New method of measuring the cross-section $e^+e^- \to n\bar{n}$ near threshold at SCTF

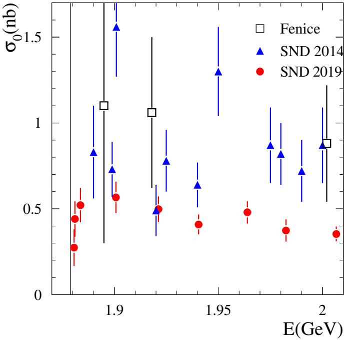
Alex Bondar, Alex Bobrov (speaker)

BINP SB RAS, NSU

Cross sections $e^+e^- \to N\bar{N}$ and partial widths $\frac{d\Gamma}{dm_{N\bar{N}}}$ $N\bar{N}$ do not tend to 0 at the threshold.

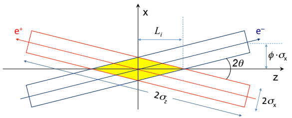
The low energy range nonperturbative QCD.

Invariant mass resolution σ_W for reactions $e^+e^- \to N\bar{N}$ depends on the beam energy spread $\sigma_W/m_n \propto \delta E_b/E_b$, $\sigma_W \sim$ MeV.



The figure is take from an article of SND collaboration EPJ Web Conf., 212 (2019) 07007

To reconstruct the cross-section, angular distribution as a function beam energy and beam energy spread function was studied.
The velocity of C.M.S. in lab. frame > 0.



Crab Waist collision scheme
$\Theta \sim 0.05$, $v_{c.m.s.} = \tan\Theta \sim 0.05$

$$W = \sqrt{(P_0 + \delta P)^2} = W_0 + (P_0; \delta P_{\|})/W_0 + (\delta P_{\perp}^2 + \delta P_{\|}^2)/2W_0 = W_0 + (P_0; \delta P_{\|})/W_0 + O(\delta P^2)$$

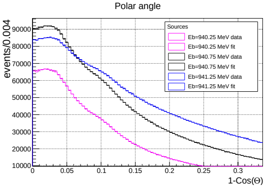
$$v_{c.m.s.} = \frac{P_0 + \delta P_{\|} + \delta P_{\perp}}{\sqrt{(P_0 + \delta P_{\|} + \delta P_{\perp})^2}} = v_{c.m.s.}^0 + \frac{\delta P_{\perp}}{W_0}$$

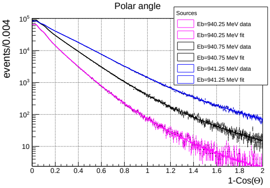
Approximation: W invariant mass can vary , but velocity is the same. At the threshold, all particles have singular angular distribution: $\delta(\vec{n} - \vec{v}/|\vec{v}|)$.

Simulation

1. $\delta E_b/E_b = 10^{-3}$
2. Angular dispersion is 10^{-3} radian
3. Half of the beam intersection angle is 0.05 radian
4. Neutron/antineutron angular distribution is isotropic
5. Beam energy E_b varies from 939.75 to 942.25 MeV, with step 0.25 MeV (threshold $E_b = 940.74$ MeV, $\sigma_W \sim 1.3$ MeV.)
6. Radiative correction is absent
7. Antineutron angle reconstruction is ideal
8. Antineutron/neutron time life is ∞
9. Cross section (1 nb) is simulated by the Heaviside function with step value at the energy threshold, except for cases with hypothetical resonances.

Cross section reconstruction





In case $W < 2m_n/\sqrt{1 - v_{c.m.s}^2} = W^\star$ (critical invariant mass), there is maximum scattering angle.

$v_{c.m.s.} = \tanh\psi$ $W = 2m_n\sqrt{1 + \sinh^2\psi \sin^2\alpha} \sim$. $\sim 2m_n + m_n \sinh^2\psi\alpha^2$

$R = (W^\star - 2m_n)/\sigma_W$. If $R \ll 1$, large fraction events are $W > W^\star$, good resolution is unavailable for $W < W^\star$ region.

In our simulation $R = 1.75$.

Invariant mass resolution due to beam energy spread $\sigma_W = m_n \sinh^2\psi\sigma_\alpha^2(0)$.

$\sigma_\alpha(0)$ angular spread at $\alpha = 0$, $\sigma_\alpha(0) = \delta p_{c.m.s.}/p_{c.m.s.} \propto \delta E_b/(E_b \sinh\psi)$.

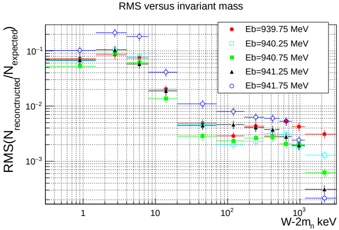
So $\sigma_W \propto m_n(\delta E_b/E_b)^2 \sim$ keV!

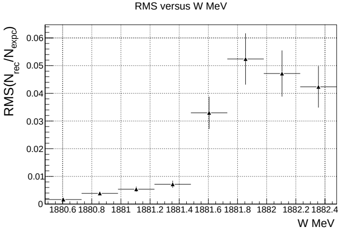
Invariant mass resolution due to angular resolution $\sigma_W = m_n \sinh^2\psi\sigma_\alpha^2$

$\sigma_\alpha = 5cm/1m = 1/20$

$\sigma_W \sim 10$ keV.

Accuracy of the cross section reconstruction





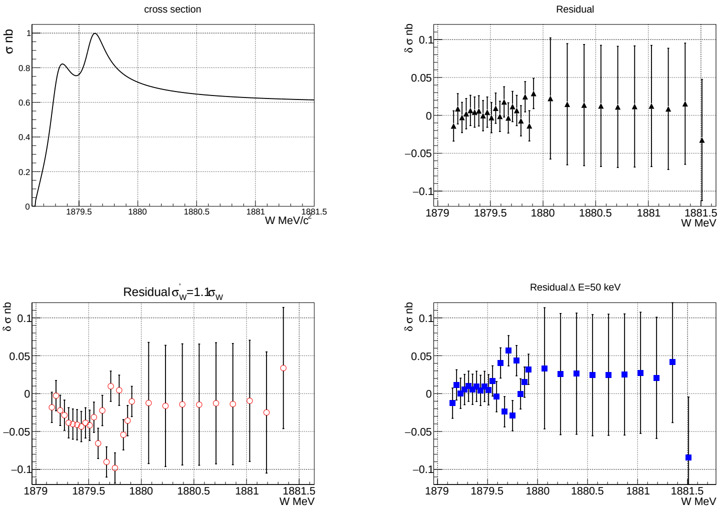
Cross section (1 nb) is simulated by the Heaviside function with a step value at the energy threshold.

Angular distribution was fitted by a linear function. Cross-sections in invariant mass intervals were minimized. The shapes of the angular distribution in each interval of the invariant mass were obtained from independent simulation.

Relative accuracy varies from ~ 0.1 at keV scale up to $\sim 3 \times 10^{-3}$ at MeV scale.

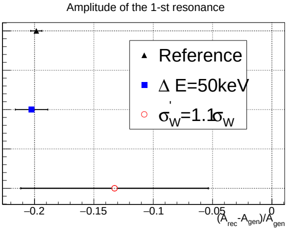
Accuracy drops down after passing the critical invariant mass.

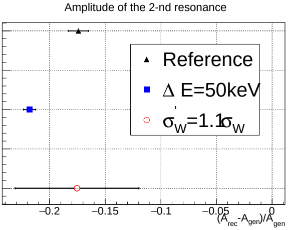
Reconstruction of the resonances

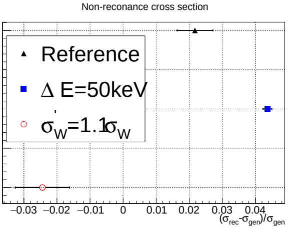


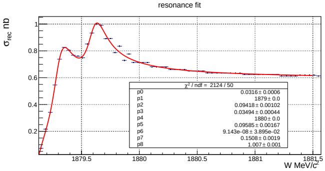
10 pseudo experiments (700 fb^{-1}), simulated cross section and a residual errors as an invariant mass function.

Reconstruction of the resonances



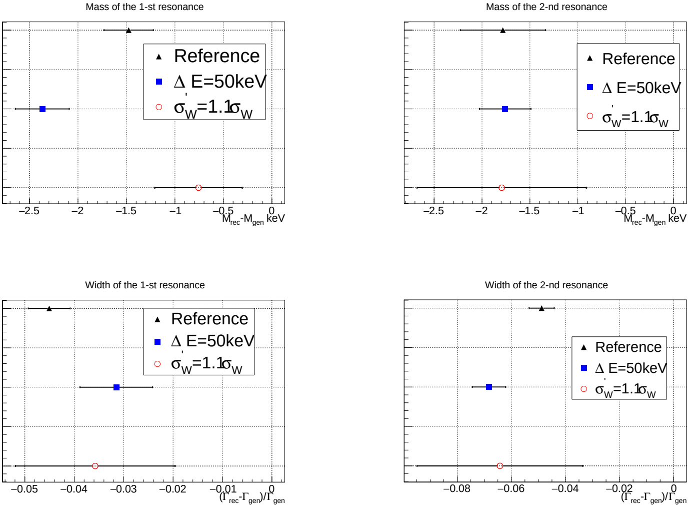






Amplitude consists of two resonances with $\Gamma = 0.1$ MeV ($\phi = 0$) and continuum contribution.

Reconstruction of the resonances



Three conditions: reference $\delta E = 0, \delta\sigma_W = 0$; $\delta E = 50$ keV$\delta\sigma_W = 0$; $\delta E = 0$ $\delta\sigma_W = 0.1\sigma_W$.

Analytical solution

To find approximation that allows one to obtain analytical solution?
How to obtain the distribution of invariant mass $\frac{dN}{dW}$ from the angular distribution ($\frac{dN}{d\Omega}$) distribution of invariant mass?

- Angular resolution is ideal, efficiency is ideal;
- The velocity of the C.M.S. is constant $\delta P_{\perp} = 0$;
- Events with $W > W^{\star}$ are absent;
- Radiation corrections are absent ;

$$\frac{dS}{d\cos\alpha} = \int \frac{dN(W)}{dW} K(\cos\alpha, W) dW$$

Fredholm integral equation. New variables are

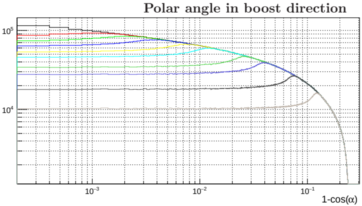
$$z = \frac{\sinh\psi}{\sinh\beta} = \frac{2m_n \sinh\psi}{\sqrt{W^2 - 4m_n^2}}.\ t = \frac{1}{\sin\alpha} z \in [1, \infty)\ t \in [1, \infty).$$

$$\frac{dS(t)}{dt} = \int_1^t \frac{dN(z)}{dz} \frac{\rho(t, z)_{reg}}{\sqrt{t - z}} dz$$

Analytical solution

$$\frac{dS(t)}{dt} = \int_1^t \frac{dN(z)}{dz} \frac{\rho(t,z)_{reg}}{\sqrt{t-z}} dz$$

$\rho(t,z)_{reg}$ is non-singular part, $\rho(t,t)_{reg} > 0$. This is linear Volterra equation of the first kind with weak singularity.



$$\frac{dN(r)}{dr} = \frac{1}{\pi\rho(r,r)_{reg}} \Big[\frac{\partial}{\partial r} \int_1^r \frac{dS(t)}{dt} \frac{dt}{\sqrt{r-t}} - \\ - \int_1^r \frac{dN(z)}{dz} \Big\{ \frac{\partial}{\partial r} \int_z^r \frac{\rho(t,z)_{reg}}{\sqrt{t-z}\sqrt{r-t}} dt \Big\} dz \Big]$$

$$\sigma(W) = \frac{dN(W)}{dW}\frac{dW}{d\mathcal{L}} \quad \rho(W_0;W) = \frac{1}{\sqrt{2\pi}\sigma_W} e^{\frac{-(W-W_0)^2}{2\sigma_W^2}}$$

$\frac{d\mathcal{L}}{dW} = \int \frac{d\mathcal{L}(W_0)}{dW_0} \rho(W_0;W) dW_0$ If $\frac{d\mathcal{L}(W_0)}{dW_0} = const$, when $\frac{d\mathcal{L}(W)}{dW} = const$

Optimal scenario of the luminosity data taking?

Separation of the formfactors

$$\frac{d\sigma}{d\Omega} = \frac{C\alpha^2\beta}{4s}\left[(|G_m|^2 + |G_e|^2/\tau) + (|G_m|^2 - |G_e|^2/\tau)\cos^2\Theta\right]$$

Θ scattering angle in C.M.S. G_m, G_e are magnet and electric formfactors; $\sqrt{s}$ is energy β is velocity of the nucleon in C.M.S. C is final state interaction factor.

There are two contributions in cross section: isotropic and anisotropic.

Anisotropic part of the cross-section can be presented as $\rho(t,z)^{aniso}_{reg}\cos^2\phi/\sqrt{t-z}$; ϕ is the azimuthal angle counted from orbit plane.

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab.frame}(\alpha;\phi) =$$

$$\frac{C\alpha^2\beta}{4s}\left[(|G_m|^2+|G_e|^2/\tau)A^{iso}(\alpha)+(|G_m|^2-|G_e|^2/\tau)A^{aniso}(\alpha)\cos^2\phi\right]$$

Summary

1. New method of the threshold cross-section measurement by studying the angular distribution in lab. frame is presented. This approach gives an opportunity to study the fine structure of the cross-section at scale much smaller the beam energy spread.
2. The new approach was tested with MC at $e^+e^- \to n\bar{n}$
3. The area of good resolution depends on $v_{c.m.s.}$ $W^\star = 2m_n/\sqrt{1 - v^2_{c.m.s.}}$ and $R = (W^\star - 2m_n)/\sigma_W$.
4. The sensitivity of the method critically depends on the accuracy of measuring the beam energy spread and beam energy. Contributions from other sources are relatively small.
5. We found the analytical solution that allows us to directly reconstruct cross-section from the angular distribution (with some initial approximation). Magnetic and electric formfactors can be separated by studying angular distribution.

Appendix

$$\frac{d\rho(\alpha,\beta)^{iso}_{\beta\leq\psi}}{d\cos\alpha} = \frac{\Theta(\arcsin\frac{\sinh\beta}{\sinh\psi}-\alpha)(\sinh^2\beta-\sinh^2\psi\sin^2\alpha+\cos^2\alpha\cosh^2\beta\tanh^2\psi)}{\cosh^2\psi\sinh\beta(1-\cos^2\alpha\tanh^2\psi)^2\sqrt{\sinh^2\beta-\sinh^2\psi\sin^2\alpha}}$$

$$\frac{d\rho(\alpha,\beta)^{iso}_{\beta>\psi}}{d\cos\alpha} = \frac{(\sqrt{\sinh^2\beta-\sinh^2\psi\sin^2\alpha}+\cos\alpha\cosh\beta\tanh\psi)^2}{2\cosh^2\psi\sinh\beta(1-\cos^2\alpha\tanh^2\psi)^2\sqrt{\sinh^2\beta-\sinh^2\psi\sin^2\alpha}}$$

$\frac{d\rho(\cos\alpha,\beta)^{iso}}{d\cos\alpha}$ is angular distribution of the isotropic part, α is the angle between boost direction and antineutron momentum, β,ψ are pseudorapidities of the c.m.s in lab. frame and antineutron in c.m.s ($W = 2m_n\cosh\psi$).

Appendix

$$\frac{d^2\rho(\alpha,\phi,\beta)^{aniso}_{\beta\le\psi}}{d\cos\alpha d\phi} = 3G\Theta(\arcsin\frac{\sinh\beta}{\sinh\psi}-\alpha)\Big[\cos^4\alpha\cosh^4\beta\tanh^4\psi+$$

$$+(\sinh^2\beta-\sinh^2\psi\sin^2\alpha)^2+6(\sinh^2\beta-\sinh^2\psi\sin^2\alpha)\cos^2\alpha\cosh^2\beta\tanh^2\psi\Big]$$

$$\frac{d^2\rho(\alpha,\phi,\beta)^{aniso}_{\beta>\psi}}{d\cos\alpha d\phi} = \frac{3}{2}G(\sqrt{\sinh^2\beta-\sinh^2\psi\sin^2\alpha}+\cos\alpha\cosh\beta\tanh\psi)^4$$

$$G = \frac{\sin^2\alpha\cos^2\phi}{2\pi\cosh^4\psi\sinh^3\beta(1-\cos^2\alpha\tanh^2\psi)^4\sqrt{\sinh^2\beta-\sinh^2\psi\sin^2\alpha}}$$

Angular distribution of the anisotropic part ($\frac{d\sigma}{d\Omega}\propto\cos^2\Theta$). ϕ is the azimuthal angle from orbit plane.