

Status of the c - τ factory drift chamber simulation

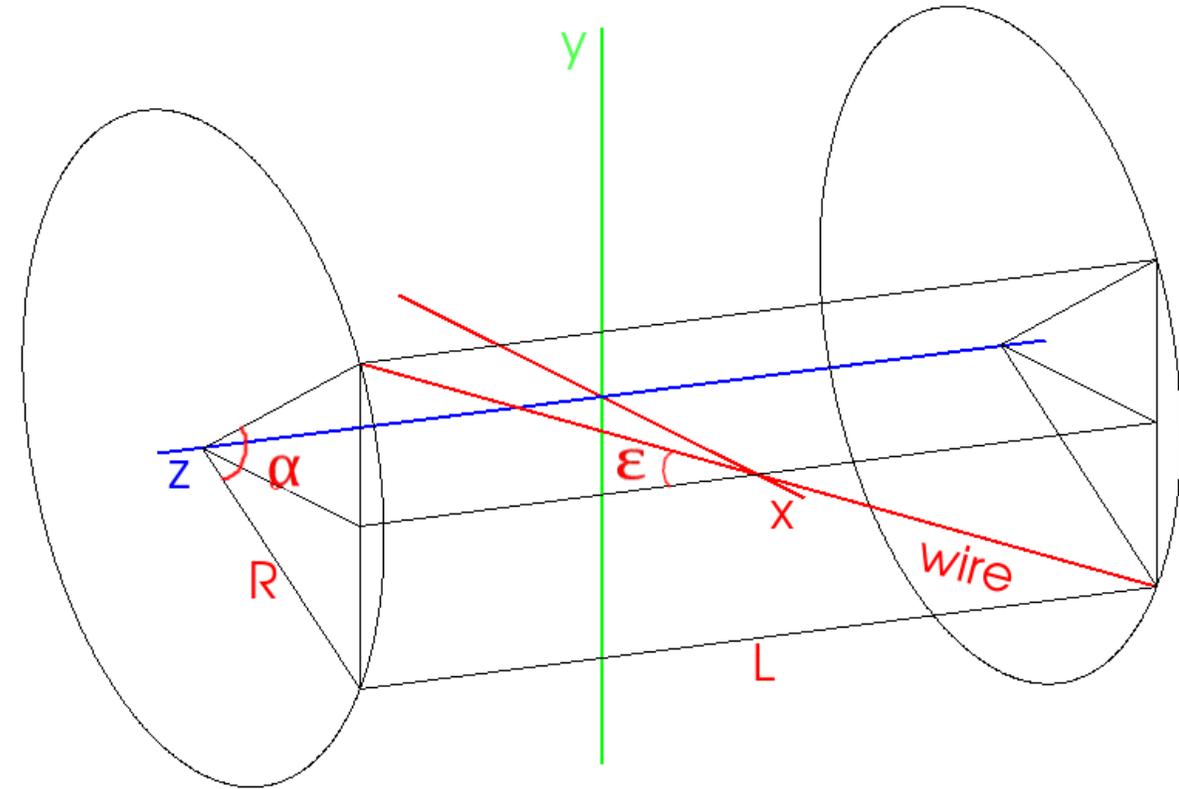
Vyacheslav Ivanov

22.06.2021

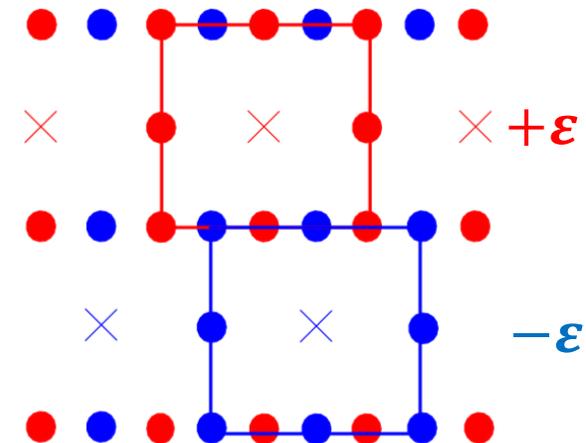
Drift Chamber Geometry

- We simulate the full-stereo drift chamber with the parameters, close to that proposed by F. Grancagnolo: https://indico.ijclab.in2p3.fr/event/4902/contributions/17030/attachments/13603/16389/SCTFDrift_Chamber.pdf

Parameter	Value
R_{in}	200 mm
R_{out}	800 mm
Length	1800 mm
Cell shape	Square
α	$\pi/6$ rad
Gas	90% He – 10% iC_4H_{10}
$N_{superlayers}$	8
Layers in superlayer	8
Signal wire diameter/material	25 μm , W
Field wire diameter/material	50 μm , Al
Cell size	$\sim 7\text{-}9$ mm
Stereoangle ϵ	$\sim 60\text{-}220$ mrad
$N_{signal\ wires}$	21824
$N_{field\ wires}$	109780

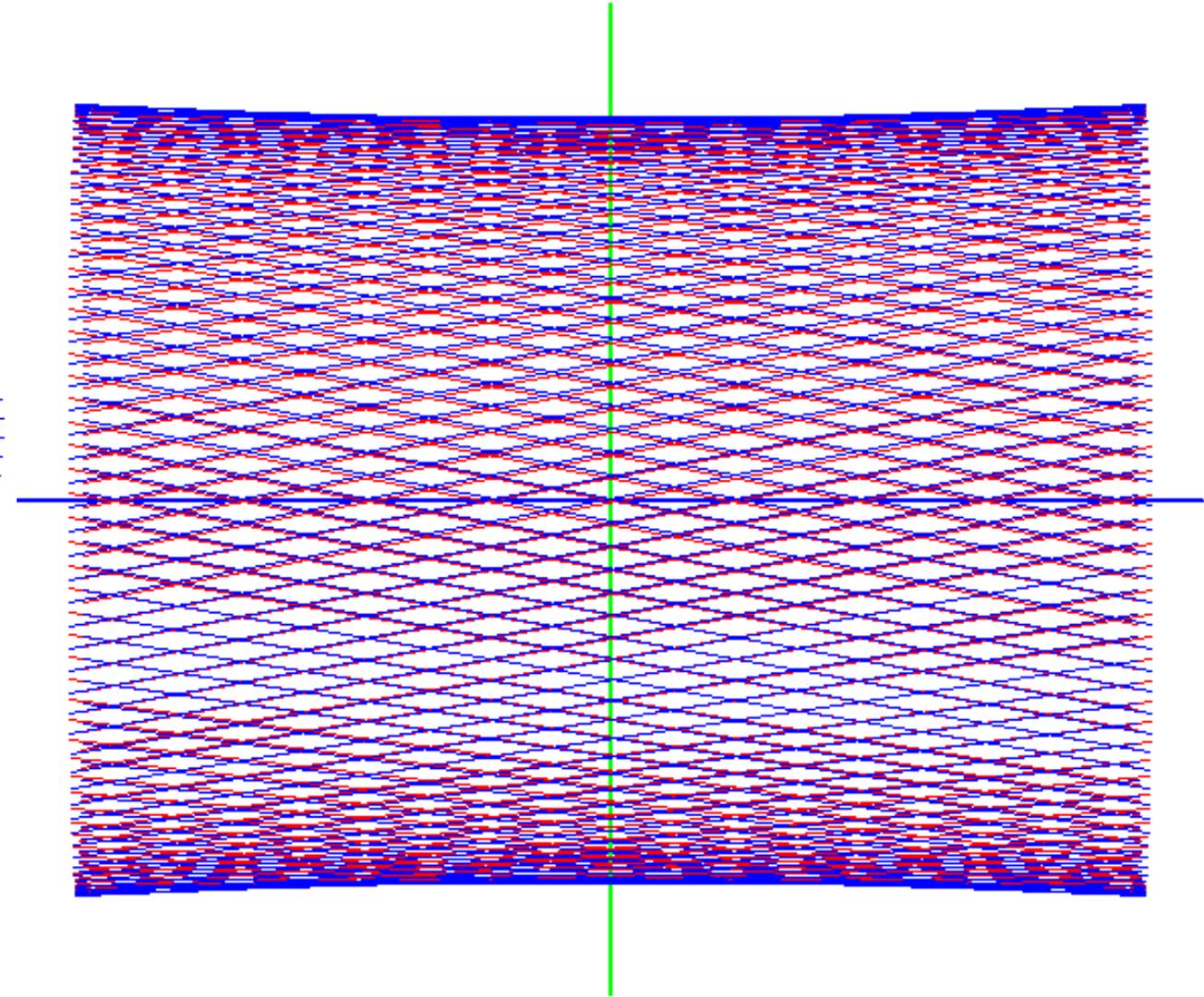
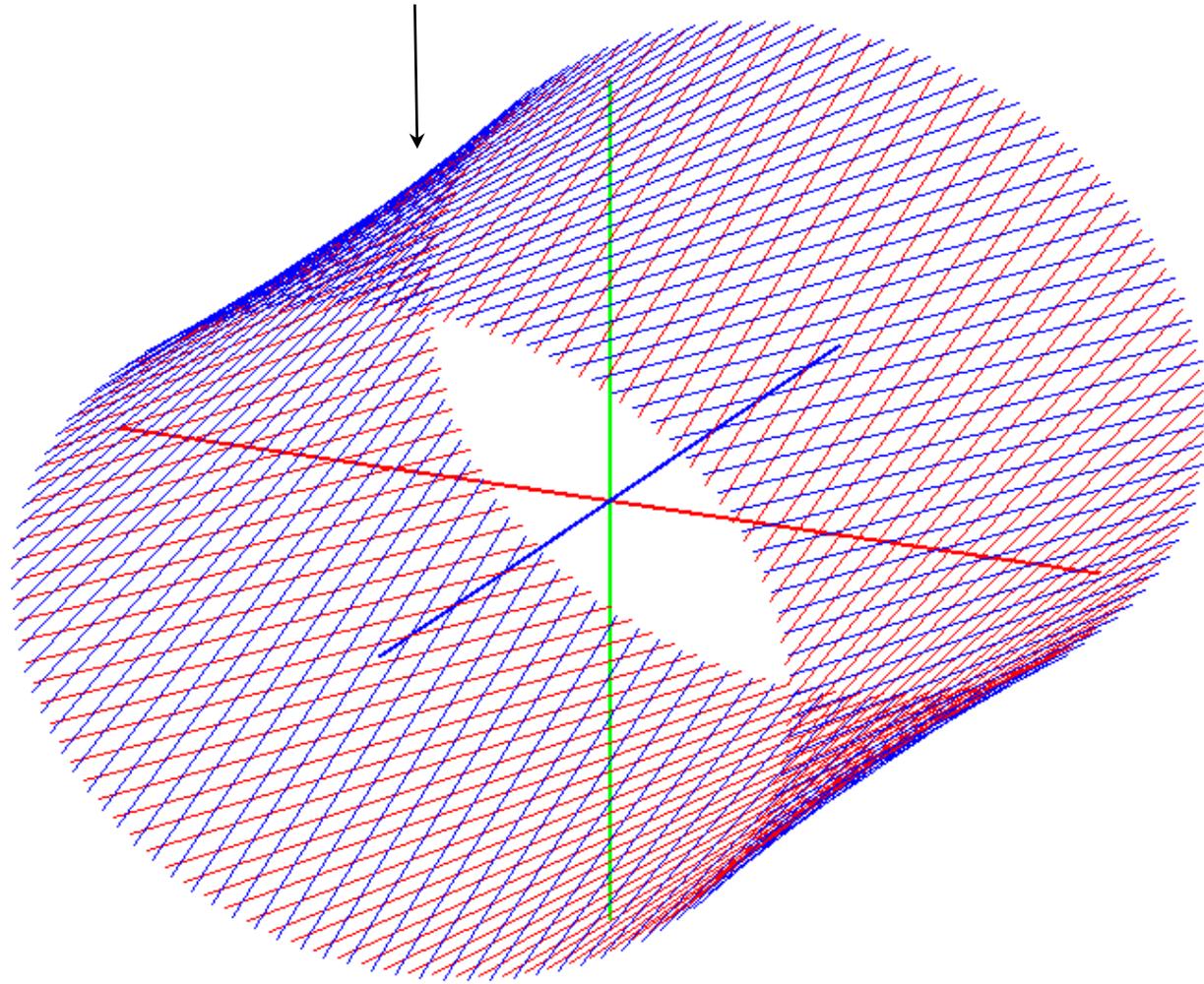


- Amount of material:
 Gas – $3.8 \cdot 10^{-4} X_0$
 Signal wires – $10 \cdot 10^{-4} X_0$
 Field wires – $7,8 \cdot 10^{-4} X_0$



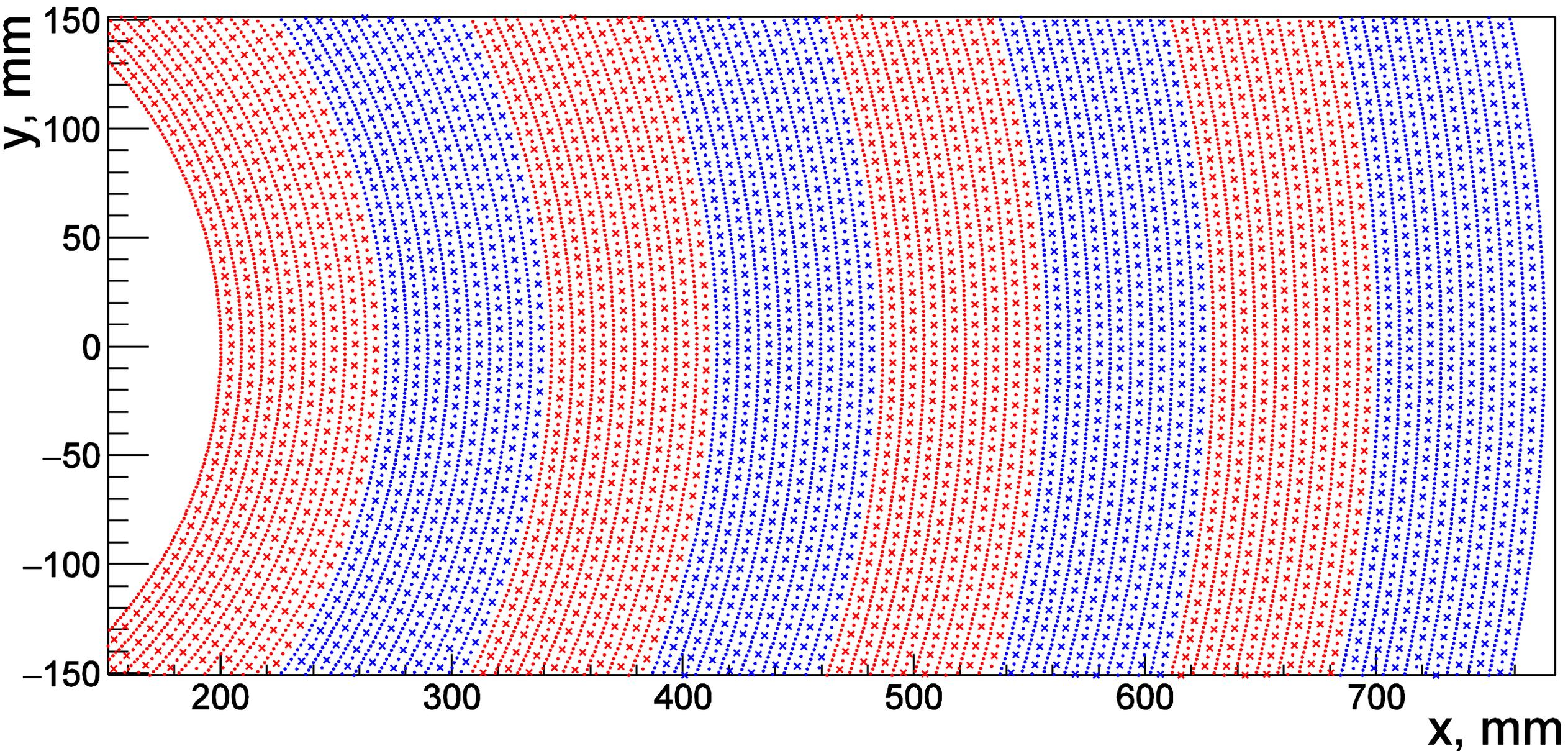
Drift Chamber Geometry

single-sheet hyperboloid



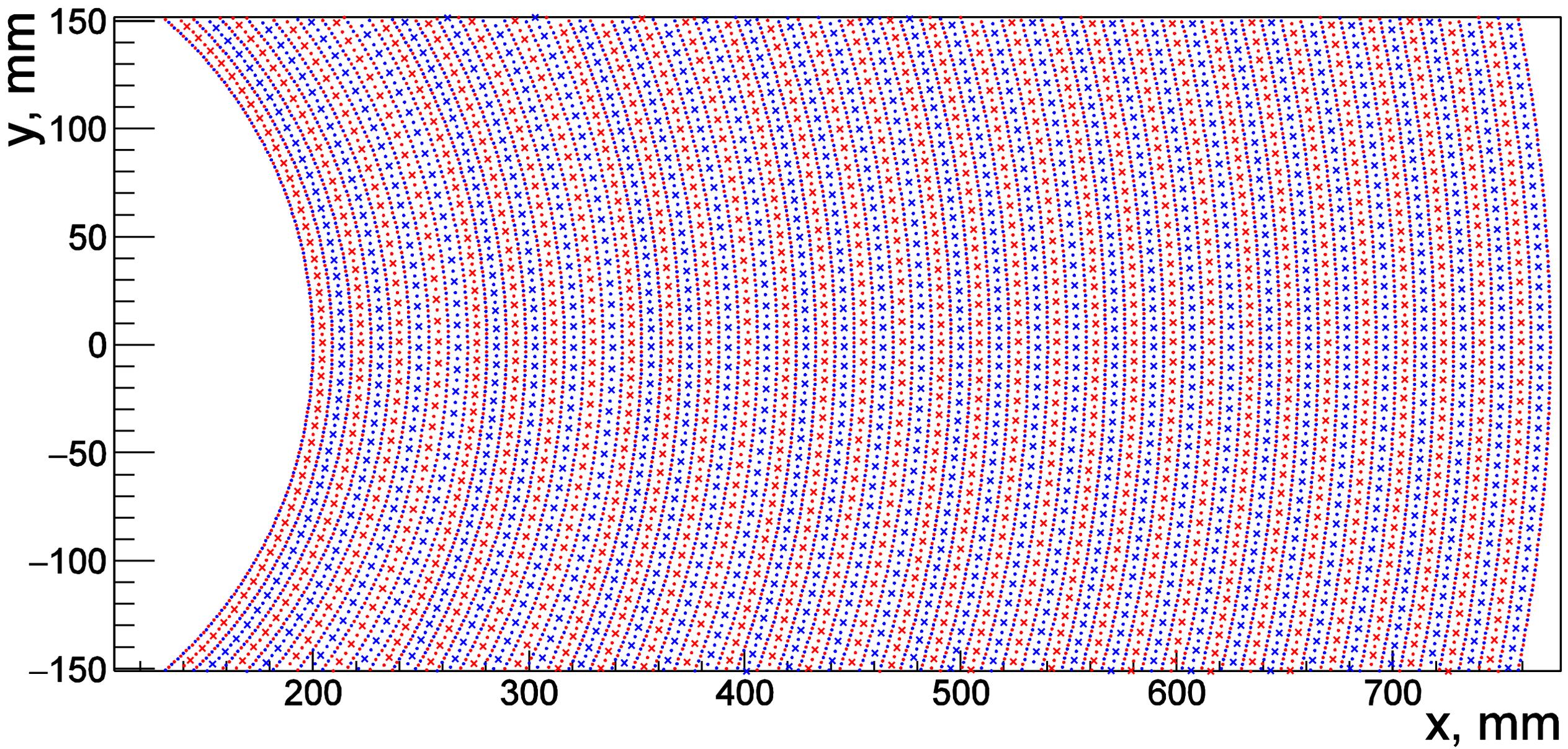
Drift Chamber Geometry

• Odd and even superlayers:



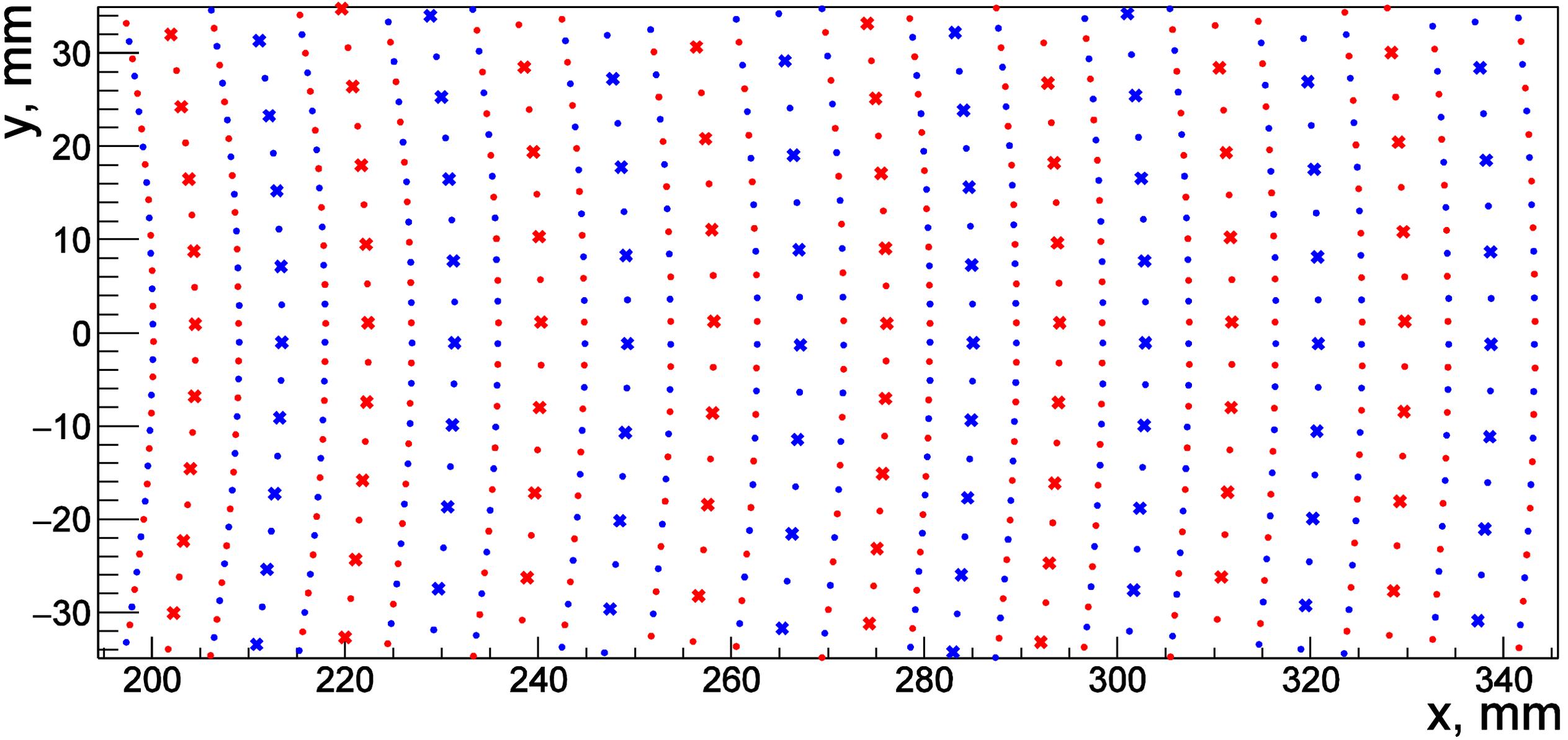
Drift Chamber Geometry

• Positive and negative streoangle:



Drift Chamber Geometry

• Positive and negative streoangle:



Drift Chamber Geometry

- The DC geometry is passed to **GEANT4** via the **DD4HEP** toolkit
- The material of wires is taken into account in two ways:
 1. Each wire is placed as a separate volume. It should be noted, that placing the wires as the child volumes directly into the gas volume of the DC would lead to a radical slowdown of the simulation (too many intersection checks during tracking). To avoid this, we place **fictitious hyperboloid daughter gas volumes** on each wire layer. The wires of this layer are then placed as daughter volumes of corresponding fictitious hyperboloid. This approach does not significantly affect the tracking time, but the amount of memory required grows up and the run initialization time becomes ~ few minutes.
 2. Instead of each wire layer we place a hyperboloid layer with width

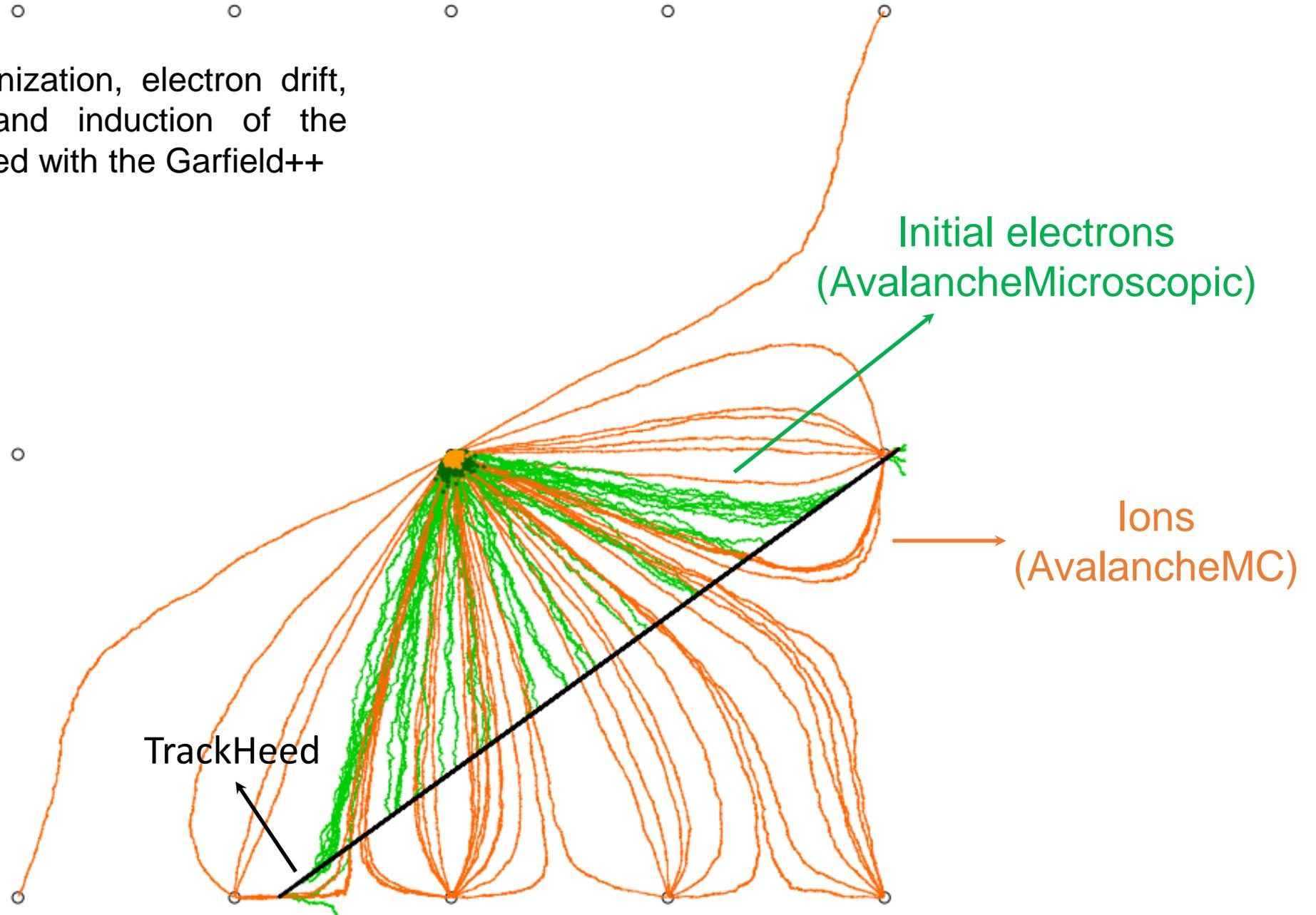
$$h = \frac{n_{wires} l_{wire} r_{wire}^2}{2L_{DC} R_{layer}(z = 0)}$$

having the same amount of material

- We save the DC geometry using **DD4HEP DataExtensions** tool and invoke the geometry whenever necessary during digitization and reconstruction via the special **GaudiTool**

Simulation of wire signals

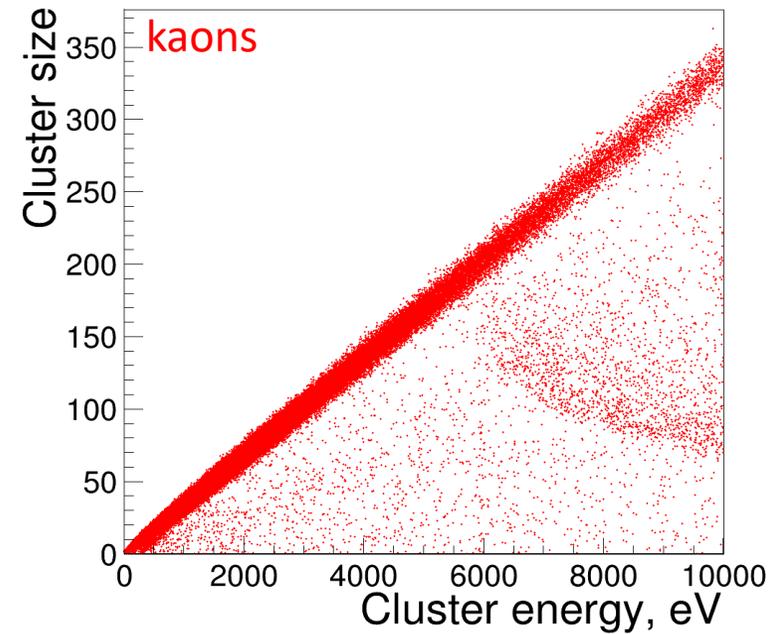
- The effects of the gas ionization, electron drift, avalanche amplification and induction of the signal on a wire were studied with the Garfield++



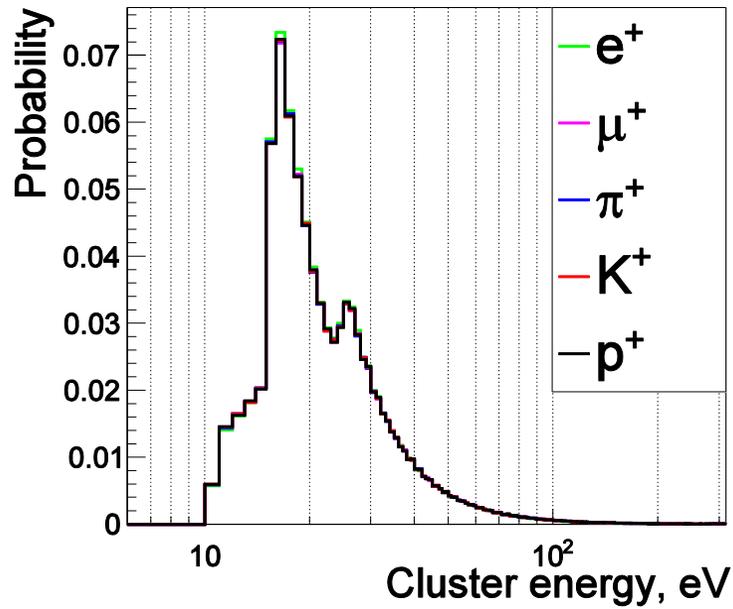
Simulation of Ionization Clusters

- On the spiral stretched between the beginning and the ending points of the GEANT4 hit (G4Step) we generate the ionization clusters until their total energy exhausts the hit energy
- According to Garfield::TrackHeed:

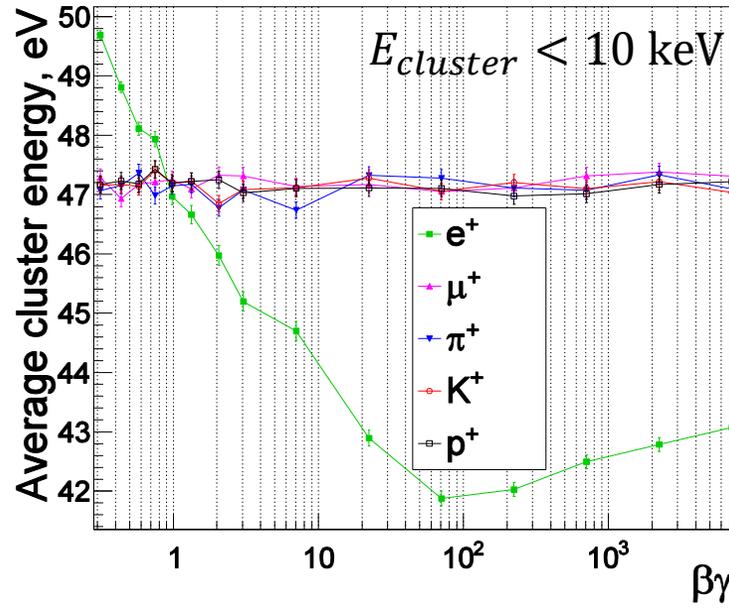
- Garfield predicts the average energy, required for single electron-ion pair production $W = 29.52 \text{ eV}$ and Fano factor $F = 0.19$. For the generated cluster energy E_{cl} the average number of electrons in cluster is E_{cl}/W and the fluctuation is $\sqrt{FE_{cl}/W}$



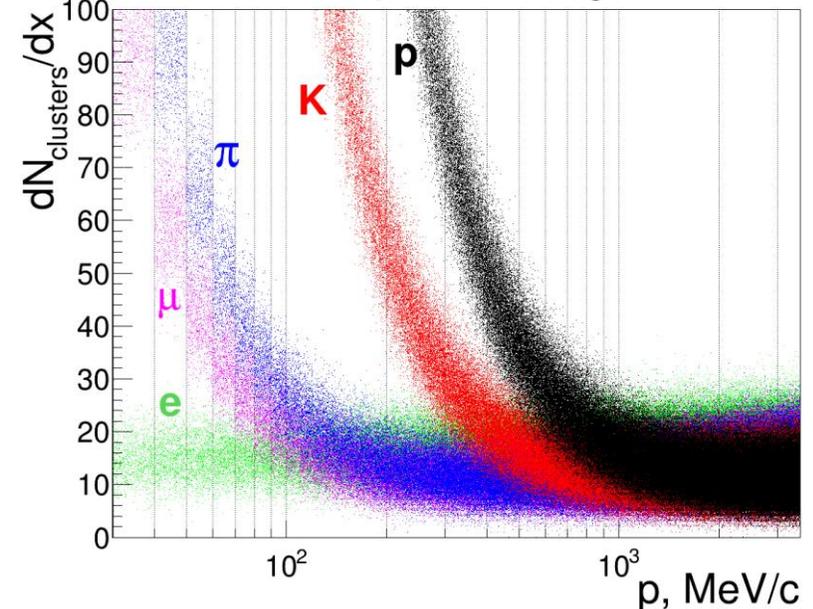
The difference between the cluster energy (=energy transfer) spectra for heavy particles appears to be very small:



The dependence of the cluster energy spectra on the velocity is also small for heavy particles:

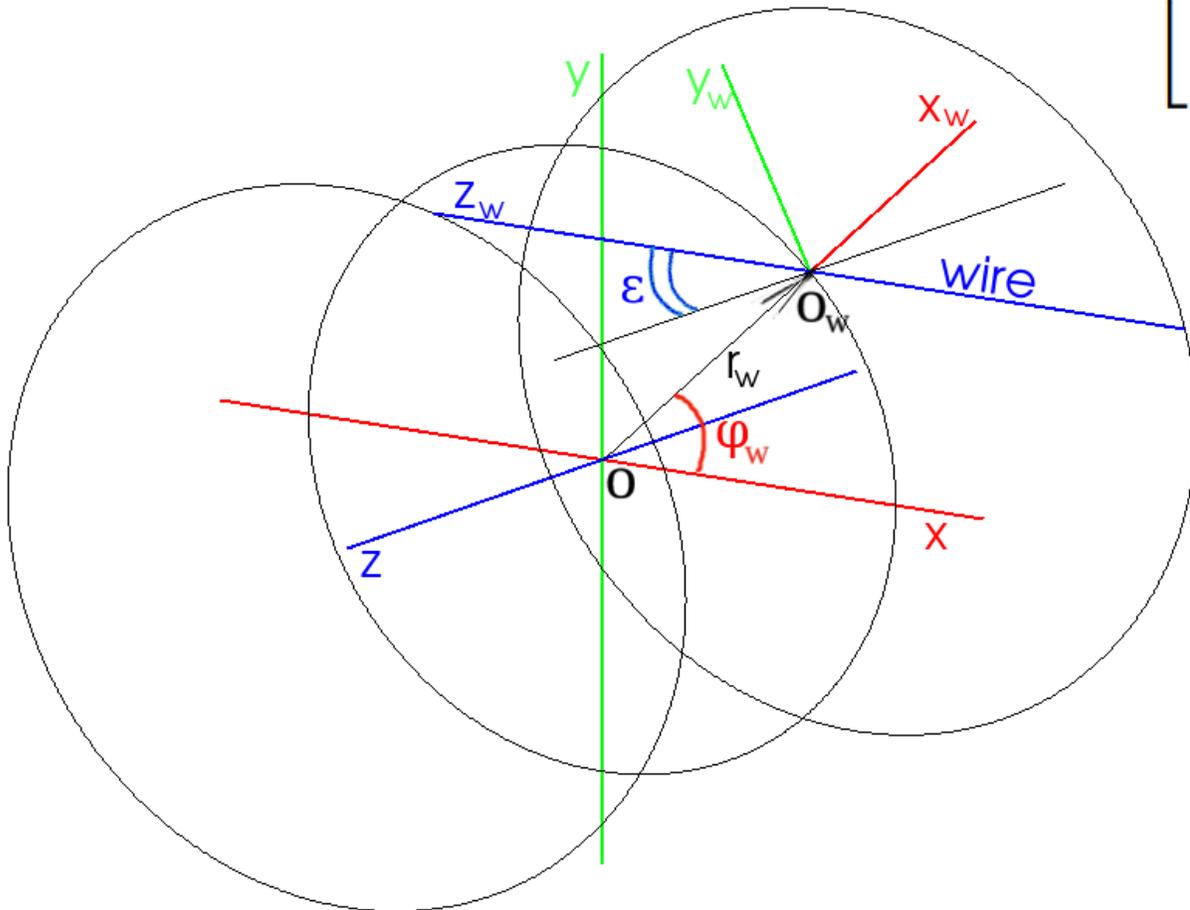


The major dependence on the particle type and momentum is seen in the number of ionization clusters per unit length:



Association of Clusters with Wires

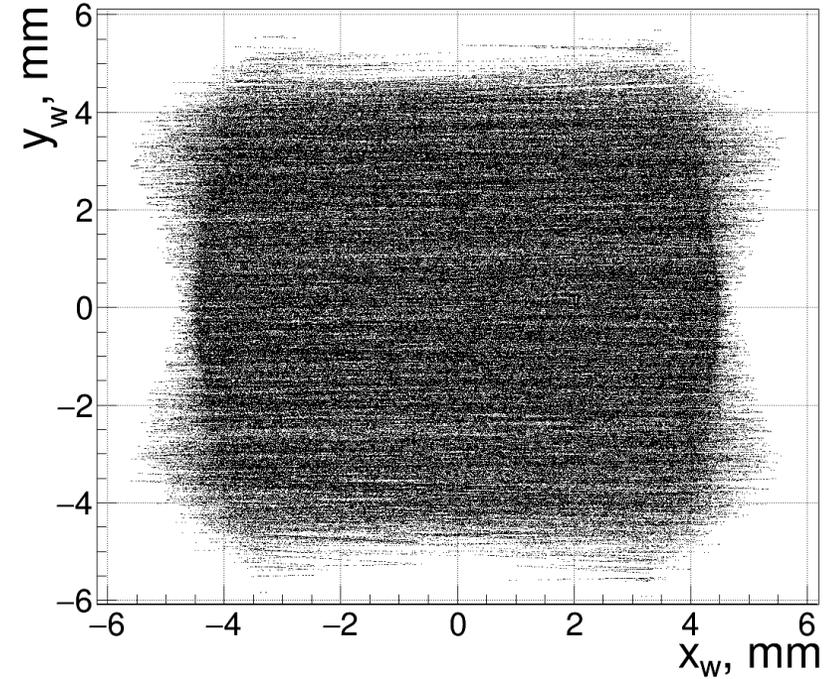
- Each ionization cluster is associated with the closest signal wire
- Cluster coordinates are translated into the wire coordinate frame $O_w x_w y_w z_w$



$$\vec{OO}_w \equiv \vec{r}_w = (r_w \cos(\varphi_w), r_w \sin(\varphi_w), 0)$$

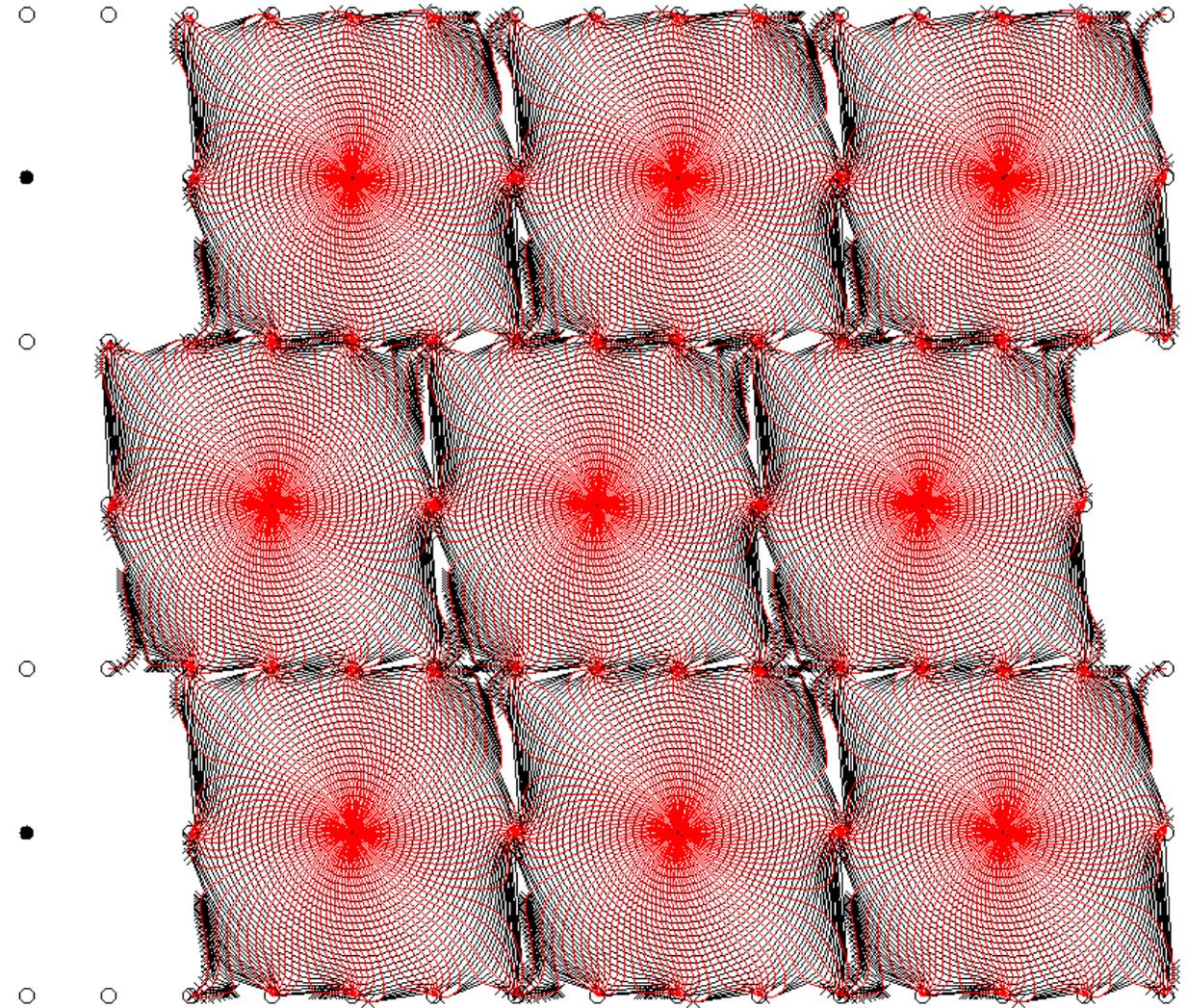
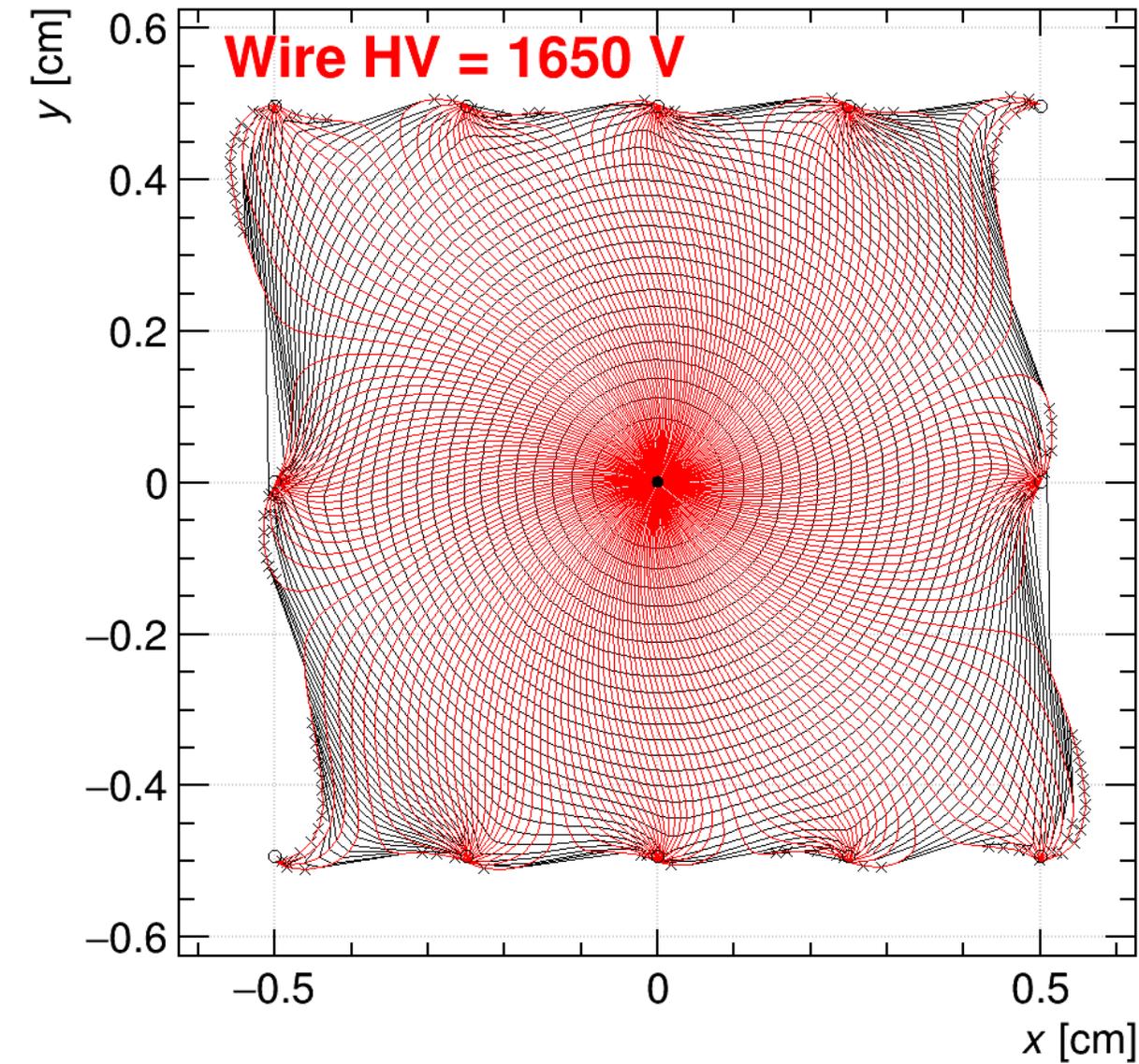
$$\vec{r}_{cl,w} = R_x(\epsilon) R_z(-\varphi_w) (\vec{r}_{cl} - \vec{r}_w) = \begin{bmatrix} \cos(\varphi_w) & \sin(\varphi_w) & 0 \\ -\sin(\varphi_w) \cos(\epsilon) & \cos(\varphi_w) \cos(\epsilon) & -\sin(\epsilon) \\ -\sin(\varphi_w) \sin(\epsilon) & \cos(\varphi_w) \sin(\epsilon) & \cos(\epsilon) \end{bmatrix} (\vec{r}_{cl} - \vec{r}_w)$$

- Clusters in the wire frame:



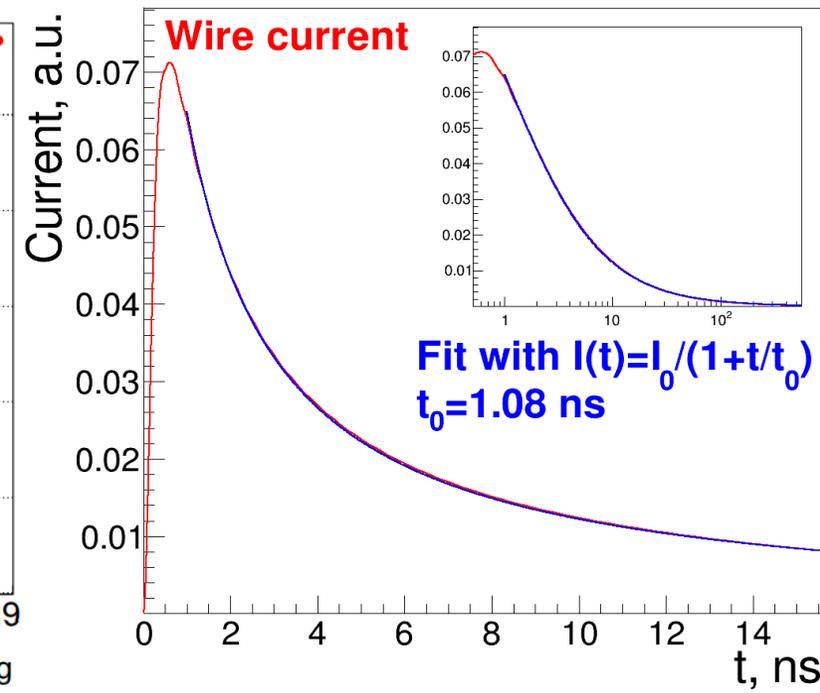
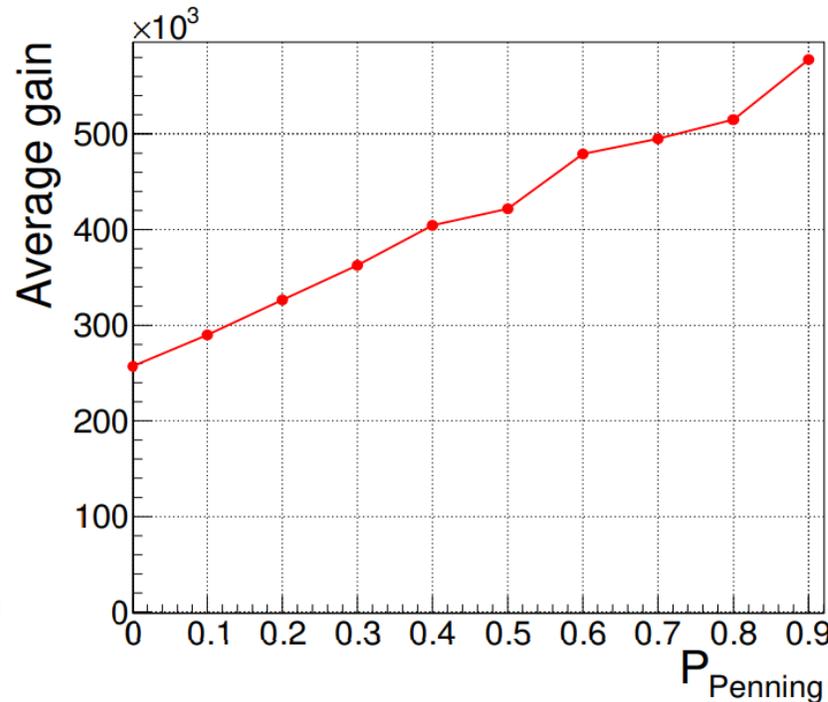
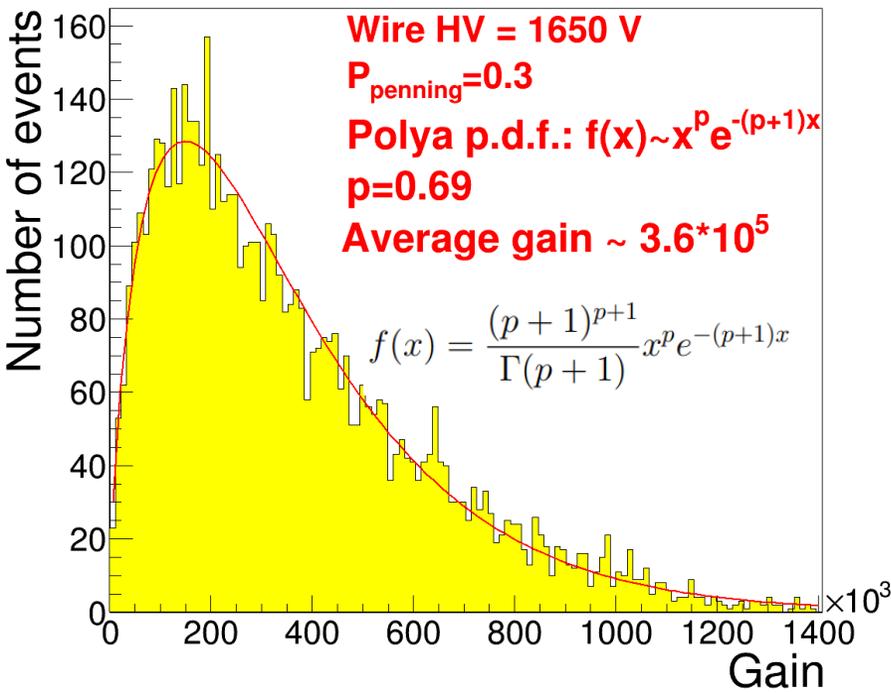
Simulation of Isochrones

- We obtained from Garfield++ the isochrones for $10 \times 10 \text{ mm}^2$ 2D square cell and use them as the first approximation for all cells at all z



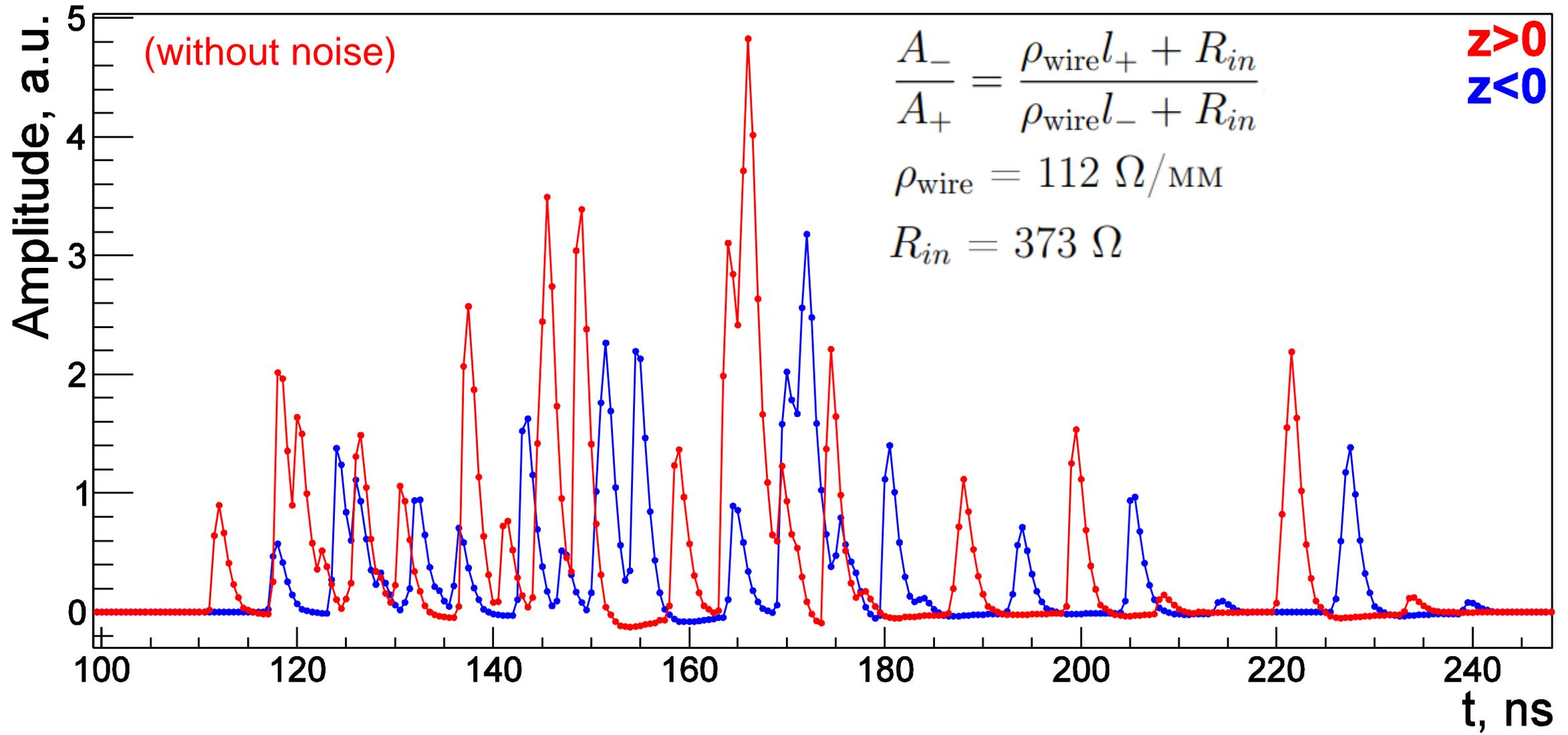
Determination of the Wire High Voltage

- The desired signal/noise ratio requires the average avalanche gain to be **few 10^5**
- The average probability P_{Penning} of the Penning/Jesse energy transfers for the mixtures with helium is unknown (we didn't find any data), so we scanned over this parameter
- We have chosen the **HV=1650 V** to be a working point, the P_{Penning} is fixed at **0.3** in further studies. The average gain in this case is **$3.6 \cdot 10^5$**
- From the fit of the gain spectrum with the Polya distribution we obtained the parameter of the latter **$p = 0.69$**
- At the given working HV by averaging the signals from many avalanches we determined the characteristic signal decrease time **$t_0 = 1.08 \text{ ns}$**



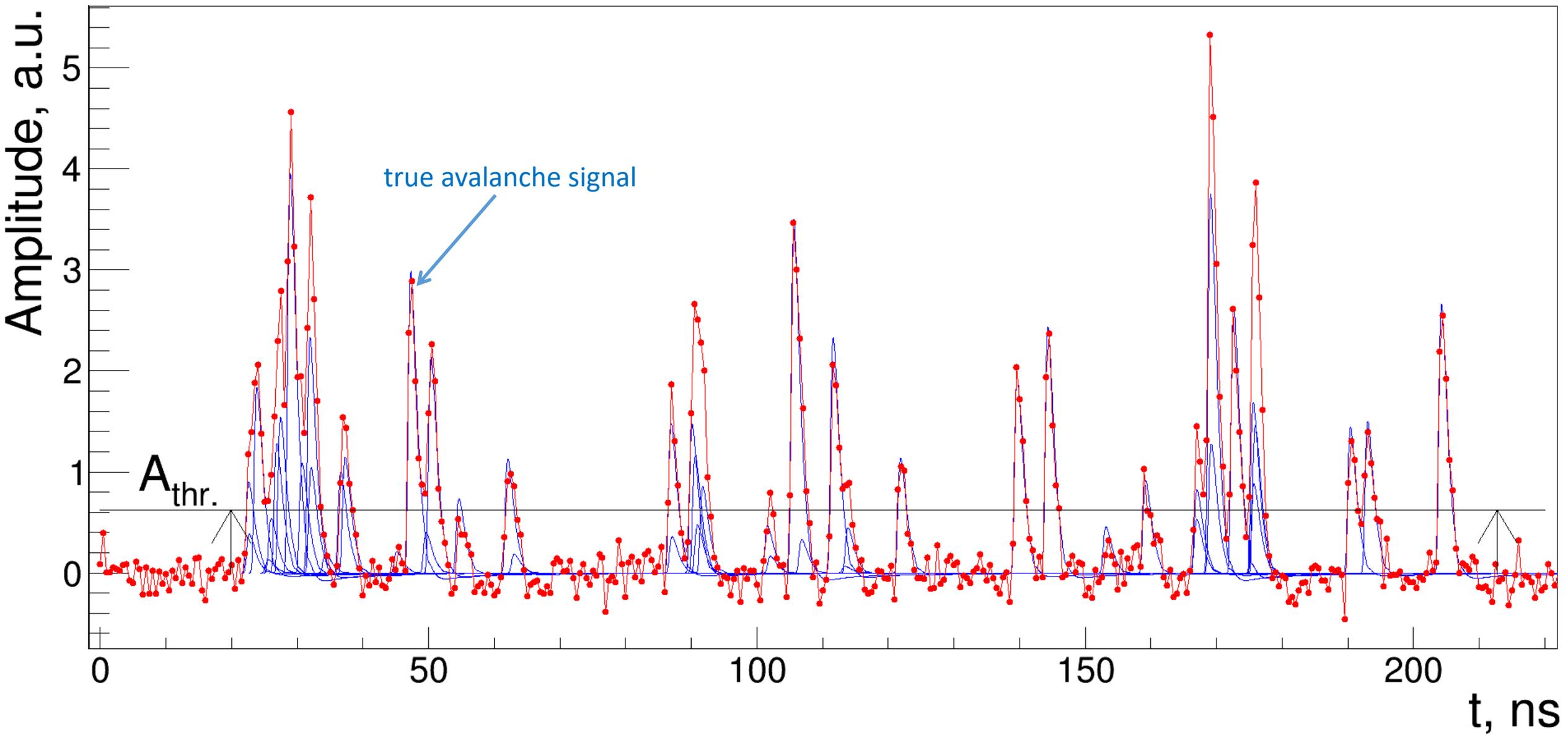
Wave Form Scan

- The waveform for all wires is scanned with 2 GHz frequency (for cluster counting)
- The signal shape is provided by the V.M. Aulchenko, signal/noise ratio is estimated to be $\sim 1/8$



Wave Form Scan

- The amplitude integration starts from 2 ns before wire triggering up to 5 ns after the last peak:

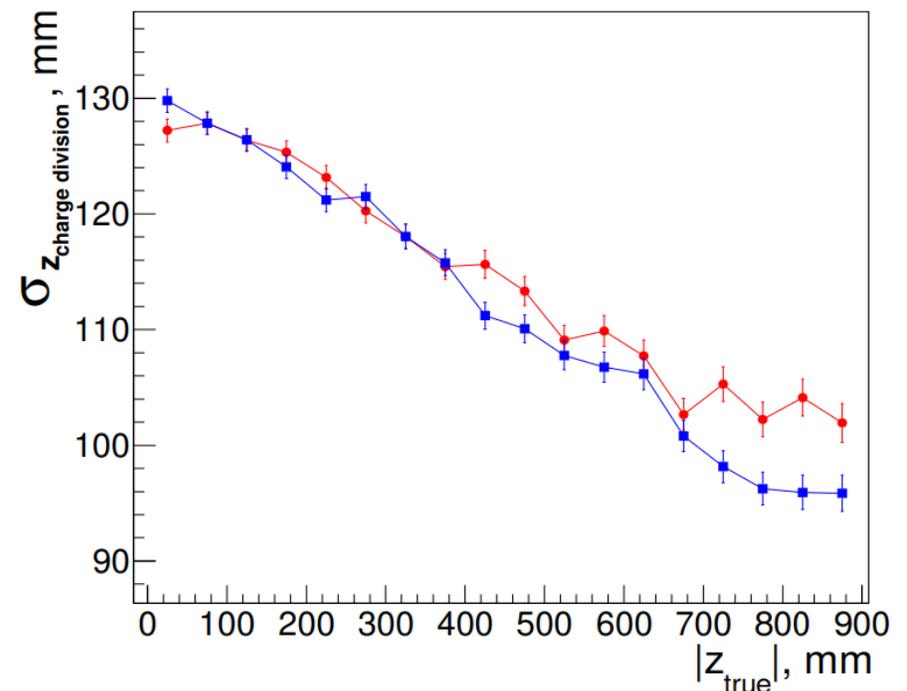
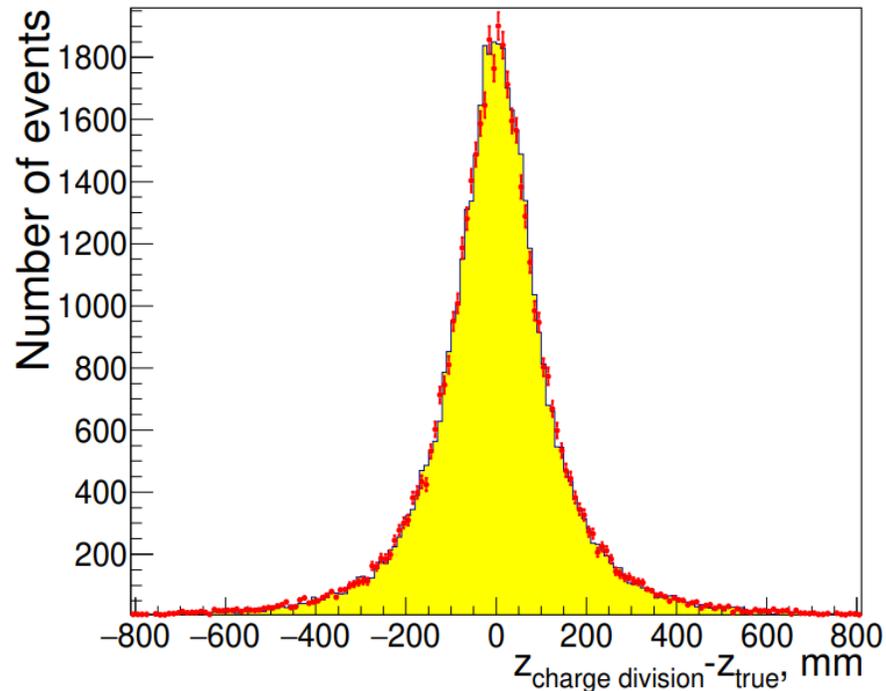


Reconstruction of Hits

- “Raw” DC hit contains: 1) integrated amplitudes from both wire ends; 2) triggering time; 3) times and amplitudes of peaks for scanned signals from both ends. From these data we obtain the collection of “reconstructed hits”
- z –coordinate of the hit is reconstructed via the charge division formula:

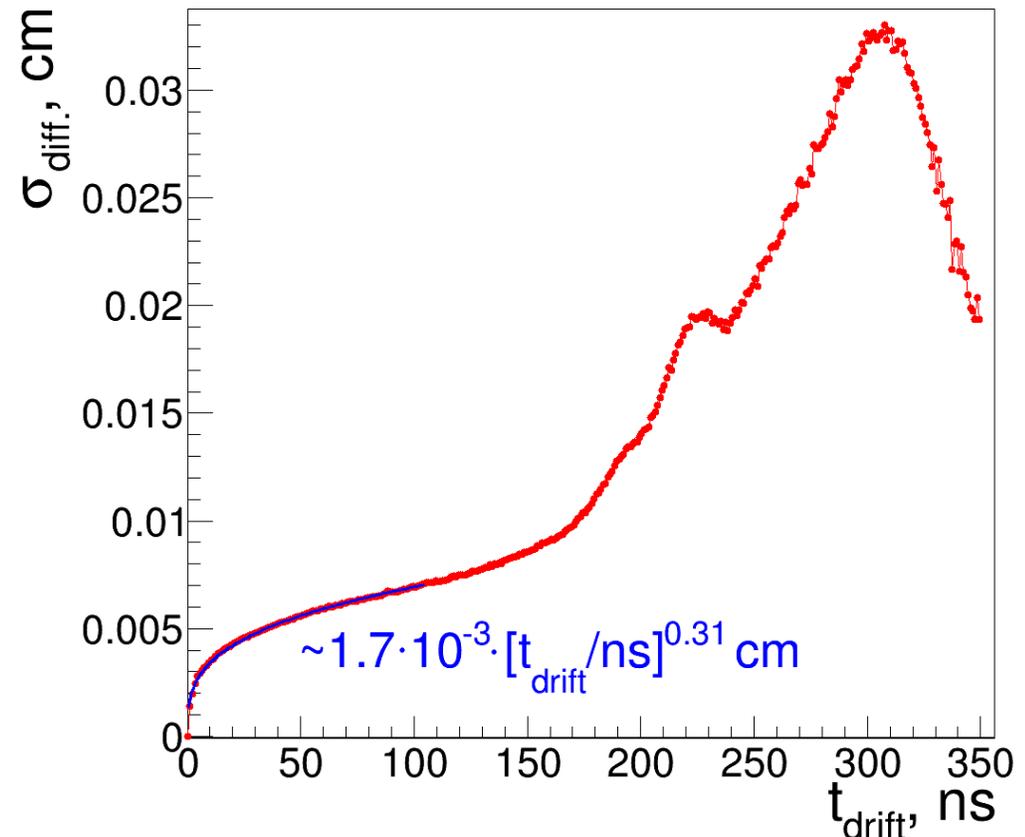
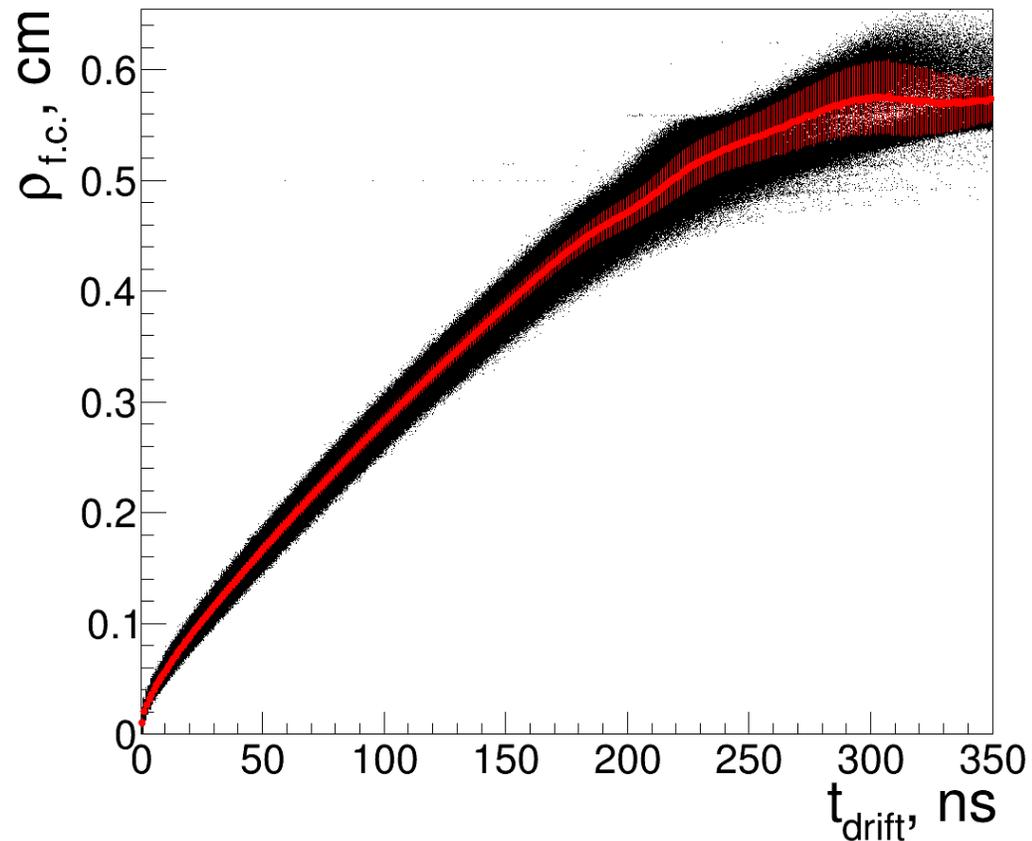
$$z = \left(\frac{l_{\text{wire}}}{2} + \frac{R_{\text{in}}}{\rho_{\text{wire}}} \right) \frac{A_+ - A_-}{A_+ + A_-}$$
$$\sigma_z = 2 \left(\frac{l_{\text{wire}}}{2} + \frac{R_{\text{in}}}{\rho_{\text{wire}}} \right) \frac{\sqrt{A_+^2 \sigma_{A_-}^2 + A_-^2 \sigma_{A_+}^2}}{(A_+ + A_-)^2}$$

$$\sigma_{A_{\pm}} = \sqrt{\sigma_{\text{noise}} t_{\text{int}} f}$$



Reconstruction of Impact Parameter

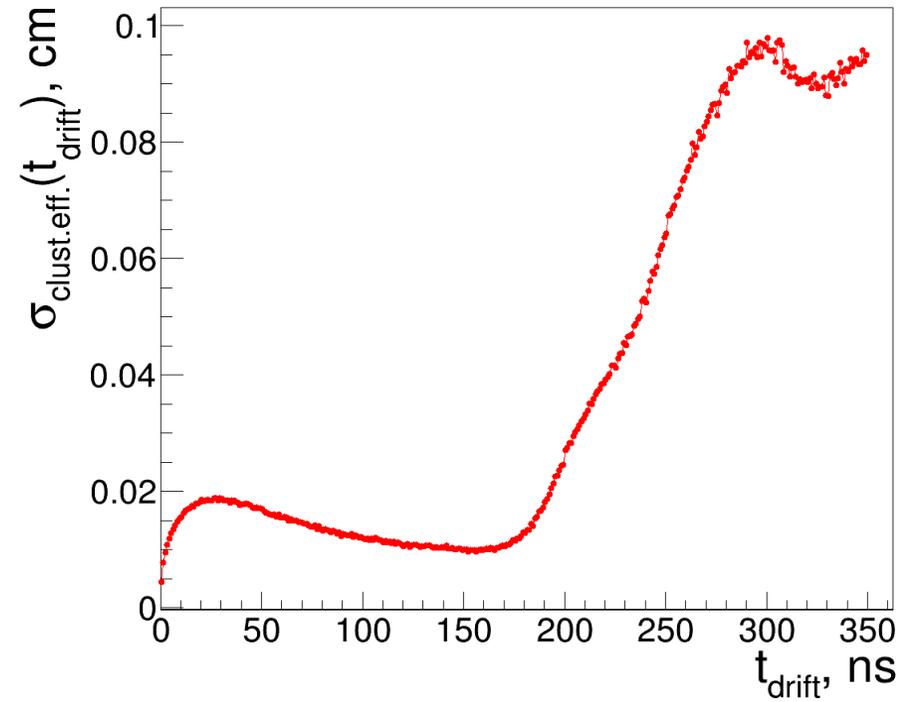
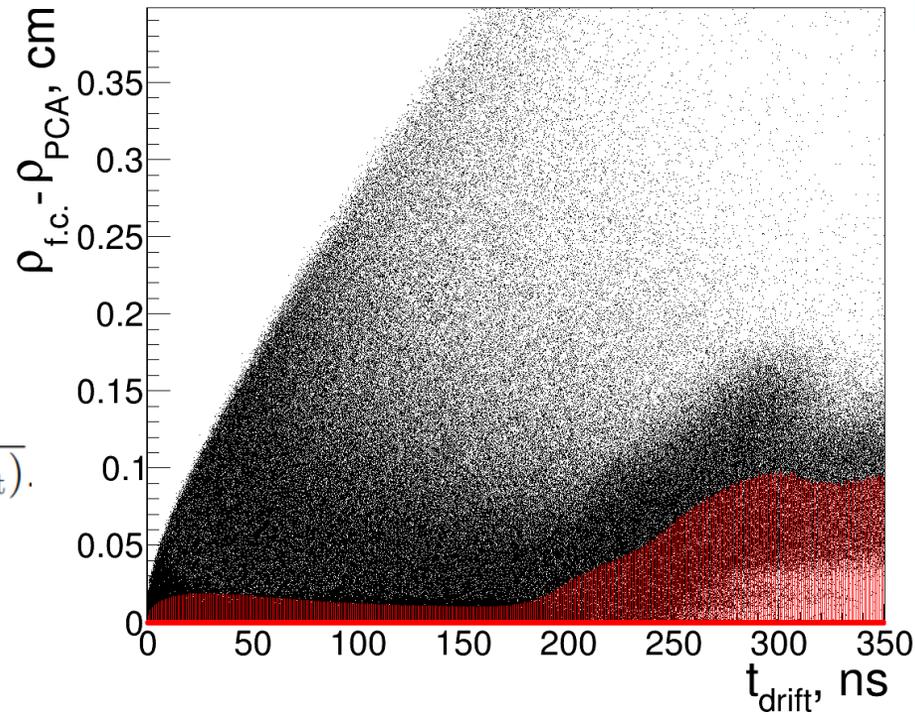
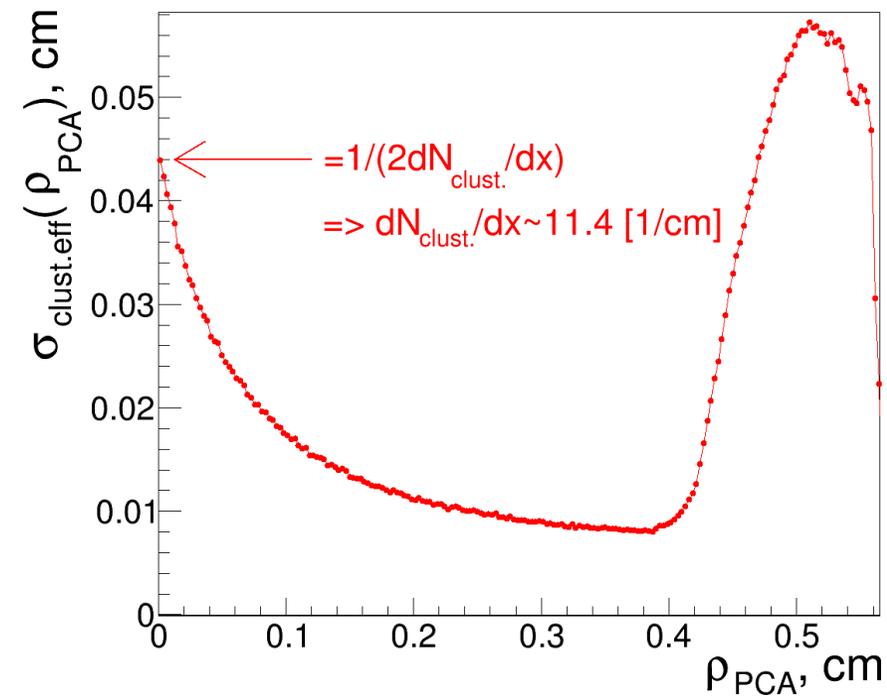
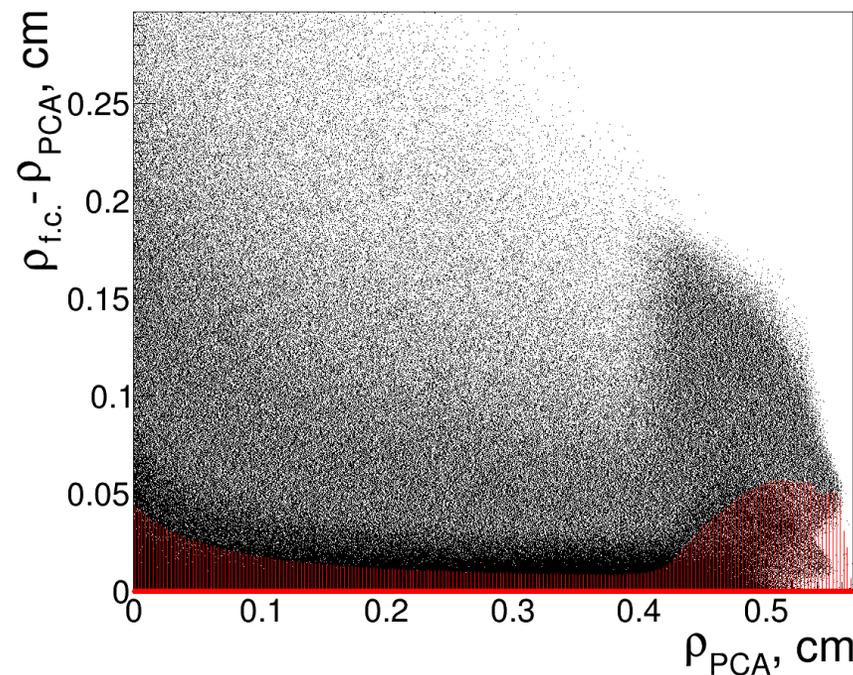
- We assume the distance to the first cluster $\rho_{f.c.}$ to be a (biased) estimation of the track impact parameter ρ_{PCA}
- The $\rho_{f.c.}$ is calculated from drift time t_{drift} via $t - \rho$ relation, obtained for the beam-originated muons
- The uncertainty of $\rho_{f.c.}$ at the given t_{drift} gives the estimate of the contribution of diffusion in the resolution on ρ_{PCA}



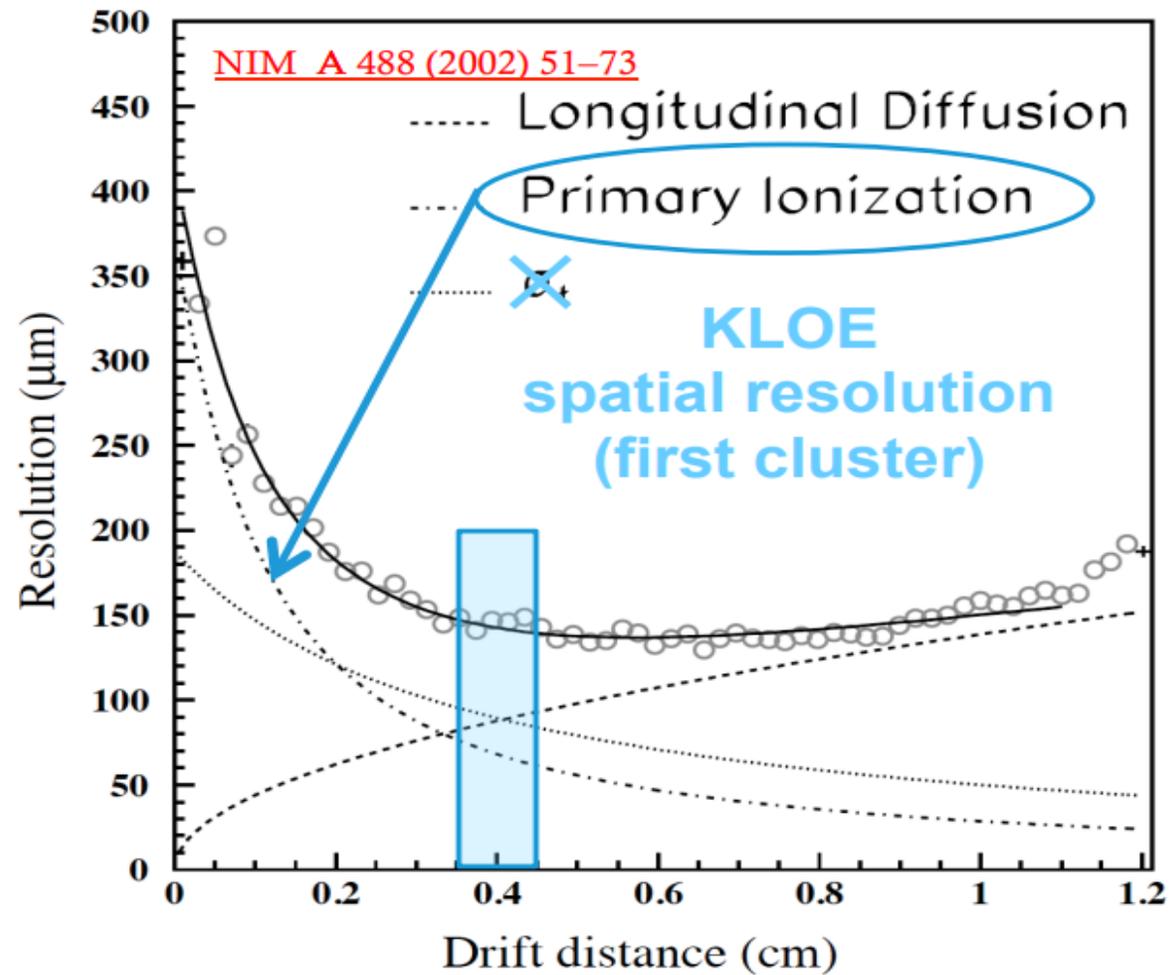
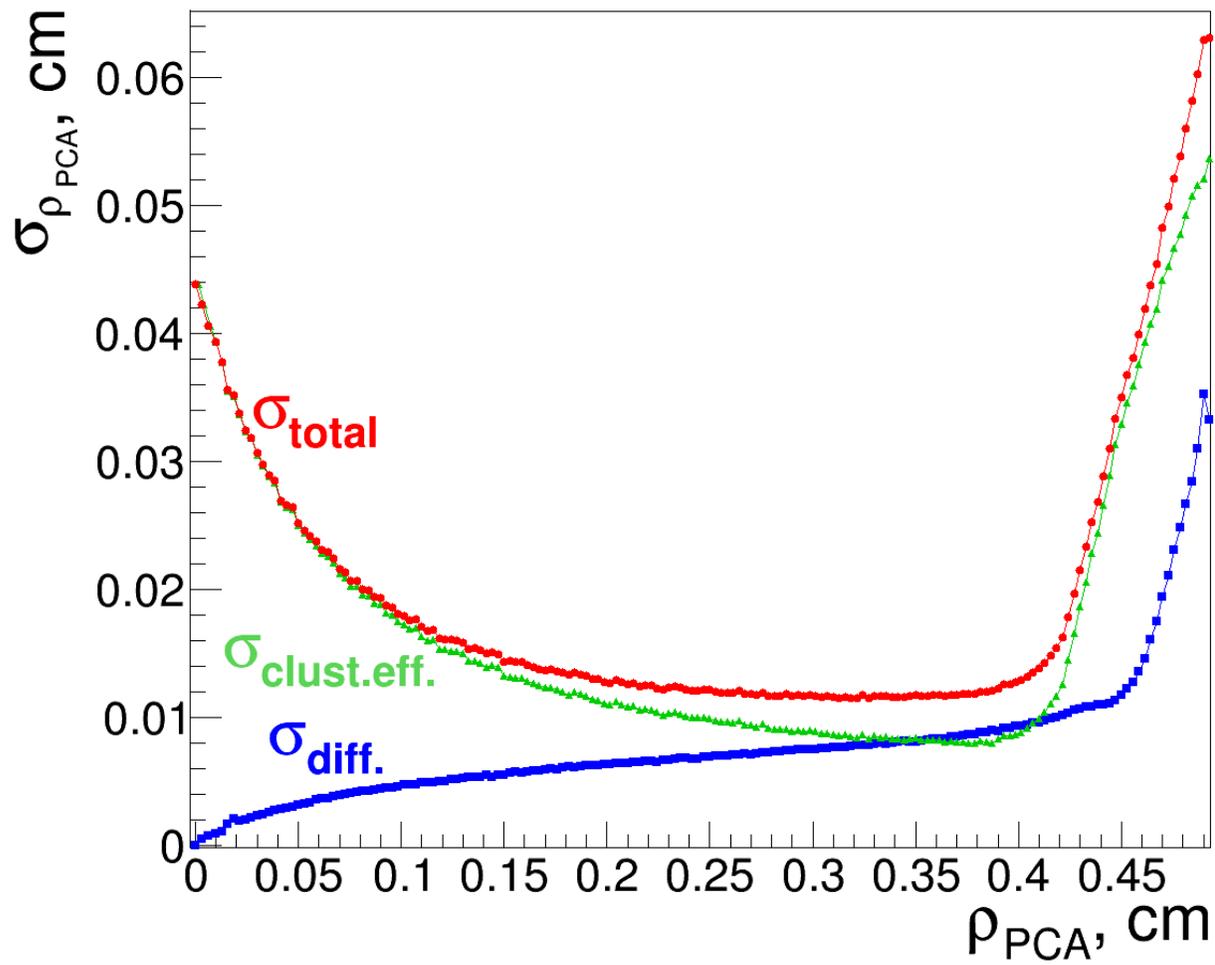
Reconstruction of Impact Parameter

- $\rho_{f.c.}$ is a biased estimate of the ρ_{PCA} due to the cluster effect (discreteness of ionization)
- We do not apply any correction to compensate this bias, but calculate the uncertainty in ρ_{PCA} due to the cluster effect as the square root from the second moment about the origin of the distribution of $\rho_{PCA} - \rho_{f.c.}$ at given t_{drift}

$$\sigma_{\rho_{PCA}} \equiv \sqrt{\sigma_{diff.}^2(t_{drift}) + \sigma_{clust. eff.}^2(t_{drift})}$$

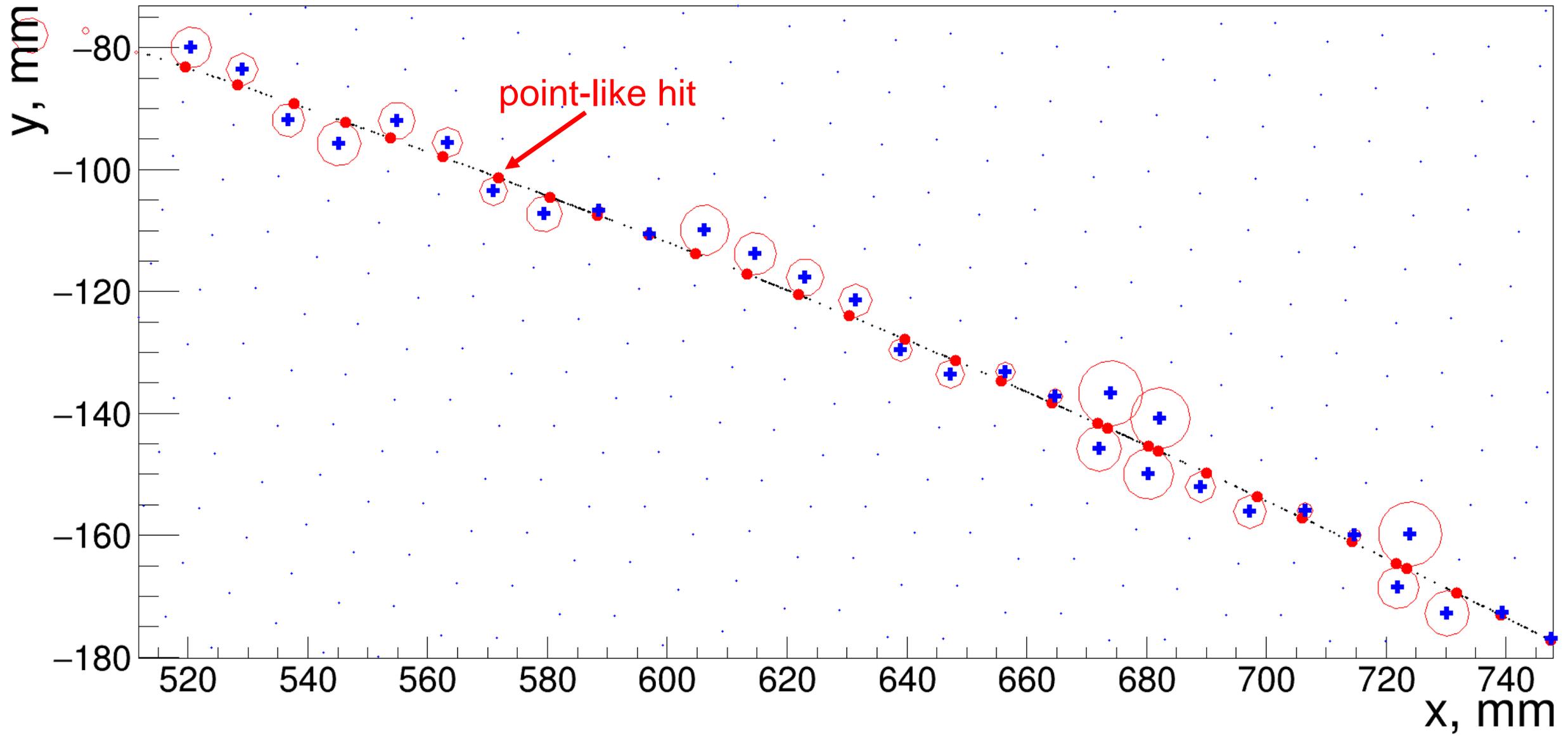


Resolution on Impact Parameter



Reconstruction of Point-like Hits

- We do not have the track finding algorithm yet, but if once we have it, we know the estimated track direction at each cell and the resolved left-right ambiguity. Knowing this we can convert the wire hit into the **point-like hit**



Point-like Hit Cov. Matrix

- Calculation of the point-like hit covariance matrix:

$$\text{cov}(x'_w, y'_w, z'_w) = \begin{bmatrix} \delta^2 & 0 & 0 \\ 0 & \sigma_{\rho_{\text{PCA}}}^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix} \quad \delta \equiv 0.01 \cdot \min\{\sigma_{\rho_{\text{PCA}}}, \sigma_z\}$$

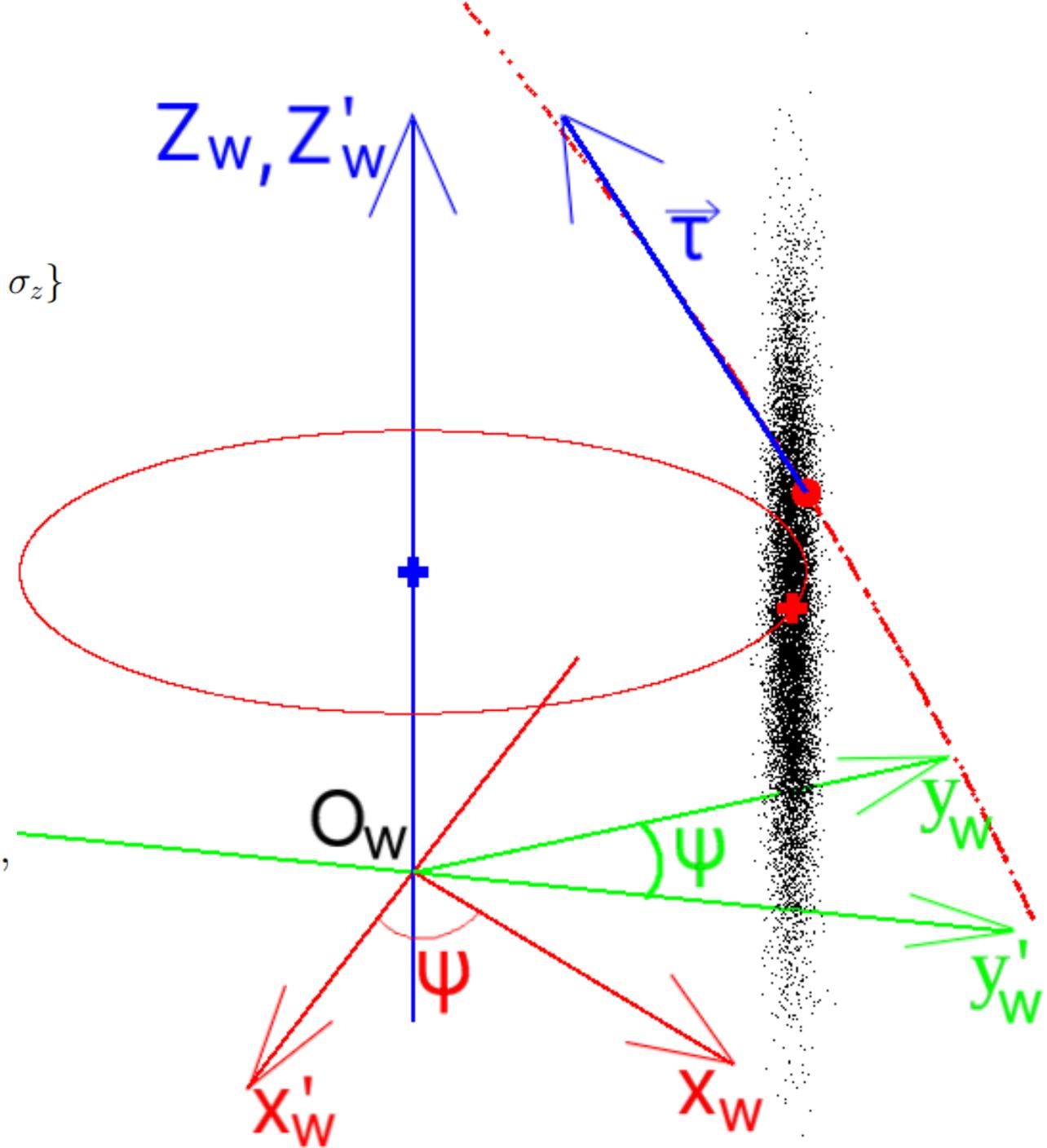
$$\text{cov}(x, y, z) = J \cdot \text{cov}(x'_w, y'_w, z'_w) \cdot J^T$$

$$J = R_z(\varphi_w)R_x(-\epsilon)R_z(\psi) = [\vec{e}_{x'_w}, \vec{e}_{y'_w}, \vec{e}_{z'_w}],$$

$$\vec{e}_{x'_w} = \begin{bmatrix} \cos(\varphi_w)\cos(\psi) - \sin(\varphi_w)\sin(\psi)\cos(\epsilon) \\ \sin(\varphi_w)\cos(\psi) + \cos(\varphi_w)\sin(\psi)\cos(\epsilon) \\ -\sin(\psi)\sin(\epsilon) \end{bmatrix},$$

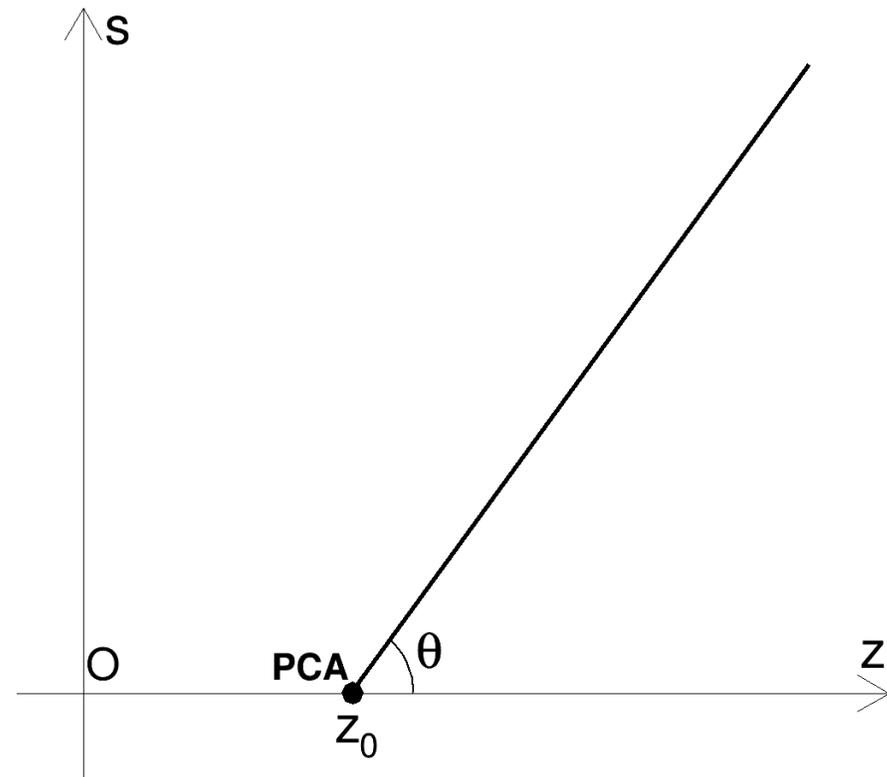
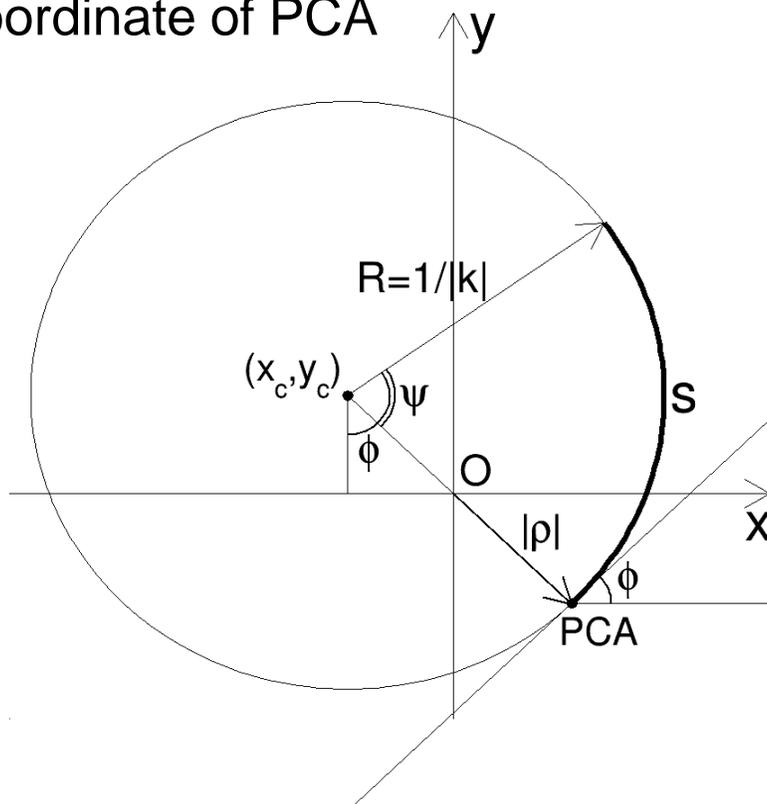
$$\vec{e}_{y'_w} = \begin{bmatrix} -\cos(\varphi_w)\sin(\psi) - \sin(\varphi_w)\cos(\psi)\cos(\epsilon) \\ -\sin(\varphi_w)\sin(\psi) + \cos(\varphi_w)\cos(\psi)\cos(\epsilon) \\ -\cos(\psi)\sin(\epsilon) \end{bmatrix},$$

$$\vec{e}_{z'_w} = \begin{bmatrix} -\sin(\varphi_w)\sin(\epsilon) \\ \cos(\varphi_w)\sin(\epsilon) \\ \cos(\epsilon) \end{bmatrix},$$



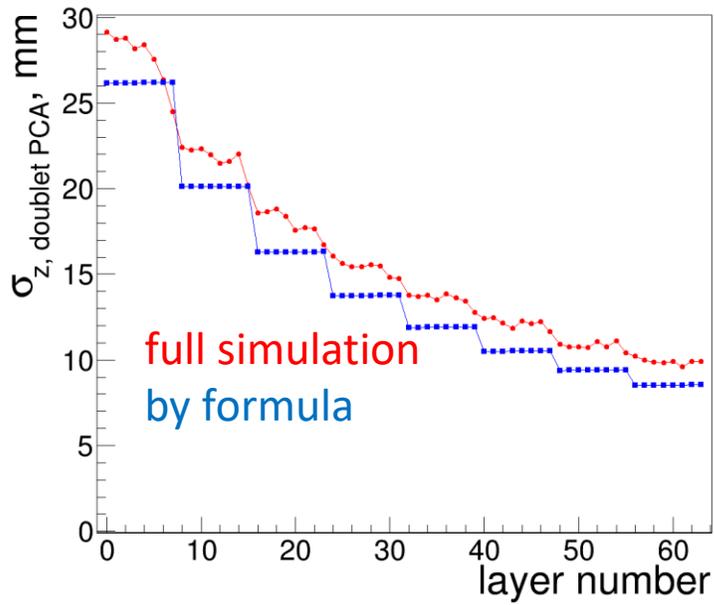
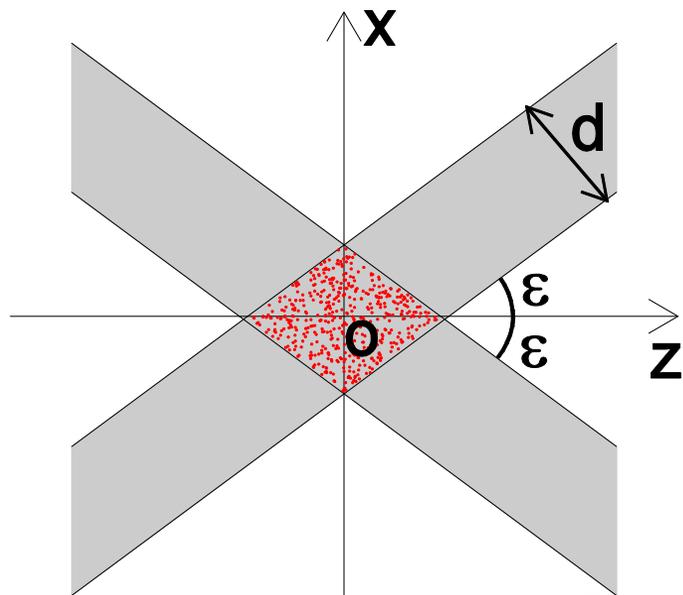
Track reconstruction

- We use usual track helix parametrization:
 1. k – signed curvature in xy -plane
 2. ρ – signed distance from z axis to the PCA on track
 3. φ – azimuthal angle of particle direction at PCA
 4. $\cot(\theta)$ – cotangent of polar angle of track direction
 5. z_0 – z coordinate of PCA



Reconstruction of hit doublets

$$\sigma_{z, \text{doublet PCA}} = \frac{d}{\sqrt{24} \sin(\epsilon)}$$



$\vec{s} = (1,1,1), (1,1,-1)$ – external tangents; $\vec{s} = (1,-1,1), (-1,1,1)$ – internal tangents

$(x_{0,1}, y_{0,1})$ – circle centers, $\rho_{0,1}$ – their radii, $\vec{d}_{0,1}$ – wire directions in 3D

$$r_{0,1} \equiv s_{0,1} \rho_{0,1}, \quad \Delta r \equiv s_0 \rho_0 - s_1 \rho_1,$$

$$\Delta x \equiv x_0 - x_1, \quad \Delta y \equiv y_0 - y_1,$$

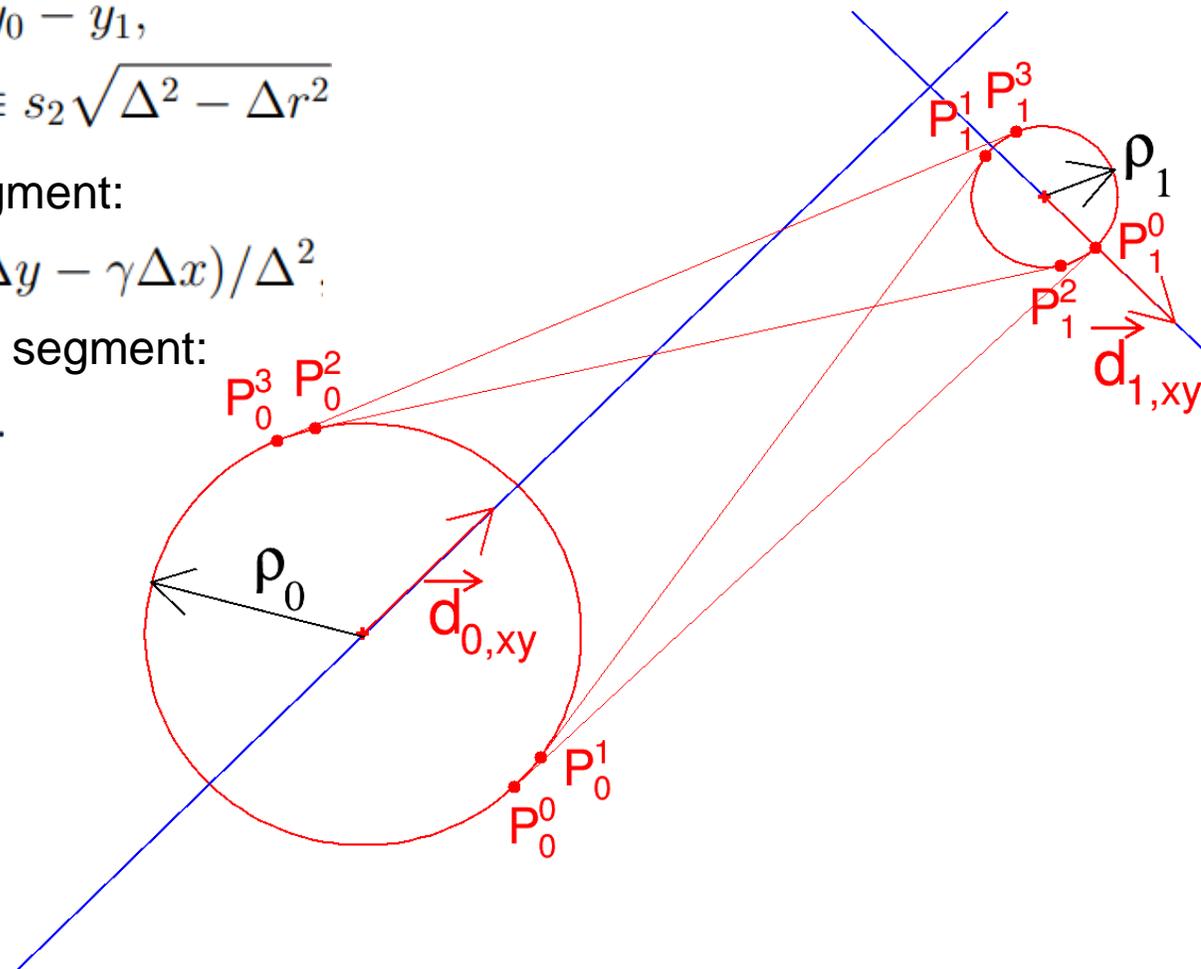
$$\Delta^2 \equiv \Delta x^2 + \Delta y^2, \quad \gamma \equiv s_2 \sqrt{\Delta^2 - \Delta r^2}$$

Normal to the tangent segment:

$$\vec{n} = (\Delta r \Delta x + \gamma \Delta y, \Delta r \Delta y - \gamma \Delta x) / \Delta^2,$$

Start/end point of tangent segment:

$$P_{0,1} = (x_{0,1}, y_{0,1}) - r_{0,1} \vec{n}.$$



Tangent points covariance matrix

$$\text{cov}(P_{i,x}, P_{i,y}) = J \begin{bmatrix} \sigma_{\rho_0}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\rho_1}^2 & 0 & 0 \\ 0 & 0 & \sigma_{z_0}^2 & 0 \\ 0 & 0 & 0 & \sigma_{z_1}^2 \end{bmatrix} J^T, i = 0, 1 \quad J = \frac{\partial(P_{i,x}, P_{i,y})}{\partial(\rho_0, \rho_1, z_0, z_1)} = \begin{bmatrix} \frac{\partial P_{i,x}}{\partial \rho_0} & \frac{\partial P_{i,x}}{\partial \rho_1} & \frac{\partial P_{i,x}}{\partial z_0} & \frac{\partial P_{i,x}}{\partial z_1} \\ \frac{\partial P_{i,y}}{\partial \rho_0} & \frac{\partial P_{i,y}}{\partial \rho_1} & \frac{\partial P_{i,y}}{\partial z_0} & \frac{\partial P_{i,y}}{\partial z_1} \end{bmatrix}, i = 0, 1$$

$$\begin{aligned} \Delta r &\equiv s_i \rho_i - s_{1-i} \rho_{1-i}, & \frac{\partial P_{i,x}}{\partial \rho_i} &= -\frac{s_i}{\Delta^2} (\Delta x \Delta r + \Delta y \gamma + r_i (\Delta x - \Delta y \Delta r / \gamma)) & \frac{\partial P_{i,x}}{\partial \rho_{1-i}} &= -\frac{r_i s_{1-i}}{\Delta^2} (\Delta x - \Delta y \Delta r / \gamma) \\ \Delta x &\equiv x_i - x_{1-i}, \quad \Delta y \equiv y_i - y_{1-i}, & & & & \\ \gamma &\equiv (-1)^i s_2 \sqrt{\Delta^2 - \Delta r^2}, \quad i = 0, 1 & \frac{\partial P_{i,y}}{\partial \rho_i} &= -\frac{s_i}{\Delta^2} (\Delta y \Delta r - \Delta x \gamma + r_i (\Delta y + \Delta x \Delta r / \gamma)) & \frac{\partial P_{i,y}}{\partial \rho_{1-i}} &= -\frac{r_i s_{1-i}}{\Delta^2} (\Delta y + \Delta x \Delta r / \gamma) \end{aligned}$$

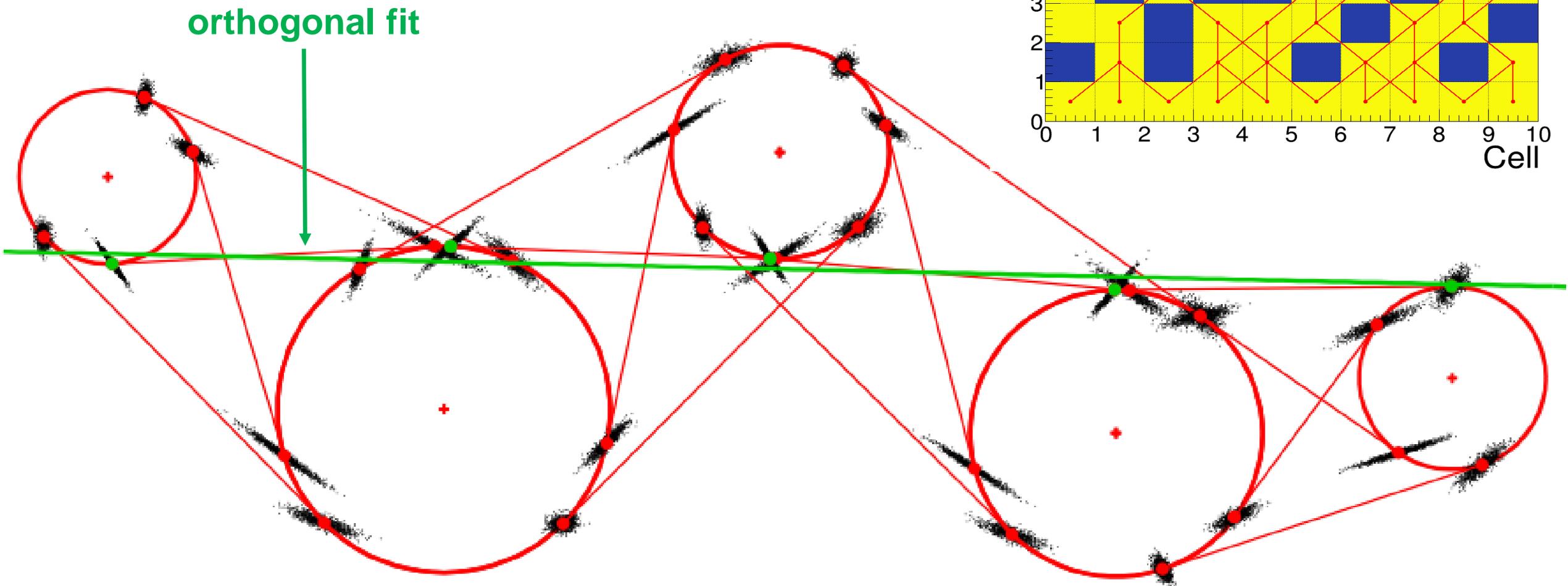
$$\frac{\partial P_{i,x}}{\partial z_0} = \frac{d_{0,x}}{d_{0,z}} (1 - i) + (-1)^i \left(\frac{2r_i}{\Delta^4} \left(\Delta x \frac{d_{0,x}}{d_{0,z}} + \Delta y \frac{d_{0,y}}{d_{0,z}} \right) (\Delta x \Delta r + \Delta y \gamma) - \frac{r_i}{\Delta^2} \left(\Delta r \frac{d_{0,x}}{d_{0,z}} + \gamma \frac{d_{0,y}}{d_{0,z}} + \frac{\Delta y}{\gamma} \left(\Delta x \frac{d_{0,x}}{d_{0,z}} + \Delta y \frac{d_{0,y}}{d_{0,z}} \right) \right) \right)$$

$$\frac{\partial P_{i,y}}{\partial z_0} = \frac{d_{0,y}}{d_{0,z}} (1 - i) + (-1)^i \left(\frac{2r_i}{\Delta^4} \left(\Delta x \frac{d_{0,x}}{d_{0,z}} + \Delta y \frac{d_{0,y}}{d_{0,z}} \right) (\Delta y \Delta r - \Delta x \gamma) + \frac{r_i}{\Delta^2} \left(-\Delta r \frac{d_{0,y}}{d_{0,z}} + \gamma \frac{d_{0,x}}{d_{0,z}} + \frac{\Delta x}{\gamma} \left(\Delta x \frac{d_{0,x}}{d_{0,z}} + \Delta y \frac{d_{0,y}}{d_{0,z}} \right) \right) \right)$$

$$\frac{\partial P_{i,x}}{\partial z_1} = \frac{d_{1,x}}{d_{1,z}} (1 - i) + (-1)^i \left(-\frac{2r_i}{\Delta^4} \left(\Delta x \frac{d_{1,x}}{d_{1,z}} + \Delta y \frac{d_{1,y}}{d_{1,z}} \right) (\Delta x \Delta r + \Delta y \gamma) + \frac{r_i}{\Delta^2} \left(\Delta r \frac{d_{1,x}}{d_{1,z}} + \gamma \frac{d_{1,y}}{d_{1,z}} + \frac{\Delta y}{\gamma} \left(\Delta x \frac{d_{1,x}}{d_{1,z}} + \Delta y \frac{d_{1,y}}{d_{1,z}} \right) \right) \right)$$

$$\frac{\partial P_{i,y}}{\partial z_1} = \frac{d_{1,y}}{d_{1,z}} (1 - i) + (-1)^i \left(-\frac{2r_i}{\Delta^4} \left(\Delta x \frac{d_{1,x}}{d_{1,z}} + \Delta y \frac{d_{1,y}}{d_{1,z}} \right) (\Delta y \Delta r - \Delta x \gamma) - \frac{r_i}{\Delta^2} \left(-\Delta r \frac{d_{1,y}}{d_{1,z}} + \gamma \frac{d_{1,x}}{d_{1,z}} + \frac{\Delta x}{\gamma} \left(\Delta x \frac{d_{1,x}}{d_{1,z}} + \Delta y \frac{d_{1,y}}{d_{1,z}} \right) \right) \right)$$

Reconstruction of hit chains



Riemann circle fit

- We use the **Riemann fit** as the pre-fit for the forthcoming fit with Kalman filter

- The hits in the transversal plane are mapped on the surface of the circular paraboloid

$$x_i = u_i$$

$$y_i = v_i$$

$$z_i = u_i^2 + v_i^2$$

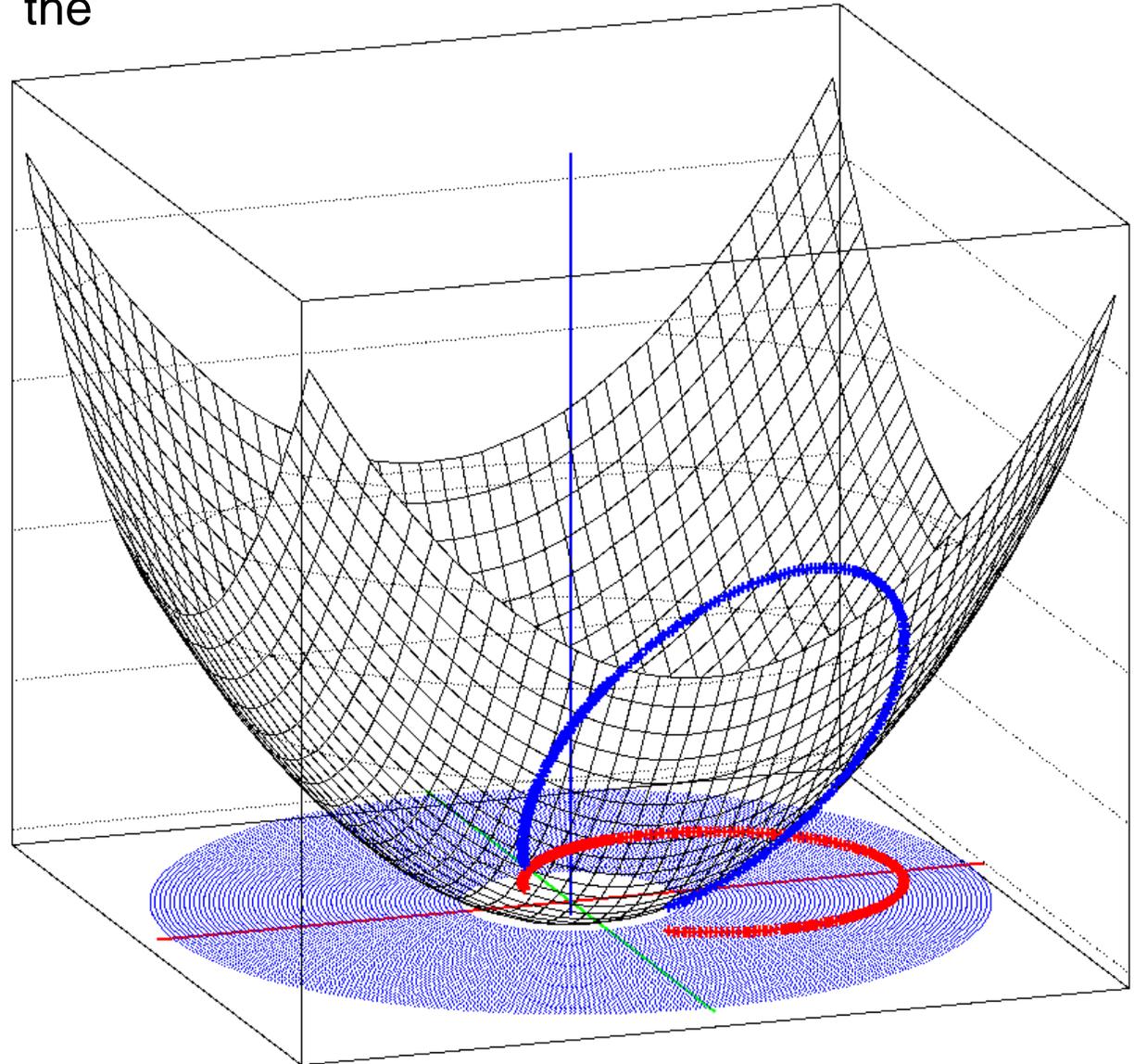
- The **circle** on plane maps to the figure on the paraboloid, which is the intersection of some **plane** $\vec{n}^T \vec{r} + c = 0$ with the paraboloid

$$(u - u_0)^2 + (v - v_0)^2 = \rho^2$$

$$z - 2xu_0 - 2yv_0 = \rho^2 - u_0^2 - v_0^2$$

- The fit is performed by the minimization of the cost function **S** – the weighted sum of squared distances from the **mapped hits** to the fitting **plane**

$$S = \sum_{i=1}^N w_i d_i^2 \quad w_i \propto 1/\sigma_i^2, \quad \sum_{i=1}^N w_i = 1$$



Riemann circle fit

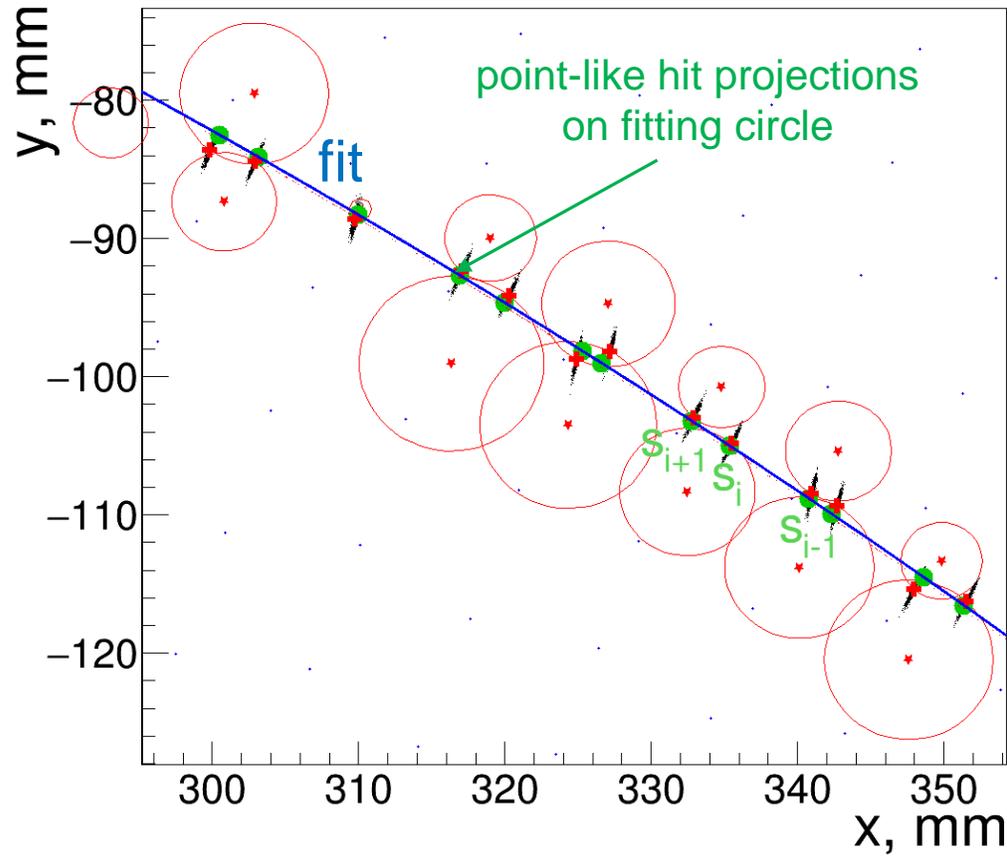
- The minimization of cost function S gives the value of $c = -\vec{n}^T \vec{r}_0$, where $\vec{r}_0 = \sum_{i=1}^N w_i \vec{r}_i$
- Then the normal to the plane \vec{n} is obtained as the unit eigenvector of the sample covariance matrix A , corresponding to the smallest eigenvalue

$$A = \sum_{i=1}^N w_i (\mathbf{r}_i - \mathbf{r}_0)(\mathbf{r}_i - \mathbf{r}_0)^T$$

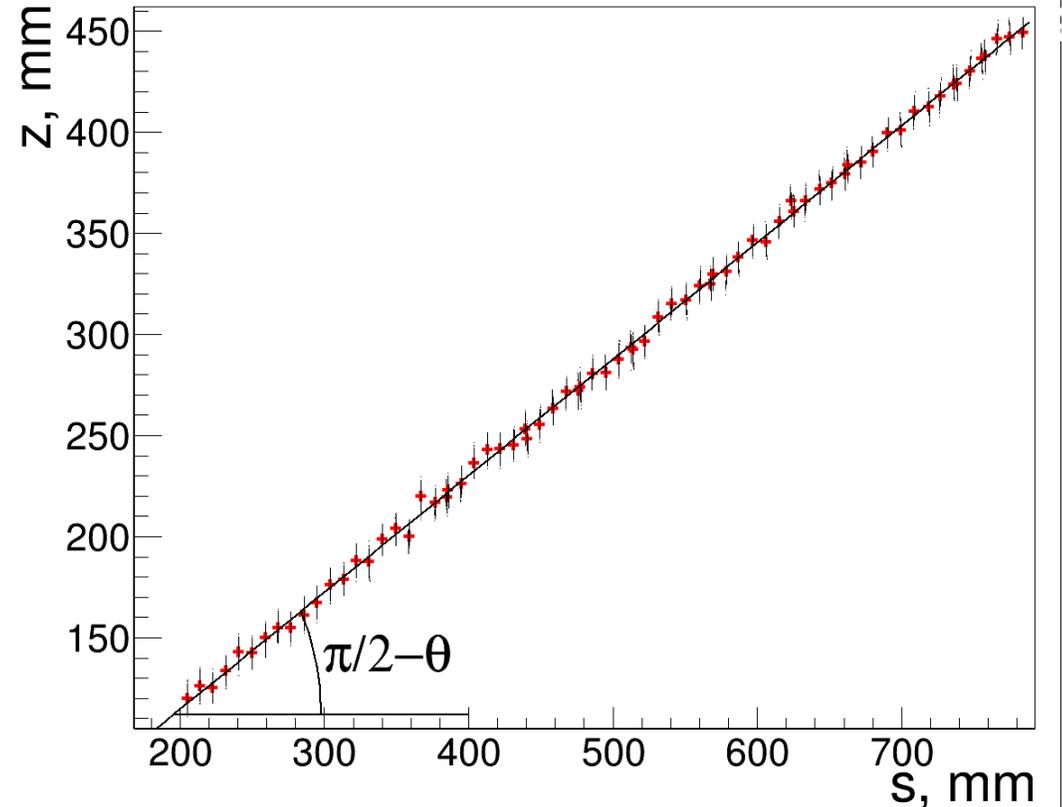
- Finally, the obtained parameters of the plane are mapped back to the parameters of the fitting circle

$$u_0 = -\frac{n_1}{2n_3} \quad v_0 = -\frac{n_2}{2n_3}$$

$$\rho = \frac{\sqrt{1 - n_3^2 - 4cn_3}}{2n_3}$$

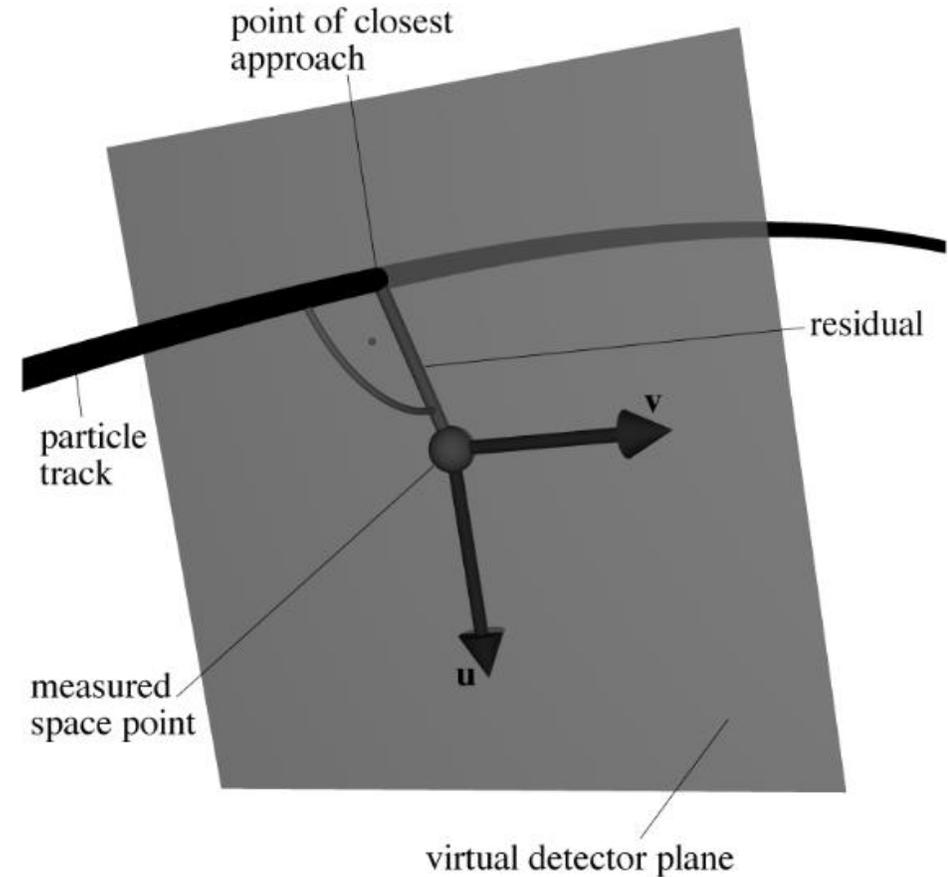
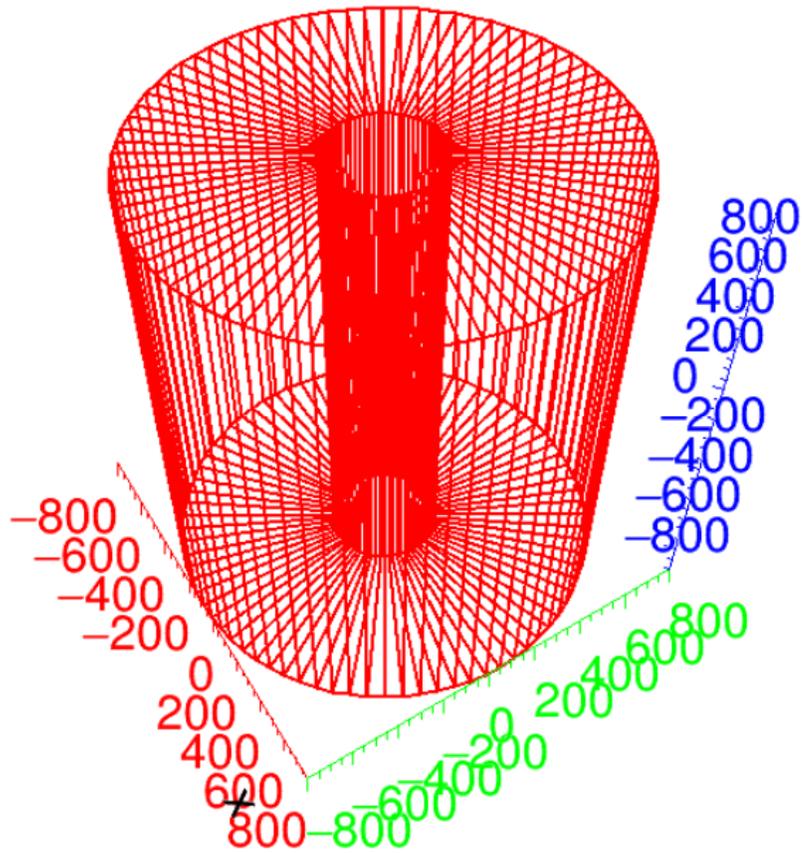


- Line fit on sz -plane:

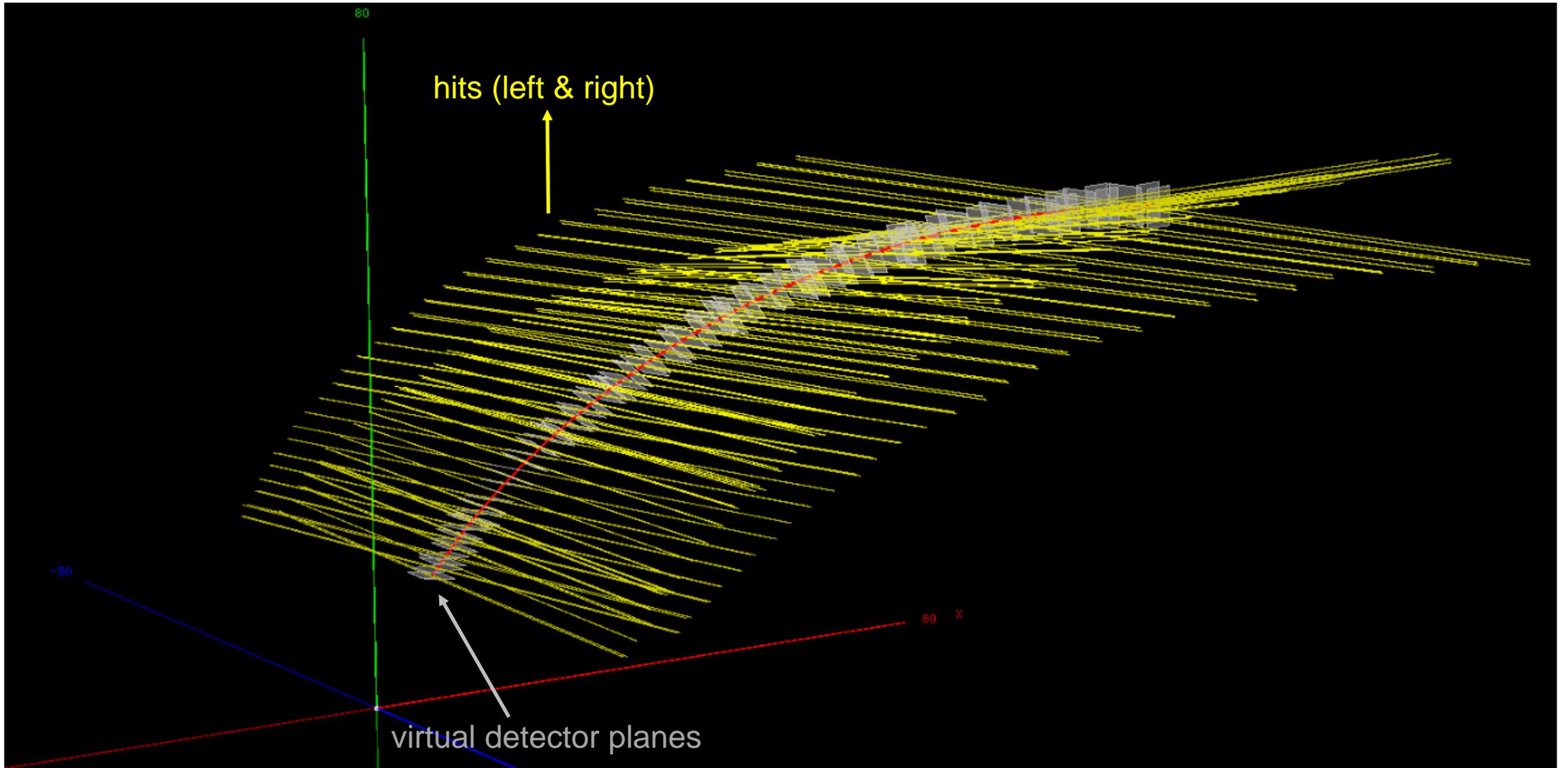


Track fit with the Kalman Filter

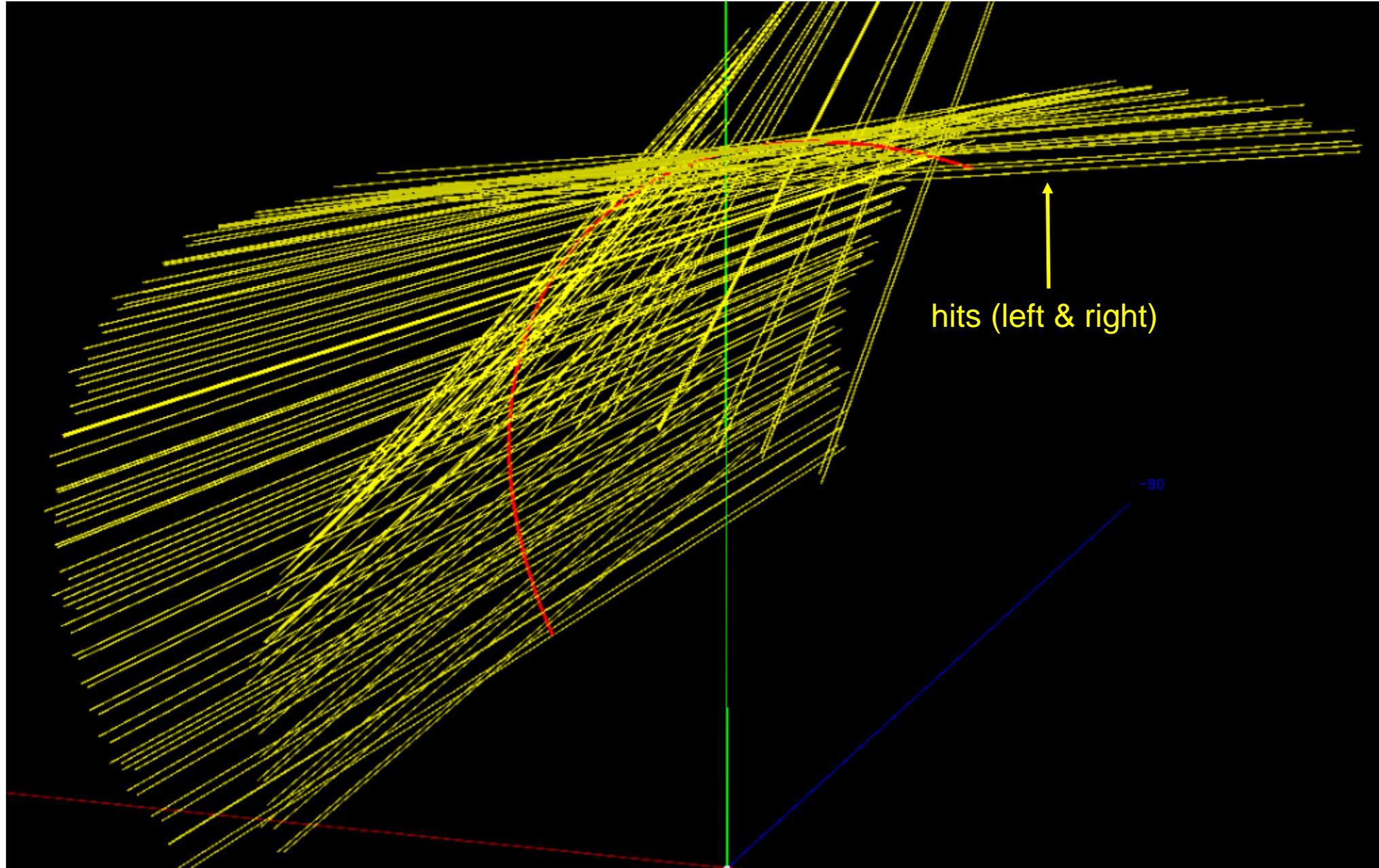
- We use the **Deterministic Annealing Filter** (iterative Kalman Filter) from **GenFit** package
- To account for the material effects we currently use only the gas volume in the fitter. In future the geometry and materials of all relevant subsystems should be added
- We use **WirePointMeasurement** class objects as a hits to be fitted



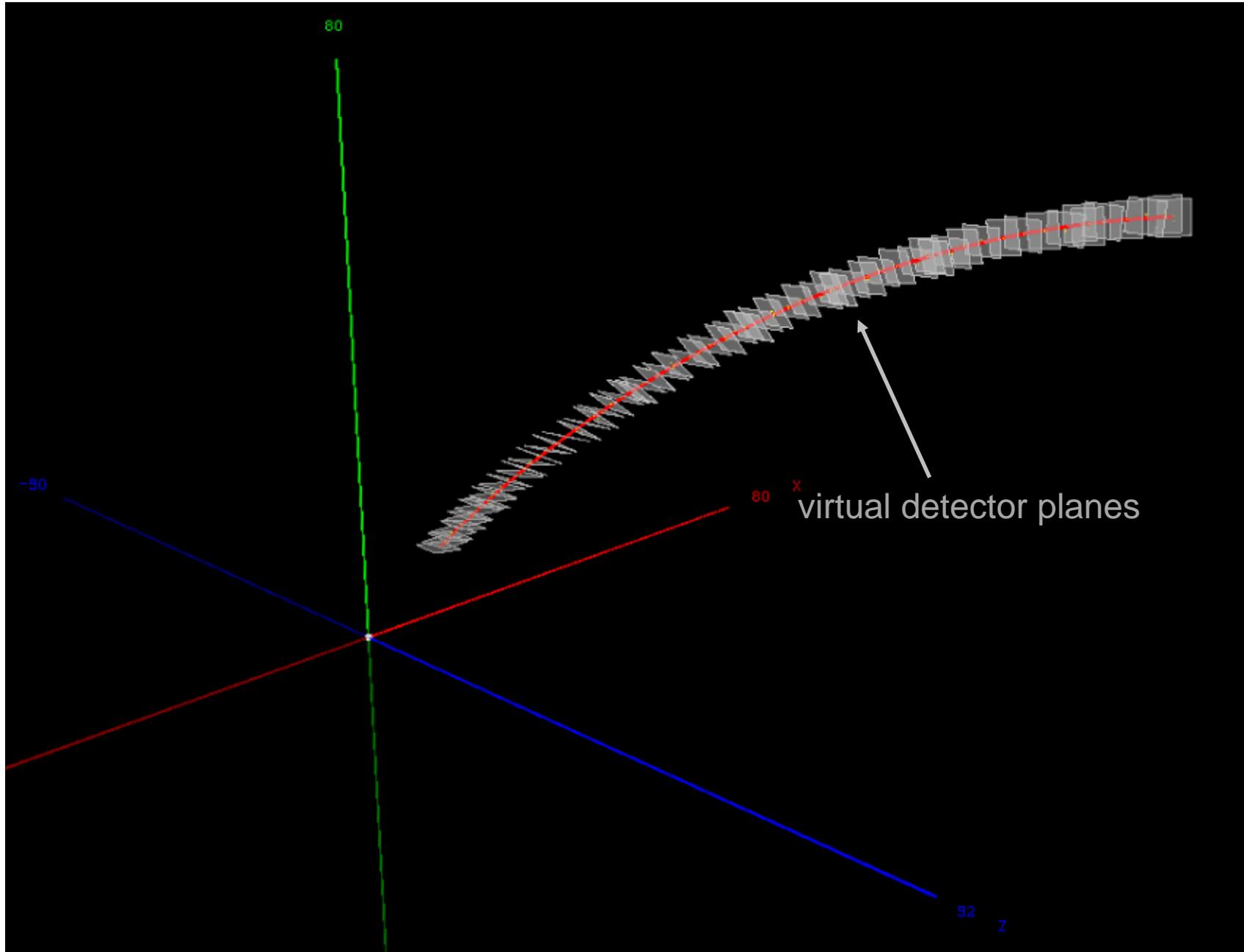
Track fit in visualizer



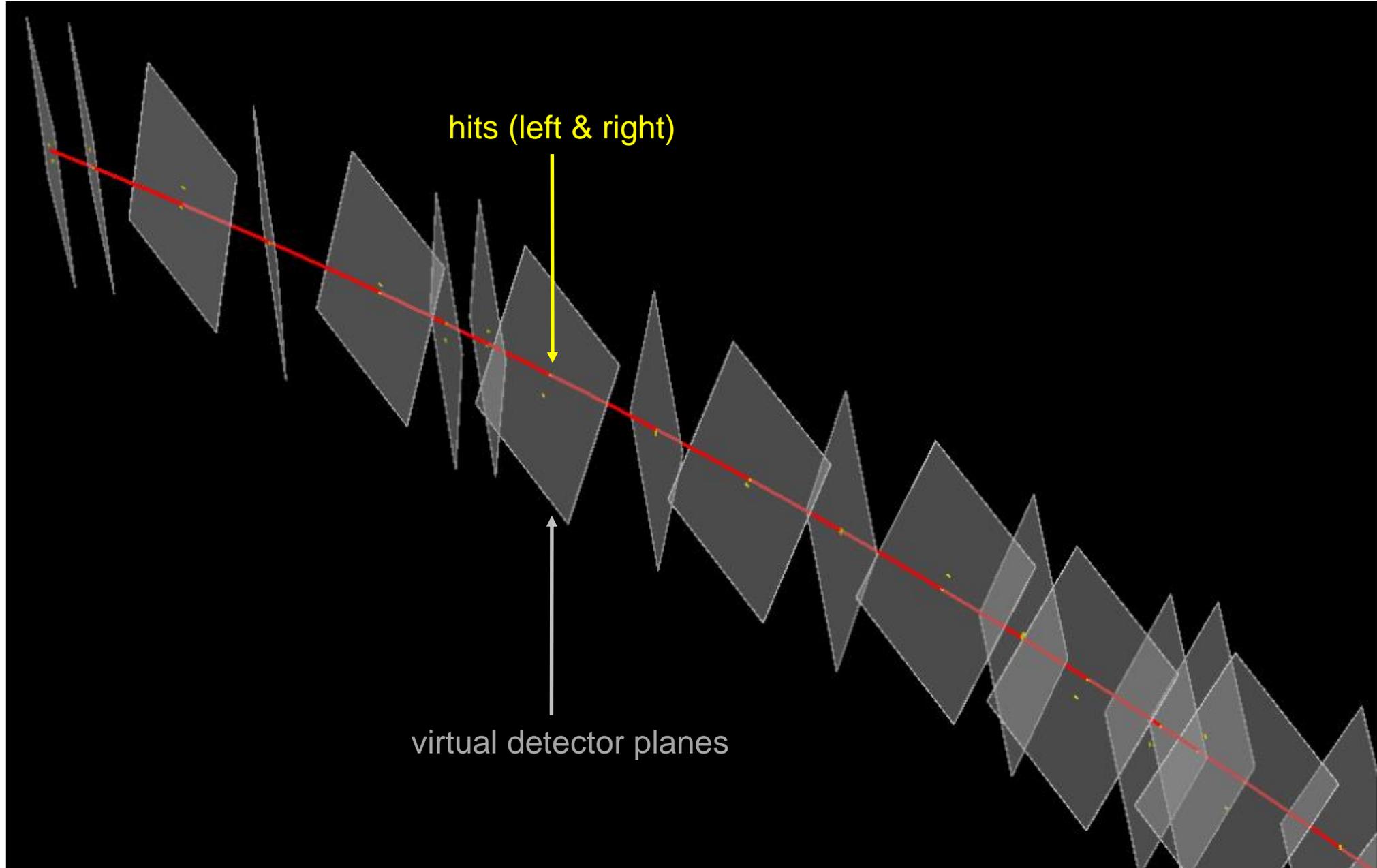
Track fit in visualizer



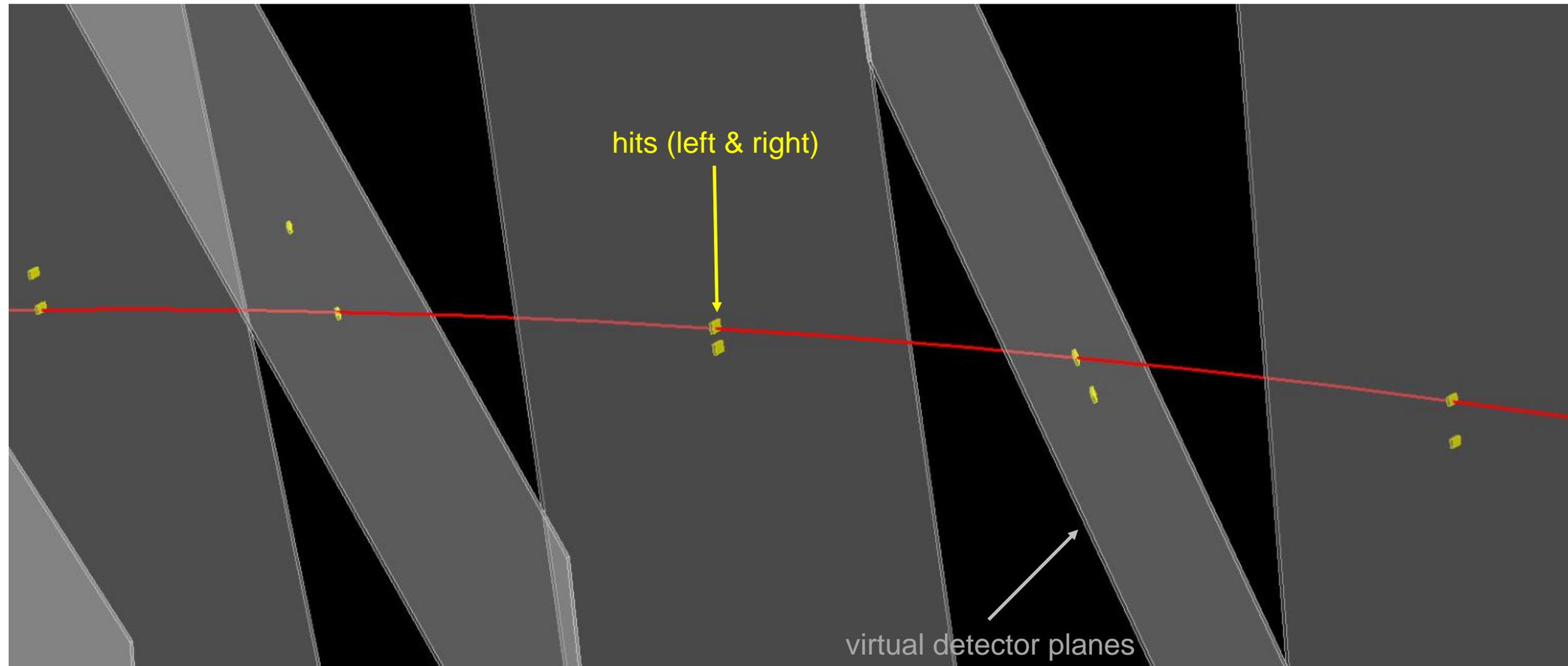
Track fit in visualizer



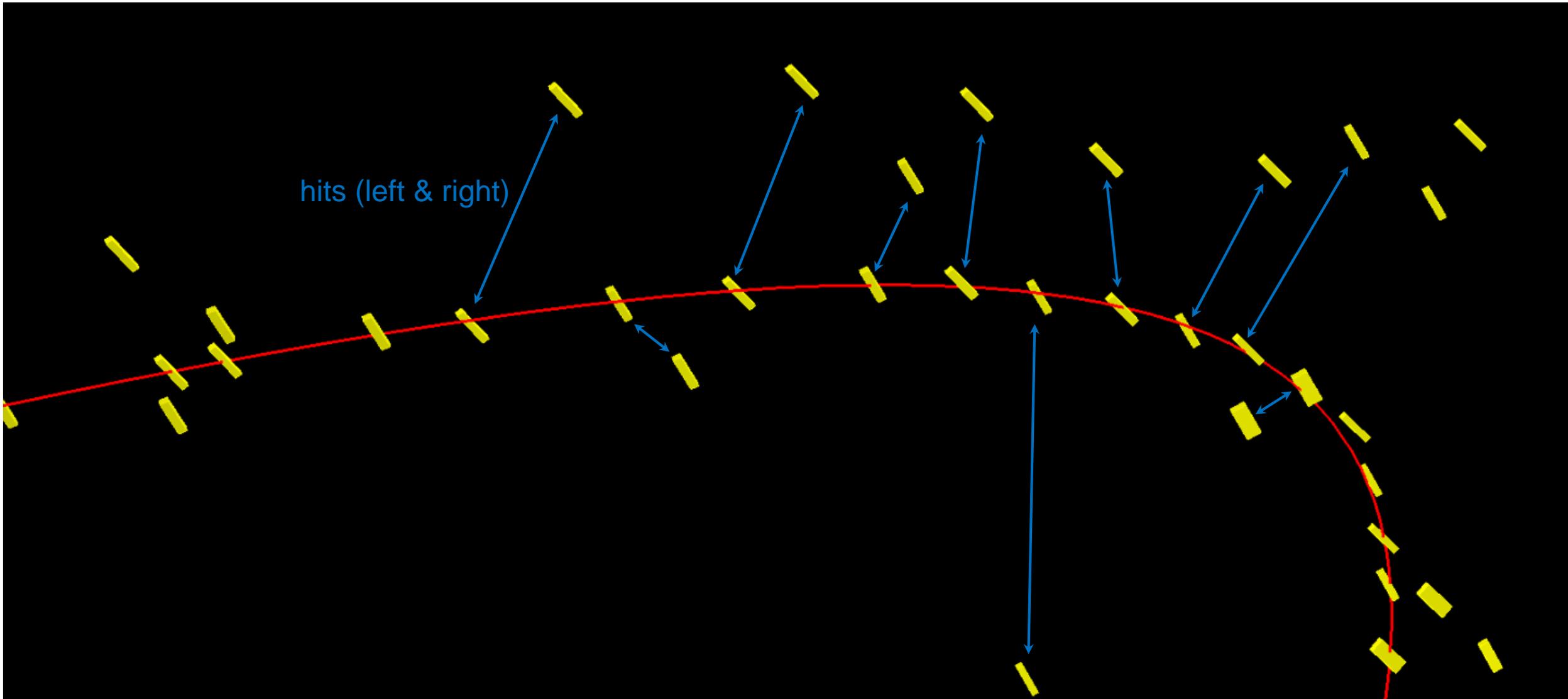
Track fit in visualizer



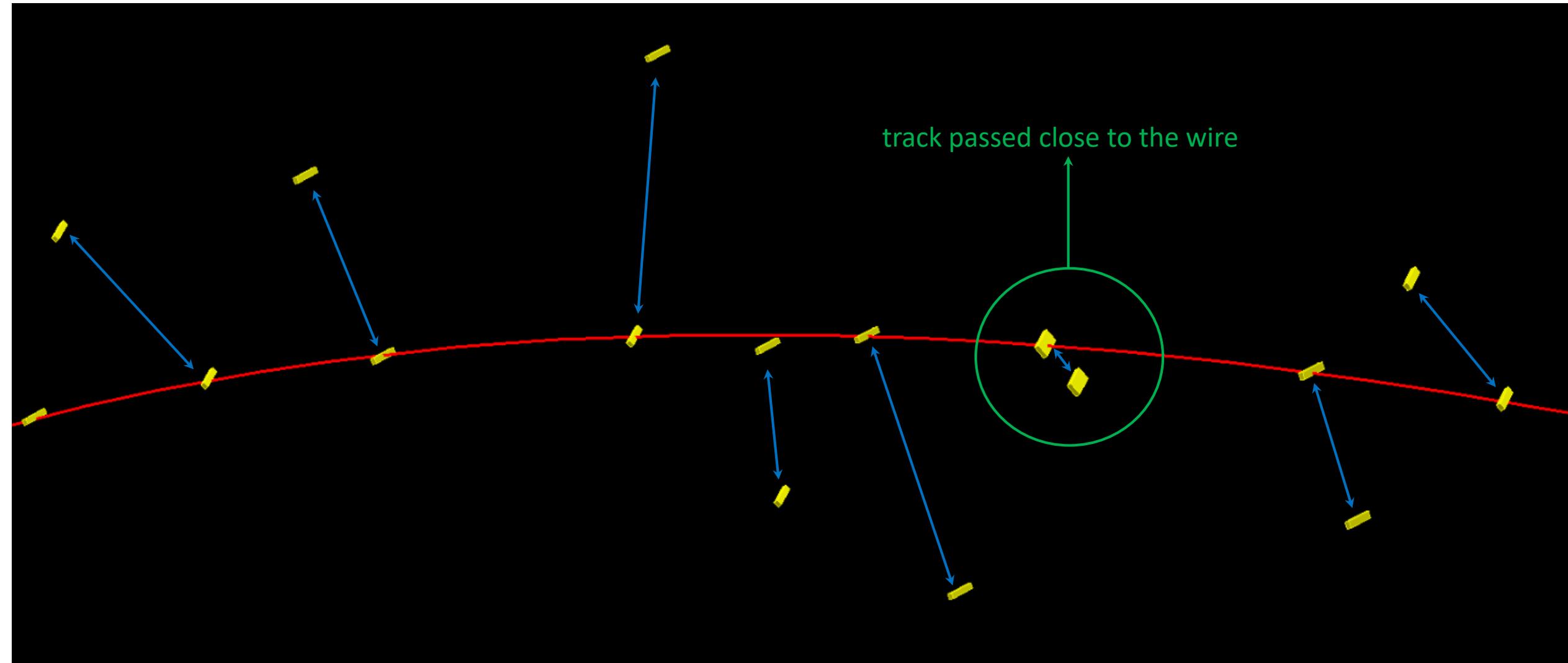
Track fit in visualizer



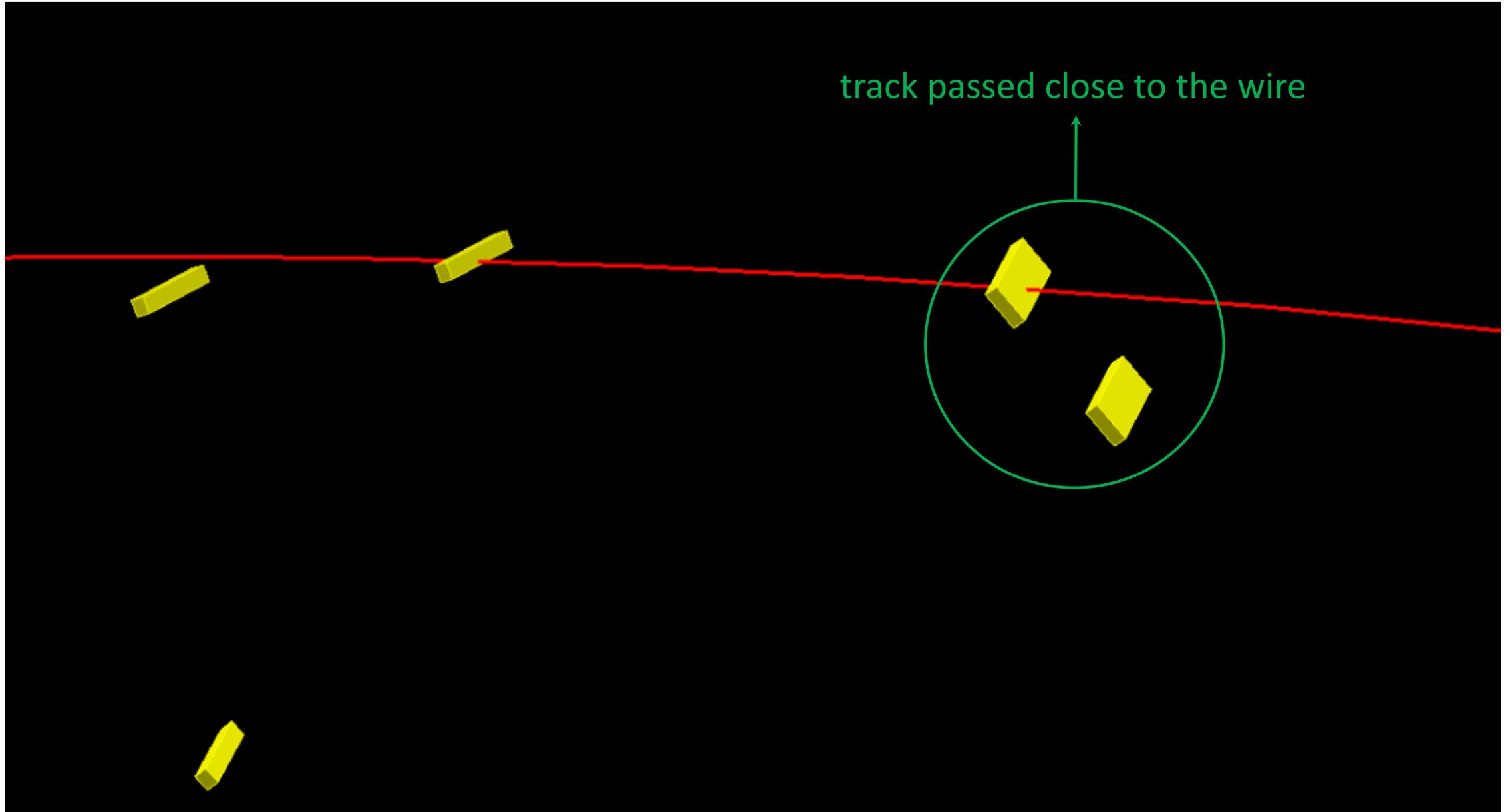
Track fit in visualizer



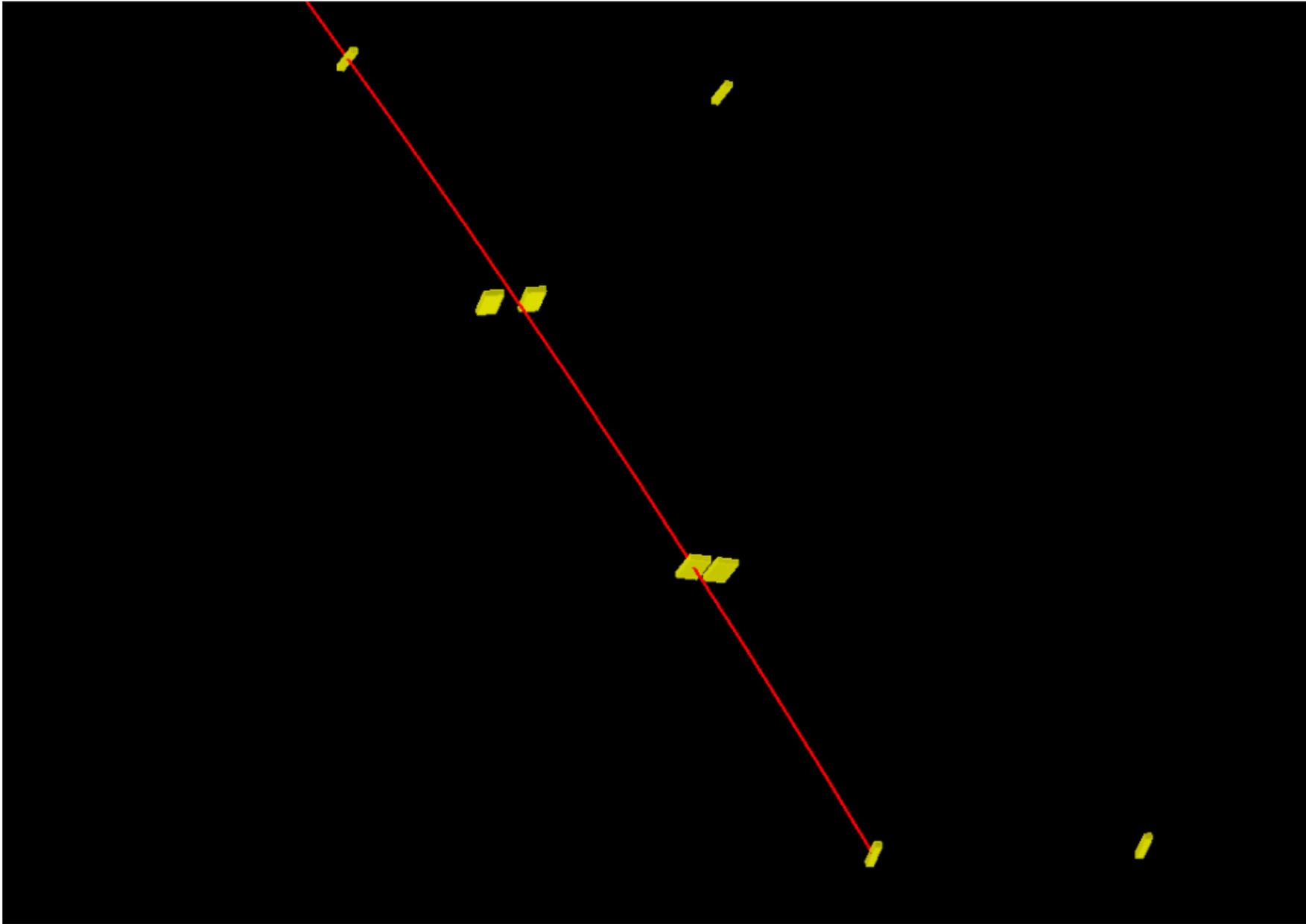
Track fit in visualizer



Track fit in visualizer



Track fit in visualizer



Fit parameters covariance matrix

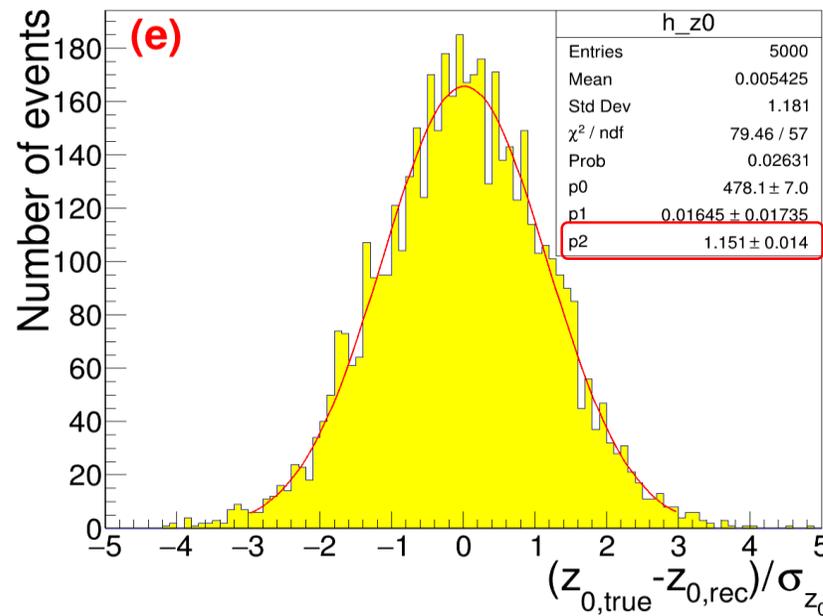
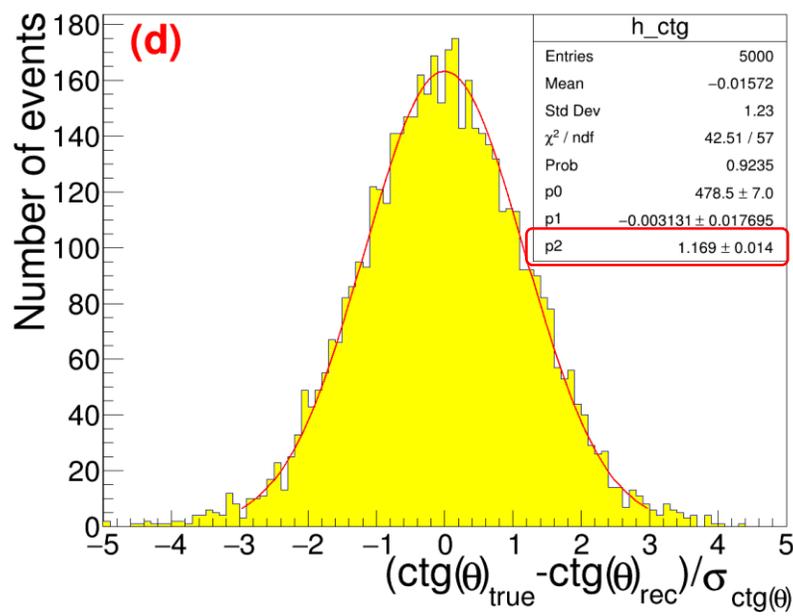
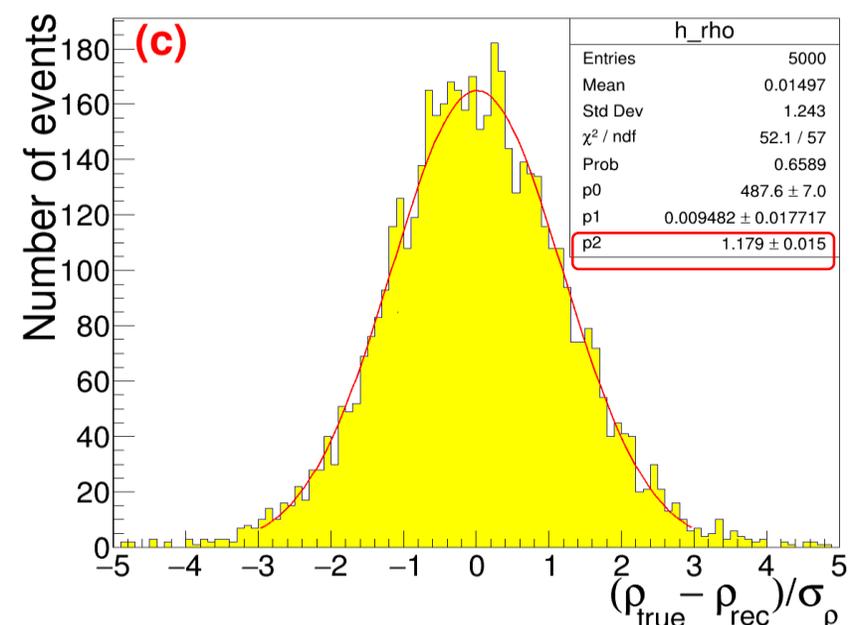
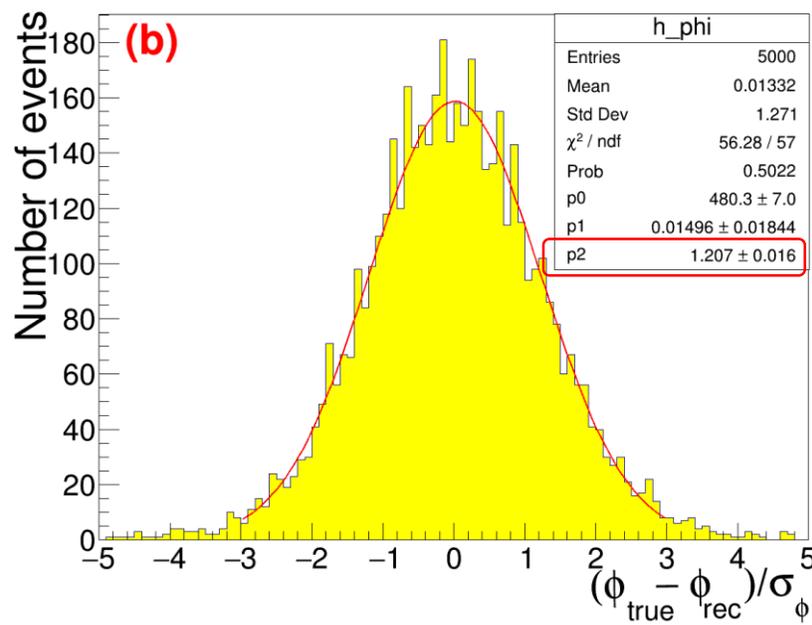
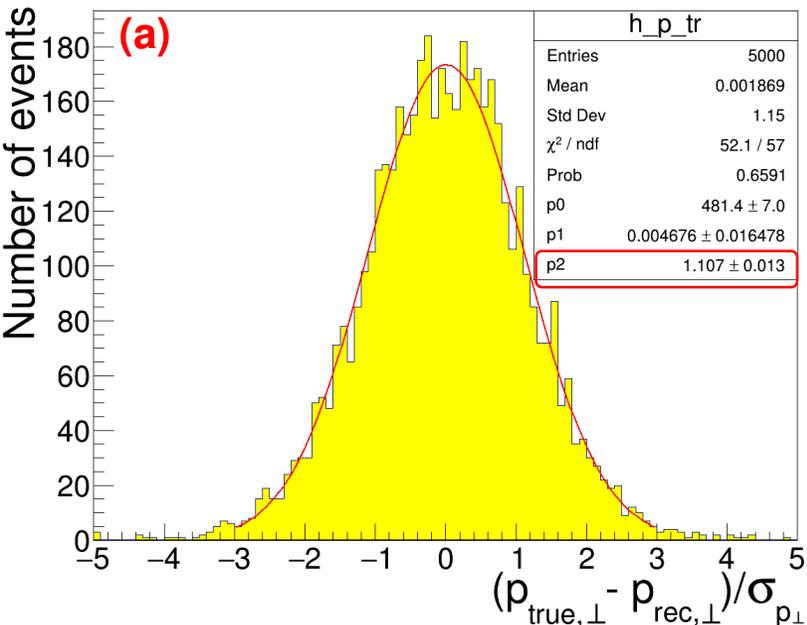
- The measured track state is propagated back to PCA to the beamline
- Since the covariance matrix is returned for the (x, y, z, p_x, p_y, p_z) state vector, we convert it to the covariance matrix for the vector $(k, \varphi, \rho, ctg(\theta), z_0)$ using the Jacobian:

$$J = \frac{\partial(k, \varphi, \rho, ctg(\theta), z_0)}{\partial(x, y, z, p_x, p_y, p_z)} = \begin{bmatrix} 0 & 0 & 0 & -k \frac{p_x}{p_{\perp}^2} & -k \frac{p_y}{p_{\perp}^2} & 0 \\ 0 & 0 & 0 & -\frac{p_y}{p_{\perp}^2} & -\frac{p_x}{p_{\perp}^2} & 0 \\ -\frac{p_y}{p_{\perp}} & \frac{p_x}{p_{\perp}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -ctg(\theta) \frac{p_x}{p_{\perp}^2} & -ctg(\theta) \frac{p_y}{p_{\perp}^2} & \frac{1}{p_{\perp}} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

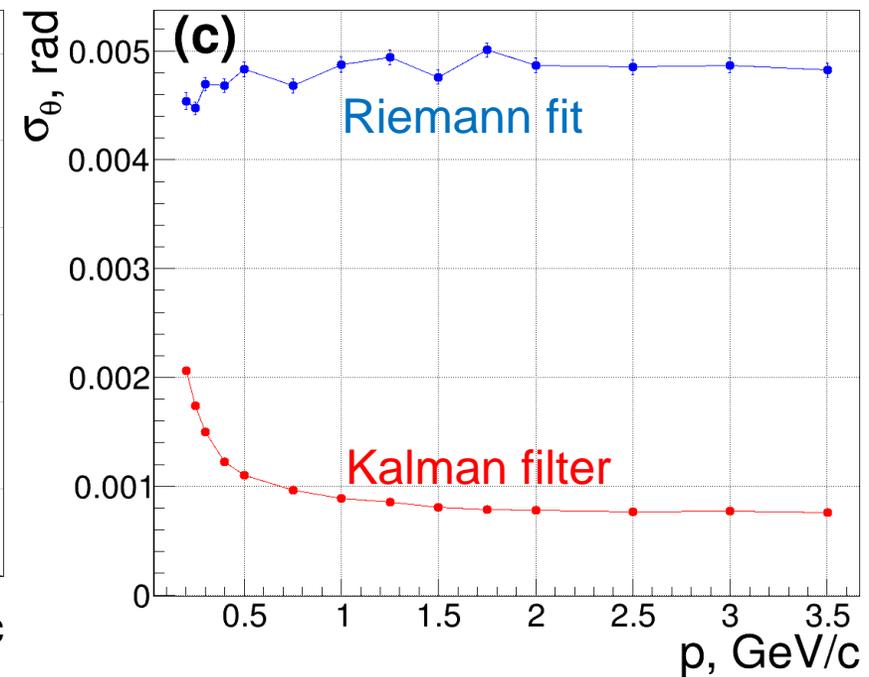
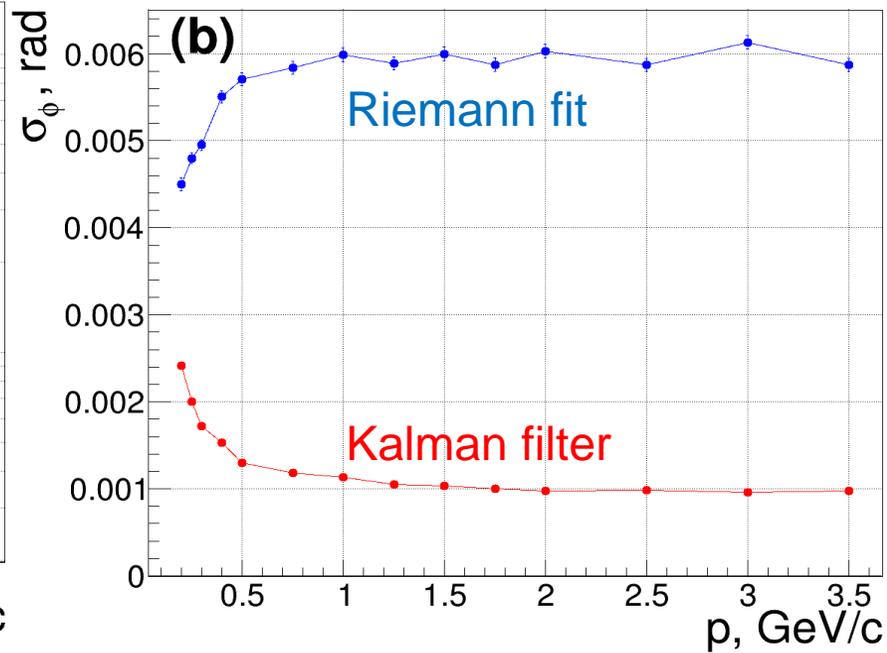
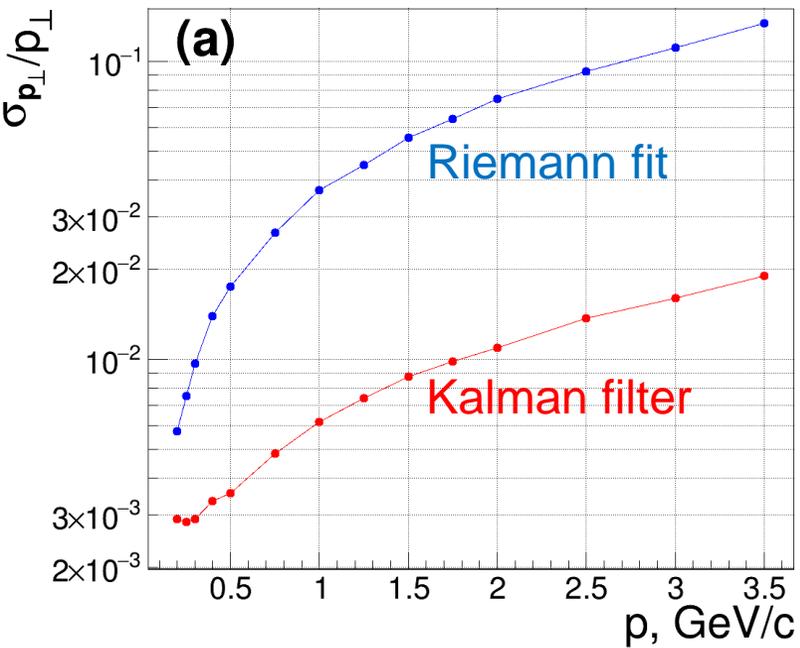
- Next, we perform the check of the correctness of the estimation of the track's parameters uncertainties

Fit parameters covariance matrix

- Muons, $p = 1 \text{ GeV}/c$:



Angular and momentum resolutions (very preliminary)



Plans

- Study of the dependence of the isochrones and $t - \rho$ relations on z and layer number
- Development of the track finding algorithm
- Peak searching algorithm, algorithm to obtain the most probable impact parameter for given set of cluster times
- Study of the possible effect of usage of the metal coated wires on the resolutions
- Vertex reconstruction
- Combining the inner tracker with DC into the united tracker system
- Development of kinematic fit with calorimeter, final estimation of resolutions in concrete physics analysis