ MeV; $\Delta Q_e \sim (1-100)$ nC

Basic Equations

$$e\mathcal{E} = -T_e \frac{d}{dx}\ln(n_e), \qquad \frac{d\mathcal{E}}{dx} = -4\pi e n_e .$$

Boundary conditions

$$n_e(0) = n_0 , \quad n_e(\infty) = 0 , \qquad \mathcal{E}(\infty) = 4\pi\sigma_e .$$

Charge balance condition

$$4\pi \int_0^\infty n_e(x)dx = \mathcal{E}(0) - \mathcal{E}(\infty), \rightarrow \mathcal{E}(0) = 4\pi\Sigma_e ;$$

Irradiation by a **high-contrast ultrashort laser pulse creates a steep**, virtually **step-like density profile at the front** target boundary, whereas **the rear target-vacuum interface,** unaffected by the laser, constitutes **a sharp boundary a priori**. **The assumption of high laser contrast** allows to use a semi-infinite medium model in the DL theory **for both sides of the target**;

Analytical solution for equations of stationary DL

Analytical solution for $\delta > 0$

$$n_e = n_0 A^2 \mathrm{sh}^{-2}\left(A\frac{x}{\sqrt{2}\lambda_D} + \mathrm{arsh}(A)\right),$$

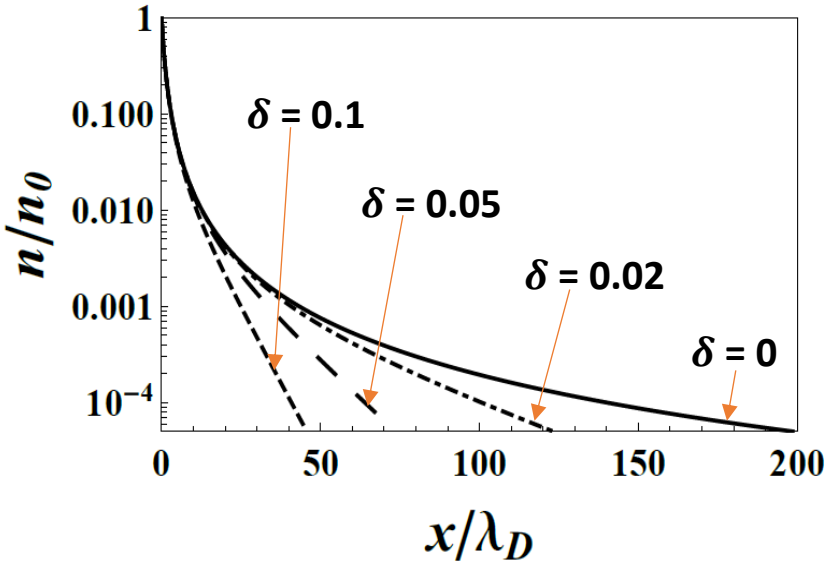
$$\mathcal{E} = \mathcal{E}_0\delta\,\mathrm{cth}\left(A\frac{x}{\sqrt{2}\lambda_D} + \mathrm{arsh}(A)\right), \qquad A = \frac{\delta}{\sqrt{1-\delta^2}}.$$

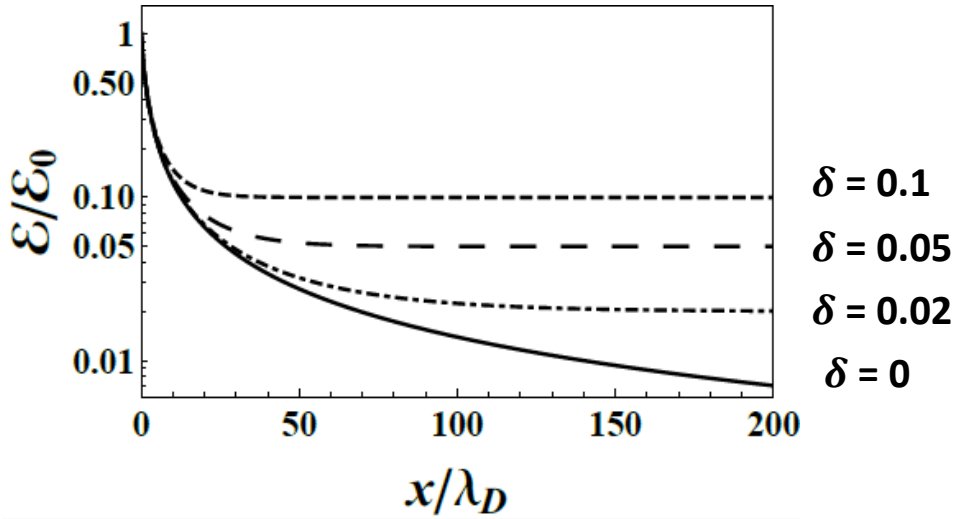
Analytical solution for $\delta = 0$

$$n_e = n_0\left(\frac{x}{\sqrt{2}\lambda_D} + 1\right)^{-2}, \qquad \mathcal{E} = \mathcal{E}_0\left(\frac{x}{\sqrt{2}\lambda_D} + 1\right)^{-1},$$

$$n_0 \equiv n_e(0) = \frac{2\pi}{T_e}\Sigma_e^2\left(1-\delta^2\right), \qquad \mathcal{E}_0 \equiv \mathcal{E}(0) = 4\pi\Sigma_e,$$

$$\lambda_{D0} = \frac{T_e}{2\sqrt{2}\pi e\Sigma_e}, \qquad \lambda_D = \frac{\lambda_{D0}}{\sqrt{1-\delta^2}}, \qquad \delta = \frac{\sigma_e}{\Sigma_e},$$





Characteristics of DL & Laser characteristics

Energy balance condition

$$\eta W_L = \int \left(n_e T_e + \mathcal{E}^2/8\pi\right) dV\,, \quad W_L \equiv \left(a_0^2/2\right) n_{c0} m_e c^3 \tau S_0\,, \quad S_0 = \pi r^2.$$

W_L - total energy of a laser pulse; η - conversion efficiency of laser energy into hot-electron energy,

τ - pulse duration; r - radius of a focal spot; a_0 – dimensionless amplitude of a laser pulse.

Integration over volume V: $S \simeq \pi\,(r + c\tau)^2 = S_0(1 + c\,\tau/r)^2\,, \quad x \simeq R = r + c\tau.$

S – expanded hot spot ; V. Yu. Bychenkov, A. V. Brantov, *Phys.Plasmas* 28, 063106 (2021).

Equation for surface charge density

$$\delta^2 G^2 + \frac{G}{\pi^2}\left(1 - \delta\,\mathrm{cth}\left(2\pi^2\delta G + \mathrm{arsh}(A)\right)\right) = \kappa; \quad \kappa = \left(\frac{m_e c^2}{T_e}\right)^2 \frac{R}{\lambda_0}\,\frac{\eta W_L}{2\pi^3 m_e c^2 n_{c0}\lambda_0^3}\,, \quad \frac{\Sigma_e}{e n_{c0}\lambda_0} = \frac{T_e}{m_e c^2}\,\frac{\lambda_0}{R}\,G\,.$$

$$\frac{\mathcal{E}_0}{e n_{c0}\lambda_0} = 4\pi\,\frac{T_e}{m_e c^2}\,\frac{\lambda_0}{R}\,G\,, \quad \frac{n_0}{n_{c0}} = 2\pi^2\left(1 - \delta^2\right)\frac{T_e}{m_e c^2}\left(\frac{\lambda_0}{R}\,G\right)^2\,, \quad \frac{\lambda_0}{\lambda_D} = 2\pi^2\sqrt{2(1-\delta^2)}\,\frac{\lambda_0}{R}\,G.$$

For the so-called $J \times B$ mechanism of laser electron heating: $T_e = m_e c^2(\gamma - 1)$, where $\gamma = \sqrt{1 + a_0^2/2}$.

S. C. Wilks, ... Phys. Rev. Lett. Phys. Rev. Lett. v.69, 1383 (1992). Hui Chen, ... *Phys. Rev. Lett.* 114, 215001 (2015).

$$\frac{\mathcal{E}_0}{e n_c \lambda_0} = 4\pi\,\frac{\lambda_0}{R}\,G, \quad \frac{n_0}{n_c} = 2\pi^2\left(1 - \delta^2\right)\left(\frac{\lambda_0}{R}\,G\right)^2, \quad \frac{\lambda_0}{\lambda_D} = 2\pi^2\sqrt{2(1-\delta^2)}\,\frac{\lambda_0}{R}G, \quad \frac{\Sigma_e}{e n_c \lambda_0} = \frac{\lambda_0}{R}\,G, \quad \kappa = \frac{\eta}{2\pi^2}\left(\frac{r}{\lambda_0}\right)^2\frac{c\tau}{R}, \quad n_c = \gamma n_{c0}.$$

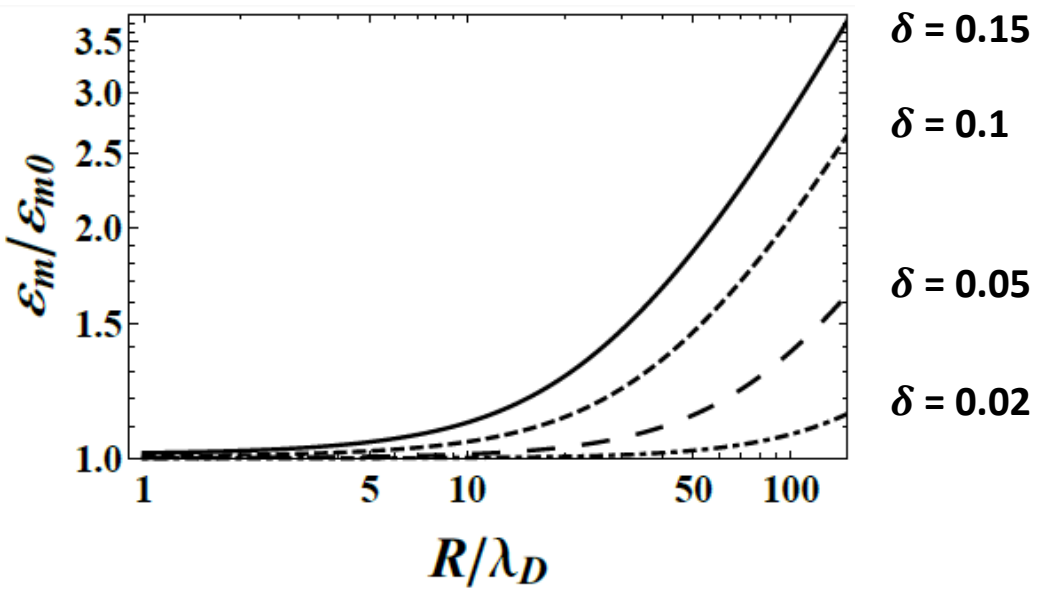
Ion acceleration in DL

Test particle approximation

$$\varepsilon_m = e_i[\phi(0) - \phi(R)] = ZT_e \ln\left(n_e(0)/n_e(R)\right)$$

$$\varepsilon_m = 2ZT_e \ln\left[\frac{1}{A}\,\mathrm{sh}\left(A\frac{R}{\sqrt{2}\lambda_D} + \mathrm{arsh}(A)\right)\right], \quad Z = e_i/e\,,$$

$$\varepsilon_{m0} = 2ZT_e \ln\left(\frac{R}{\sqrt{2}\lambda_D} + 1\right), \quad \sigma_e = 0, \qquad R = r + c\tau.$$



$$\varepsilon_m = 2ZT_e \ln\left[\frac{1}{A}\,\mathrm{sh}\left(2\pi^2\delta\, G + \mathrm{arsh}(A)\right)\right].$$

$$\frac{\varepsilon_m}{m_e c^2} = \sqrt{2}Za_0 \ln\left[\frac{1}{A}\,\mathrm{sh}\left(2\pi^2\delta\, G + \mathrm{arsh}(A)\right)\right], \quad a_0^2 \gg 1$$

$\delta \to 0$

$$G = \frac{\kappa\pi^2}{2}\left(1 + \sqrt{1 + \frac{2}{\kappa\pi^4}}\right), \qquad \kappa = \frac{\eta}{2\pi^2}\frac{r^2}{\lambda_0^2}\frac{c\tau}{R},$$

$$\frac{\varepsilon_{m0}}{m_e c^2} = \sqrt{2}Za_0 \ln\left[\kappa\pi^4\left(1 + \sqrt{1 + \frac{2}{\kappa\pi^4}}\right) + 1\right].$$

$2\pi^2\delta G + \mathrm{arsh}(A) > 1$

$$G = \frac{1}{2\pi^2\delta}\left[-\frac{1-\delta}{\delta} + \sqrt{4\pi^4\kappa + \left(\frac{1-\delta}{\delta}\right)^2}\right],$$

$$\frac{\varepsilon_m}{m_e c^2} = \sqrt{2}Za_0\left(\ln\frac{1}{2A} + \mathrm{arsh}(A) - \frac{1-\delta}{\delta} + \sqrt{4\pi^4\kappa + \left(\frac{1-\delta}{\delta}\right)^2}\right),$$

Examples of ion acceleration in DL

Laser pulse compression -- problem of shortening high-power, multi-terawatt femtosecond pulses to increase their intensity while keeping the total energy virtually constant.

$$\frac{\varepsilon_m}{m_e c^2} \simeq 10^2 Z\sqrt{W_L}\,K, \qquad K = \frac{\lambda_0/r}{\sqrt{c\,\tau/\lambda_0}} \ln\left[\frac{1}{A}\,\mathrm{sh}\left(2\pi^2\delta G + \mathrm{arsh}(A)\right)\right],$$

W_L *in* J, $\lambda_0 = 0.8\mu m$, $a_0 > 2$,

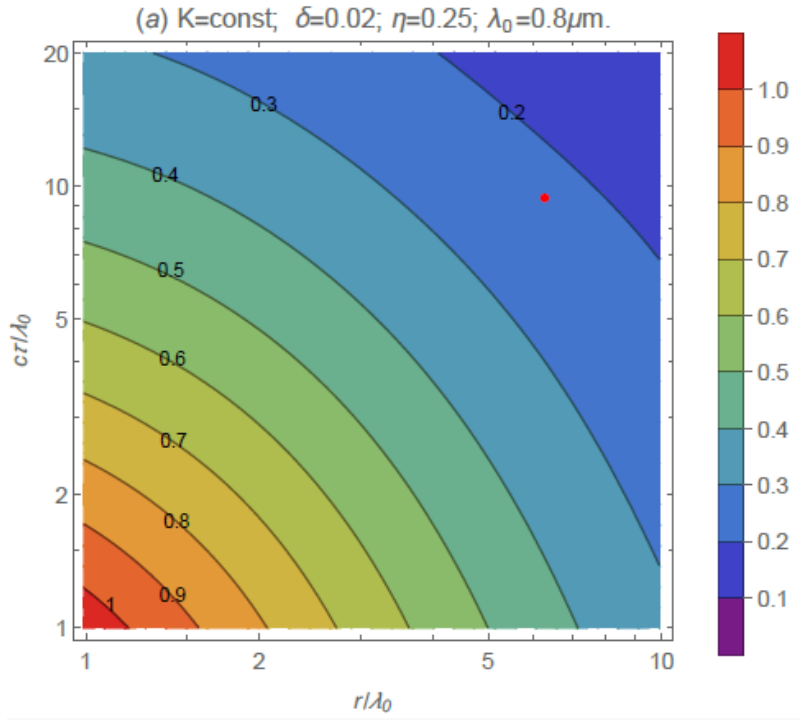
$$4.6\times 10^{-4}(c\,\tau/\lambda_0)\left(r/\lambda_0\right)^2 < W_L[\mathrm{J}]$$

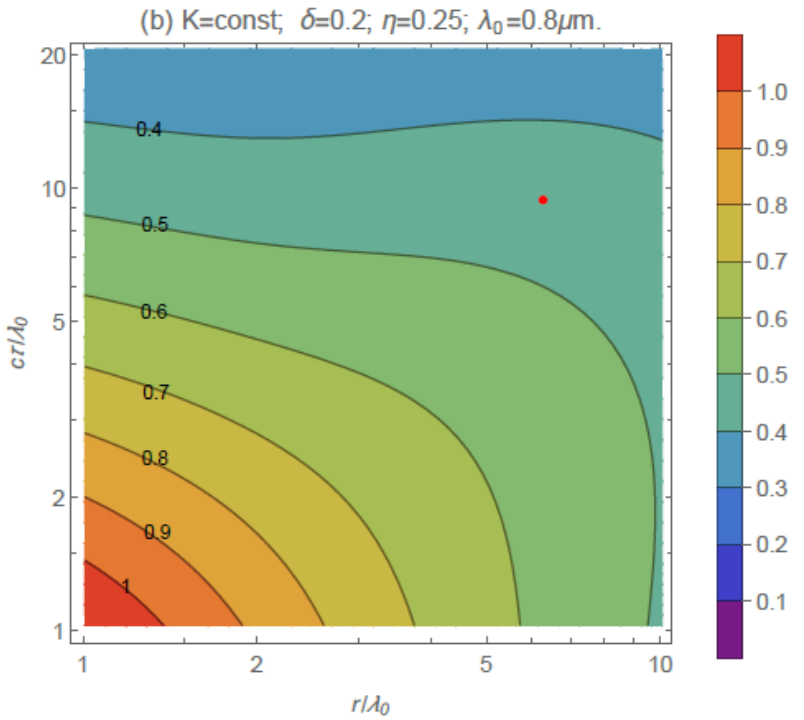
τ =25 fs, r =5 μm,
$r/\lambda_0 = 6.25$, $c\tau/\lambda_0 = 9.375$;
$\delta = 0.2$, $K \approx 0.448$;
$\delta = 0.02$, $K \approx 0.225$;

W_L =1 J (40 TBт, $a_0 = 4$),
$\varepsilon_m \approx 20$ MeV for $\delta = 0.2$,
$\varepsilon_m \approx 10$ MeV for $\delta = 0.02$,

W_L =10 J (400 TW, $a_0 = 12.4$)
$\varepsilon_m \approx 70$ MeV for $\delta = 0.2$,
$\varepsilon_m \approx 35$ MeV for $\delta = 0.02$

Experiment: $\tau = 30$ fs





K. V. Safronov, *et al,* JETP Letters, 121, Issue 7, 562 (2025). W_L =1.56 J , r =2 μm, $\varepsilon_{pr} \approx 10$ MeV; $\varepsilon_m \approx 20$ MeV;
S. Fourmaux, *et al,* Physics of Plasmas, 20, 013110 (2013). $W_L = 0.54$ J , r =3 μm, $\varepsilon_{pr} \approx 4$ MeV; $\varepsilon_m \approx 10$ MeV;

? Electron cooling
$R \to Rl_c/(R + l_c)$

Conclusions & What's next?

- The resulting solutions, which are of a novel form, are presented as analytical formulas.
- The theory demonstrates how the dependence of the Debye length on plasma parameters is modified.
- A significant achievement of the theory is its inclusion of the charge lost by the target; this is crucial for describing the DL in realistic experimental conditions and for understanding TNSA ion acceleration.
- We emphasize that the estimates presented here regarding ion energy gain in the DL are qualitative and problem-oriented, derived within the test-particle approximation.

This paves the way for developing a consistent theory of ion acceleration that accounts for the self-consistent redistribution of energy between the field and the particles. An analysis of the double-layer structure for cylindrical and spherical targets—including submicron targets with a thickness less than the Debye length, where field penetration into the target must be taken into account—is also of interest.

Stationary DL in Cylindrical Geometry

Basic Equations

$$e\mathcal{E} = -T_e \frac{d}{dr}\ln(n_e), \qquad \frac{d(r\mathcal{E})}{dr} = -4\pi e r n_e .$$

Analytical solution

$$n_e = n_0 \frac{C_1\lambda_D^2}{r^2}\,\mathrm{sh}^{-2}\left(\sqrt{\frac{C_1}{2}}\ln\frac{r}{\rho} + \mathrm{arsh}\left(\sqrt{\frac{C_1\lambda_D^2}{\rho^2}}\right)\right), \quad C_1 \geq 0 ,$$

$$\mathcal{E} = \frac{2T_e}{er}\left[1 + \sqrt{\frac{C_1}{2}}\,\mathrm{cth}\left(\sqrt{\frac{C_1}{2}}\ln\frac{r}{\rho} + \mathrm{arsh}\left(\sqrt{\frac{C_1\lambda_D^2}{\rho^2}}\right)\right)\right] .$$

$$\sqrt{C_1} = \frac{2\sqrt{2}\pi e\rho\sigma}{T_e}\left[\delta - \frac{T_e}{2\pi e\rho\sigma}\right], \qquad \delta = \frac{\sigma_e}{\sigma} ,$$

$$n_0 = \frac{2\pi\sigma^2}{T_e}(1-\delta)\left[1 + \delta - \frac{T_e}{\pi e\rho\sigma}\right] .$$

Boundary + Charge balance conditions

$$n_e(r=\rho) = n_0 , \quad n_e(\infty) = 0 , \qquad \mathcal{E}(r=\rho) = 4\pi\sigma.$$

$$\frac{2\sqrt{2}\pi e\lambda_D}{T_e}\sigma = \sqrt{1 + C_1\lambda_D^2/\rho^2} + \sqrt{2}\,\lambda_D/\rho , \quad \sigma - \sigma_{in} = \sigma_e ,$$

$$\frac{2\sqrt{2}\pi e\lambda_D}{T_e}\sigma_{in} = \sqrt{1 + C_1\lambda_D^2/\rho^2} - \sqrt{C_1\lambda_D^2/\rho^2} .$$

Characteristics of DL & Laser characteristics

- Energy balance condition
- Equation for surface charge density
- Specification of mechanism of laser electron heating
- Limitations on losses of electrons $\delta \ll \frac{\lambda_D}{\rho}$, ???

Analytical approach to stationary DL : self-consistent losses

Analytical solution

$$n_e = n_0 A^2 \mathrm{sh}^{-2}\left(A\frac{x}{\sqrt{2}\lambda_D} + \mathrm{arsh}(A)\right),$$

$$\mathcal{E} = \frac{\sqrt{2}T_e}{e\lambda_D} A \,\mathrm{cth}\left(A\frac{x}{\sqrt{2}\lambda_D} + \mathrm{arsh}(A)\right).$$

A =???

Boundary + Charge balance conditions

$$4\pi\Sigma_{in} = 4\pi e\int_0^R n_e(x)dx = -\mathcal{E}(R) + \mathcal{E}(0), \qquad \Sigma_{in} < \Sigma_e$$

$$\mathcal{E}_0 \equiv \mathcal{E}(0) = 4\pi\Sigma_e, \qquad \Sigma_{out} = \Sigma_e - \Sigma_{in}$$

$$4\pi\Sigma_e = \left(\sqrt{2}\,T_e/e\,\lambda_D\right)\sqrt{1+A^2},$$

$$4\pi\Sigma_{in} = \frac{\sqrt{2}T_e}{e\lambda_D}\left[\sqrt{1+A^2} - A\,\mathrm{cth}\left(A\frac{R}{\sqrt{2}\lambda_D} + \mathrm{arsh}(A)\right)\right],$$

$$4\pi\Sigma_{out} = \frac{\sqrt{2}T_e}{e\lambda_D} A \,\mathrm{cth}\left(A\frac{R}{\sqrt{2}\lambda_D} + \mathrm{arsh}(A)\right).$$

Equation for A(δ)

$$\delta = \frac{A}{\sqrt{1+A^2}}\,\mathrm{cth}\left(A\frac{R}{\sqrt{2}\lambda_D} + \mathrm{arsh}(A)\right), \qquad \delta = \frac{\Sigma_{out}}{\Sigma_e}.$$

$R \to \infty$

$$n_0 \equiv n_e(0) = \frac{2\pi}{T_e}\Sigma_e^2\left(1-\delta^2\right), \qquad \mathcal{E}_0 \equiv \mathcal{E}(0) = 4\pi\Sigma_e,$$

$$\lambda_{D0} = \frac{T_e}{2\sqrt{2}\pi e\Sigma_e}, \qquad \lambda_D = \frac{\lambda_{D0}}{\sqrt{1-\delta^2}}, \qquad A = \frac{\delta}{\sqrt{1-\delta^2}}.$$

$A \to 0, \qquad R < \infty$

$$\delta = \left(\frac{R}{\sqrt{2}\lambda_D} + 1\right)^{-1},$$

Small finite losses for limited values of R/ $\sqrt{2}\lambda_D$!!!

Thank you for attention!

Questions?