Calculation of radiative corrections and estimation of luminosity at low energies with the Monte Carlo generator `ReneSANCe`

Vitaly Yermolchyk
on behalf of SANC group

JINR, Dubna

Workshop on future Super c-tau factories
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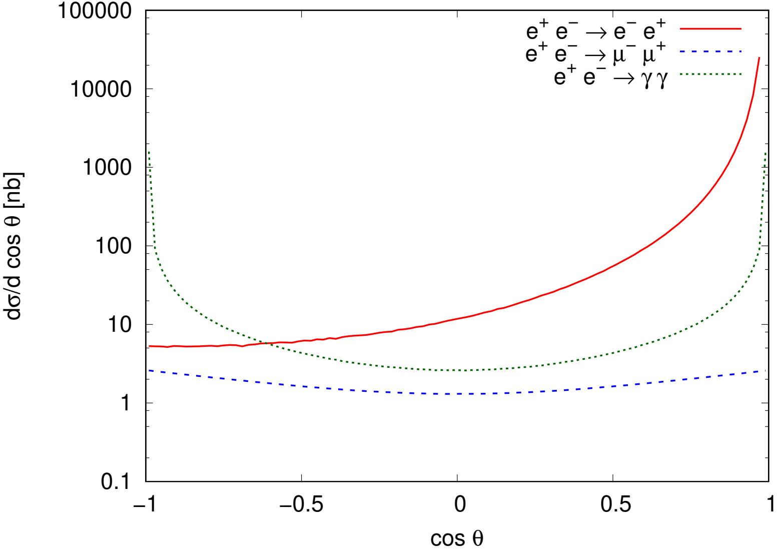
SANC, theoretical uncertainty:

- **one-loop level,**
- **higher order corrections,**
- **massive case,**
- **accounting for polarization effects,**
- **MC generator/integrator,**
- **full-phase operation,**
- **output format.**

SANC: Precise determination of luminosity, NLO QED/EW+ho RC for polarized scattering

- **Bhabha scattering**
  $e^+e^- \longrightarrow e^+e^-$
  A.B. Arbuzov, S.G. Bondarenko, Ya.V. Dydyshka, L.V. Kalinovskaya, L.A. Rumyantsev
  (Phys.Rev.D 98, 013001)
- **annihilation into a fermion pair**
  $e^+e^- \longrightarrow f\bar{f}$
  S. Bondarenko, Ya. Dydyshka, L. Kalinovskaya, R. Sadykov, V. Yermolchyk
  (Phys.Rev.D 102 (2020) 3, 033004. 2005.04748 [hep-ph])
- **photon-pair production**
  $e^+e^- \longrightarrow \gamma + \gamma,$
  S. Bondarenko, Ya. Dydyshka, L. Kalinovskaya, L. Rumyantsev, V. Yelmolchyk,
  Done, massive case, to be prepared for publication
- **Moeller&Bhabha scattering**
  $e^-e^- \longrightarrow e^-e^-$
  L. Kalinovskaya, L. Rumyantsev, V. Yelmolchyk,
  Done, massive case, to be prepared for publication
- **Processes at low $\mathbf{Q}^2$**
  In preparation

Basic processes for lumi, $\sqrt{s} = 5$ GeV



Photon-pair production advantage: no $\Pi_{\gamma\gamma}$ at 1-loop

• The cross section value estimated for large angles is of **the same order as that of Bhabha scattering**. Events of this process have two collinear photons at large angles providing **a clean signature** for their selection among other detected particles.

• This process gives **large background** while studying hadronic processes with neutral particles in the final state.

• **No vacuum polarization effects.**
No source of uncertainty $\Pi_{\gamma\gamma}$.

Photon-pair production, required accuracy

Cross section with radiative corrections (RC) at the level of per mill accuracy is needed:

- **1-loop level**
  - **polarization – done.**
  - **massive case – done.**
- **matching,** $e^+e^- \to \gamma\gamma + n\gamma$ – **soon.**

  Implementation of works:

  - Frits A. Berends, R. Kleiss, et al., Nucl. Phys., B239, (1984)
  - E.A. Kuraev, V.S. Fadin, Sov.J.Nucl.Phys, **41**, 466 (1985)

Cross-section structure at 1-loop

The cross-section of this processes at the one-loop level can be divided into four parts:

$$\sigma^{\text{1-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

Contributions due to:
σ^{Born} — **Born** level cross-section,
σ^{virt} — **virtual** (loop) corrections,
σ^{soft} — **soft** photon emission,
σ^{hard} — **hard** photon emission (with energy $E_\gamma > \omega$).
Auxiliary parameters λ ("photon mass") and ω cancel out after summation.

From analytical results to numbers

- **Virtual corrections**

$$d\sigma = \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} \left| \mathcal{H}\left(\mathcal{F}^{\textbf{Born+1loop+2loop+...}}\right)_{\lambda_1\lambda_2\lambda_3\lambda_4} \right|^2$$

- **Real corrections**
  - **Soft bremsstrahlung**

$$d\sigma^{Soft} = K d\sigma^{Born}$$

  - **Hard bremsstrahlung**

Scheme of FF calculation

The calculations are organized in a way to control consistency of result.

- All calculations at the one-loop precision level are realized in the R_ξ gauge with three gauge parameters: ξ_A, ξ_Z and $\xi \equiv \xi_\mathsf{W}$
- To parameterize ultraviolet divergences, **dimensional regularization** is used
- Loop integrals are expressed in terms of standard scalar **Passarino-Veltman functions**: A_0, B_0, C_0, D_0

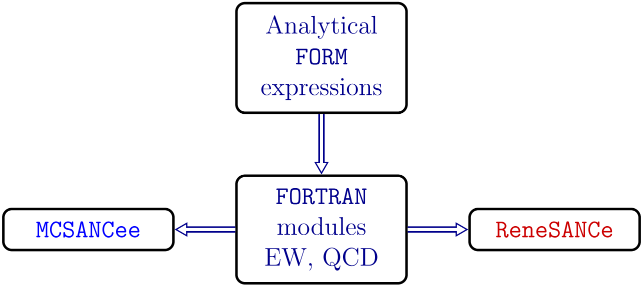
These features make it possible to carry out several important checks at the level of analytical expressions, e.g., checking the gauge invariance by eliminating the dependence on the gauge parameter, checking cancellation of ultraviolet poles, as well as checking various symmetry properties and the Ward identities.

Helicity approach

Helicity approach provides us possibility to describe in the future:

- **any initial** (not only longitudinal) polarization,
- polarization of **final states**,
- **spin correlations**, polarization transfer from initial to final states.

SANC framework



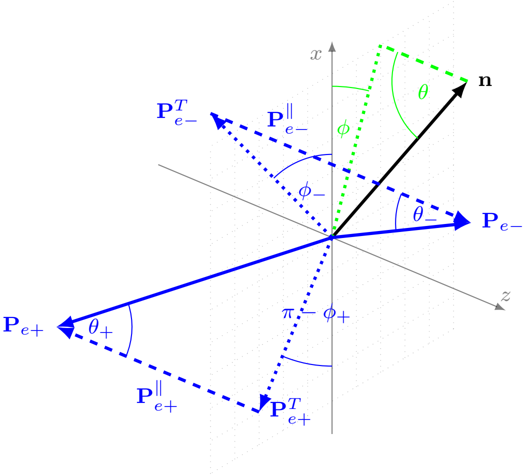
Publications:

SANC – Comput.Phys.Commun. 174 (2006), 481-517.
MCSANC (pp-mode) – Comput.Phys.Commun. 184 (2013),2343-2350; JETP Letters 103 (2016), 131-136.
ReneSANCe – Comput.Phys.Commun. 256 (2020), 107445.

ReneSANCe generator and MCSANCee integrator

- CMAKE build system (generator) and AUTOCONFIG system (integrator)
- Modular architecture
- c++ & FORTRAN
- For sampling we used adaptive algorithm mFOAM Jadach, S. and Sawicki, P., Comp. Phys. Comm. 177 (2007), pp. 441–458 (generator)
  and Vegas from CUBA multithread integration system (integrator)
- The events are written in root/LHEF format and then are analysed (produce histograms) (generator) and write the preseted histograms (integrator)

Decomposition of the $e^{\pm}$ polarization vectors



Matrix element squared

$$
\begin{aligned}
|\mathcal{M}|^2 = {} & L^{\shortparallel}_{e^-} R^{\shortparallel}_{e^+} |\mathcal{H}_{-+}|^2 + R^{\shortparallel}_{e^-} L^{\shortparallel}_{e^+} |\mathcal{H}_{+-}|^2 + L^{\shortparallel}_{e^-} L^{\shortparallel}_{e^+} |\mathcal{H}_{--}|^2 + R^{\shortparallel}_{e^-} R^{\shortparallel}_{e^+} |\mathcal{H}_{++}|^2 \\
& - \frac{1}{2} P^{\perp}_{e^-} P^{\perp}_{e^+} \,\mathrm{Re}\Big[e^{i(\Phi_+ - \Phi_-)} \mathcal{H}_{++} \mathcal{H}^*_{--} + e^{i(\Phi_+ + \Phi_-)} \mathcal{H}_{+-} \mathcal{H}^*_{-+} \Big] \\
& + P^{\perp}_{e^-} \,\mathrm{Re}\Big[e^{i\Phi_-} \Big(L^{\shortparallel}_{e^+} \mathcal{H}_{+-} \mathcal{H}^*_{--} + R^{\shortparallel}_{e^+} \mathcal{H}_{++} \mathcal{H}^*_{-+} \Big) \Big] \\
& - P^{\perp}_{e^+} \,\mathrm{Re}\Big[e^{i\Phi_+} \Big(L^{\shortparallel}_{e^-} \mathcal{H}_{-+} \mathcal{H}^*_{--} + R^{\shortparallel}_{e^-} \mathcal{H}_{++} \mathcal{H}^*_{+-} \Big) \Big],
\end{aligned}
$$

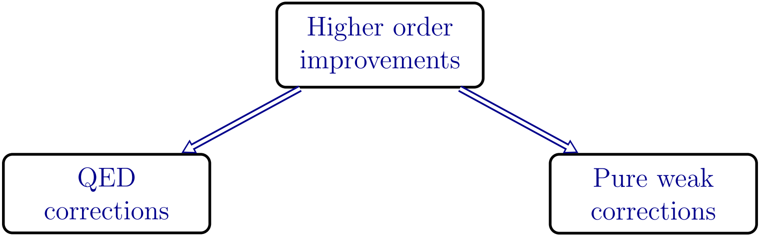
where

$$
L^{\shortparallel}_{e^\pm} = \frac{1}{2}(1 - P^{\shortparallel}_{e^\pm}), \quad R^{\shortparallel}_{e^\pm} = \frac{1}{2}(1 + P^{\shortparallel}_{e^\pm}), \quad \Phi_\pm = \phi_\pm - \phi,
$$

$\mathcal{H}_{--}$, $\mathcal{H}_{++}$,$\mathcal{H}_{-+}$,$\mathcal{H}_{+-}$ — helicity amplitudes.

Moortgat-Pick, G. et al. Phys.Rept. 460 (2008) 131-243

Higher order improvements



- Leading logarithmic (LL) approximation
- Corrections to $\Delta\alpha$
- Shower with matching

- Corrections to $\Delta\rho$
- Leading Sudakov logarithms

Higher order improvements, QED

The leading log in the annihilation channel is $L = \ln \frac{s}{m_l^2}$.

$O(1)$	1		
$O(\alpha)$	αL	α	
$O(\alpha^2)$	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
$O(\alpha^3)$	$\frac{1}{6}\alpha^3 L^3$	$\frac{1}{6}\alpha^3 L^2$	...

In the LL approximation we can separate pure photonic (marked "γ") and the rest corrections which include pure pair and mixed photon-pair effects (marked as "pair").

Higher order improvements, weak

Higher order improvements added throw $\Delta\rho$ parameter:
$s_W^2 \to \bar{s}_W^2 \equiv s_W^2 + \Delta\rho\, c_W^2$.

At the two-loop level, the quantity $\Delta\rho$ contains two contributions:

$$\Delta\rho = N_c X_t \left[1 + \rho^{(2)}\left(M_H^2/m_t^2\right) X_t\right]\left[1 - \frac{2\alpha_s(M_Z^2)}{9\pi}(\pi^2+3)\right],$$

where $X_t = \frac{\sqrt{2} G_F m_t^2}{16\pi^2}$.

Production of resonance

$$\mathcal{A}(z) = \mathcal{A}\left(p_1 + \zeta N_{13}, p_2, p_3 - \zeta N_{13}, \dots\right).$$

$$A(0) = \oint \frac{d\zeta}{2\pi i \zeta} A(\zeta) = \sum_{\zeta_k} \mathrm{res}_{\zeta=\zeta_k} \frac{A(\zeta)}{\zeta}.$$

$$p_1 + p_2 \to -p_{res} - p_X, \quad p_{res} \to p_3 + p_Y.$$

$$(p_{12X} + \zeta_{res} N_{13})^2 = M_{res}^2 \quad \Rightarrow \quad \zeta_{res} = \frac{p_{12X}^2 - M_{res}^2}{2p_{12X} \cdot N_{13}}.$$

$$A_{h_1,h_2,h_3,\dots}^{Prod+Dec}(p_1, p_2, p_3, p_X, p_Y) =$$

$$\sum_{h_{res}} A_{h_1,h_2,h_{res},\dots}^{Prod}(p_1 + \zeta_{res} N_{13}, p_2, -p_{12X} - \zeta_{res} N_{13}, p_X)$$

$$\times \frac{1}{p_{12X}^2 - M_{res}^2} A_{-h_{res},h_3,\dots}^{Dec}(p_{12X} + \zeta_{res} N_{13}, p_3 - \zeta_{res} N_{13}, p_Y).$$

Numerical results: presentation

- separation of gauge-invariant contributions: QED, EW, higher order corrections,
- modern state of $\Delta\alpha_5^{hadr}$ used for $\alpha(s)$ calculation,
- scheme dependence – $\alpha(0)$, G_F,
- evaluation of polarization effects.

Setup for tuned comparison

We performed a tuned comparison of our results for polarized Born and hard Bremsstrahlung with the results WHIZARD and CalcHEP programs.

Initial parameters

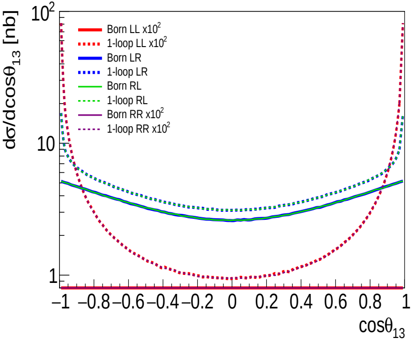
$$\begin{aligned}
\alpha^{-1}(0) &= 137.03599976, & M_W &= 80.451495 \text{ GeV}, & \Gamma_W &= 2.0836 \text{ GeV},\\
M_H &= 125.0 \text{ GeV}, & M_Z &= 91.1876 \text{ GeV}, & \Gamma_Z &= 2.49977 \text{ GeV},\\
m_e &= 0.5109990 \text{ MeV}, & m_\mu &= 0.105658 \text{ GeV}, & m_\tau &= 1.77705 \text{ GeV},\\
m_d &= 0.083 \text{ GeV}, & m_s &= 0.215 \text{ GeV}, & m_b &= 4.7 \text{ GeV},\\
m_u &= 0.062 \text{ GeV}, & m_c &= 1.5 \text{ GeV}, & m_t &= 173.8 \text{ GeV}.
\end{aligned}$$

$$\text{with cuts } |\cos\theta| < 0.9, \quad E_\gamma > 1 \text{ GeV}$$

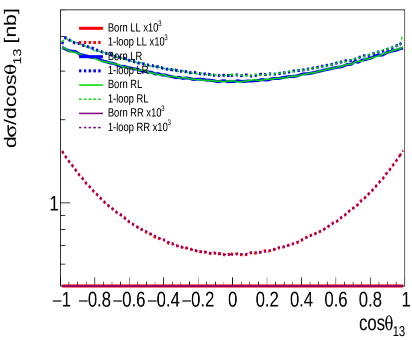
WHIZARD and CalcHEP

- W. Kilian, T. Ohl, J. Reuter, Eur.Phys.J.C71 (2011) 1742,
- A.Belyaev, N.Christensen,A.Pukhov, Comp. Phys. Comm. 184 (2013), pp. 1729-1769

$e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$, $\cos(\vartheta)$, $\sqrt{s} = 5$ GeV



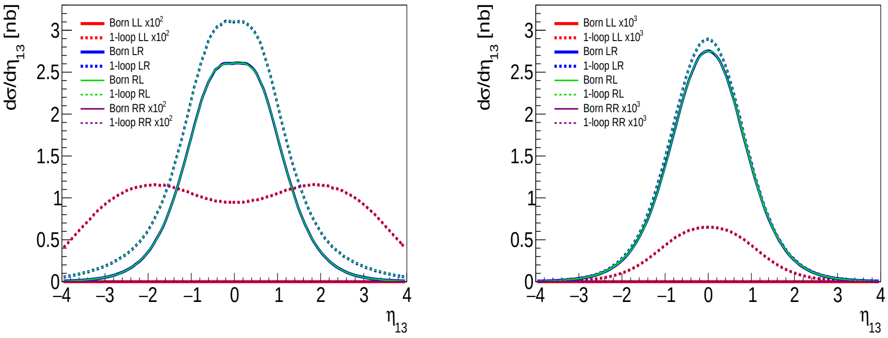
$e^+e^- \to \mu^-\mu^+$



$e^+e^- \to \tau^-\tau^+$

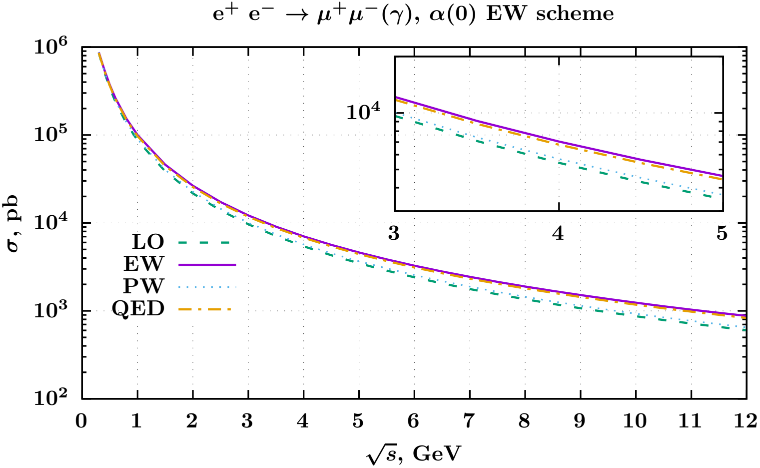
$e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$, η, $\sqrt{s} = 5$ GeV

$$\eta = -\log(\tan\frac{\vartheta}{2})$$

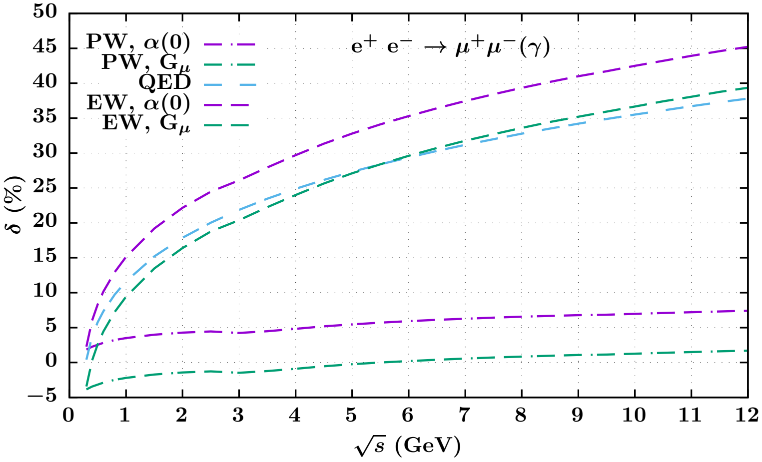


$e^+e^- \to \mu^-\mu^+$ $\qquad$ $e^+e^- \to \tau^-\tau^+$

$e^+e^- \to \mu^+\mu^-$, σ (pb)

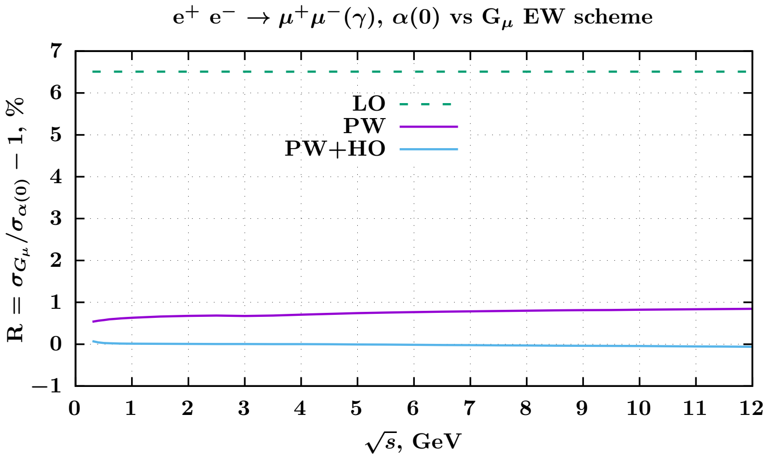


$e^+e^- \to \mu^+\mu^-$, δ (%)

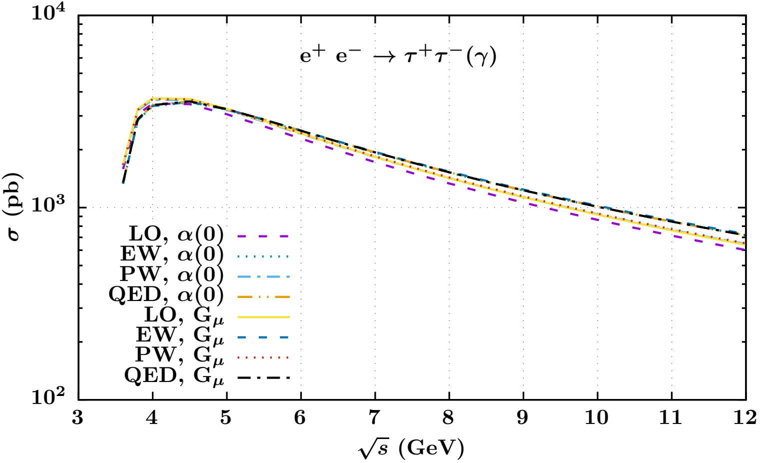


$$\mathrm{EW} = \mathrm{QED} + \mathrm{PW}$$

$e^+e^- \to \mu^+\mu^-$, scheme dependence, R for LO, NLO and NLO+h.o.



$e^+e^- \to \tau^+\tau^-$, σ (pb)



$e^+e^- \to \mu^+\mu^-$, $\sqrt{s} = 5$ GeV

P_{e^+},P_{e^-}	σ^{Born}, pb	σ^{hard}, pb	$\sigma^{\text{B+v+s}}$, pb	$\sigma^{\text{1-loop}}$, pb	δ, %
−1, −1	0	81.51(1)	0	81.51(1)	–
−1, +1	6946.5(1)	8299.8(6)	842.6(1)	9142.4(6)	31.61(1)
+1, −1	6949.9(1)	8301.4(5)	841.8(1)	9143.2(5)	31.56(1)
+1, +1	0	81.51(1)	0	81.51(1)	–

P_{e^+}	P_{e^-}	σ^{Born}, pb	$\sigma^{\text{1-loop}}$, pb	δ, %
0	0	3474.1(1)	4612.2(2)	32.76(1)
0	−0.8	3473.4(1)	4612.0(3)	32.78(1)
0	0.8	3474.8(1)	4612.3(2)	32.76(1)

$e^+e^- \to \mu^+\mu^-$, cuts: $|\cos\theta_{\mu^-}| < 0.9$, $|\cos\theta_{\mu^+}| < 0.9$, $\sqrt{s} = 5$ GeV

P_{e^+},P_{e^-}	σ^{Born}, pb	σ^{hard}, pb	$\sigma^{\text{B+v+s}}$, pb	$\sigma^{\text{1-loop}}$, pb	δ, %
-1, -1	0	13.10(1)	0	13.10(1)	–
-1, $+1$	5955.6(1)	7670(2)	-813(1)	6858(2)	15.1(1)
$+1$, -1	5958.6(1)	7670(2)	-811(1)	6859(2)	15.2(1)
$+1$, $+1$	0	13.11(1)	0	13.11(1)	–

P_{e^+}	P_{e^-}	σ^{Born}, pb	$\sigma^{\text{1-loop}}$, pb	δ, %
0	0	2978.6(1)	3436(1)	15.4(1)
-0.6	-0.8	1548.7(1)	1793(1)	15.8(1)
0	-0.8	2978.0(1)	3436(1)	15.4(1)
0.6	-0.8	4407.2(1)	5079(1)	15.2(1)

$e^+e^- \to \tau^+\tau^-$, $\sqrt{s} = 5$ GeV

P_{e^+}, P_{e^-}	σ^{Born}, pb	σ^{hard}, pb	$\sigma^{\text{B+v+s}}$, pb	$\sigma^{\text{1-loop}}$, pb	δ, %
-1, -1	0	1.7618(1)	0	1.7618(1)	–
-1, $+1$	6123.34(1)	5449.5(1)	988.22(2)	6437.8(1)	5.135(2)
$+1$, -1	6120.30(1)	5446.8(1)	994.51(2)	6441.3(1)	5.245(2)
$+1$, $+1$	0	1.7616(1)	0	1.7616(1)	–

P_{e^+}	P_{e^-}	σ^{Born}, pb	$\sigma^{\text{1-loop}}$, pb	δ, %
0	0	3060.9(1)	3220.6(1)	5.218(1)
0	-0.8	3060.3(1)	3221.3(1)	5.262(1)
0	0.8	3061.5(1)	3219.9(1)	5.175(1)

$e^+e^- \to \tau^+\tau^-$, cuts: $|\cos\theta_{\tau^-}| < 0.9$, $|\cos\theta_{\tau^+}| < 0.9$, $\sqrt{s} = 5$ GeV

P_{e^+}, P_{e^-}	σ^{Born}, pb	σ^{hard}, pb	$\sigma^{\text{B+v+s}}$, pb	$\sigma^{\text{1-loop}}$, pb	δ, %
−1, −1	0	1.303(1)	0	1.303(1)	–
−1, +1	5407.6(1)	3704.6(2)	1929.4(1)	5634.0(2)	4.186(3)
+1, −1	5405.0(1)	3703.2(2)	1929.7(1)	5632.8(2)	4.216(2)
+1, +1	0	1.302(1)	0	1.302(1)	–

P_{e^+}	P_{e^-}	σ^{Born}, pb	$\sigma^{\text{1-loop}}$, pb	δ, %
0	0	2703.1(1)	2817.3(1)	4.227(1)
0	−0.8	2702.6(1)	2817.1(1)	4.218(1)

SANC software: `ReneSANCe`&`MCSANCee`

- provide complete 1-loop and leading higher-order corrections,
- gives the results in the full phase space if needed,
- useful for luminosity measurements at low Q^2 region,
- account of longitudinal polarization,
- provide calculations in three input parameter schemes $\alpha(0)$, $\alpha(M_Z)$ and G_μ.

Summary

SANC products are available at http://sanc.jinr.ru/download.php

ReneSANCe v1.2.0

is available at

http://sanc.jinr.ru/download.php

https://renesance.hepforge.org

MCSANCee

will be available soon

Thank you for attention!

Schemes: $\alpha(0)$, G_μ

• $\alpha(0)$: The fine-structure constant $\alpha(0)$ and all particle masses define the complete input. The relative corrections to the cross sections sensitively depend on the light-quark masses via $\alpha \ln m_q$ terms that enter the charge renormalization.

• G_μ: The Fermi constant G_μ and all particle masses define the basic input. Tree-level couplings are derived from the effective coupling $\alpha_{G_\mu} = \sqrt{2} G_\mu M_W^2 (1 - M_W^2/M_z^2)/\pi$, and the relative corrections receive contributions from the quantity Δr, which describes the radiative corrections to muon decay. Since $\Delta\alpha(M_Z)$ is contained in Δr, there is no large effect on the l^+l^- channels induced by the running of the electromagnetic coupling in the G_μ-scheme either.

Since light-quark masses are perturbatively ill-defined and can only play the role of phenomenological fit parameters, the G_μ-schemes is preferable over the $\alpha(0)$-scheme for the l^+l^- annihilation processes.

QED and PW subsets

QED subset.
It describes electromagnetic (QED) corrections including QED vertices and fermionic self-energies, $\gamma\gamma$ and $Z\gamma$ boxes, and the QED bremsstrahlung.
PW subset.
It describes pure weak corrections which are not included in the first subset. The total electroweak amplitude is a sum of ‘dressed’ γ and Z exchange amplitudes, plus the contribution from *the weak box* diagrams (WW and ZZ boxes), i.e., four EW separately gauge-invariant subgroups.

The improved γ exchange amplitude (IBA)

The improved γ exchange amplitude with running QED-coupling where only fermion loops are taken into account. The ‘dressed’ γ exchange amplitude reads

$$A_\gamma^{\mathrm{IBA}} = i\frac{4\pi Q_e Q_f}{s}\alpha(s)\gamma_\mu \otimes \gamma_\mu \,.$$

It is identical to the Born amplitude modulo the replacement of $\alpha(0)$ by the running electromagnetic coupling $\alpha(s)$:

$$\alpha(s) = \frac{\alpha}{1 - \frac{\alpha}{4\pi}\left[\Pi_{\gamma\gamma}^{\mathrm{fer}}(s) - \Pi_{\gamma\gamma}^{\mathrm{fer}}(0)\right]} \,.$$

The improved Z exchange amplitude (IBA)

The improved Z exchange amplitude with four complex-valued in general EW form factors $\rho_{ef}, \kappa_e, \kappa_f, \kappa_{ef}$:

$$\begin{aligned}
\mathcal{A}_Z^{IBA}(s,t) &= i\, e^2\, 4\, I_e^{(3)} I_f^{(3)} \frac{\chi_Z(s)}{s} \rho_{ef}(s,t) \\
& \Big\{ \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu(1+\gamma_5) \\
& +4|Q_e| s_W^2 \kappa_e(s,t) \gamma_\mu \otimes \gamma_\mu(1+\gamma_5) \\
& -4|Q_f| s_W^2 \kappa_f(s,t) \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu \\
& +16|Q_e Q_f| s_W^4 \kappa_{e,f}(s,t) \gamma_\mu \otimes \gamma_\mu \Big\}.
\end{aligned}$$

The improved Z exchange amplitude (IBA)

These form factors are directly related to the one-loop form factors

$$\begin{aligned}
\rho_{ef} &= 1 + F_{LL}(s,t) - s_W^2 \Delta r, \\
\kappa_e &= 1 + F_{QL}(s,t) - F_{LL}(s,t), \\
\kappa_f &= 1 + F_{LQ}(s,t) - F_{LL}(s,t), \\
\kappa_{ef} &= 1 + F_{QQ}(s,t) - F_{LL}(s,t).
\end{aligned}$$

II) Traditional parametrization of corrections to the running QED coupling

As well known physical sense of the running effective QED coupling with momentum transfer, $\alpha(0) \to \alpha(s)$, is fermion-pair loop insertions to the photon propagator.
(The contribution of the W-boson loop is gauge-dependent. We shift these contributions to the gauge invariant expression sums of own energies, counter terms and vertices)
Dyson summation leads to the change of $\alpha(0)$ into $\alpha(s)$:

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha^{\text{fer}}(s)} = \frac{\alpha(0)}{1 - \Delta\alpha^{(5)}(s) - \Delta\alpha^{t}(s) - \Delta\alpha^{\alpha\alpha_s}(s)},$$

where we explicitly disentangled the one-loop top-quark contribution $\Delta\alpha^{t}(s)$ and the two-loop irreducible higher-order correction $\Delta\alpha^{\alpha\alpha_s}(s)$.

The 5-flavor part $\Delta\alpha^{(5)}$

The 5-flavor part $\Delta\alpha^{(5)}(s)$ is a sum of leptonic and hadronic contributions

$$\Delta\alpha^{(5)}(s) = \Delta\alpha_l(s) + \Delta\alpha_{hadr}^{(5)}(s).$$

They are normally calculated SEPARATELY.

leptonic contributions ← perturbation theory

hadronic contributions ← experimental data as input

The leptonic part $\Delta\alpha_l(s)$

The leptonic (where lepton $l = e, \mu, \tau$) one-loop contribution, $\Delta\alpha_l(s)$, is defined by

$$\Delta\alpha_l(s) = \frac{\alpha}{1 - \frac{\alpha}{4\pi}\left[\Pi^{\text{fer}}_{\gamma\gamma}(s) - \Pi^{\text{fer}}_{\gamma\gamma}(0)\right]},$$

with

$$\begin{aligned}
\Pi^{\text{fer}}_{\gamma\gamma}(s) &= 4\sum_f c_f Q_f^2 B_f\left(-s; m_f, m_f\right), \\
B_f\left(p^2; M_1, M_2\right) &= 2\left[B_{21}\left(p^2; M_1, M_2\right) + B_1\left(p^2; M_1, M_2\right)\right] \\
&= -\frac{1}{\bar{\varepsilon}} + 2\int_0^1 dx x(1-x)\ln\frac{p^2 x(1-x) + M_1(1-x) + M_2 x}{\mu^2}.
\end{aligned}$$

The contributions are:

$$\begin{aligned}
\Delta\alpha_l &= 314.97637 \cdot 10^{-4} \\
&= \left[314.18942_{1-\text{loop}} + 0.77616_{2-\text{loop}} + 0.01079_{3-\text{loop}}\right] \cdot 10^{-4}.
\end{aligned}$$

The hadronic part $\Delta\alpha^{(5)}_{hadr}(s)$

The contribution of light quark loops cannot be calculated theoretically since the quark mass in the logarithm does not have a strict theoretical definition, i.e. at these low energy scales perturbative QCD is not applicable. Therefore, the total contribution from the five light quark flavors to the hadronic vacuum polarization, $\Delta\alpha^{(5)}_{hadr}(s)$, is more accurately obtained from a dispersion integral over the measured hadronic cross-section in electron-positron annihilations at low center-of-mass energies:

$$\Delta\alpha^{(5)}_{hadr}(s) = -\frac{\alpha}{3\pi} s \, \mathrm{Re} \int_{4m_\pi^2}^{\infty} ds' \frac{R_\gamma(s')}{s'(s'-s-i\epsilon)},$$

$$R_\gamma(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

as an experimental input. For the hadronic contribution at M_Z reference value is:

$$\Delta\alpha^{(5)}_{hadr}(M_Z^2) = 275.76 \cdot 10^{-4}.$$

This value corresponds to version of the parametrization provided by F.Jegerlehner (2020), $\Delta\alpha^{(5)}_{hadr}(s)$ might be also treated as an **input parameter** for the fit.

The top contribution $\Delta\alpha^t(M_Z^2)$

This correction is given by

$$\Delta\alpha^t(M_z^2) = \frac{\alpha}{4\pi}\Big[\Pi_{\gamma\gamma}^{t,F}\left(M_z^2\right) - \Pi_{\gamma\gamma}^{t,F}\left(0\right)\Big].$$

it depends on the mass of the top quark, and we present its numerical value for $m_t = 173.8$ GeV:

$$\Delta\alpha^t(M_z^2) = -0.585844 \cdot 10^{-4}.$$

The mixed two-loop $\mathcal{O}(\alpha\alpha_s)$ correction

This contribution arises from $t\bar{t}$ loops with gluon exchange. Its numerical value at $m_t = 173.8$ and $\alpha_s = 0.119$ is

$$\Delta\alpha^{\alpha\alpha_s}(M_Z^2) = -0.103962 \cdot 10^{-4}.$$