Coupling Electromagnetic and Quasi-Electrostatic Waves in Electron Cyclotron Frequency Range in High-β Devices

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Abstract. The transformation of high-frequency electromagnetic waves in quasi-electrostatic waves in a high-β open trap shows a number of peculiar features which significantly distinguish this process both from the case a strong magnetic field, which is used for heating of a dense plasma in toroidal traps and other low-β devices, and from the case of Langmuir wave coupling in nonmagnetized plasmas. These features are studied theoretically in the present work.

INTRODUCTION AND THE PROBLEM SET-UP

Microwave heating of electrons under the electron cyclotron resonance conditions is one of the most efficient ways to increase the electron temperature of magnetically confined plasmas. Recent pprogress plasma confinement in axially symmetric magnetic traps led to the achievement of high-β regimes, in which the ratio between plasma kinetic pressure and the magnetic field pressure (β) is of the order of unity [1-5]. Under these circumstances the Langmuir plasma frequency is much greater than the electron cyclotron frequency, $\omega_{pe} >> \omega_{ce}$. Therefore, the use of common schemes of electron cyclotron heating based on direct launch of electromagnetic waves from the vacuum window meets obvious difficulties because the resonance region is screened by dense plasma for all electromagnetic modes except waves propagating strictly along the external magnetic field. However, in many cases strictly longitudinal propagation is not technically possible. One way to overcome this problem is to use the linear transformation of electromagnetic waves in quasi-electrostatic plasma oscillations in the vicinity of the plasma upper hybrid (UH) resonance $\omega_{UH}^2 = \omega_{pe}^2 + \omega_{ce}^2 \approx \omega_{pe}^2$ [6-9]. Once exited, the quasi-electrostatic oscillations can be effectively damped by electrons, in particular, in overdense plasmas. Such microwave heating schemes at low cyclotron harmonics have been implemented in magnetic traps with low kinetic pressure: stellarators, classical and spherical tokamaks [7]. The heating frequency for this systems is of the order of the electron cyclotron frequency, $\omega \sim \omega_{ce} < \omega_{pe}$. On the other hand, the opposite ordering $\omega_{ce} << \omega_{pe} \sim \omega$ is more natural for high-β devices, otherwise the linear coupling and the plasma heating are localized at the very periphery of the plasma column [8]. As quasi-electrostatic waves are well absorbed even at high cyclotron harmonics [10], the key factor controlling the efficiency of the heating is the excitation efficiency of quasi-electrostatic waves by electromagnetic waves launched from vacuum.

In magnetized plasma excitation of the quasi-electrostatic oscillations is possible as a straight-forward XB tunneling of the fast extraordinary wave (X) into the electron Bernstein wave (B) in the vicinity of the UH resonance, and as so called OXB process, a consequent linear transformations of the ordinary wave (O) into the slow extraordinary wave, and then to the then Bernstein wave [6,7].

The efficiency of the XB and OX coupling (the later in major extend defines the efficiency on the entire OXB process) is traditionally evaluated with Budden–Ginzburg approach as an integral of the purely imaginary wave vector over the evanescent coupling region in the vicinity of the UH resonance [11,12]. This approach is based entirely on the dispersion relation of waves in locally homogeneous plasma and do not take into account the peculiarities of the vector nature of Maxwell's equations in weakly inhomogeneous gyrotropic media. The resulting formulas work fairly well in case of strong magnetic field $\omega_{ce} \sim \omega$ [7, 13, 14], however do not allow correct transition to the isotropic plasma, in which the efficiency of the linear coupling of electromagnetic waves to Langmuir oscillations is also known [15]. The formal reason is that the phase integral technique (underlying the Budden–Ginzburg approach) works only when the singular points of the dispersion equations are spaced far apart, what requires either the strong anisotropy, or the isotropic degeneracy. Thus, the existing theory do not work in the domain of finite but low magnetic fields, $\omega_{ce} << \omega_{pe} \sim \omega$, that is of interest for high-β plasmas. In the present paper we fill this gap. We examine the XB and OX efficiency in the case of weak anisotropy, and compare the results with both known limiting cases of isotropic and strongly anisotropic cold plasma.

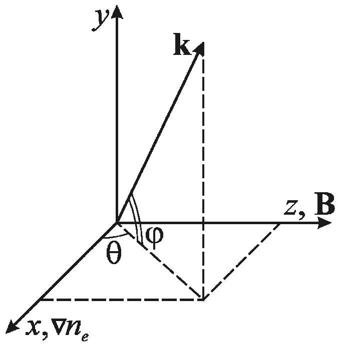

FIGURE 1. Coordinate system

We use the following simplifications. First, we consider one-dimensional geometry in which the external magnetic field is uniform and directed along the *z* axis, and the plasma density changes by parabolic law along the *x* axis, $n_e = n_0(1 - x^2 / L^2)$. It should be noted that non-one-dimensional effects may be important in the considered problem [9], however it is out of scope of the present communication. Second, we consider cold but weakly collisional plasma, i.e. neglecting effects of spatial dispersion due to thermal motion but introducing the effective collision rate $\nu_{eff} << \omega$. Here we rely on well-known conclusion of the equality of the weak electromagnetic energy losses due the collisional and resonant collisionless wave absorption in the vicinity of the UH resonance [16], for the particular XB and OXB case see [13]. In this frame, the efficiency of coupling to the quasi-electrostatic mode is calculated simply as the absorption coefficient of the electromagnetic wave. Third, we consider a plane electromagnetic wave $\propto \exp(i\omega t - i\mathbf{kr})$ launched from the vacuum at certain direction characterized by two angles θ, φ as shown in Fig. 1. With these assumption we solve numerically the full set of Maxwell's equations using the impedance technique developed in [13,16].

X-B COUPLING

When an electromagnetic wave propagates in a plane orthogonal to the magnetic field ($\theta = 0$ in Fig.1), the ordinary and extraordinary waves are independent, and only the extraordinary wave may be absorbed in the vicinity of the UH resonance. In this case Maxwell's equations may be reduced to a single second-order wave equation on the magnitude of the magnetic field in the X wave.

Originally, one can select five dimensionless parameters that totally define the studied problem. They are: (1) the maximum plasma density $X_0 = \omega_{pe}^2(0)/\omega^2$, (2) the magnetic field strength $Y = \omega_{ce} / \omega$, (3) the width of the density distribution $\chi = k_0 L$ with $k_0 = \omega / c$, (4) the wave propagation angle φ, and (5) the collision rate ν_{eff} / ω. We can reduce the number of these parameters noting that the efficiency of XB conversion is only affected by the density variation in the local vicinity of the UH resonance. Therefore free parameters X_0 and χ should be included only in combination $\chi_{\mathrm{UH}} = k_0 L_{\mathrm{UH}}$, where L_{UH} is the inhomogeneity scale of the plasma density in the vicinity of the UH resonance. Aslo the collision rate may be set to $\nu_{eff} / \omega \to 0$ since the total absorption does not depend on collisionality when $\nu_{eff} / \omega << 1$. Thus, this parameter is needed for numerical calculations, however it can be formally excluded from a theoretical consideration. With these assumptions and taking into account the proximity of the Langmuir and UH resonances ($\omega_{pe} \approx \omega_{UH}$ for $Y << 1$), we obtain the following equation for the wave magnetic field in the vicinity of the UH resonance:

$$\frac{d^2 H_z}{d\xi^2} - \frac{1}{\xi}\frac{\xi^2 + \tilde{Y}^2}{\xi^2 - \tilde{Y}^2}\frac{dH_z}{d\xi} - \left(\xi + \tilde{\varphi}^2 - \frac{\tilde{Y}^2}{\xi} - \frac{\tilde{Y}\tilde{\varphi}}{\xi^2 - \tilde{Y}^2}\right)H_z = 0, \quad (1)$$

where $\xi = \chi_{UH}^{2/3}(x - x_{\mathrm{UH}})/L_{\mathrm{UH}}$, $\tilde{Y} = \chi_{UH}^{2/3} Y$ and $\tilde{\varphi} = \chi_{UH}^{1/3} \sin\varphi$. Therefore, the final result for the XB coupling efficiency or, equivalently, the X wave absorption should depend on only two parameters, $\tilde{Y}$ that describes the external magnetic field, and $\tilde{\varphi}$ that is determined by the wave incident angle with respect to the layer. Both parameters include a large (for microwaves) parameter of geometric optics $k_0 L_{UH} >> 1$, for this reason we can not immediately answer how to correlate these parameters with the unit.

A new and important feature described by (1) is the asymmetry with respect to substitution $\tilde{\varphi} \to -\tilde{\varphi}$, see last term in the brackets. This implies that solutions of this equation also do not posses symmetry relative to the sign of wave incident angle. This feature differs essentially from the all known results obtained either for the isotropic, or for the strongly anisotropic plasmas (in both cases the evaluated dispersion relation is symmetric with respect to the sign of incident angle). To clarify a physical reason for such asymmetry let us consider the resonant component of the electric field parallel to plasma density gradient: $E_x \propto H_z \sin\varphi + iY E_y$. The interplay of the polarization of an incident wave (proportional to $\sin\varphi$) and the gyrotropy (proportional to magnetic field strength *Y*) can either suppress, or enhance the resonant field, which in turn, determines the coupling efficiency.

Here it is necessary to make a point, that far away from the UH resonance, equation (1) can be reduced to the so-called parabolic cylinder equation $d^2F/dx^2 + (a + b/x)F = 0$ used previously for analysis of the strongly anisotropic case [10]. However, inside the UH resonance zone, the singular points and the topology of the Stokes lines [18] of Eq.(1) and the parabolic cylinder equation differ dramatically from what follows that the properties of the solutions of these equations will differ significantly (outside the WKB approximation).

The above qualitative conclusions are confirmed with numerical simulations. As an example, several dependences of the XB coupling efficiency T_{XB} over normalized incidence angle $\tilde{\varphi}$ and external magnetic field strength $\tilde{Y}$ are shown in Fig. 2 (left). The coupling efficiency is defined as the total absorption coefficient of the incident X wave at the UHR. With the absence of a magnetic field, $\tilde{Y} = 0$, we reproduce the well-known result obtained by Ginzburg for the isotropic plasma and characterized with two symmetric extrema at $\varphi \approx \pm 0.7\chi_{UH}^{1/3}$ [12]. With the increase of the magnetic field the dependence of the coupling efficiency over the incident angle becomes asymmetric—the right maximum increases, reaching unity at $\tilde{Y} \approx 0.4$, and the left maximum decreases. With further increase of the magnetic field up $\tilde{Y} \approx 1$ the right maximum shifts to $\tilde{\varphi} \approx 0$ and the left maximum vanishes, thus the plot again becomes (approximately) symmetrical. At very high magnetic fields, $\tilde{Y} > 1$, the efficiency tends to zero, in accordance with the predictions of the previous theory of the linear X mode coupling [12]. It is interesting to note that there is always one incident angle for which the coupling is exactly zero, and the very existence of this angle breaks the symmetry with respect to the sign of the wave incidence.

O-X-B COUPLING

If the wave vector of the incident wave has a component along the magnetic field, the ordinary and extraordinary waves cease to propagate independently. In this case the most effective coupling to the electrostatic modes is possible at a specific angle to the magnetic field defined as $\sin^2\theta = Y/(1+Y)$ [6,14]. This is the OXB coupling regime. In this case, equations for the wave field becomes rather cumbersome, however the numerical modeling shows that the OXB coupling efficiency is defined by the similar two parameters as in previous case, namely $\tilde{Y}$ and $\tilde{\theta} = \chi_{UH}^{1/3} \sin^2\theta$. Here we consider a wave propagating in the $x - z$ plane, i.e. $\varphi = 0$ in Fig.1

Figure 2 (right) shows the results of the numerical calculation of the OXB coupling efficiency T_{OXB} defined as the total absorption coefficient of the incident O wave. One can see that in the region $\tilde{Y} \sim 1$ there is a good agreement with the widely used approximation for the OX tunneling efficiency obtained by Mjølhus [14]; in our notation the Mjølhus formula is $T = \exp[-\pi(2\tilde{Y})^{1/2}(|\tilde{\theta}| - \tilde{Y}^{1/2})^2]$. However, with the magnetic field decrease the coupling efficiency also decreases as compared to the Mjølhus result. The physical reason is that there are the X waves that propagates towards the incident O wave behind the UH resonance. In a weak magnetic field such waves may partially tunnel through the evanescent region near the UH resonance, resulting in additional reflection of the incident radiation. In the limiting case $\tilde{Y} << 1$ we find a clear transition the isotropic case, where the efficiency of the transformation is determined by the presence of linear TM polarization in the O mode.

Note that the increasing ability of the X mode to tunnel through the UH region in low magnetic field results not only in degradation of the OXB coupling, but also it increases the efficiency of the direct XB coupling. In this case, the maximum coupling efficiency is realized when both XB and OXB mechanisms are active, i.e. for the specially chosen mixture of X and O polarizations in the incident wave. Such optimization can significantly increase the efficiency of excitation of a quasi-electrostatic waves in high-β devices.

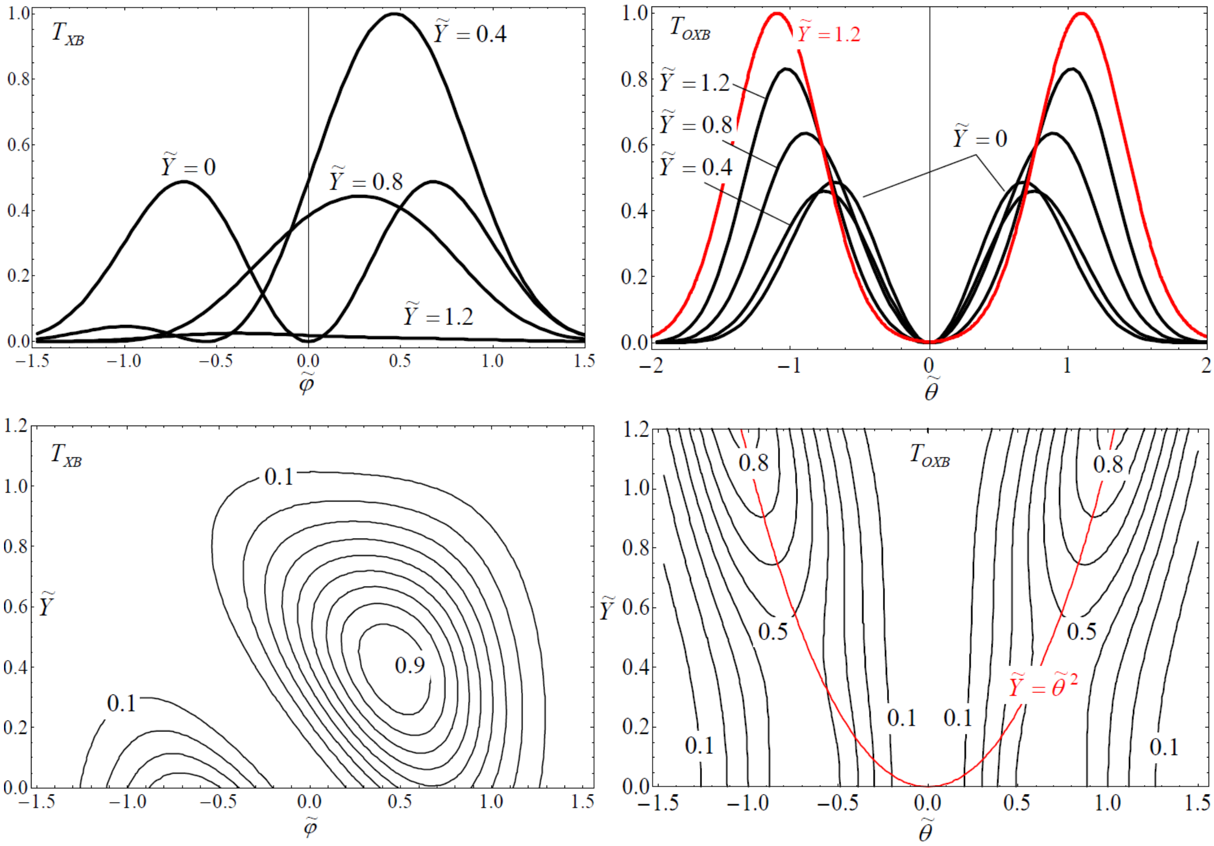

FIGURE 2. Top plots: XB (left) and OXB (right) coupling efficiencies versus the normalized incidence angle for different values of the external magnetic field strength. The red line corresponds to Mjølhus formula for $\tilde{Y} \approx 1.2$. Bottom plots: contour lines of XB (left) and OXB (right) coupling efficiencies in the incident angle – magnetic field strength plane. Contour levels are spaced with 0.1 increment, the red line corresponds to the parameters for which Mjølhus formula predicts 100% coupling. Coupling efficiencies are defined as the total absorption in the vicinity of upper-hybrid resonance

In conclusion, we note the coupling of high-frequency electromagnetic waves to the quasi-electrostatic waves under condition of weakly magnetized plasmas can not be described by common (for fusion plasmas) theories even within the simplest plane geometry and at least in the parameter range $\omega_{ce} / \omega \leq (k_0 L_{UH})^{2/3}$. Important new feature is pronounced asymmetry of the coupling process with respect to the plane defined by the gradient the plasma density and the external magnetic field. Most previous theories inherently lead to symmetrical about this plane results because they rely only on the spatial dependence of refractive index. On the other hand, straight-forward approach based on realistic wave equations not only correct the existing theory, but shows new options for the effective transformation of electromagnetic waves into quasi-electrostatic oscillations in weakly magnetized high-β plasmas.

Therefore, the further studies in this direction are of prior importance for the development of effective electron heating scenarios in high-β magnetic traps.

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