AFAD-2021

16-18 March 2021
Budker INP

# Laguerre Gaussian laser beam driven electron acceleration


***Harjit Singh Ghotra***

*Associate Professor*

***Advanced Study Hub** (ASH)*

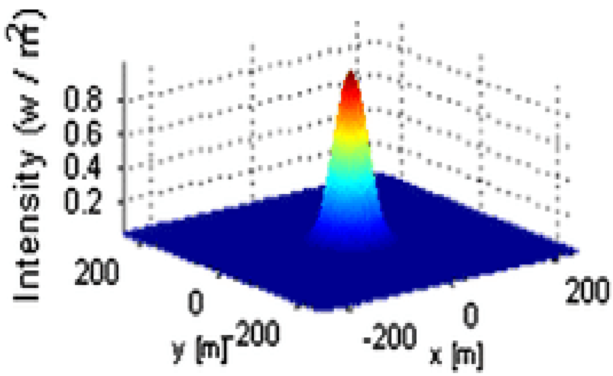
*Theoretical and Computational Research Form (TCRF)*

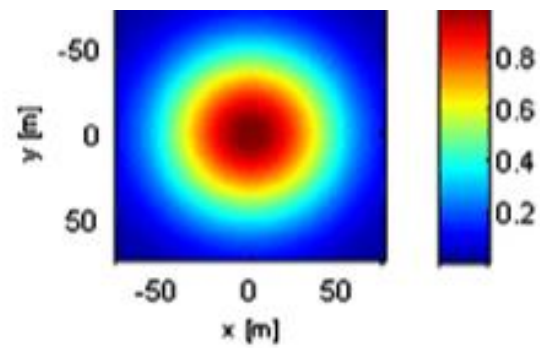
*Punjab, India*

# Outline

- Beam characteristics
  - Gaussian Beam
  - Laguerre Gaussian (LG) Beam
- A few benchmarks (theory and experiments)
- Transverse and Longitudinal field components of LG Beam
- Momentum and Energy gain equations
- Analysis with parameters
  - Low and High intensity
  - Presence and Absence of longitudinal fields
  - Mode index considerations
- Conclusion.

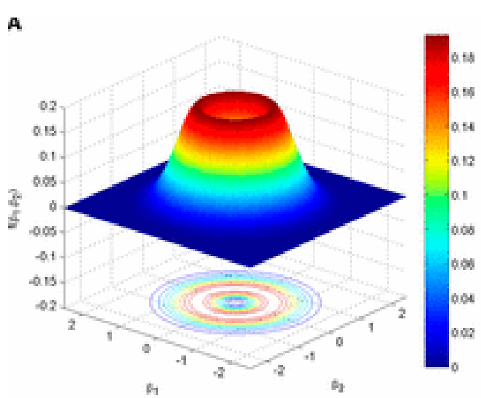
# Gaussian Beam vs Laguerre Gaussian Beam

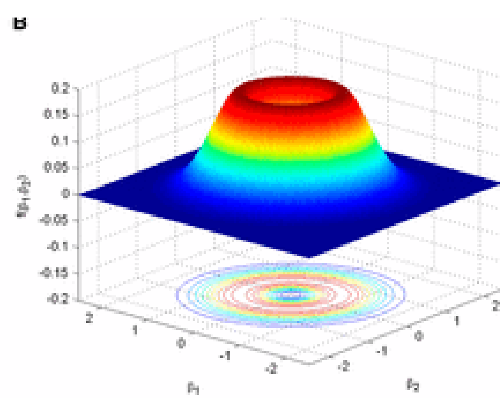



**Gaussian Beam**

- **Gaussian Intensity profile**
  $$I(r) = I_0 \exp(-2r^2 / w_0^2)$$
- **Possess Linear momentum.**
- **Remain Gaussian after reflection or refraction.**




**Laguerre Gaussian Beam**

- **Optical Vortices**
- **Helical beams**
- **Bottle beams**
- **Twisted beams**
- **Doughnut beams**

# Laguerre Gaussian Beam
## ( properties..)

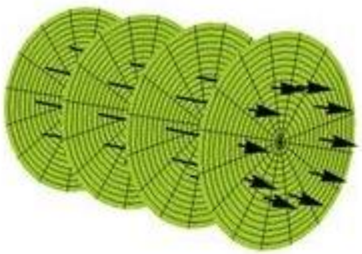
- Represented by $LG_p^l$
- Having azimuthal phase term
  $\exp(-il\phi)$
- Helical wavefront
- Have phase singularity

- Possesses
  - Spin Angular momentum due $\hbar$ to polarization
  - Orbital angular momentum of $l\hbar$ due to phase profile of the beam.
- Field amplitude is given by

$$LG_0^1(r,\phi) = \frac{2}{\sqrt{\pi}} \frac{r}{w_0^2} \exp\left(\frac{r^2}{w_0^2}\right) \exp(i\phi)$$

**Gaussian Beam**

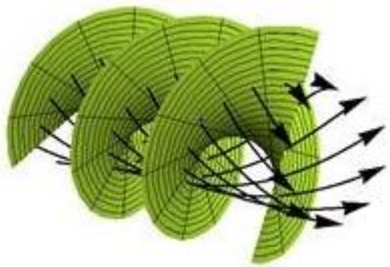
- Planar Wavefront



- Same Phase along the cross-section

**Laguerre Gaussian Beam**

- Helical Wavefront



- Phase varies $l$ times in a $LG^l$ beam

# Laguerre Gaussian Beam driven Electron Energy Gain (Some theoretical and Experimental works……)

- *C. Baumann and A. Pukhov (Phys. Plasmas **25**, 2018)* presented the electron dynamic in twisted light modes of relativistic intensity with **p=0** and **p≠0** LG modes. An efficient transfer of angular momentum from the twisted electromagnetic field to beam electrons has been observed.

- *J. Vieira et al. (Phys. Rev Letters **121**, 2018*) the reports on the generation of a discrete number of helical electron beams by using a superposition of linearly polarized and frequency shifted LG lasers (**p=0** and **m≠0** ) in a laser-wakefield setup. In their proposed scheme, the authors find that the number of distinct helices is given by the effective LG mode index m that is acquired by the wakefield.

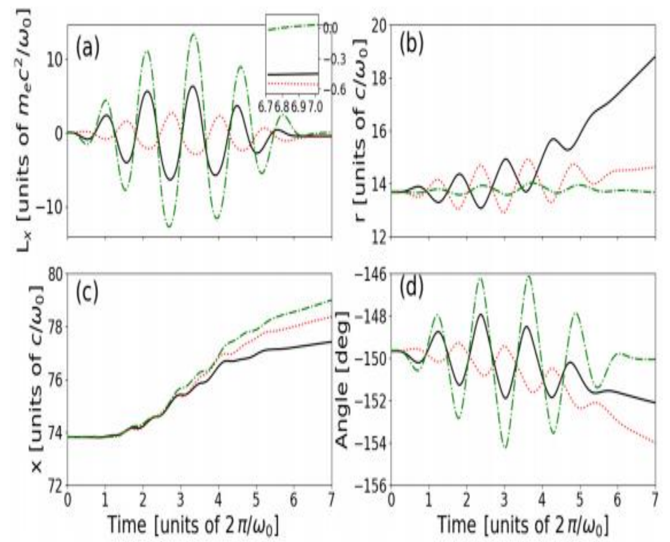
- *L.X. Hu et al. (Sci Reports **8**, 2018)* presented Attosecond electron bunches generation from a nanofiber driven by Laguerre Gaussian laser pulses. They observed the transfer of the laser OAM to the electron BAM is observed because of the circularly-symmetric non-zero azimuthal components of the LG laser field.

- *Z. Lin et al. (Appl. Sci. **8**, 2018)* investigated the Propagation Characteristics of High-Power Vortex Laguerre-Gaussian Laser Beams in Plasma. The propagation characteristics of the x-polarized high-power LG(2 1) and doughnut-shaped LG(1 0) laser beam.

# Laguerre Gaussian Beam driven Electron Energy Gain (Some theoretical and Experimental works……)

**Gain of electron orbital angular momentum in a DLA process** ( by R. *Nuter et al.* ) Physical Review E **101**, 053202 (**2020**)

- Similar to DLA where Gaussian laser beam transfer its axial momentum to electrons, the LG beam transfer a part of their OAM to electrons through dephasing process
- Quasistatic magnetic field generation in a plasma by a LP LG beam with a non zero OAM.
- The constant part of the longitudinal ponderomotive force as a dephasing process as it increases the electron veloci




**Polarized electon-beam acceleration driven by vortex laser pulses** (by Y. *Wu et al.*)

New Journal of Physics **21**, 073052 (**2019**)

- The vortex beam produces a donut-shaped electron bunche,
- The intensity of the LG laser beam is maximized off-axis, leaving a hollow space in the propagation center,
- There is generation of polarized electron beams via LWFA driven by a vortex LG Laser.

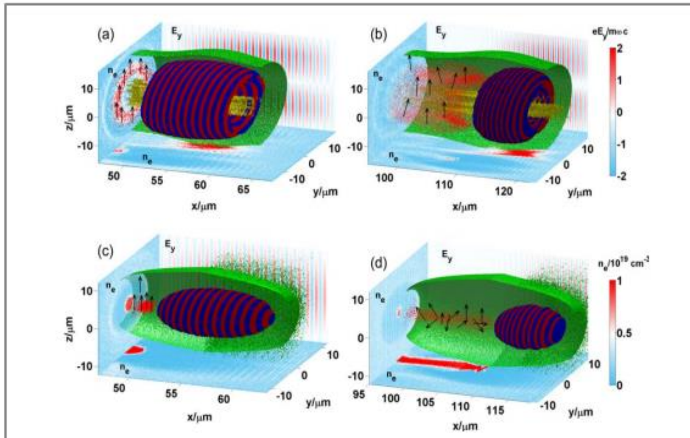



**Figure 2.** Electron density and laser field distributions (iso-surface) with $a = eE_y/m\omega c = 2$ and $x_f = 250\lambda$ at $75T_0$ and $150T_0$ for the LG laser (a) and (b) and for the Gaussian laser (c) and (d). The black arrows denote the electron spin directions. The electron density is projected onto the $x$–$y$ (at $z = 0$) and $y$–$z$ planes, while the laser electric field is projected onto the $x$–$z$ plane (at $y = 0$). For the LG laser, we choose $w_0 = 12.5\lambda$. The Gaussian beam also has a beam radius of $12.5\lambda$. Therefore at the same laser amplitude, the pulse energy of the former is 2.7 times of that for the latter.

# Laguerre Gaussian Beam profile

- Along the polarization axes, the complex-valued electric field of such a CP LG pulse in **transverse electric field** components: $\vec{E}_{\perp}^{LG}(r,z,t) = C_p^{|\ell|}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|\ell|} \exp[-il\vartheta)]\, L_p^{|\ell|}\left(\frac{2r^2}{w^2(z)}\right) E_{\perp}(r,z,t)$

  where, $C_p^{|\ell|}$ and $L_p^{|\ell|}$ are the normalization constant, and generalized Laguerre polynomial of LG laser beam respectively, $E_{\perp}(r,z,t)$ is the transverse component of Gaussian beam.

- The **longitudinal component** is obtained by using $E_z^{LG}(r,z,t) \cong -\left(\frac{i}{k}\right)\left(\frac{\partial E_x^{LG}}{\partial x} + \frac{\partial E_y^{LG}}{\partial y}\right)$

- The magnetic field components can be obtained by $\vec{B}^{LG}(r,z,t) = -\left(\frac{i}{\omega}\right)\left(\vec{\nabla}\times\vec{E}^{LG}\right)$

$$\vec{E}_z^{LG}(r,z,t) = i\left[\left(\frac{\sqrt{2}r}{w(z)}\right)^2(1+i\xi) - \frac{1}{L_p^{|l|}}\frac{\partial L_p^{|l|}}{\partial r} - \frac{|l|}{r}\right](\cos\phi\, E_x^{LG} + \sin\phi\, E_y^{LG}) - i\left(\frac{|l|}{r}\right)(\sin\phi\, E_x^{LG} - \cos\phi\, E_y^{LG})$$

$$B_z^{LG}(r,z,t) = i\left[\left(\frac{\sqrt{2}r}{w(z)}\right)^2(1+i\xi) - \frac{1}{L_p^{|l|}}\frac{\partial L_p^{|l|}}{\partial r} - \frac{|l|}{r}\right](\cos\phi\, B_x^{LG} + \sin\phi\, B_y^{LG}) + i\left(\frac{|l|}{r}\right)(\cos\phi\, B_y^{LG} - \cos\phi\, B_x^{LG})$$

# Momentum and Energy transformation equations

The relativistic equations governing the momentum and energy of the accelerated electron are expressed as:

$$\frac{dp_x}{dt} = -eE_x^{LG} + e\beta_z B_y^{LG} - e\beta_y B_z^{LG}$$

$$\frac{dp_y}{dt} = -eE_y^{LG} - e\beta_z B_x^{LG} + e\beta_x B_z^{LG}$$

$$\frac{dp_z}{dt} = -eE_z^{LG} - e\beta_x B_y^{LG} + e\beta_y B_x^{LG})$$

$$\frac{d\gamma}{dt} = -e(\beta_x E_x^{LG} + \beta_y E_y^{LG} + \beta_z E_z^{LG})$$

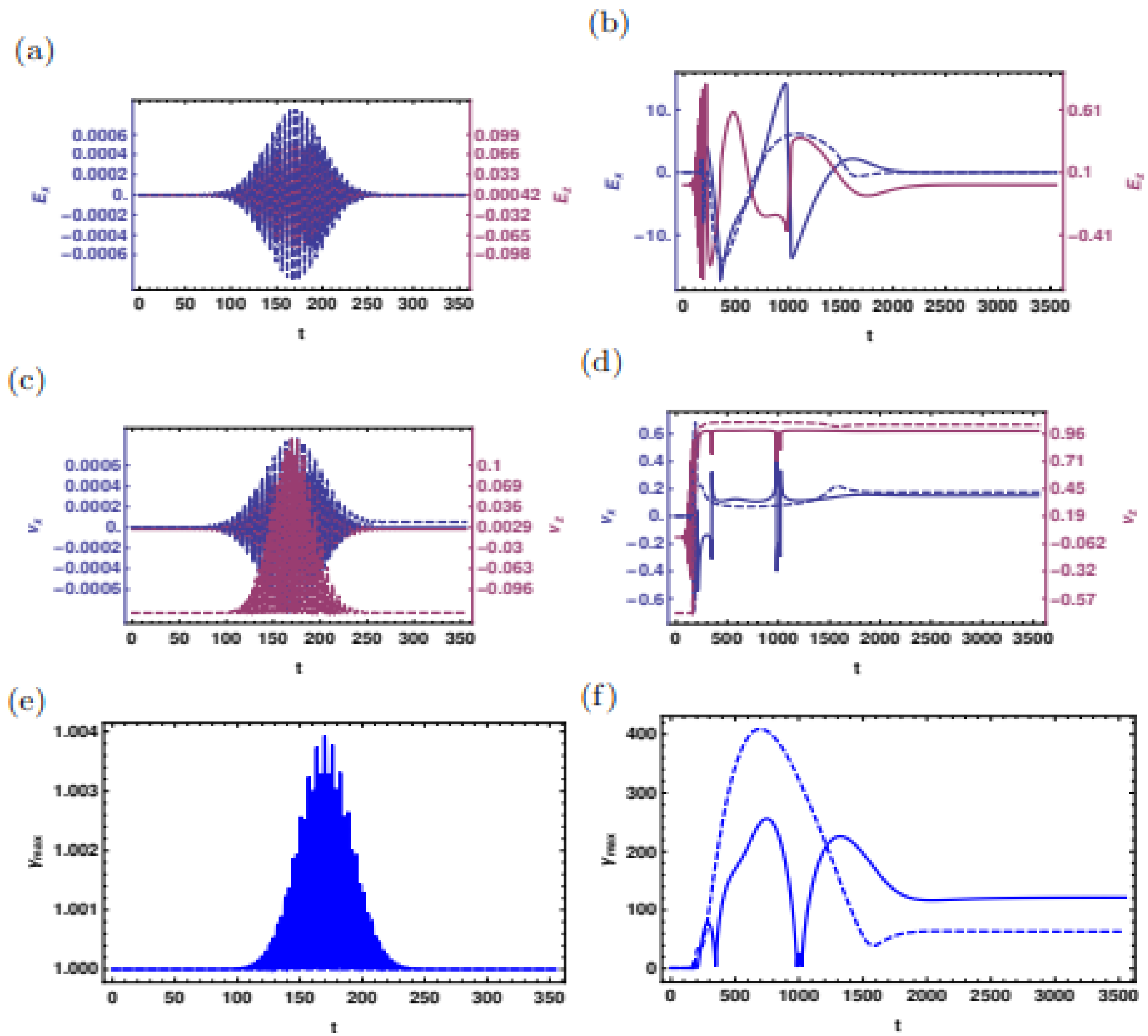
where $\gamma^2 = 1 + (p_x^2 + p_y^2 + p_z^2)/(m_0 c)^2$ is the relativistic factor,

$p_{x,y,z}$ and $\beta_{x,y,z}$ are the coordinates of normalized momentum and velocity respectively, $-e$ and $m_0$ are the electron's charge and mass respectively.

These equations form a set of coupled ordinary differential equations.

These equations have been solved numerically with a computer simulation code for electron trajectory and energy.

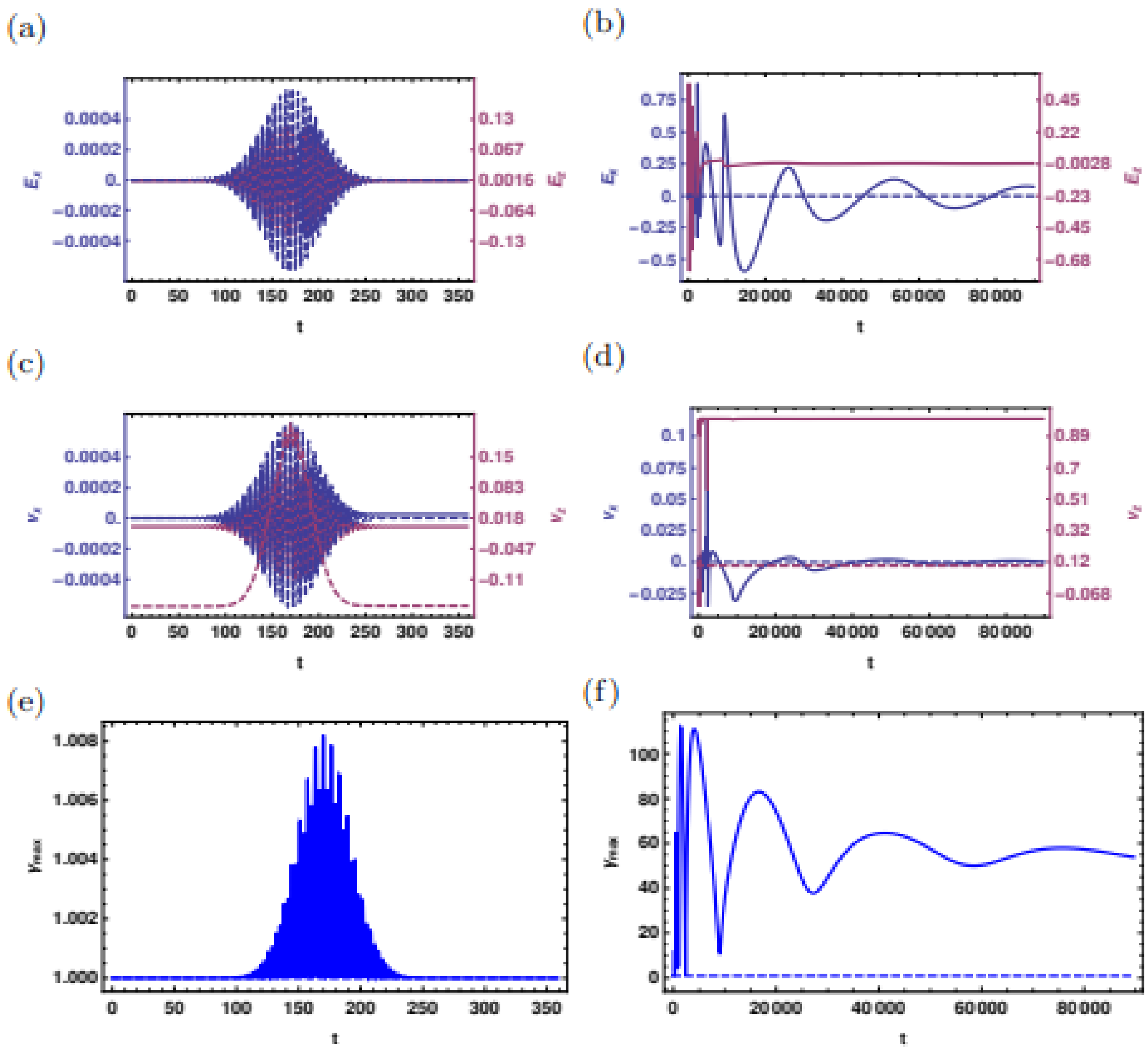
**Laguerre Gaussian (0,1) mode of LP laser beam**: Variation of longitudinal and transverse fields, velocities, and the maximum electron energy gain with (thick lines) and without (dash lines) longitudinal field as a function of "t"




- $a_0 = 3.0$,
- $I_0 = 19.46 \times 10^{18}$ W/cm$^2$,
- Ez > Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 12.38 TW
- Energy gain small
- Variations are same with and without Ez

- $a_0 = 30.0$,
- $I_0 = 19.46 \times 10^{20}$ W/cm$^2$,
- Ez ~ Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 1.23 PW
- Energy gain high
- Variations are different with and without Ez

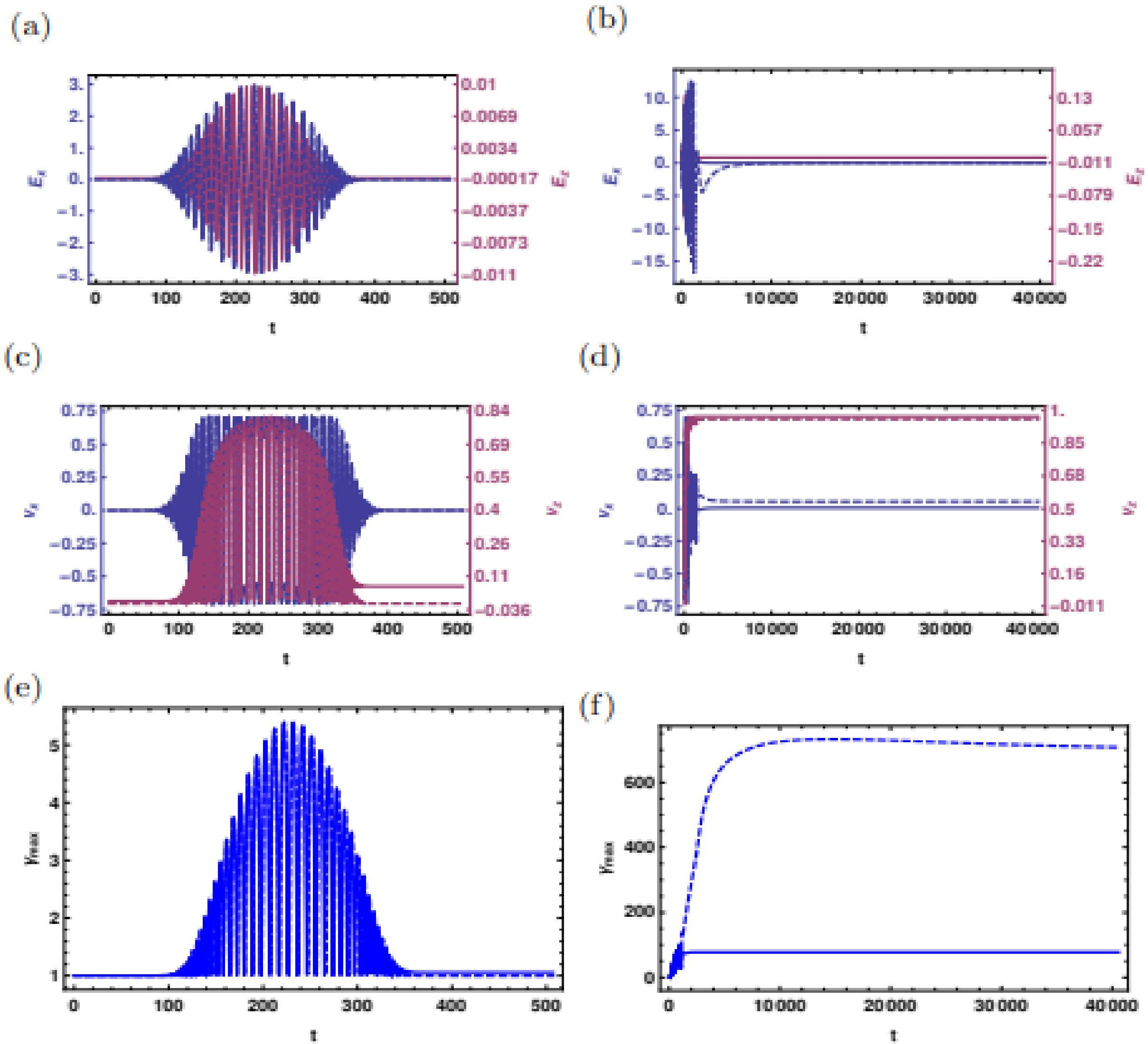
**Laguerre Gaussian (0,1) mode of CP laser beam**: Variation of longitudinal and transverse fields, velocities, and the maximum electron energy gain with (thick lines) and without (dash lines) longitudinal field as a function of "t"




- $a_0 = 3.0/\sqrt{2}$,
- $I_0 = 19.46 \times 10^{18}$ W/cm$^2$,
- Ez > Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 12.38 TW
- Energy gain small
- Variations are same with and without Ez

- $a_0 = 30.0/\sqrt{2}$,
- $I_0 = 19.46 \times 10^{20}$ W/cm$^2$,
- Ez < Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 1.23 PW
- Energy gain high
- Variations are different with and without Ez

**Laguerre Gaussian (1,0) mode of LP laser beam**: Variation of longitudinal and transverse fields, velocities, and the maximum electron energy gain with (thick lines) and without (dash lines) longitudinal field as a function of "t"

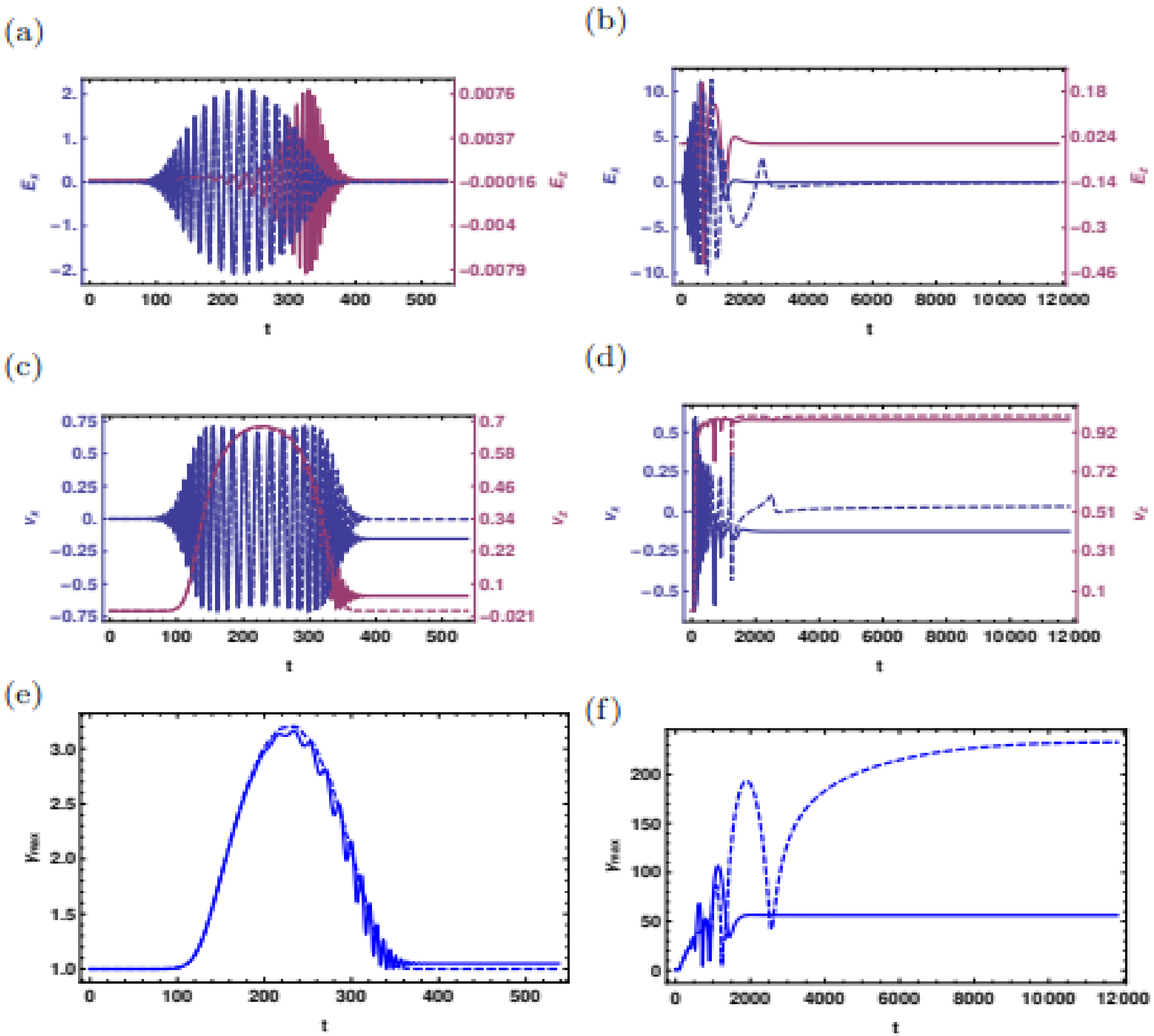



- $a_0 = 3.0$,
- $I_0 = 19.46 \times 10^{18}$ W/cm$^2$,
- Ez < Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 12.38 TW
- Energy gain small
- Variations are same with and without Ez

- $a_0 = 30.0$,
- $I_0 = 19.46 \times 10^{20}$ W/cm$^2$,
- Ez < Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 1.23 PW
- Energy gain high
- Variations are different with and without Ez

**Laguerre Gaussian (1,0) mode of CP laser beam**: Variation of longitudinal and transverse fields, velocities, and the maximum electron energy gain with (thick lines) and without (dash lines) longitudinal field as a function of ”t”

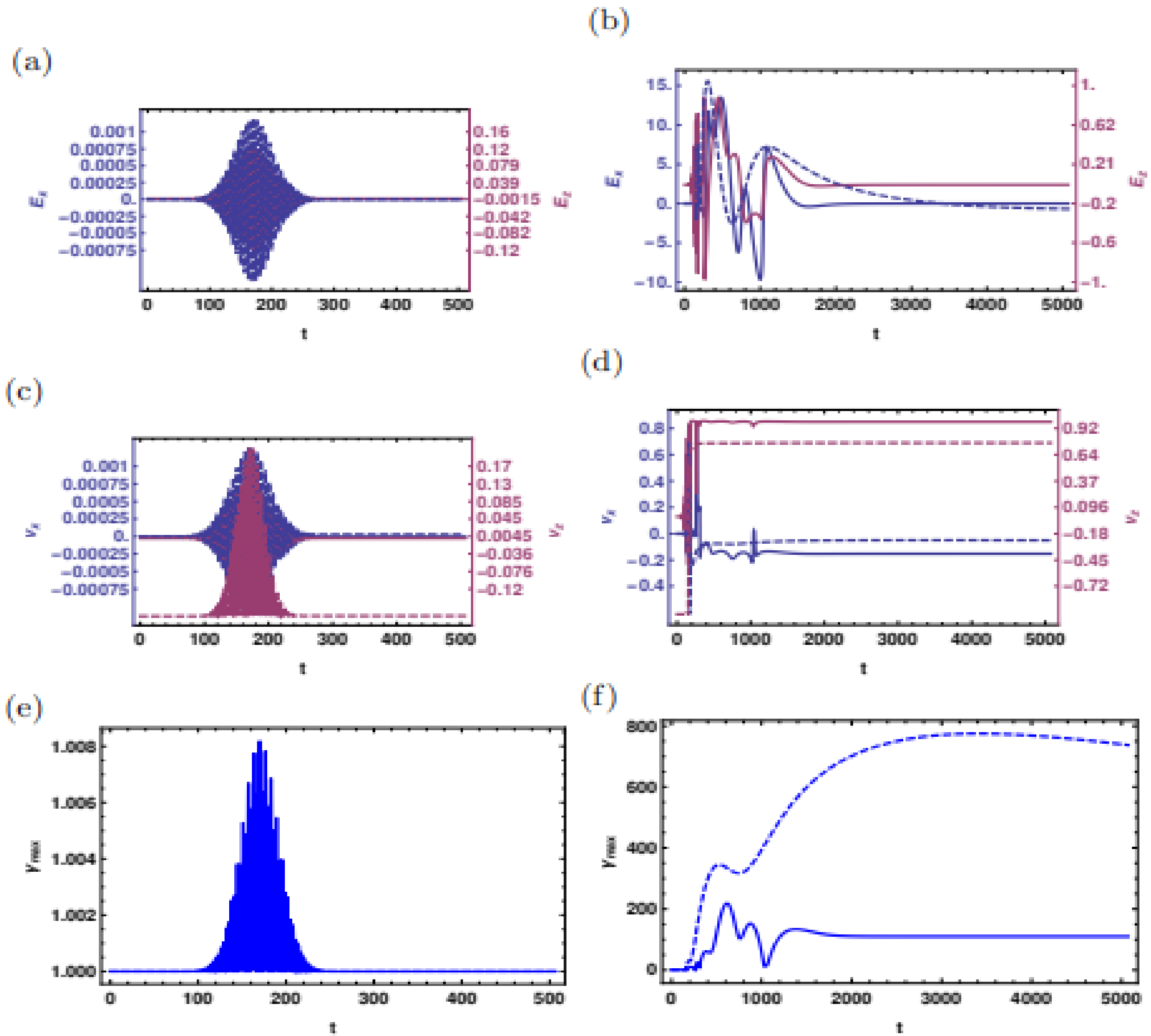
---




- $a_0 = 3.0/\sqrt{2}$,
- $I_0 = 19.46 \times 10^{18}$ W/cm$^2$,
- Ez << Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 µm
- $P_L$ = 12.38 TW
- Energy gain small
- Variations are same with and without Ez

- $a_0 = 30.0/\sqrt{2}$,
- $I_0 = 19.46 \times 10^{20}$ W/cm$^2$,
- Ez << Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 µm
- $P_L$ = 1.23 PW
- Energy gain high
- Variations are different with and without Ez

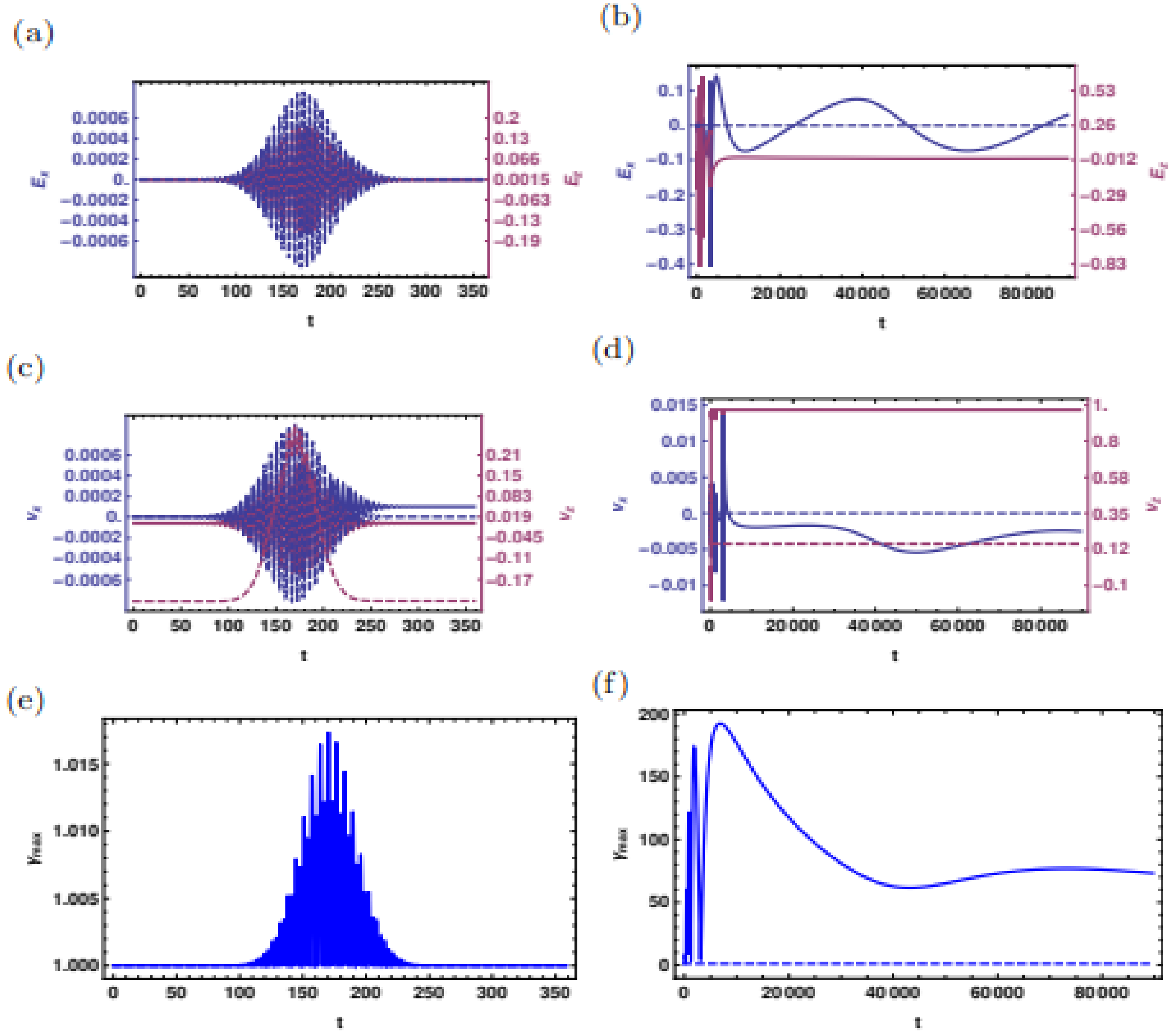
**Laguerre Gaussian (1,1) mode of LP laser beam**: Variation of longitudinal and transverse fields, velocities, and the maximum electron energy gain with (thick lines) and without (dash lines) longitudinal field as a function of "t"




- $a_0$ = 3.0,
- $I_0$ = 19.46 X $10^{18}$ W/cm$^2$,
- Ez > Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 12.38 TW
- Energy gain small
- Variations are same with and without Ez

- $a_0$ = 30.0,
- $I_0$ = 19.46 X $10^{20}$ W/cm$^2$,
- Ez < Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 1.23 PW
- Energy gain high
- Variations are different with and without Ez

**Laguerre Gaussian (1,1) mode of CP laser beam**: Variation of longitudinal and transverse fields, velocities, and the maximum electron energy gain with (thick lines) and without (dash lines) longitudinal field as a function of "t"




- $a_0 = 3.0/\sqrt{2}$,
- $I_0 = 19.46 \times 10^{18}$ W/cm$^2$,
- Ez > Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 12.38 TW
- Energy gain small
- Variations are same with and without Ez

- $a_0 = 30.0/\sqrt{2}$,
- $I_0 = 19.46 \times 10^{20}$ W/cm$^2$,
- Ez < Ex
- $\tau_L$ = 30fs
- $r_L$ = 6.4 μm
- $P_L$ = 1.23 TW
- Energy gain high
- Variations are different with and without Ez

# Conclusion

The electron laser interaction is found to be sensitive with the radial (p) and azimuthal (l) mode indices for a polarized LG laser beam. Depending upon the order of mode indices (p,l), it is possible to derive the electron dynamics and energy gain with the variations of electric and magnetic fields characteristics of LG laser beam.

Thanks…