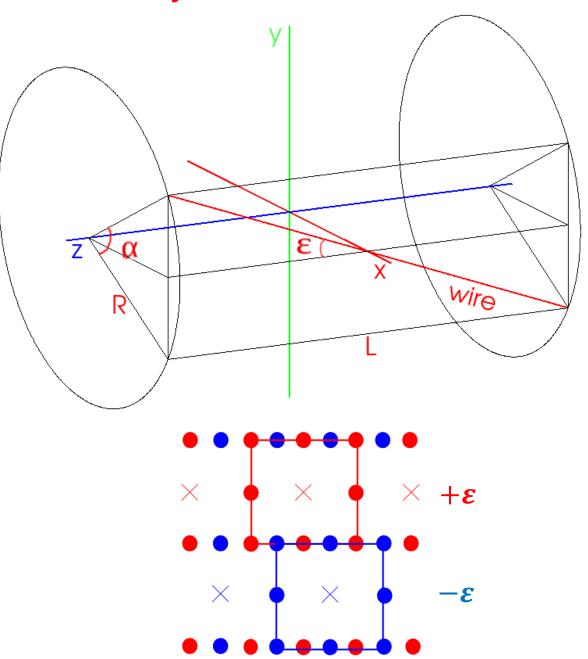
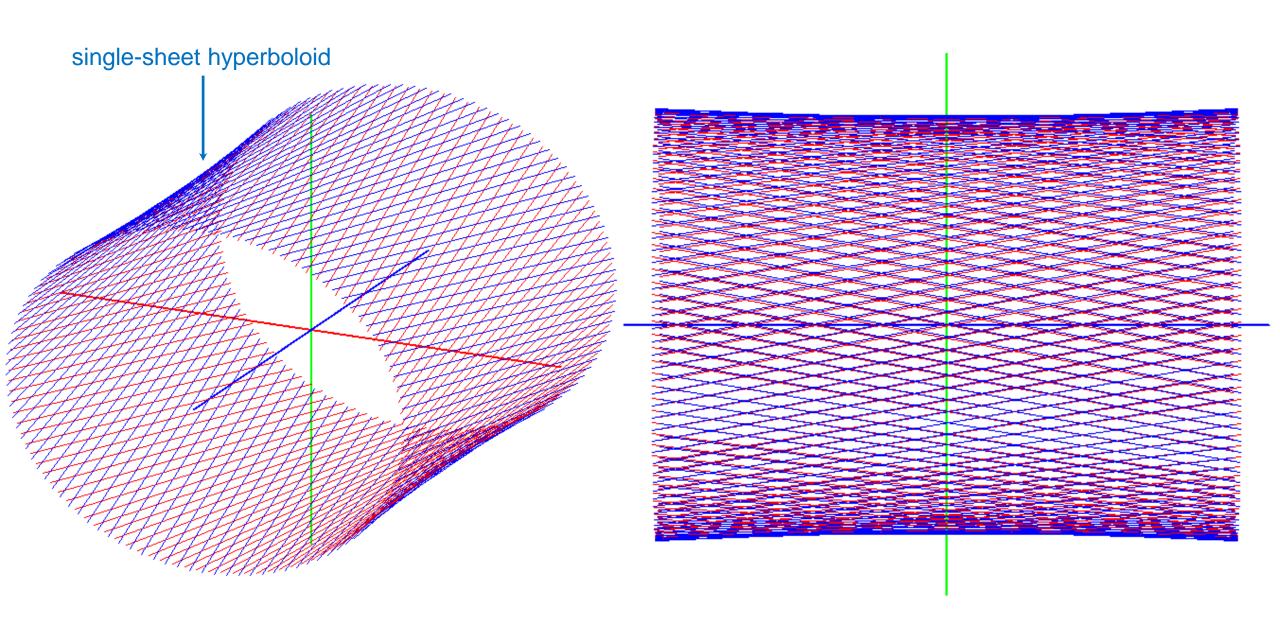
Status of the c- τ factory drift chamber simulation

Vyacheslav Ivanov

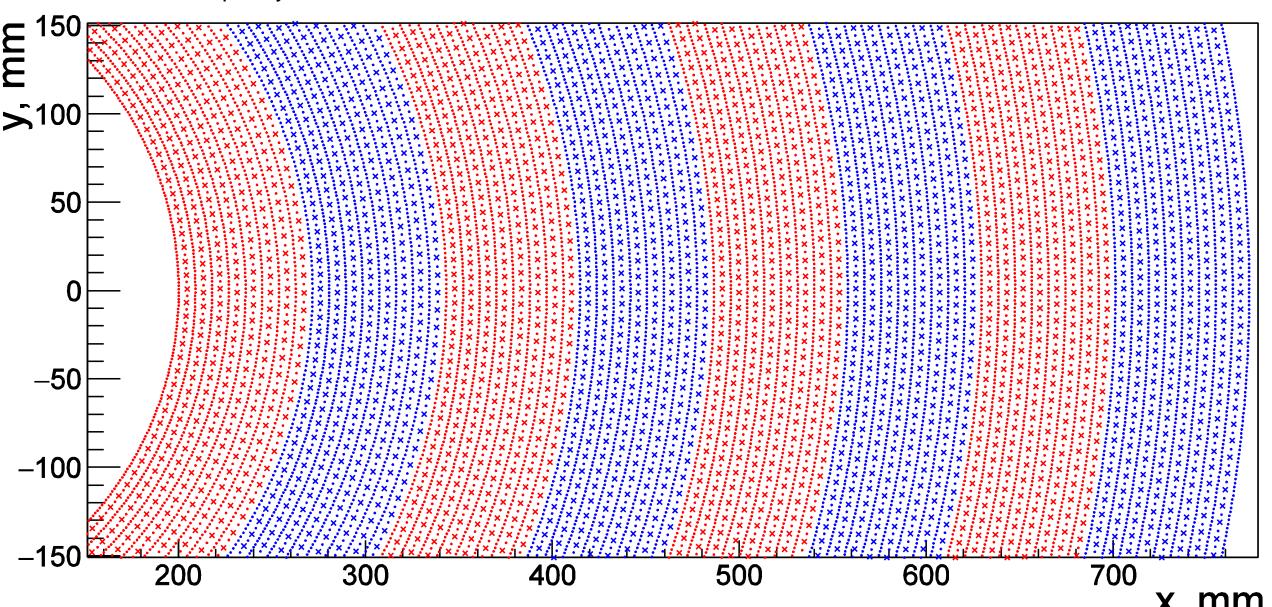
We simulate the full-stereo drift chamber with the parameters, close to that proposed by F. Grancagnolo: https://indico.ijclab.in2p3.fr/event/4902/contributions/17030/attachments/13603/16389/SCTFDrift_Chamber.pdf

Parameter	Value
R_{in}	200 mm
R_{out}	800 mm
Length	1800 mm
Cell shape	Square
α	$\pi/6$ rad
Gas	90% He $-$ 10% iC ₄ H ₁₀ $3.8 \cdot 10^{-4}$ X ₀ for $\theta = \pi/2$
$N_{superlayers}$	8
Layers in superlayer	8
Signal wire diameter/material	25 μm, W
Field wire diameter/material	50 μm, Al
Cell size	~7-9 mm
Stereoangle ε	~60-220 mrad
$N_{signal\ wires}$	21824, $10 \cdot 10^{-4} X_0$ for $\theta = \pi/2$
$N_{field\ wires}$	109780, 7,8 · 10^{-4} X ₀ for $\theta = \pi/2$

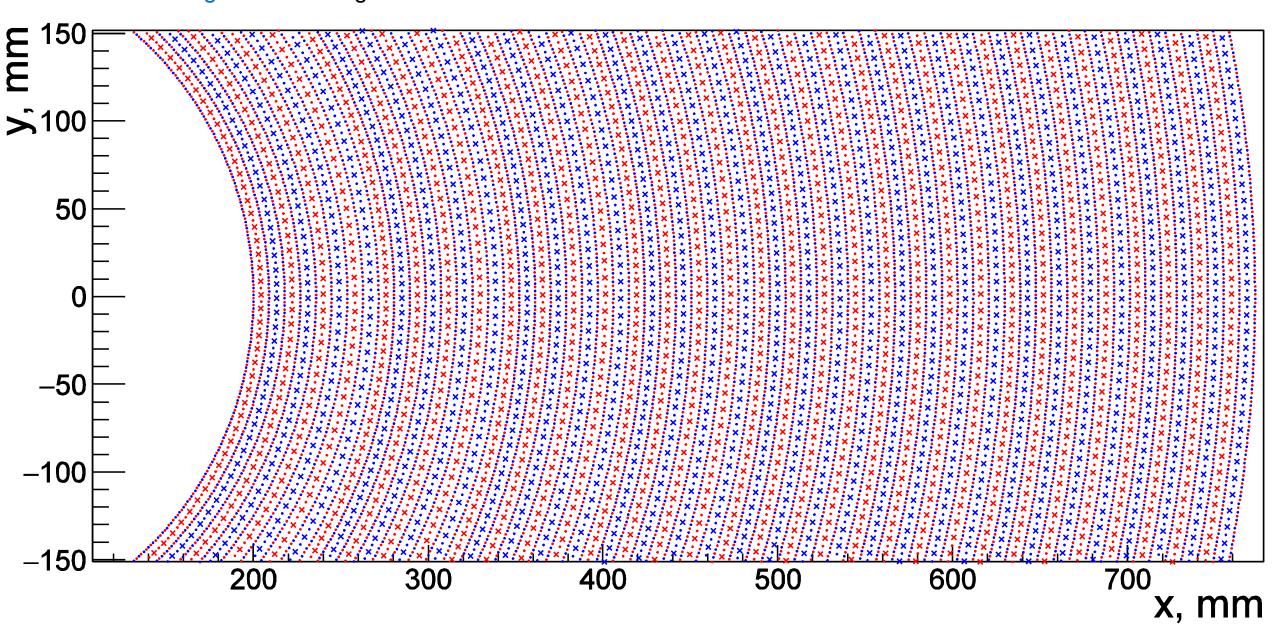




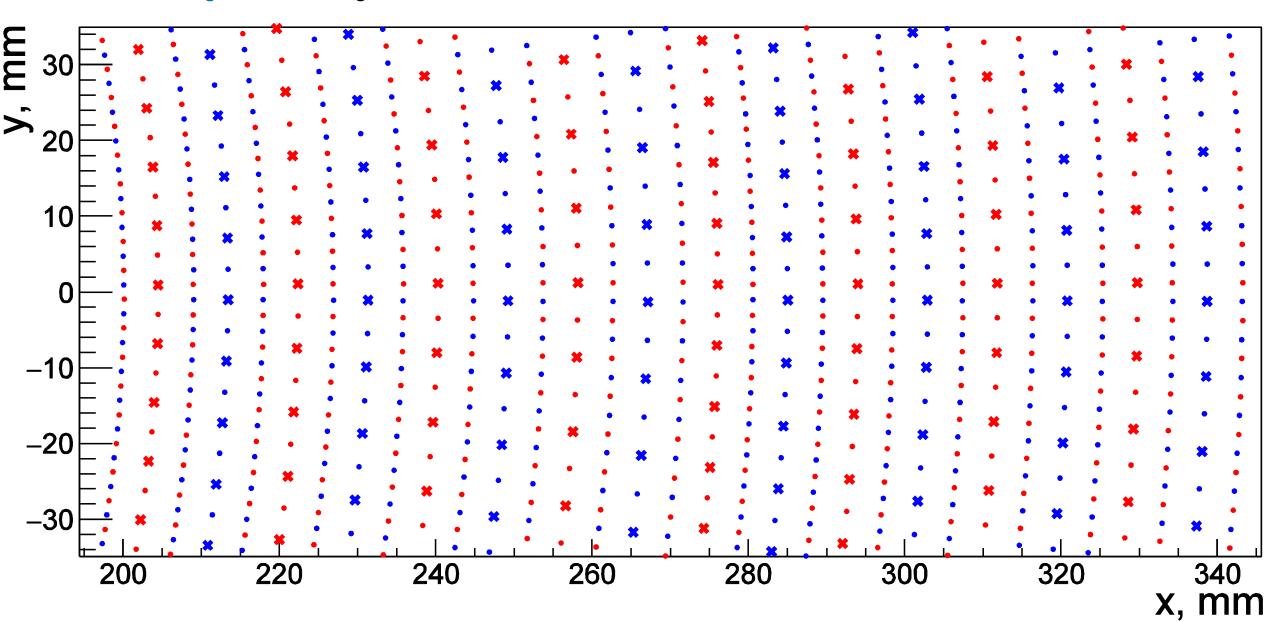
Odd and even superlayers:



• Positive and negative streoangle:



• Positive and negative streoangle:



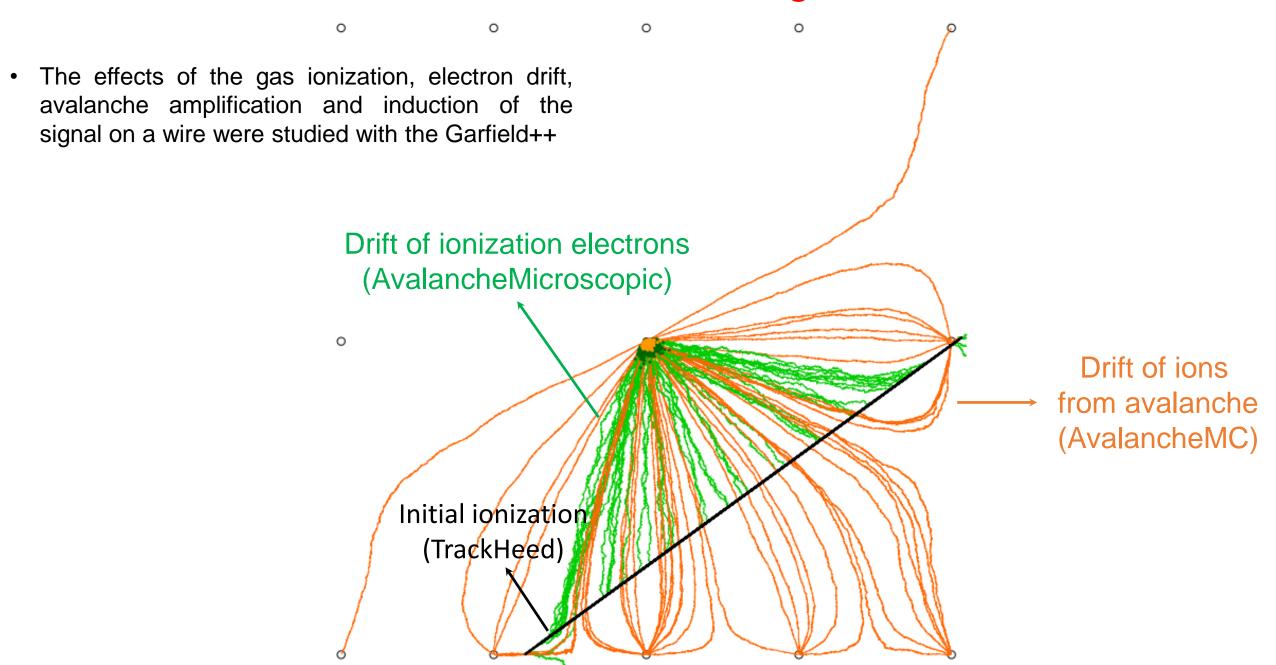
- The DC geometry is passed to GEANT4 via the DD4HEP toolkit
- The material of wires is taken into account in two ways:
 - 1. Each wire is placed as a separate volume. It should be noted, that placing the wires as the child volumes directly into the gas volume of the DC would lead to a radical slowdown of the simulation (too many intersection checks during tracking). To avoid this, we place fictitious hyperboloid daughter gas volumes on each wire layer. The wires of this layer are then placed as daughter volumes of corresponding fictitious hyperboloid. This approach does not significantly affect the tracking time, but the amount of memory required grows up and the run initialization time becomes ~ few minutes.
 - 2. Instead of each wire layer we place a hyperboloid layer with width

$$h = \frac{n_{wires}l_{wire}r_{wire}^2}{2L_{DC}R_{layer}(z=0)}$$

having the same amount of material

• We save the DC geometry using **DD4HEP DataExtensions** tool and invoke the geometry whenever necessary during digitization and reconstruction via the special **GaudiTool**

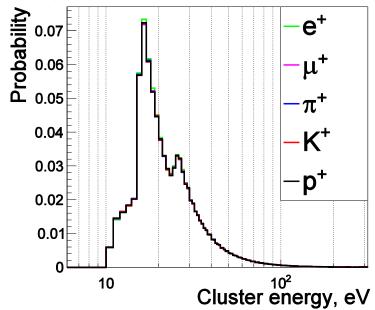
Simulation of Wire Signals



Simulation of Ionization Clusters

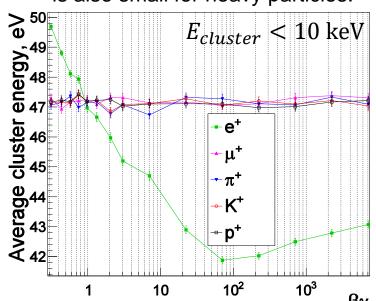
- On the spiral stretched between the beginning and the ending points of the GEANT4 hit (G4Step) we generate the ionization clusters until their total energy exhausts the hit energy
- Cluster energies are generated according to the spectra, predicted by Garfield::TrackHeed

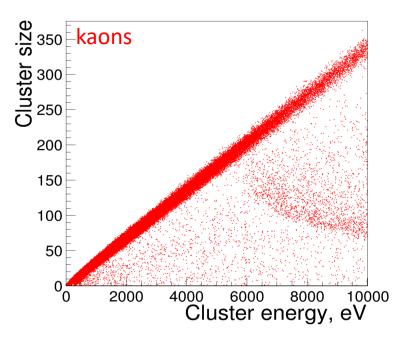
The difference between the cluster energy (=energy transfer) spectra for heavy particles appears to be very small:



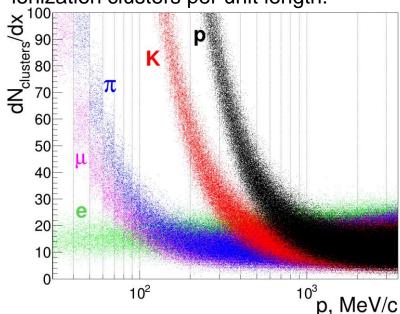
• Garfield predicts the average energy, required for single electronion pair production $W=29.52~\mathrm{eV}$ and Fano factor F=0.19. For the generated cluster energy E_{cl} the average number of electrons in cluster is E_{cl}/W and the fluctuation is $\sqrt{FE_{cl}/W}$

The dependence of the cluster energy spectra on the velocity is also small for heavy particles:



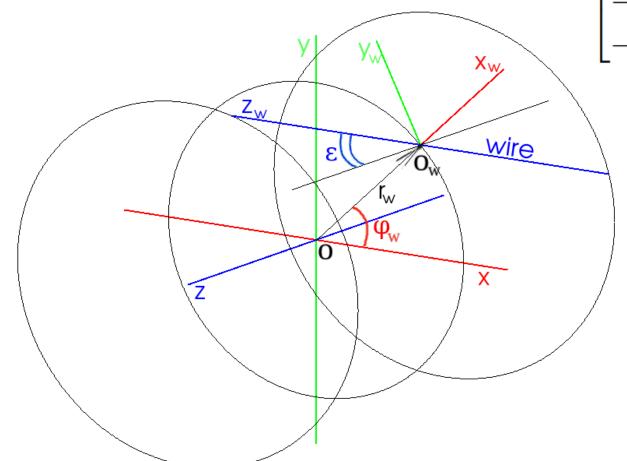


The major dependence on the particle type and momentum is seen in the number of ionization clusters per unit length:



Association of Clusters with Wires

- Each ionization cluster is associated with the closest signal wire
- Cluster coordinates are translated into the wire coordinate frame $O_w x_w y_w z_w$



$$\overrightarrow{OO}_w \equiv \overrightarrow{r}_w = (r_w cos(\varphi_w), r_w sin(\varphi_w), 0)$$

$$\overrightarrow{r}_{cl,w} = R_x(\epsilon) R_z(-\varphi_w) (\overrightarrow{r}_{cl} - \overrightarrow{r}_w) =$$

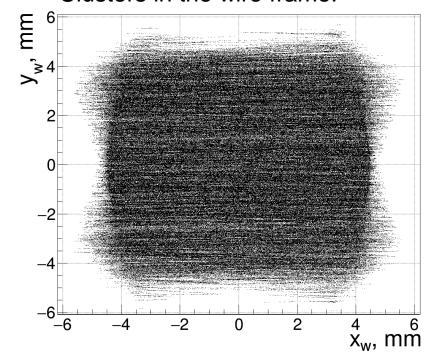
$$cos(\varphi_w) \qquad sin(\varphi_w) \qquad 0$$

$$-sin(\varphi_w) cos(\epsilon) \qquad cos(\varphi_w) cos(\epsilon) \qquad -sin(\epsilon)$$

$$-sin(\varphi_w) sin(\epsilon) \qquad cos(\varphi_w) sin(\epsilon) \qquad cos(\epsilon)$$

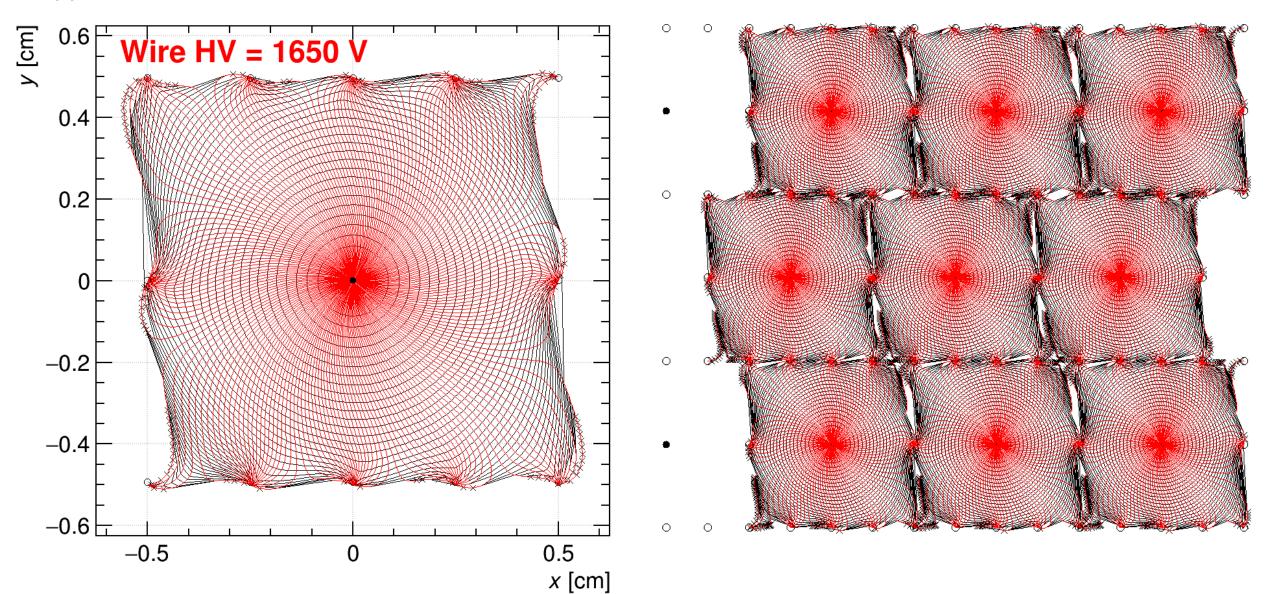
$$(\overrightarrow{r}_{cl} - \overrightarrow{r}_w)$$

Clusters in the wire frame:



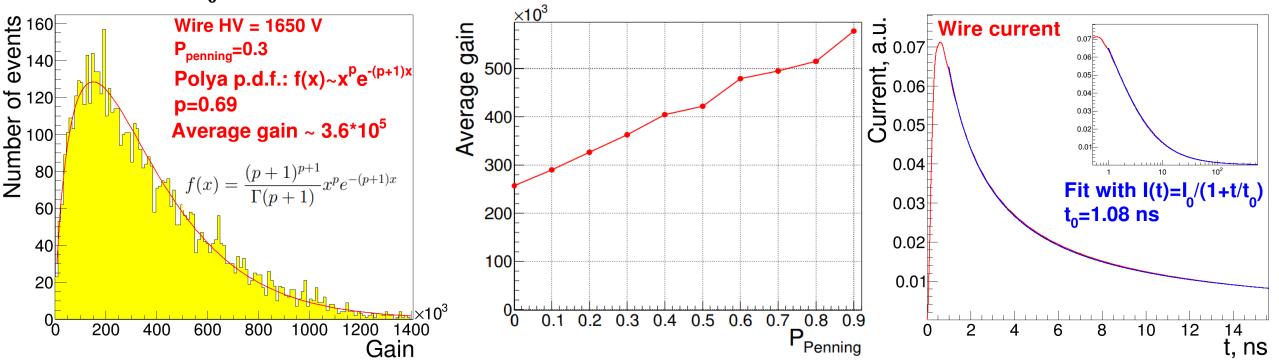
Simulation of Isochrones

• We obtained from Garfield++ the isochrones for $10 \times 10~\mathrm{mm^2}$ 2D square cell and use them as the first approximation for all cells at all z



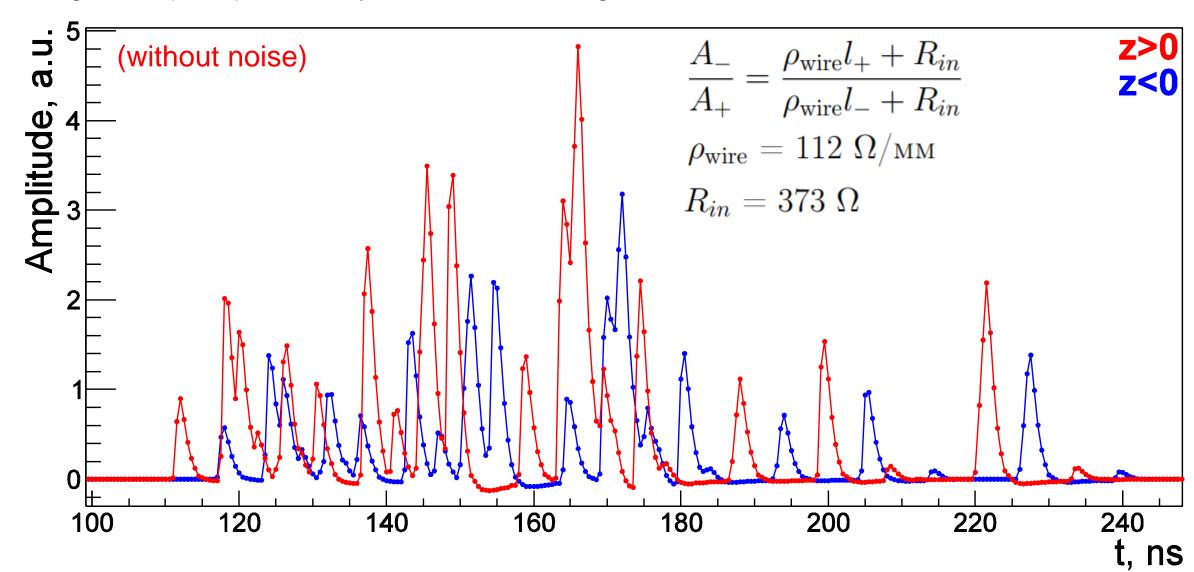
Determination of the Wire High Voltage

- The desired signal/noise ratio requires the average avalanche gain to be few 10⁵
- The average probability P_{Penning} of the Penning/Jesse energy transfers for the mixtures with helium is unknown (we didn't find any data), so we scanned over this parameter
- We have chosen the HV=1650 V to be a working point, the P_{Penning} is fixed at 0.3 in further studies. The average gain in this case is $3.6 \cdot 10^5$
- From the fit of the gain spectrum with the Polya distribution we obtained the parameter of the latter p = 0.69
- At the given working HV by averaging the signals from many avalanches we determined the characteristic signal decrease time $t_0 = 1.08 \text{ ns}$

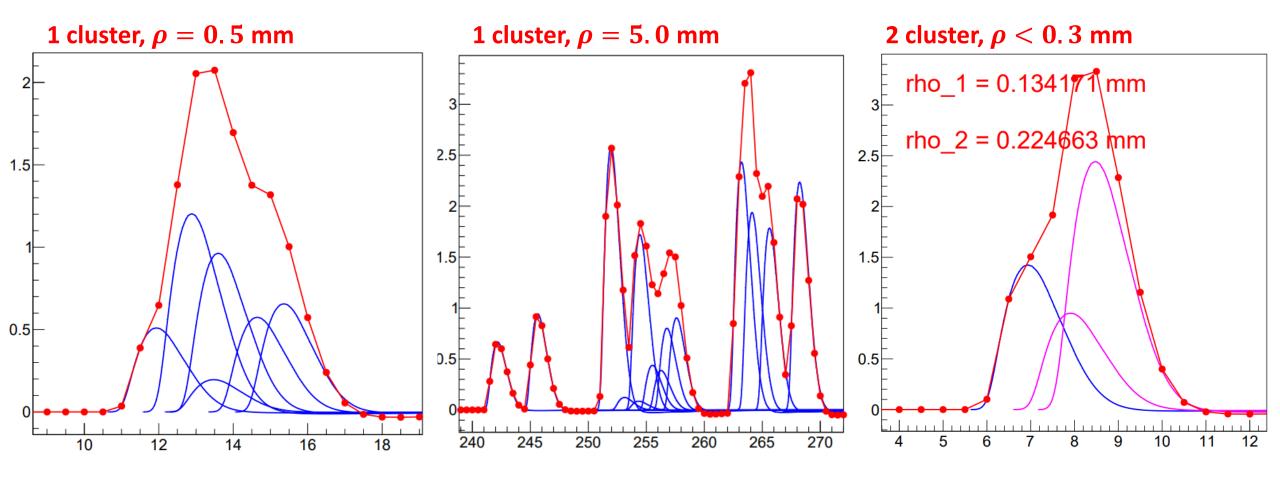


Wave Form Scan

- The waveform for all wires is scanned with 2 GHz frequency (for cluster counting & timing)
- The signal shape is provided by V.M. Aulchenko, signal/noise ratio is estimated to be ~1/8

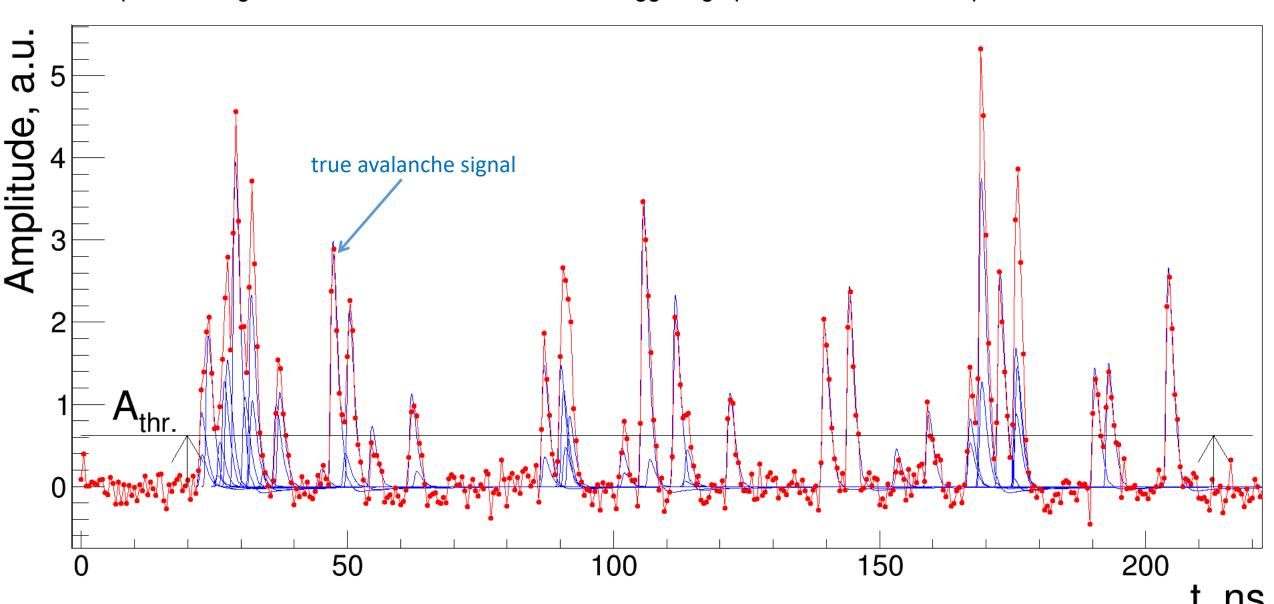


Wave Form Scan



Wave Form Scan

• The amplitude integration starts from 2 ns before wire triggering up to 5 ns after the last peak:



Reconstruction of Hits & Tracks

DC channel digitization

Raw Hits:

- Channel triggering time(s)
- Integrated amplitudes from both sides
- Peaks times and amplitudes (not implemented yet)

hit reconstruction

Reconstructed Hits:

- Track impact parameter
- z-coordinate (via charge division)

track finding

«Track candidates»:

- Set of reconstructed hits, supposedly produced by one particle
- Estimated track direction at each hit & left-right ambiguity resolved

Kalman filter (final fit):

Final estimation of track parameters

track fit

Riemann (preliminary) fit:

First approximation of track parameters,
 then used in Kalman Filter

track fit

Reconstructed Point Hits:

 (x, y, z) coordinates of track's PCA to the wire and their covariance matrix

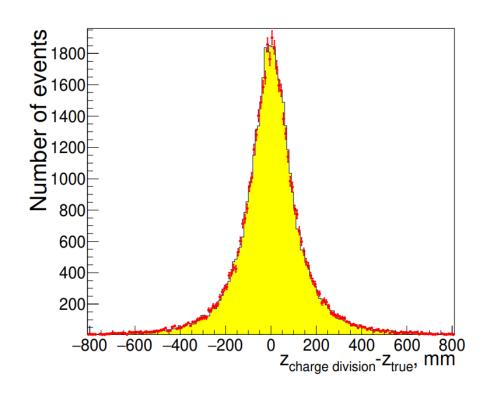
Reconstruction of Hits

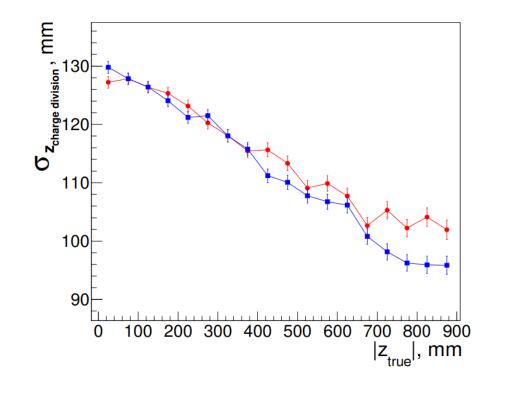
z —coordinate of the hit is reconstructed via the charge division formula:

$$z = \left(\frac{l_{\text{wire}}}{2} + \frac{R_{in}}{\rho_{\text{wire}}}\right) \frac{A_{+} - A_{-}}{A_{+} + A_{-}}$$

$$z = \left(\frac{l_{\text{wire}}}{2} + \frac{R_{in}}{\rho_{\text{wire}}}\right) \frac{A_{+} - A_{-}}{A_{+} + A_{-}} \qquad \sigma_{z} = 2\left(\frac{l_{\text{wire}}}{2} + \frac{R_{in}}{\rho_{\text{wire}}}\right) \frac{\sqrt{A_{+}^{2}\sigma_{A_{-}}^{2} + A_{-}^{2}\sigma_{A_{+}}^{2}}}{(A_{+} + A_{-})^{2}}$$

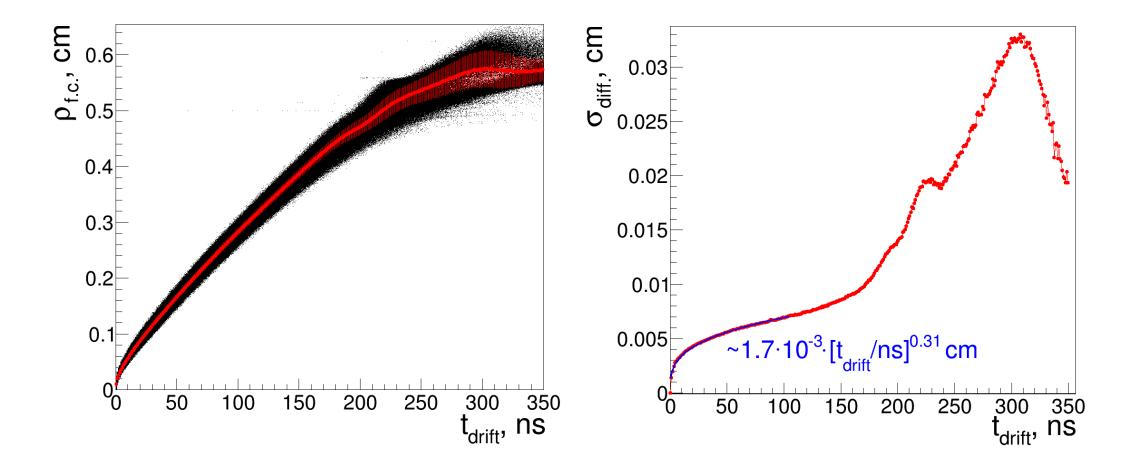
$$\sigma_{A_{\pm}} = \sqrt{\sigma_{\text{noise}} t_{\text{int}} f}$$





Reconstruction of Impact Parameter

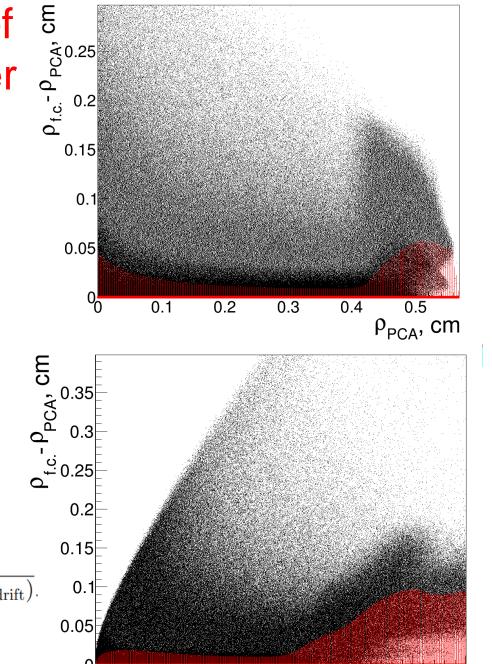
- We assume the distance to the first cluster $\rho_{f.c.}$ to be a (biased) estimation of the track impact parameter ρ_{PCA}
- The $\rho_{f.c.}$ is calculated from the drift time t_{drift} via $t \rho$ relation, obtained for the beam-originated muons
- The uncertainty of $\rho_{f.c.}$ at the given t_{drift} gives the estimate for the contribution of diffusion in the resolution on ρ_{PCA}



Reconstruction of Impact Parameter

- $ho_{f.c.}$ is a biased estimate of the ho_{PCA} due to the cluster effect (discreteness of ionization)
- We do not apply any correction to compensate this bias, but calculate the uncertainty in ρ_{PCA} due to the cluster effect as the square root from the second moment about the origin of the distribution of ρ_{PCA} - $\rho_{f.c.}$ at given t_{drift}

$$\sigma_{
ho_{
m PCA}} \equiv \sqrt{\sigma_{
m diff.}^2(t_{
m drift}) + \sigma_{
m clust.eff.}^2(t_{
m drift})}$$



150

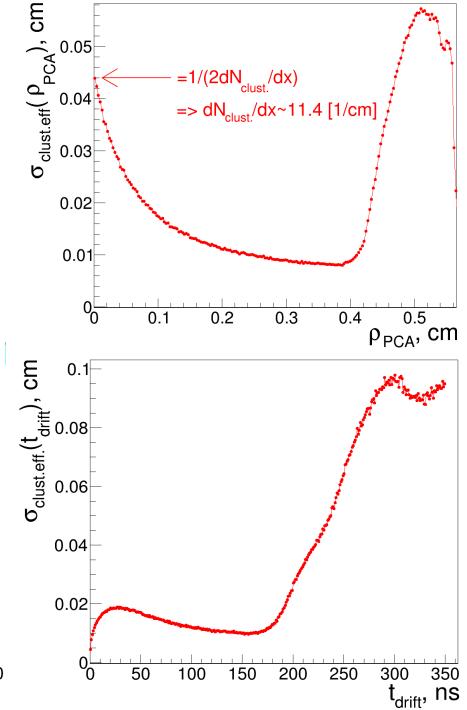
200

250

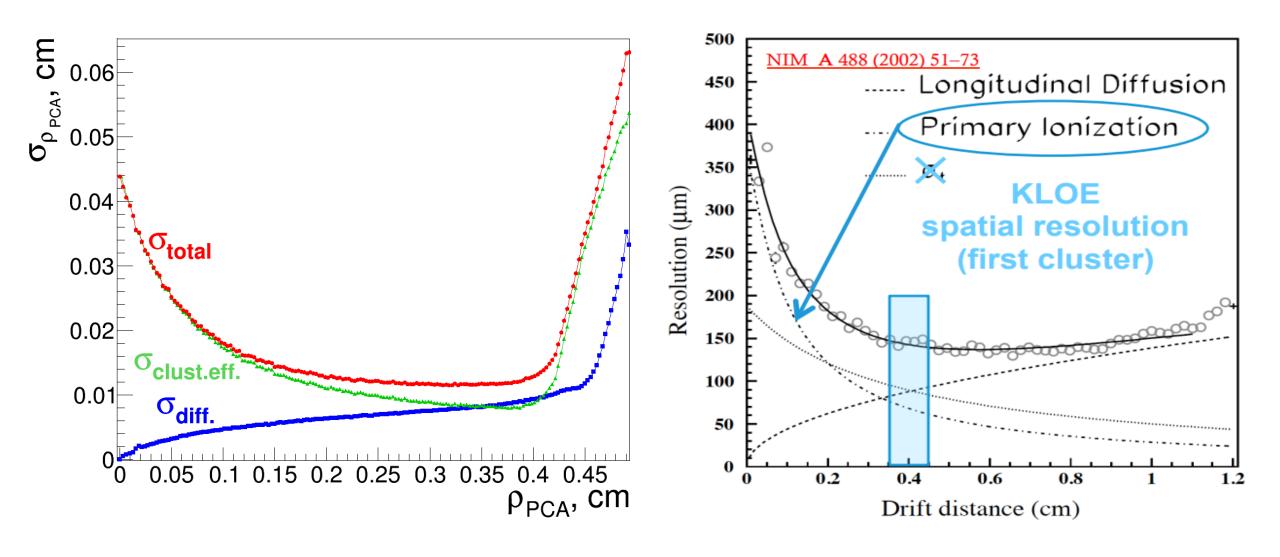
300

ι_{drift}, ns

350

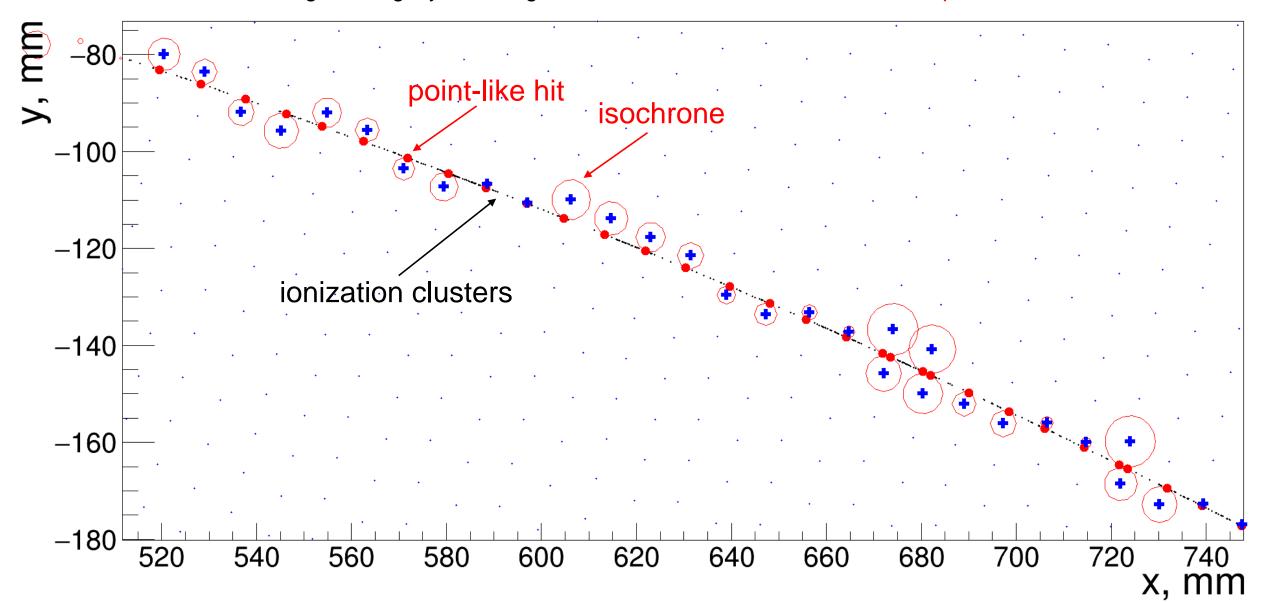


Resolution on Impact Parameter



Reconstruction of Point-like Hits

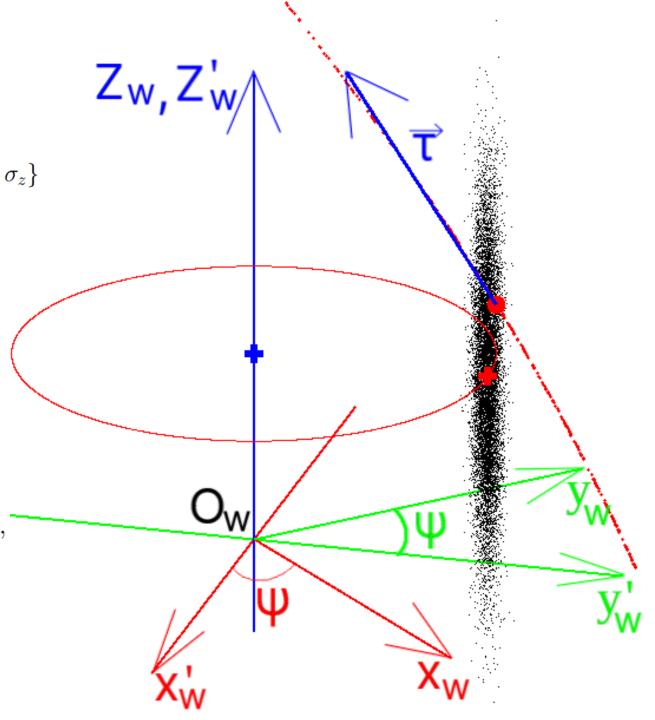
• We do not have the track finding algorithm yet, but once we had it, we would know the estimated track direction at each cell and the resolved left-right ambiguity. Knowing this we can convert the wire hit into the point-like hit



Point-like Hit Cov. Matrix

Calculation of the point-like hit covariance matrix:

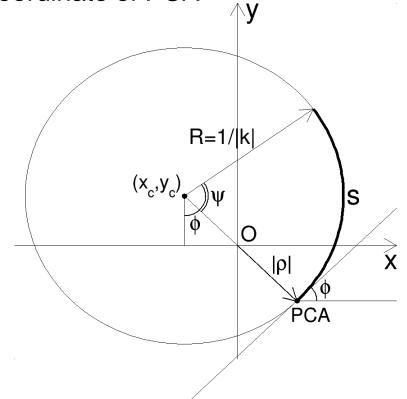
$$\begin{aligned} cov(x_w',y_w',z_w') &= \begin{bmatrix} \delta^2 & 0 & 0 \\ 0 & \sigma_{\rho_{\text{PCA}}}^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix} \quad \delta \equiv 0.01 \cdot \min\{\sigma_{\rho_{\text{PCA}}},\sigma_z\} \\ &cov(x,y,z) &= J \cdot cov(x_w',y_w',z_w') \cdot J^T \\ J &= R_z(\varphi_w)R_x(-\epsilon)R_z(\psi) &= [\vec{e}_{x_w'},\vec{e}_{y_w'},\vec{e}_{z_w'}], \\ \vec{e}_{x_w'} &= \begin{bmatrix} cos(\varphi_w)cos(\psi) - sin(\varphi_w)sin(\psi)cos(\epsilon) \\ sin(\varphi_w)cos(\psi) + cos(\varphi_w)sin(\psi)cos(\epsilon) \\ -sin(\psi)sin(\epsilon) \end{bmatrix}, \\ \vec{e}_{y_w'} &= \begin{bmatrix} -cos(\varphi_w)sin(\psi) - sin(\varphi_w)cos(\psi)cos(\epsilon) \\ -sin(\varphi_w)sin(\psi) + cos(\varphi_w)cos(\psi)cos(\epsilon) \\ -cos(\psi)sin(\epsilon) \end{bmatrix}, \\ \vec{e}_{z_w'} &= \begin{bmatrix} -sin(\varphi_w)sin(\epsilon) \\ cos(\varphi_w)sin(\epsilon) \\ \end{bmatrix}, \end{aligned}$$

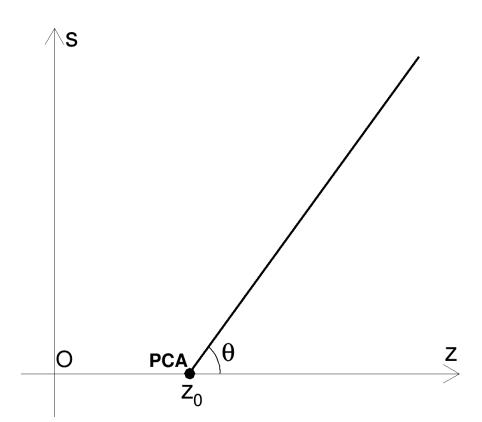


Track Reconstruction

- We use usual track helix parametrization:
 - 1. k signed curvature in xy-plane
 - 2. ρ signed distance from z axis to the PCA on track
 - 3. φ azimuthal angle of particle direction at PCA
 - 4. $cot(\theta)$ cotangent of polar angle of track direction

5. $z_0 - z$ coordinate of PCA





Track Finding (first steps)

Track Finding Algorithms

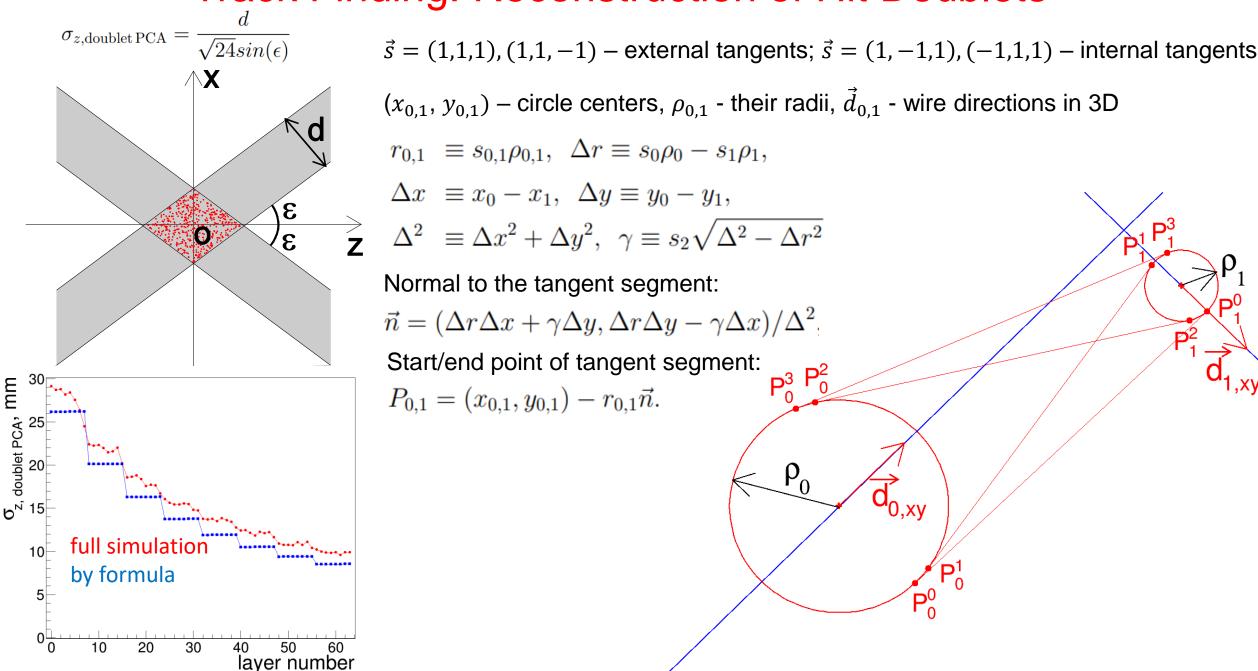
Global

- Dealing with all hits simultaneously
- Histogramming, Legendre and Hough transforms
- BaBar, Belle etc. (strereo and axial layers)

Local

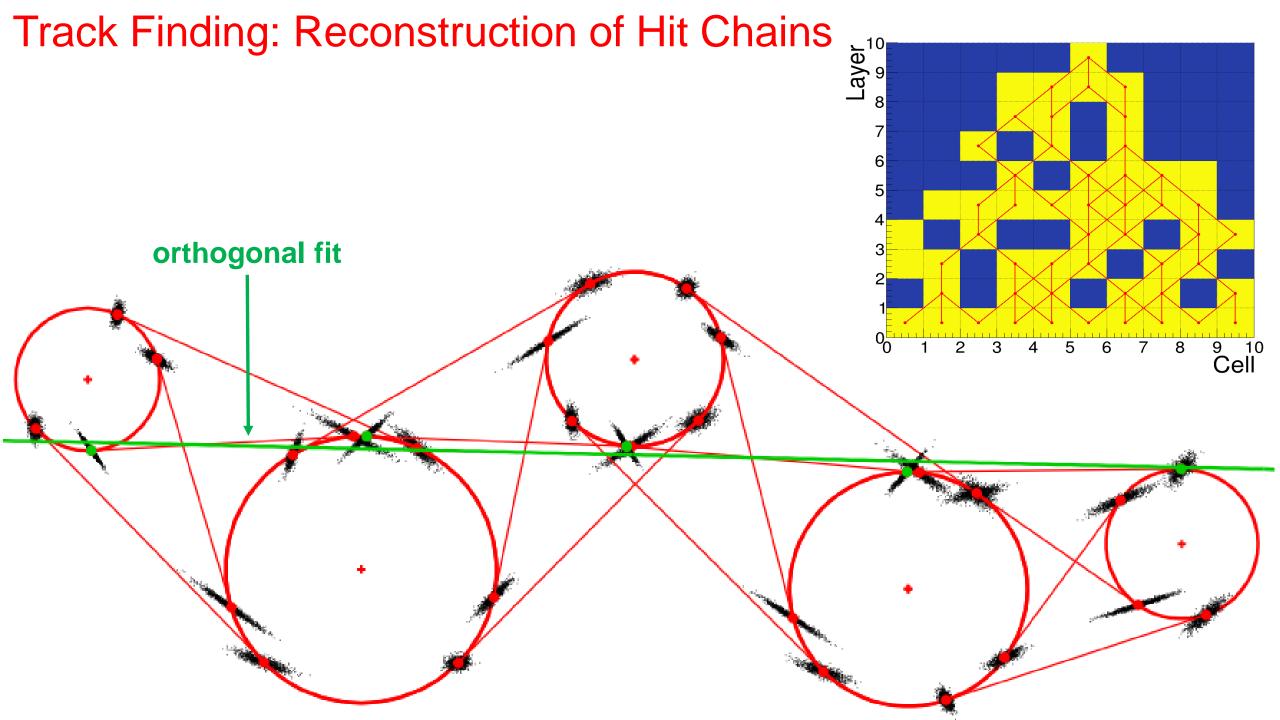
- Track Following and Cellular automaton
- KLOE, MEG2 (no axial layers)
- We are developing the local track following algorithm with the following steps:
 - 1. Reconstruction of Hit Doublets (pairs of intersecting triggered wires in consecutive layers)
 - 2. Reconstruction of Hit Chains (chains of doublets) initial track segment for extrapolation
 - Track Extrapolation and association of new hits. Since the track parameters have uncertainties, there may be incompatible variants of hit association. Is such cases the extrapolation process forks => tree of extrapolation sceneries
 - 4. For each scenario the quality function is calculated. When the extrapolation possibilities are finished we choose the best quality scenario
- Up to now only first 2 steps are implemented

Track Finding: Reconstruction of Hit Doublets



Track Finding: Tangent Points Covariance Matrix

$$cov(P_{i,x},P_{i,y}) = J \begin{bmatrix} \sigma_{\rho_0}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\rho_1}^2 & 0 & 0 \\ 0 & 0 & \sigma_{z_0}^2 & 0 \\ 0 & 0 & 0 & \sigma_{z_1}^2 \end{bmatrix} J^T, i = 0, 1 \quad J = \frac{\partial(P_{i,x},P_{i,y})}{\partial(\rho_0,\rho_1,z_0,z_1)} = \begin{bmatrix} \frac{\partial P_{i,x}}{\partial \rho_0} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial z_0} & \frac{\partial P_{i,x}}{\partial z_1} \\ \frac{\partial P_{i,y}}{\partial \rho_0} & \frac{\partial P_{i,y}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,y}}{\partial \rho_2} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,y}}{\partial \rho_2} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,y}}{\partial \rho_2} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial \rho_1} \\ \frac{\partial P_$$



Track Fit: Riemann Circle Fit

 We use the Riemann fit as the pre-fit for the final fit with Kalman filter

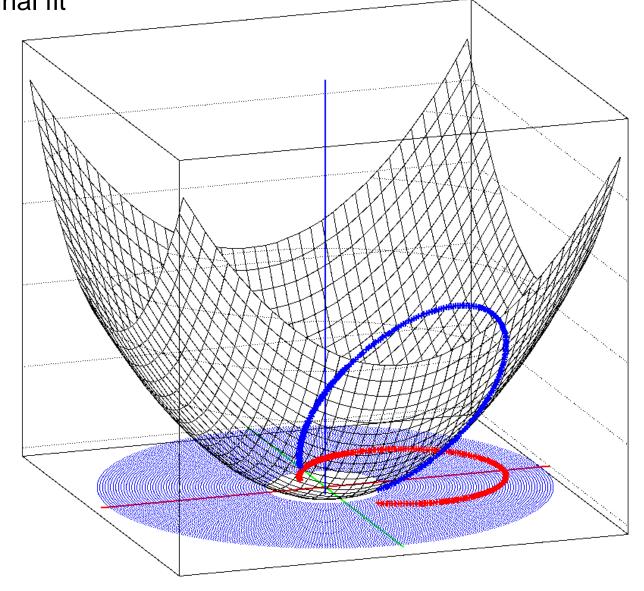
- The hits in the transversal $x_i = u_i$ plane are mapped on the $y_i = v_i$ surface of the circular paraboloid $z_i = u_i^2 + v_i^2$
- The **circle** on plane maps to the figure on the paraboloid, which is the intersection of some plane $\vec{n}^T \vec{r} + c = 0$ with the paraboloid

$$(u - u_0)^2 + (v - v_0)^2 = \rho^2$$

$$z - 2xu_0 - 2yv_0 = \rho^2 - u_0^2 - v_0^2$$

 The fit is performed by the minimization of the cost function S – the weighted sum of squared distances from the mapped hits to the fitting plane

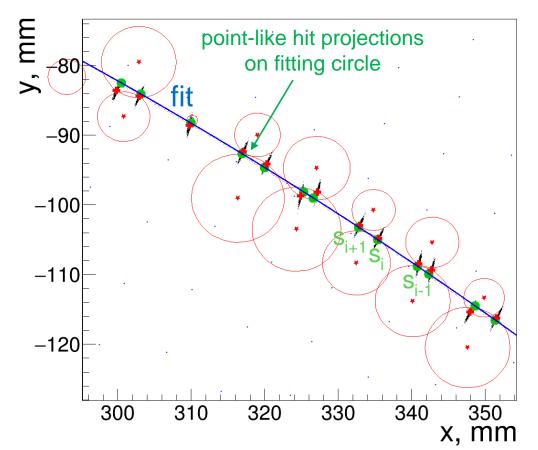
$$S = \sum_{i=1}^{N} w_i d_i^2 \quad w_i \propto 1/\sigma_i^2, \quad \sum_{i=1}^{N} w_i = 1$$



Riemann Circle Fit

- The minimization of cost function S gives the value of $c = -\vec{n}^T \vec{r}_0$, where $\vec{r}_0 = \sum_{i=1}^N w_i \vec{r}_i$
- Then the normal to the plane \vec{n} is obtained as the unit eigenvector of the sample covariance matrix A, corresponding to the smallest eigenvalue

$$\boldsymbol{A} = \sum_{i=1}^{N} w_i (\boldsymbol{r}_i - \boldsymbol{r}_0) (\boldsymbol{r}_i - \boldsymbol{r}_0)^{\mathsf{T}}$$

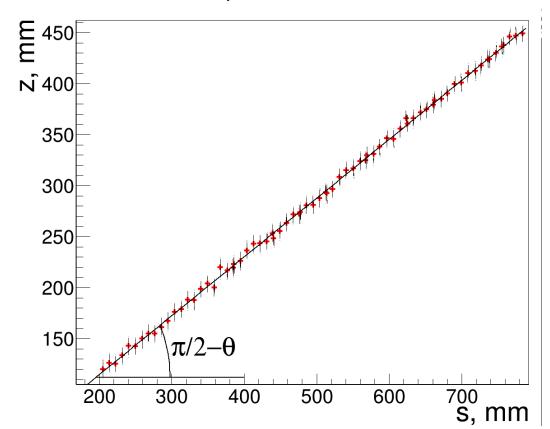


Finally, the obtained parameters of the plane are mapped back to the parameters of the fitting circle

$$u_0 = -\frac{n_1}{2n_3} \quad v_0 = -\frac{n_2}{2n_3}$$

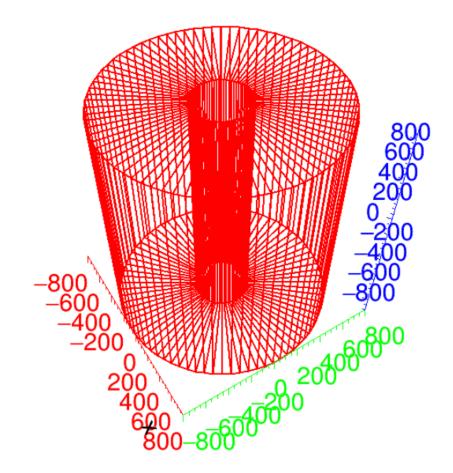
$$\rho = \frac{\sqrt{1 - n_3^2 - 4cn_3}}{2n_3}$$

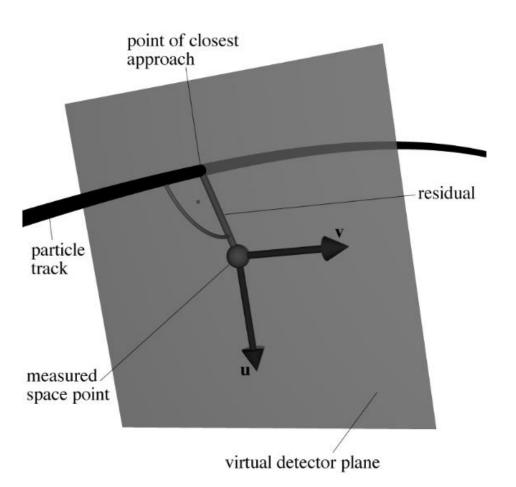
• Line fit on sz-plane:

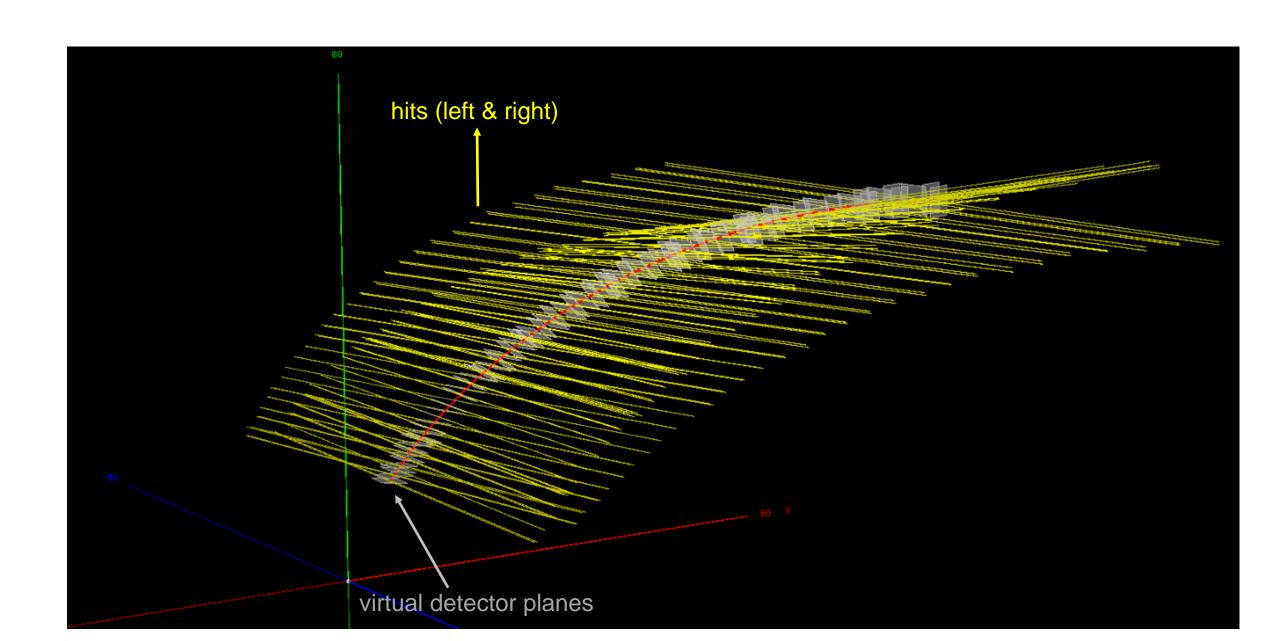


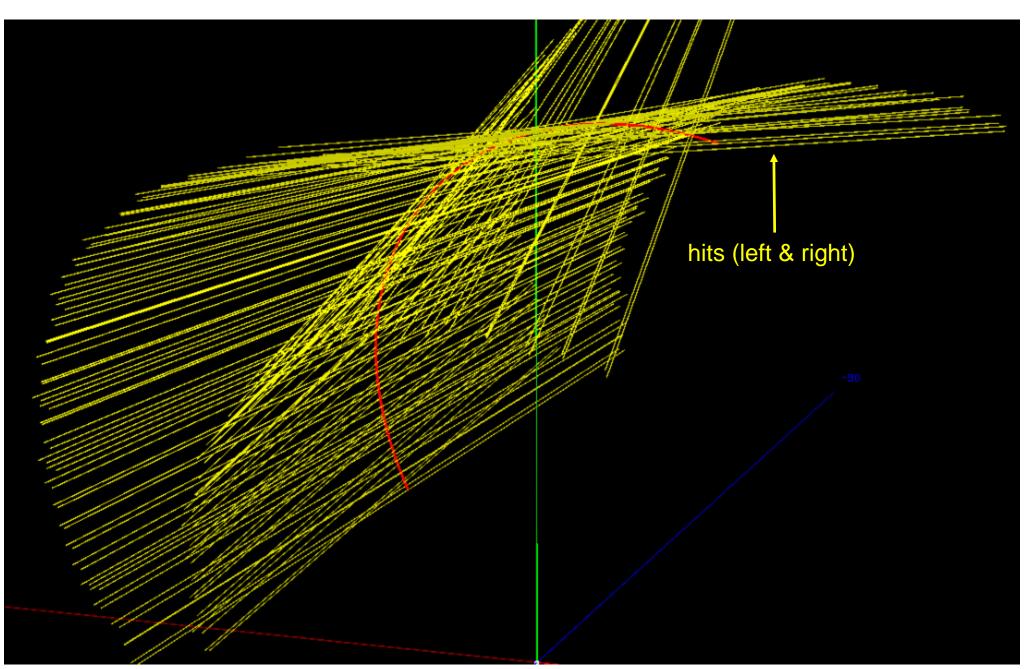
Track Fit With the Kalman Filter

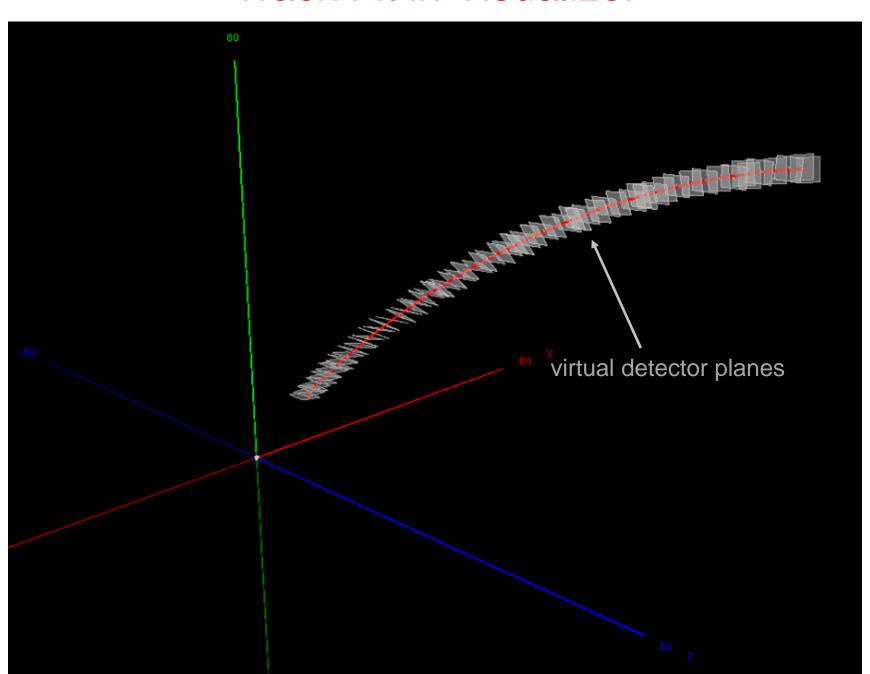
- We use the **Determenistic Annealing Filter** (iterative Kalman Filter) from **GenFit** package
- To account for the material effects we currently use only the gas volume in the fitter. In future the geometry and materials of all relevant subsystems should be added
- We use WirePointMeasurement class objects as a hits to be fitted

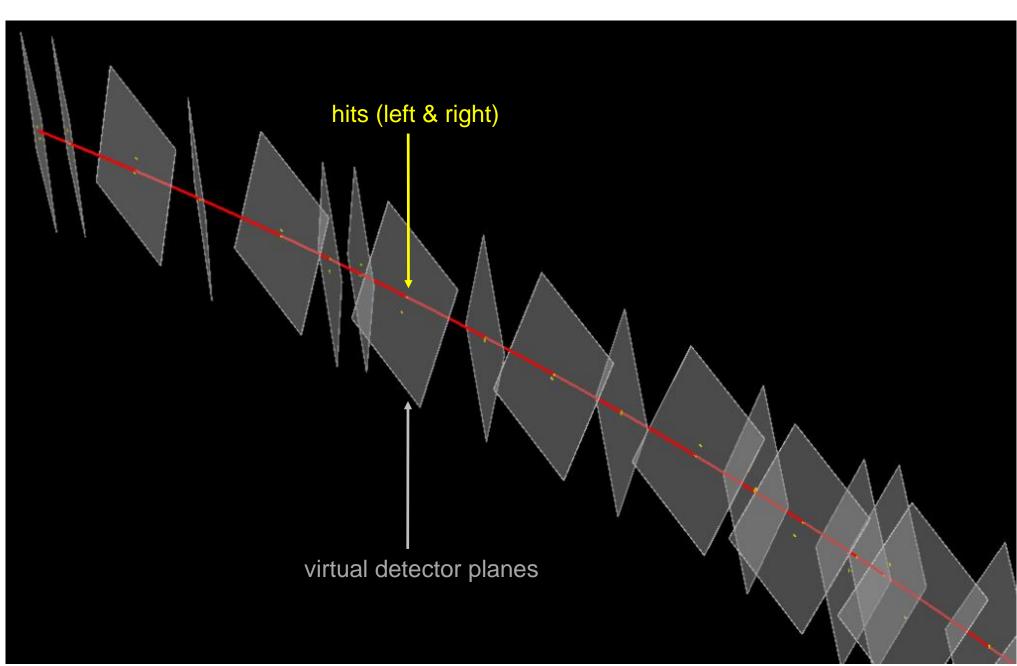


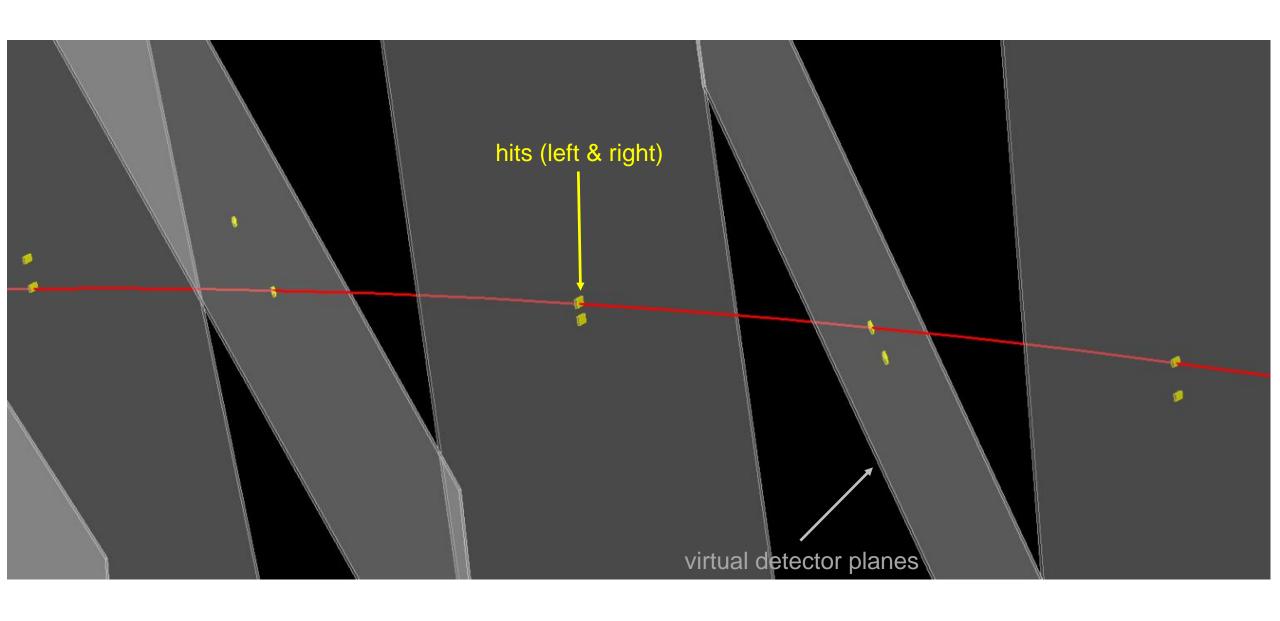


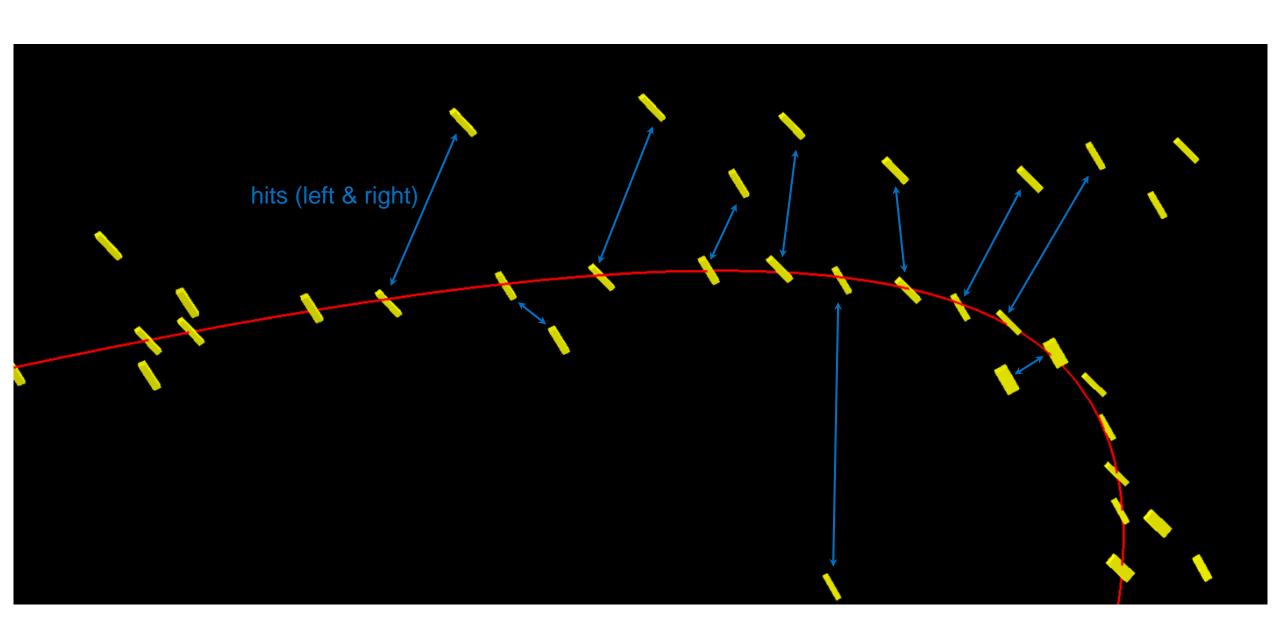


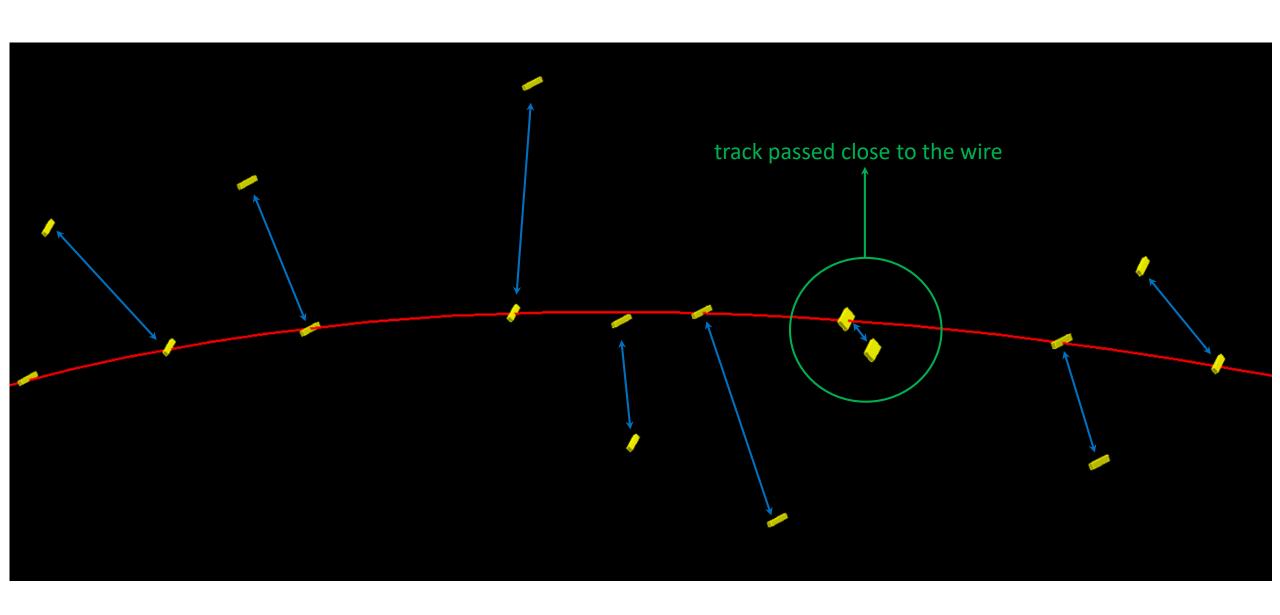


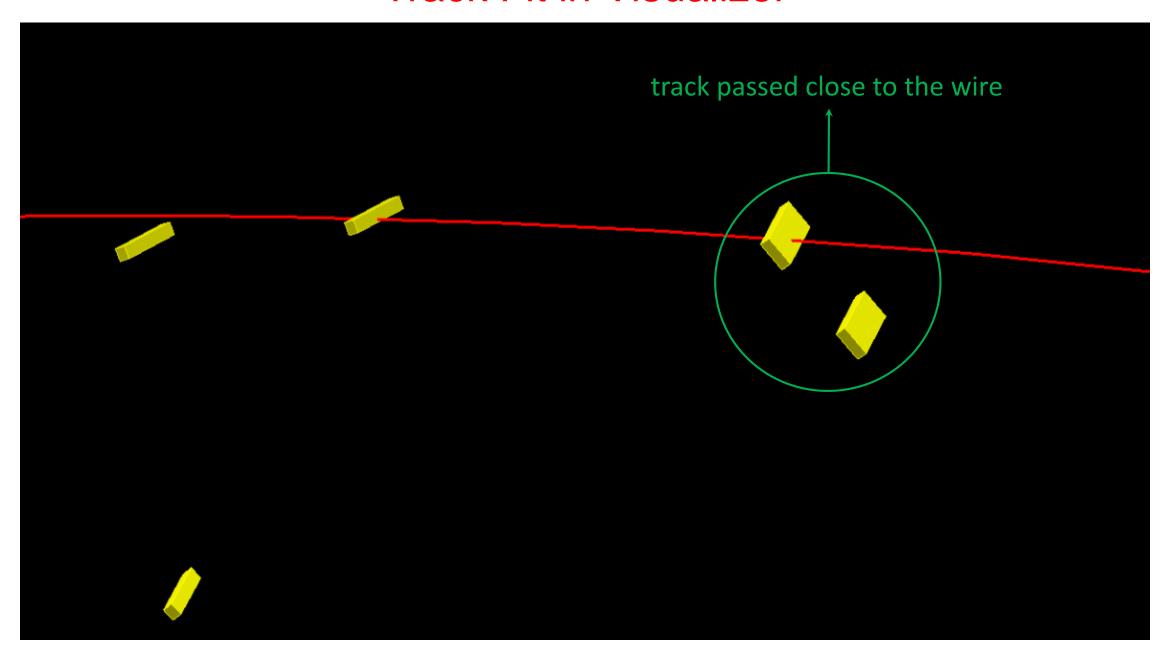


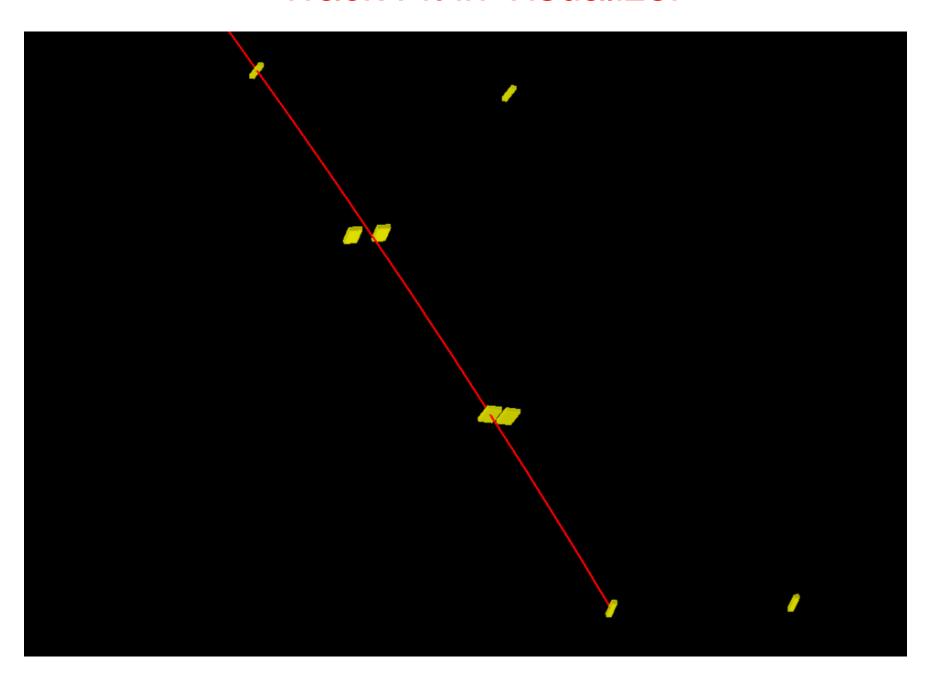












Track Parameters Covariance Matrix

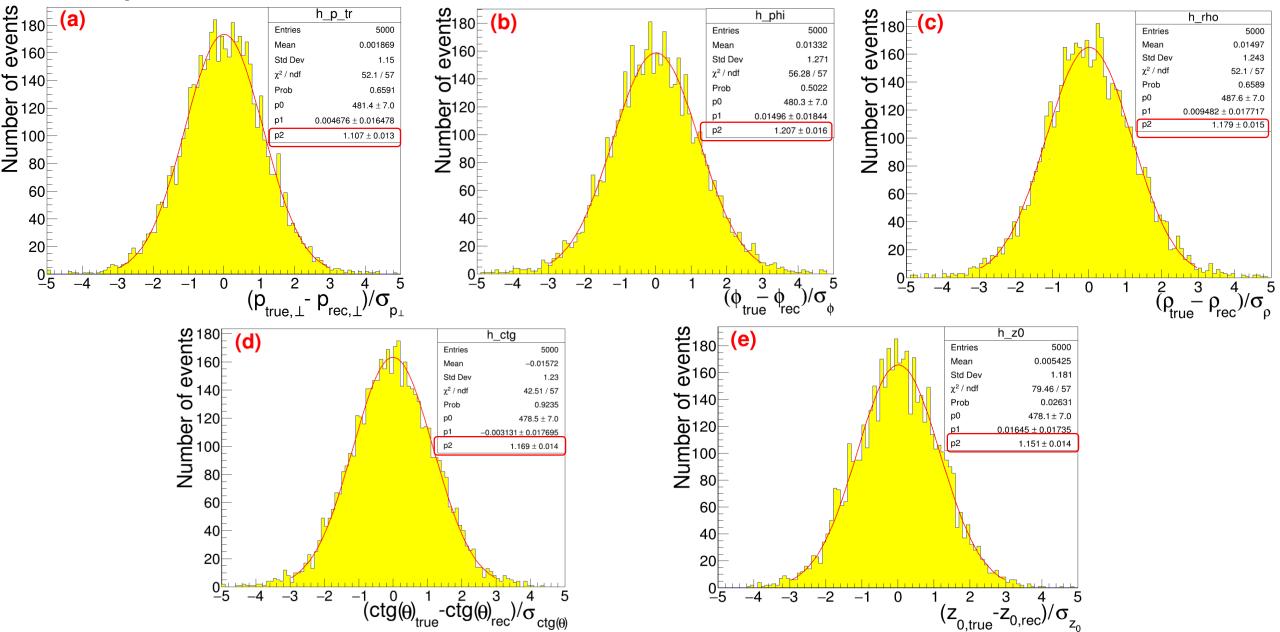
- The measured track state is propagated back to PCA to the beamline
- Since the covariance matrix is returned for the (x, y, z, p_x, p_y, p_z) state vector, we convert it to the covariance matrix for the vector $(k, \varphi, \rho, ctg(\theta), z_0)$ using the Jacobian:

$$J = \frac{\partial(k, \varphi, \rho, ctg(\theta), z_0)}{\partial(x, y, z, p_x, p_y, p_z)} = \begin{bmatrix} 0 & 0 & 0 & -k\frac{p_x}{p^2_{\perp}} & -k\frac{p_y}{p^2_{\perp}} & 0\\ 0 & 0 & 0 & -\frac{p_y}{p^2_{\perp}} & -\frac{p_x}{p^2_{\perp}} & 0\\ -\frac{p_y}{p_{\perp}} & \frac{p_x}{p_{\perp}} & 0 & 0 & 0\\ 0 & 0 & 0 & -ctg(\theta)\frac{p_x}{p^2_{\perp}} & -ctg(\theta)\frac{p_y}{p^2_{\perp}} & \frac{1}{p_{\perp}}\\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

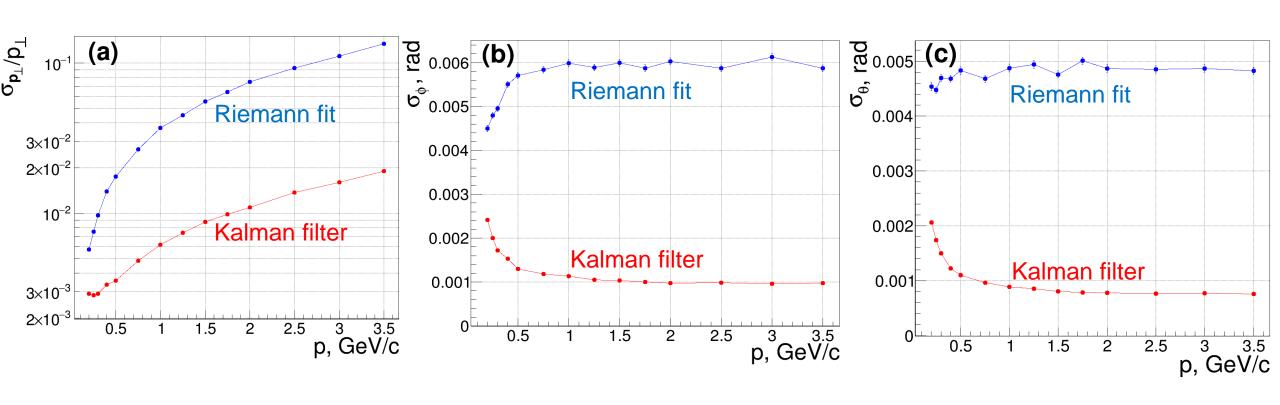
• Next, we perform the check of the correctness of the estimation of the track's parameters uncertainties

Track Parameters Covariance Matrix

• Muons, p = 1 GeV/c:



Angular and Momentum Resolutions (very-very preliminary)



Plans

- Study of the dependence of the isochrones and $t \rho$ relations on z and layer number
- Development of the track finding algorithm
- Peak and cluster searching algorithms, algorithm to obtain the most probable impact parameter for given set of cluster times
- Study of the possible effect of usage of the metal coated wires on the resolutions
- Vertex reconstruction
- Combining the inner tracker with DC into the united tracker system
- Development of kinematic fit with calorimeter, final estimation of resolutions in concrete physics analysis