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STRONG PHASE MEASUREMENT IN $D \rightarrow K_S h$ DECAYS AT SCT

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WHY DO WE NEED TO MEASURE δ_{K^0h}

Calculation of QCD contribution to charm decays is not a straightforward task. Long distance contribution (FSI) could mimic NP in charm. Pursuing the goal of the most precise measurements and searches for NP clear understanding of SM contribution is needed.

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-15.4 \pm 2.9) \times 10^{-4}$$

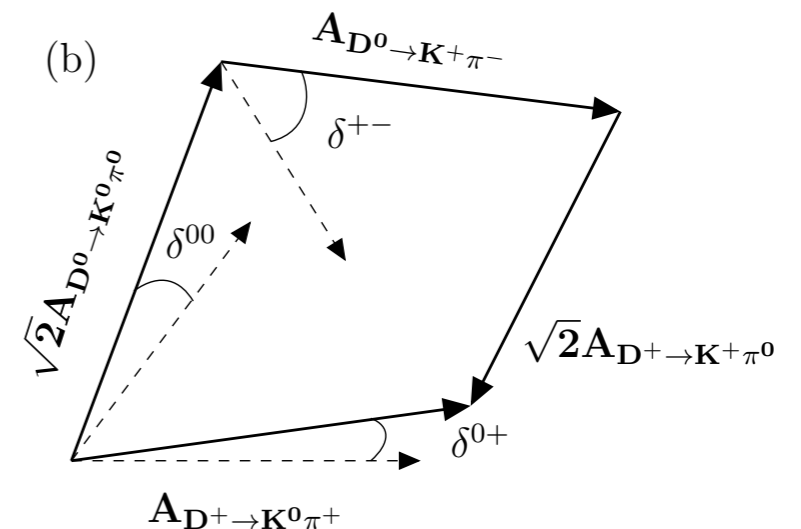
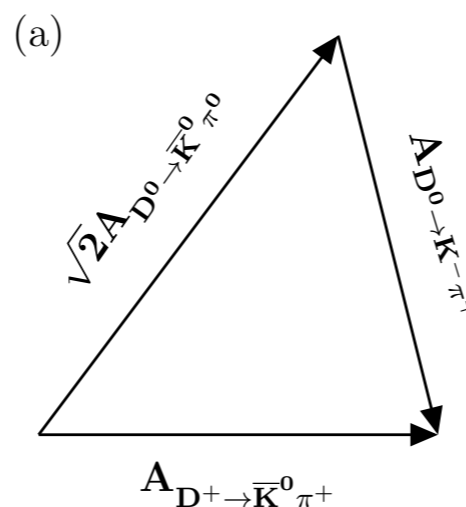
Phys. Rev. Lett. **122**, no.21,211803(2019)

Naive SM prediction: $\mathcal{O}(\alpha_s/\pi) \frac{V_{ub} V_{cb}^*}{V_{us} V_{cs}^*} \sim 10^{-4}$

SM or NP ?

Approach based on flavour symmetries allows us to get around QCD calculations.

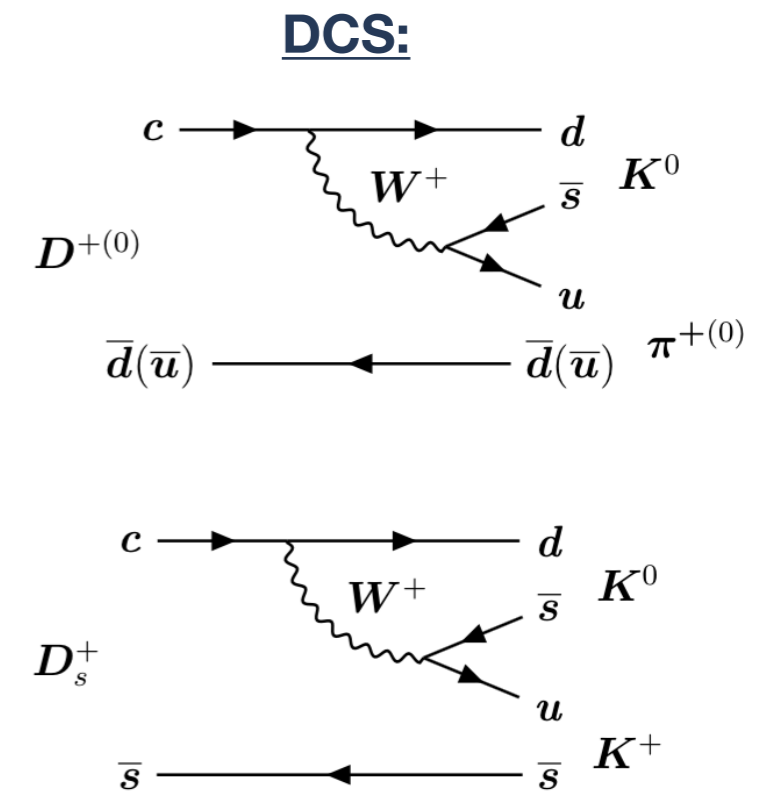
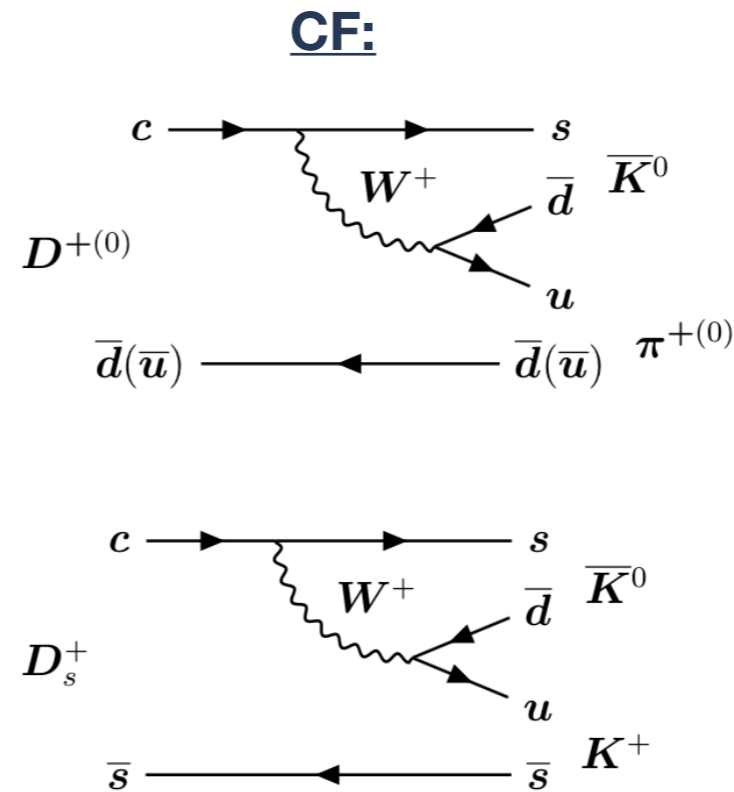
- $SU(3)_f$ allows to obtain predictions for strong phase values;
- $SU(2)_f$ sum rules allow us to flavour symmetry approach



FRAMEWORK

Initial state – $a|K^0\rangle + b|\bar{K}^0\rangle$:

- D^+/D^- , $D^+ \rightarrow \bar{K}^0\pi^+$,
- D^0/\bar{D}^0 , $D^0 \rightarrow \bar{K}^0\pi^0$,
- D_s^+/D_s^- , $D_s^+ \rightarrow \bar{K}^0K^+$,



$K^0 - \bar{K}^0$ evolution:

$$i\frac{\partial}{\partial t} \begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma} \right) \begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix}$$

CPV

$$|K^0(t)\rangle = \frac{1-\varepsilon}{\sqrt{2}}e^{-i\lambda_S t}|K_S\rangle + \frac{1-\varepsilon}{\sqrt{2}}e^{-i\lambda_L t}|K_L\rangle$$

$$|\bar{K}^0(t)\rangle = \frac{1+\varepsilon}{\sqrt{2}}e^{-i\lambda_S t}|K_S\rangle - \frac{1+\varepsilon}{\sqrt{2}}e^{-i\lambda_L t}|K_L\rangle$$

Mixing

$$|K^0(t)\rangle = g_+(t)|K^0\rangle + \left(\frac{q}{p}\right)g_-(t)|\bar{K}^0\rangle$$

$$|\bar{K}^0(t)\rangle = g_+(t)|\bar{K}^0\rangle - \left(\frac{p}{q}\right)g_-(t)|K^0\rangle$$

MEASUREMENT WITH DECAY $K^0 \rightarrow \pi \ell \nu_\ell$

 JHEP **02**,160(2020)

For the initial admixture $a |K^0\rangle + b |\bar{K}^0\rangle$ time-dependent decay rates:

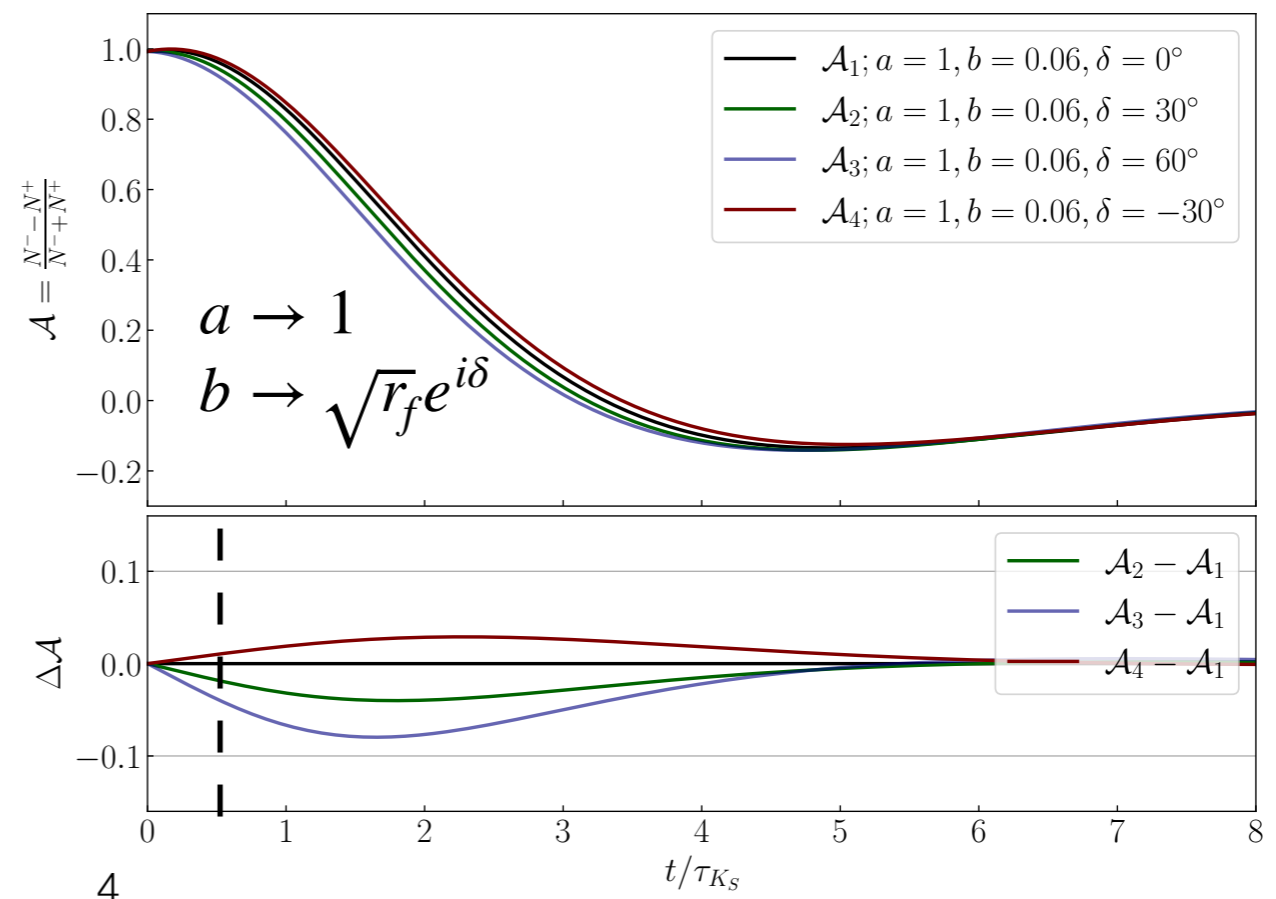
$$R(t) = \frac{1}{4} e^{-\Gamma t} |A_{t+}|^2 \left[|a|^2 K_+ + \left| b \frac{p}{q} \right|^2 K_- + 2 \operatorname{Re} \left\{ ab \frac{p}{q} (1 - e^{\Delta\Gamma t} + 2i \sin(\Delta m t) e^{\frac{1}{2}\Delta\Gamma t}) \right\} \right]$$

$$\bar{R}(t) = \frac{1}{4} e^{-\Gamma t} |A_{t-}|^2 \left[|a|^2 K_- + \left| b \frac{q}{p} \right|^2 K_+ + 2 \operatorname{Re} \left\{ ab \frac{q}{p} (1 - e^{\Delta\Gamma t} + 2i \sin(\Delta m t) e^{\frac{1}{2}\Delta\Gamma t}) \right\} \right]$$

$$K_\pm = 1 \pm 2 \cos(\Delta m t) e^{\frac{1}{2}\Delta\Gamma t} + e^{\Delta\Gamma t}$$

The third term represents an interference of CF and DCS decay amplitudes and allows us to extract the strong phase difference $-\delta$.

Both $\cos \delta$ and $\sin \delta$ could be measured, so there is no trigonometrical ambiguity in such measurement.



MEASUREMENT WITH DECAY $K_S^0 \rightarrow \pi^+ \pi^-$

Amplitudes:

$$|K^0(t)\rangle = \frac{1 - \varepsilon}{\sqrt{2}} e^{-i\lambda_S t} |K_S\rangle + \frac{1 - \varepsilon}{\sqrt{2}} e^{-i\lambda_L t} |K_L\rangle$$

$$|\bar{K}^0(t)\rangle = \frac{1 + \varepsilon}{\sqrt{2}} e^{-i\lambda_S t} |K_S\rangle - \frac{1 + \varepsilon}{\sqrt{2}} e^{-i\lambda_L t} |K_L\rangle$$

Time-dependent decay rates:

$$\bar{\mathcal{R}}(\mathcal{R}) \equiv \frac{1 \pm 2\text{Re}(\varepsilon)}{2} |A_{fS}|^2 \left[e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} - \mp 2|\eta_{+-}| e^{-\frac{1}{2}(\Gamma_L + \Gamma_S)t} \cos(\Delta m t - \varphi_{+-}) \right]$$

CPLEAR results:

Phys.Lett.B 456 (1999)

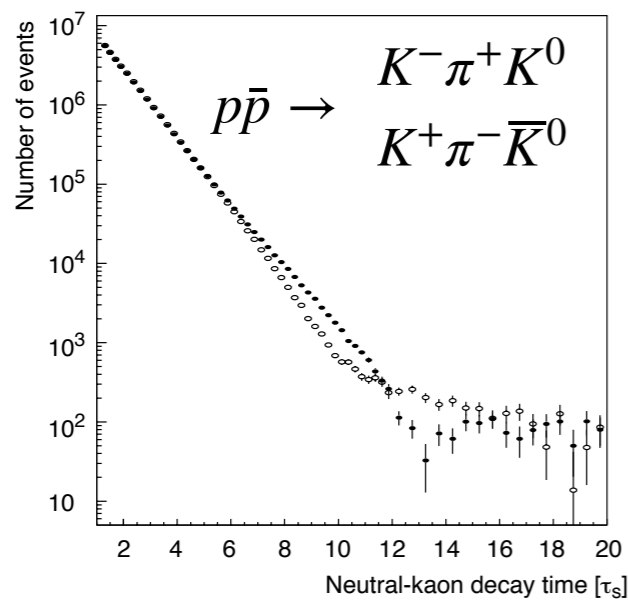
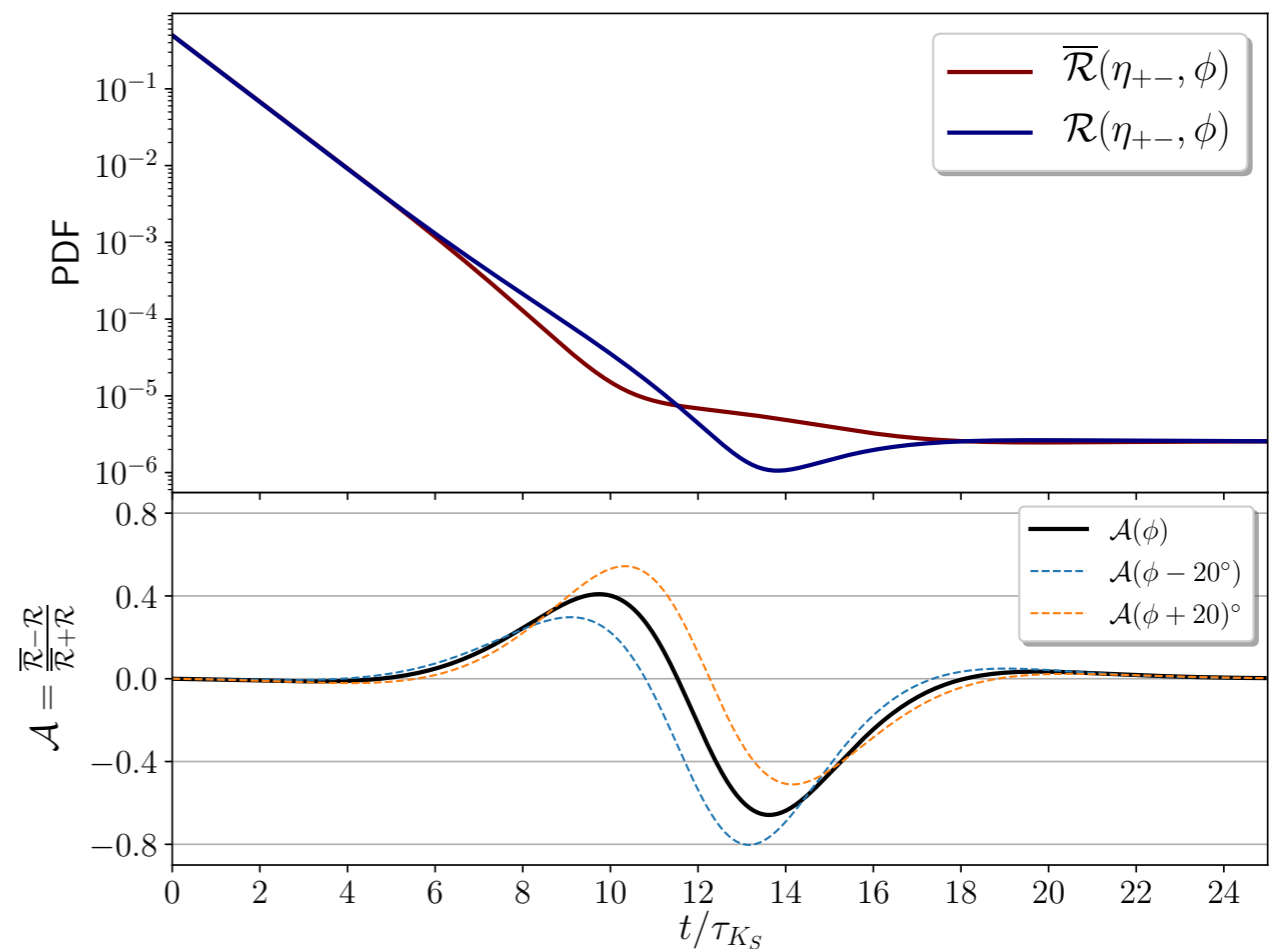


Fig. 16. The measured decay rates for K^0 (\circ) and \bar{K}^0 (\bullet) after acceptance correction and background subtraction

World average:

$$|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3}$$

$$\varphi_{+-} = 43.51 \pm 0.05$$



MEASUREMENT WITH DECAY $K_S^0 \rightarrow \pi^+ \pi^-$

JHEP 09,092(2021)

Amplitudes for the process $D \rightarrow K_S X$:

$$\langle f | H_{wk} | D^0 \rangle = \langle f | H_{wk} | \bar{K}^0 \rangle + \sqrt{r_D} e^{i\delta} \langle f | H_{wk} | K^0 \rangle$$

$$\langle f | H_{wk} | \bar{D}^0 \rangle = \sqrt{r_D} e^{i\delta} \langle f | H_{wk} | \bar{K}^0 \rangle + \langle f | H_{wk} | K^0 \rangle$$

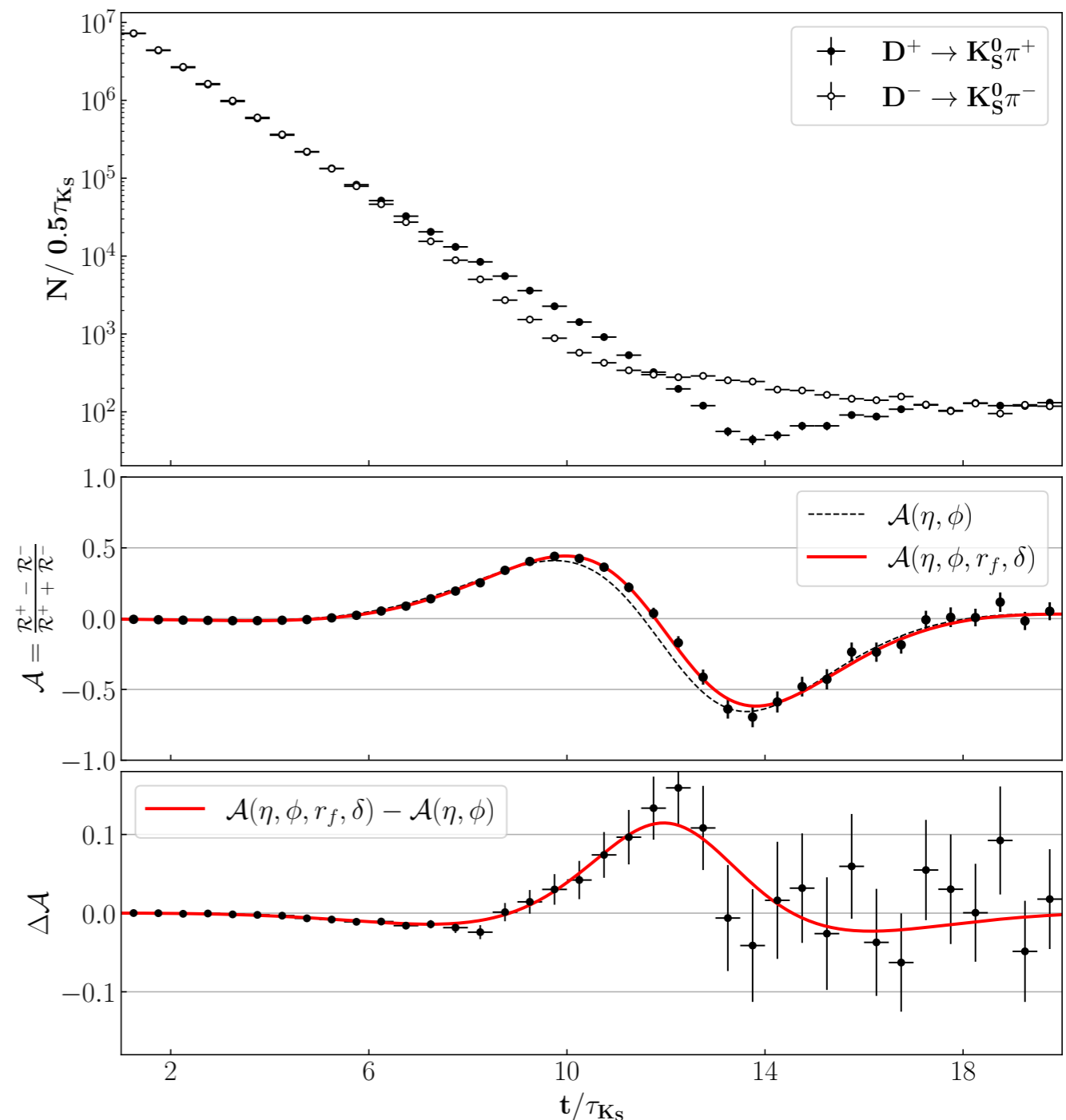
Time dependent decay rates

$K_S \rightarrow \pi^+ \pi^-$:

$$\begin{aligned} \mathcal{R}^+(t) &\equiv |\Psi^+(t)|^2 = \bar{\mathcal{R}}(t) + r_f \mathcal{R}(t) \\ &+ \sqrt{r_f} (\cos \delta + 2|\eta_{+-}| \sin \delta \sin \phi_{+-}) \times (e^{-\Gamma_S t} - |\eta_{+-}|^2 e^{-\Gamma_L t}) \\ &+ 2\sqrt{r_f} |\eta_{+-}| \left(\sin \delta + 2|\eta_{+-}| \cos \delta \sin \phi_{+-} \right) e^{-\frac{1}{2}(\Gamma_L + \Gamma_S)t} \sin(\Delta m t - \phi_{+-}), \end{aligned}$$

$$\begin{aligned} \mathcal{R}^-(t) &\equiv |\Psi^-(t)|^2 = \mathcal{R}(t) + r_f \bar{\mathcal{R}}(t) \\ &+ \sqrt{r_f} (\cos \delta - 2|\eta_{+-}| \sin \delta \sin \phi_{+-}) \times (e^{-\Gamma_S t} - |\eta_{+-}|^2 e^{-\Gamma_L t}) \\ &- 2\sqrt{r_f} |\eta_{+-}| \left(\sin \delta - 2|\eta_{+-}| \cos \delta \sin \phi_{+-} \right) e^{-\frac{1}{2}(\Gamma_L + \Gamma_S)t} \sin(\Delta m t - \phi_{+-}). \end{aligned}$$

Strong phase enters both equations. No trigonometrical uncertainty in the measurement.

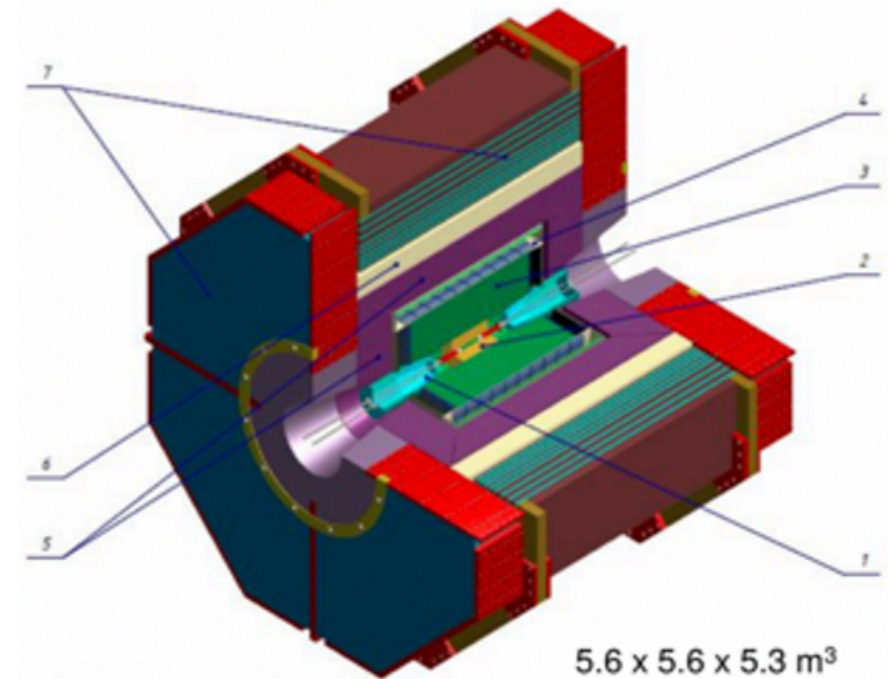


SUPER $C\text{-}\tau$ FACTORY

Proposed methods are universal however there are some requirements to achieve the best precision:

Requirement	SCTF
Good spatial resolution $\sim 100\mu\text{m}$	✓
Large tracking detector/slow kaons	✓
Good momentum resolution $\sigma_p/p < 0.01$	✓
Hadron identification	✓

SCTF



Luminosity per 1 year

	J/ψ	$\psi(2S)$	$\psi(3770)$	$\psi(4040)$	$\psi(4160)$	$\psi(4415)$
$M, \text{ GeV}$	3.097	3.686	3.773	4.039	4.191	4.421
$\Gamma, \text{ MeV}$	0.093	0.286	27.2	80	70	62
$\sigma, \text{ nb}$	~ 3400	~ 640	~ 6	~ 10	~ 6	~ 4
$L, \text{ fb}^{-1}$	300	150	300	10	100	25
N	10^{12}	10^{11}	2×10^9	10^8	6×10^8	10^8

FEASIBILITY STUDY: $K^0 \rightarrow \pi \ell \nu_\ell$

Feasibility study preformed with Monte-Carlo.

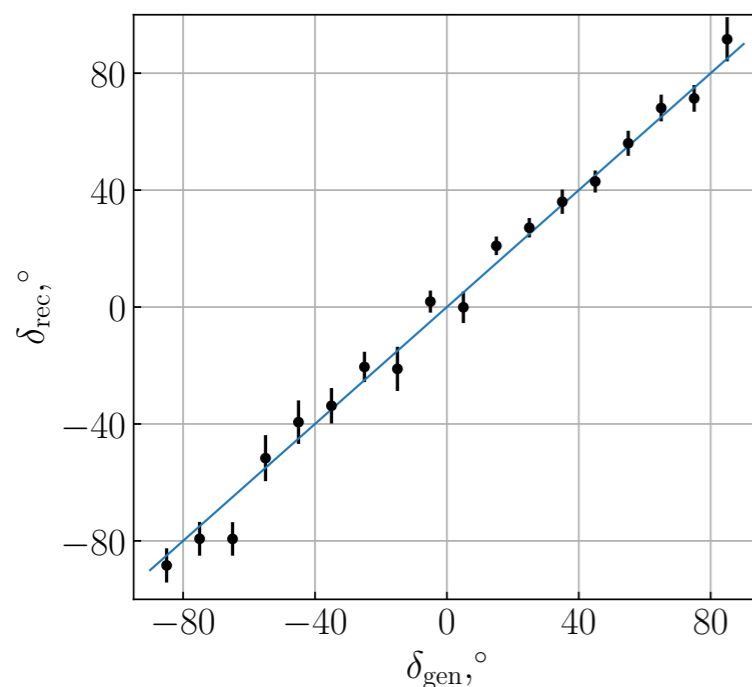
Both RS and WS distributions fitted (χ^2) simultaneously.

Studies showed no bias in such measurement.

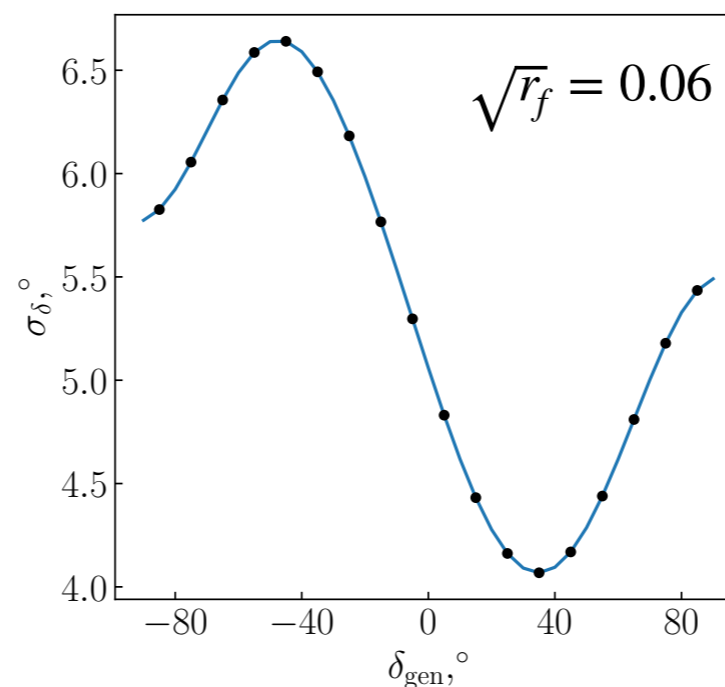
Expected number of events:

Decay channel	SCTF, $\times 10^4$
$D^0 \rightarrow \bar{K}^0 \pi^0$	6
$D^+ \rightarrow \bar{K}^0 \pi^+$	15
$D_s^+ \rightarrow \bar{K}^0 K^+$	12

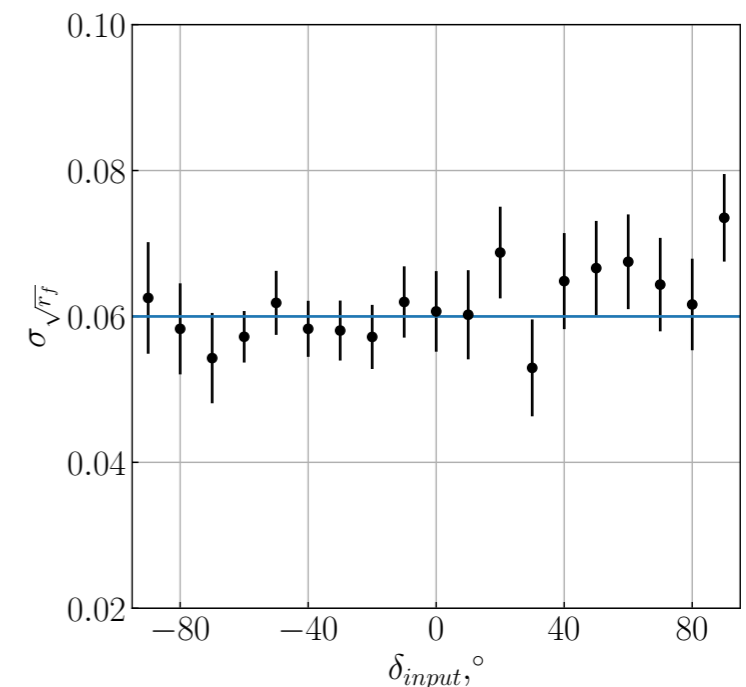
Strong phase difference



Uncertainty in δ



DCS/CF amplitude ratio

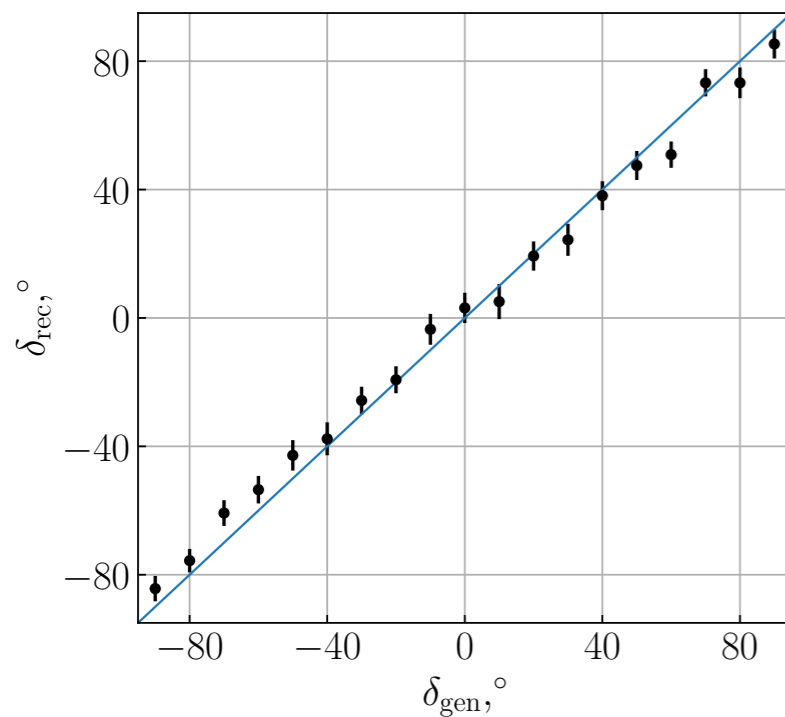


FEASIBILITY STUDY: $K_S^0 \rightarrow \pi^+ \pi^-$

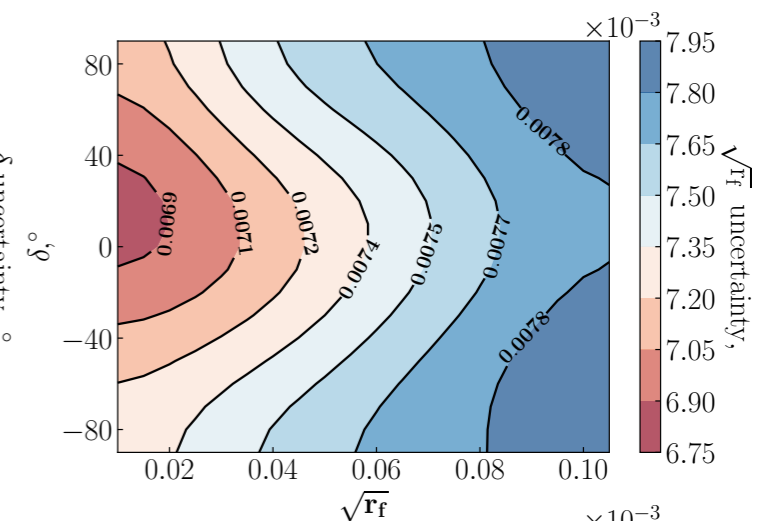
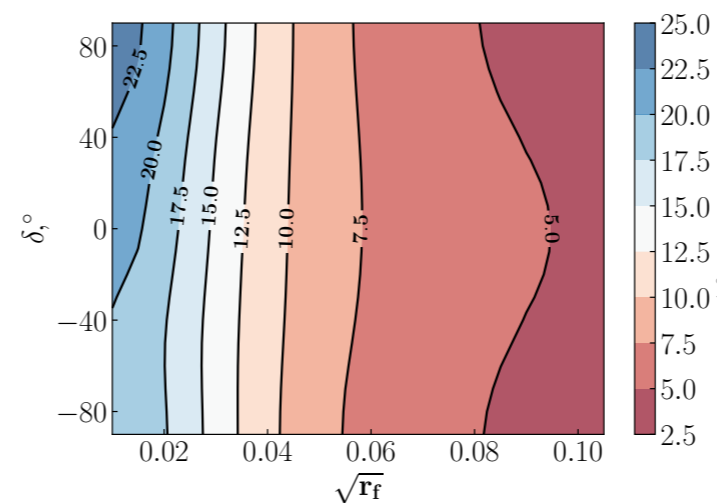
DCS/CF amplitude ratio was never measured in decay modes with K_S^0 , so scan in $\delta/\sqrt{r_f}$ performed.

Obtained with Monte-Carlo simulation samples fitted (ML) and no bias observed in proposed measurement.

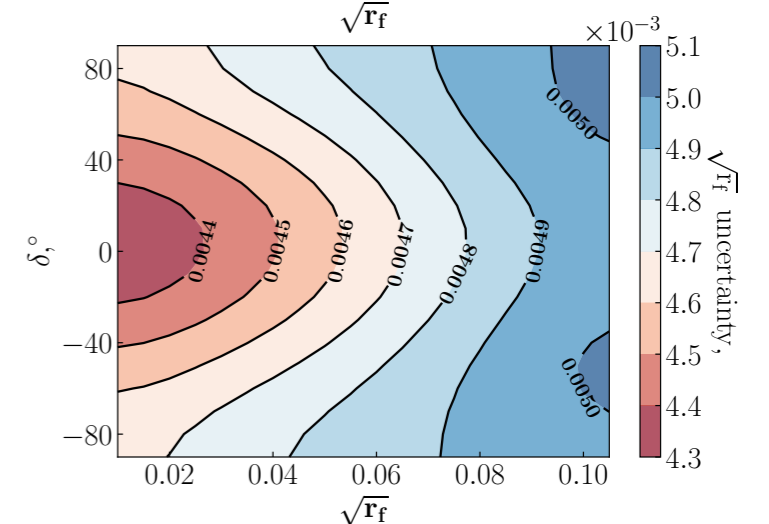
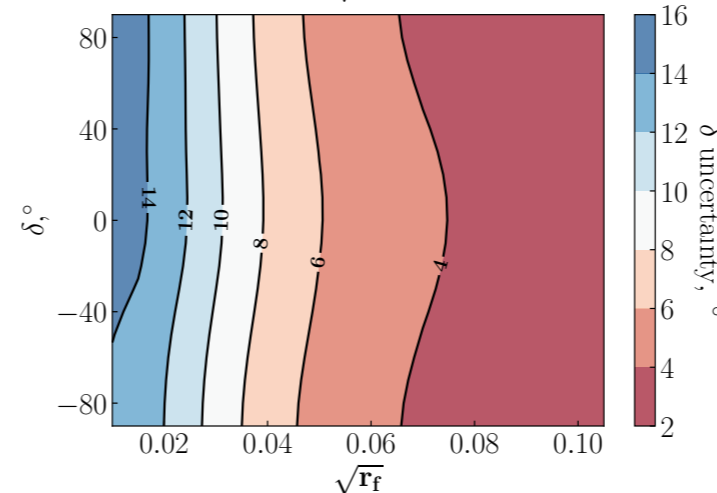
$$r_f \equiv \left| \frac{\langle K\pi | H | D \rangle}{\langle \bar{K}\pi | H | D \rangle} \right|^2 \approx \left| \frac{V_{cd} V_{us}^*}{V_{cs} V_{ud}^*} \right|^2 \sim \mathcal{O}(\tan^4 \theta_c).$$



$D^0 \rightarrow K_S^0 \pi^0$



$D^+ \rightarrow K_S^0 \pi^+$



STUDY OF CORRELATED D^0 - \bar{D}^0

$$\Psi_{D\bar{D}} = \frac{1}{\sqrt{2}} [|D_{phys}^0(t)\rangle | \bar{D}_{phys}^0(t)\rangle - | \bar{D}_{phys}^0(t)\rangle | D_{phys}^0(t)\rangle]$$

	J/ψ	$\psi(2S)$	$\psi(3770)$	$\psi(4040)$	$\psi(4160)$	$\psi(4415)$
$M, \text{ GeV}$	3.097	3.686	3.773	4.039	4.191	4.421
$\Gamma, \text{ MeV}$	0.093	0.286	27.2	80	70	62
$\sigma, \text{ nb}$	~ 3400	~ 640	~ 6	~ 10	~ 6	~ 4
$L, \text{ fb}^{-1}$	300	150	300	10	100	25
N	10^{12}	10^{11}	2×10^9	10^8	6×10^8	10^8

For correlated D^0 pair time dependent decay rate for final states f_1, f_2 :

$$R(f_1, t_1, f_2, t_2) \propto |A_{f_1}|^2 |A_{f_2}|^2 e^{-\Gamma(t_1+t_2)} \left[\frac{1}{2} |\xi + \zeta|^2 e^{-\Delta\Gamma/2(t_2-t_1)} + \frac{1}{2} |\xi - \zeta|^2 e^{\Delta\Gamma/2(t_2-t_1)} - (|\xi|^2 - |\zeta|^2) \cos(\Delta m(t_2 - t_1)) + 2 \text{Im}(\xi^* \zeta) \sin(\Delta m(t_2 - t_1)) \right]$$

$$\text{where } \zeta = \frac{\bar{A}_{f_2}}{A_{f_2}} - \frac{\bar{A}_{f_1}}{A_{f_1}}, \quad \xi = \left(\frac{p}{q} \right)_D - \left(\frac{q}{p} \right)_D \frac{\bar{A}_{f_1}}{A_{f_1}} \frac{\bar{A}_{f_2}}{A_{f_2}}$$

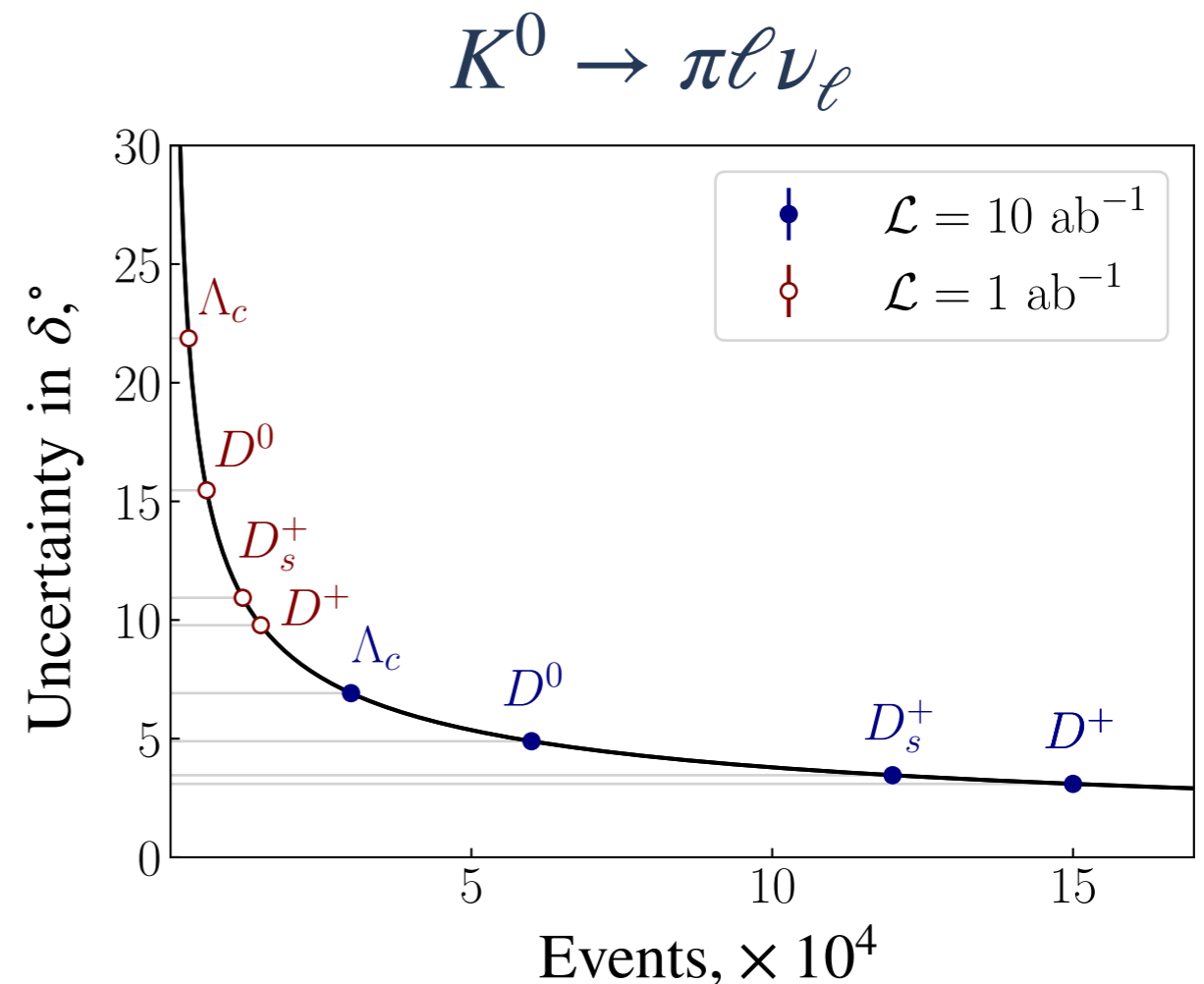
Then for combination of final states: $\{ \mathbf{D} \rightarrow \mathbf{K}^- \pi^+; \mathbf{D} \rightarrow \bar{\mathbf{K}}^0 \pi^0 \}$

$$R(t_1, t_2) \propto 2 |A_{K^- \pi^+}|^2 |A_{\bar{K}^0 \pi^0}|^2 e^{-\Gamma(t_1+t_2)} \left[(r_f^0 + r_f^- - \Delta_-) + \Gamma \Delta t \Delta_{12} + \left(\left| \frac{p}{q} \right|_D^2 + \left| \frac{q}{p} \right|_D^2 r_f^0 r_f^- - \Delta_+ \right) \frac{(\Gamma \Delta t)^2}{4} (x^2 + y^2) \right],$$

$$\Delta_{\pm} = 2 \sqrt{r_f^0 r_f^-} \cos(\delta_{00} \pm \delta_{-+}), \quad \Delta_{12} = \sqrt{r_f^-} \left(\frac{p}{q} x' + \frac{q}{p} r_f^0 y' \right) - \sqrt{r_f^0} \left(\frac{p}{q} x'' + \frac{q}{p} r_f^- y'' \right)$$

CONCLUSION

- Strong phase measurement could significantly improve our estimations for long distance QCD contribution in charm;
- The adequacy of flavour symmetry approach could be checked;
- **SCTF** is an **ideal candidate** for such measurements;
- Both methods give similar accuracy at the level of 5° ;
- Secondary vertex reconstruction is crucial for such analysis.

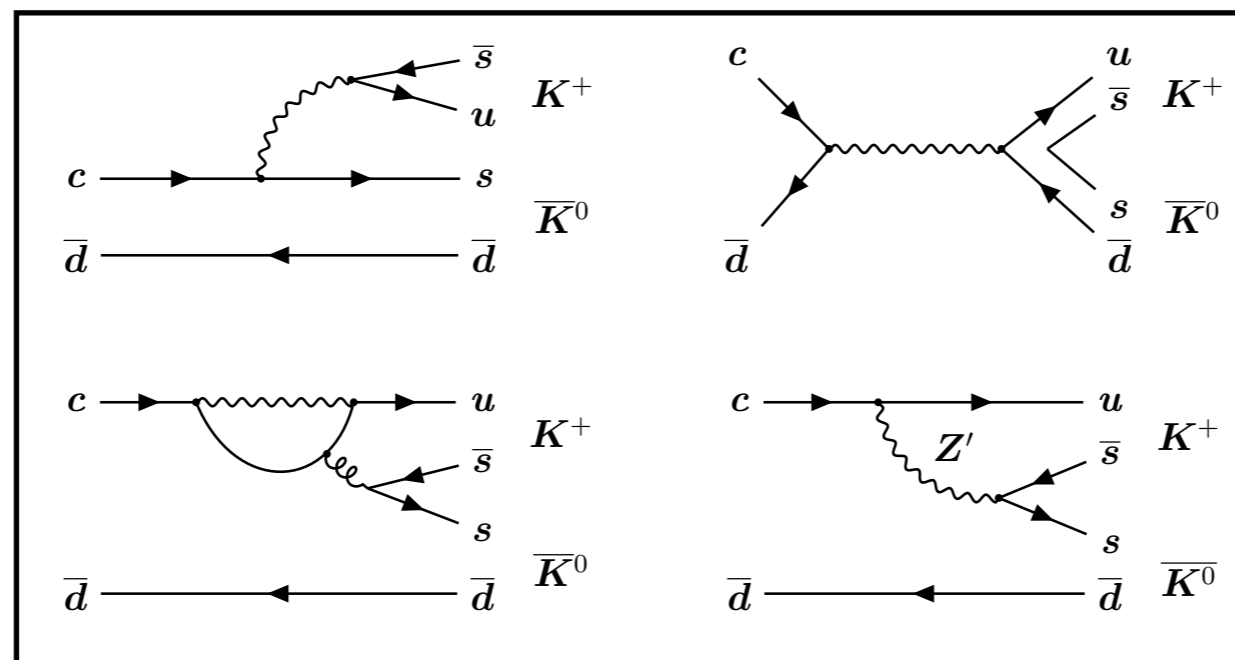




BACKUP SLIDES

OTHER APPLICATIONS

- Searches for NP in SCS decays. In the SM only one flavour is produced in SCS decays;



- Semileptonic $D^+(D^0)$ -decays could be used to improve accuracy in η_{+-} , φ_{+-} and search for NP.

K^0 REGENERATION

Strangeness conservation in strong interactions lead to inequality of rescattering amplitudes for K^0 and \bar{K}^0 on matter — $\Delta f \neq 0$. Regeneration of neutral kaons can imitate CPV and introduce a bias in strong phase measurement.

Equations for evolution should be modified as:

$$i\partial_t \begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma} \right) \begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix} - \begin{pmatrix} \chi & 0 \\ 0 & \bar{\chi} \end{pmatrix} \begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix}, \text{ где } \chi(\bar{\chi}) = \frac{2\pi N}{m} f(\bar{f})$$

Introducing regeneration parameter — r :

$$\alpha_{S,L} = e^{-i\Sigma t} \left[\alpha_{S,L}^0 \cos \left(\frac{\Delta\lambda}{2} \sqrt{1+4r^2} t \right) \pm i \frac{\alpha_{S,L}^0 \mp 2r\alpha_{L,S}^0}{\sqrt{1+4r^2}} \sin \left(\frac{\Delta\lambda}{2} \sqrt{1+4r^2} t \right) \right], \text{ где } r = \frac{1}{2} \frac{\Delta\chi}{\Delta\lambda}$$

With expansion for r :

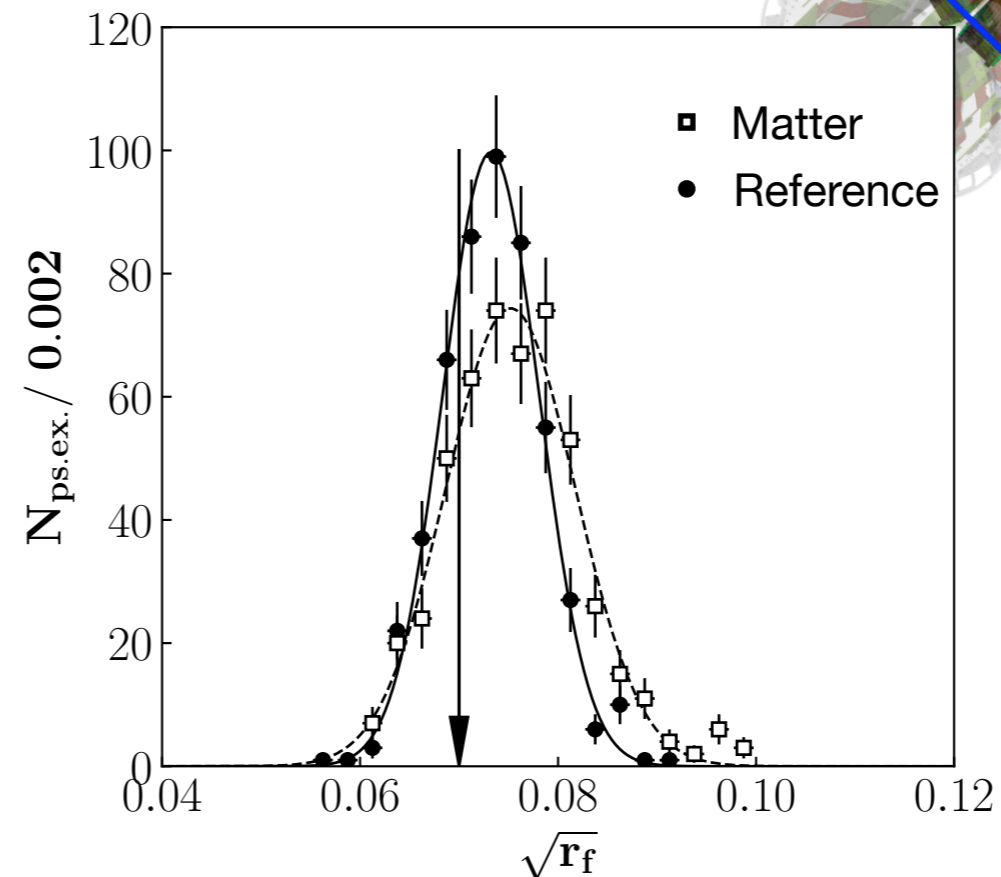
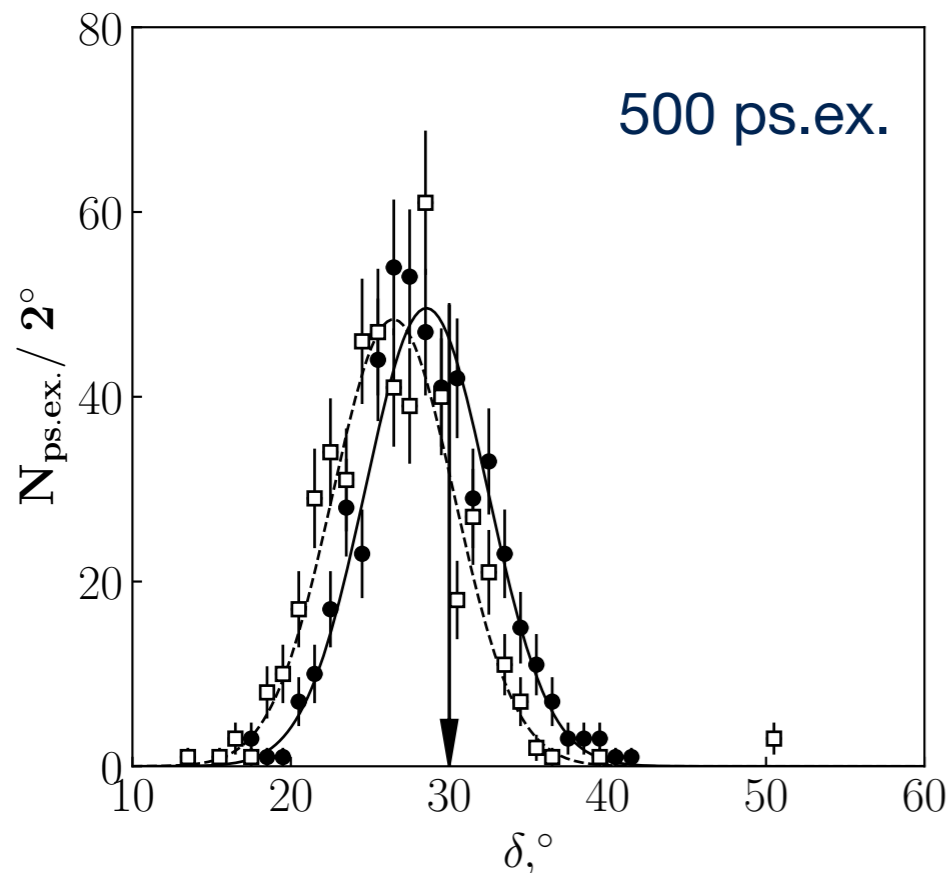
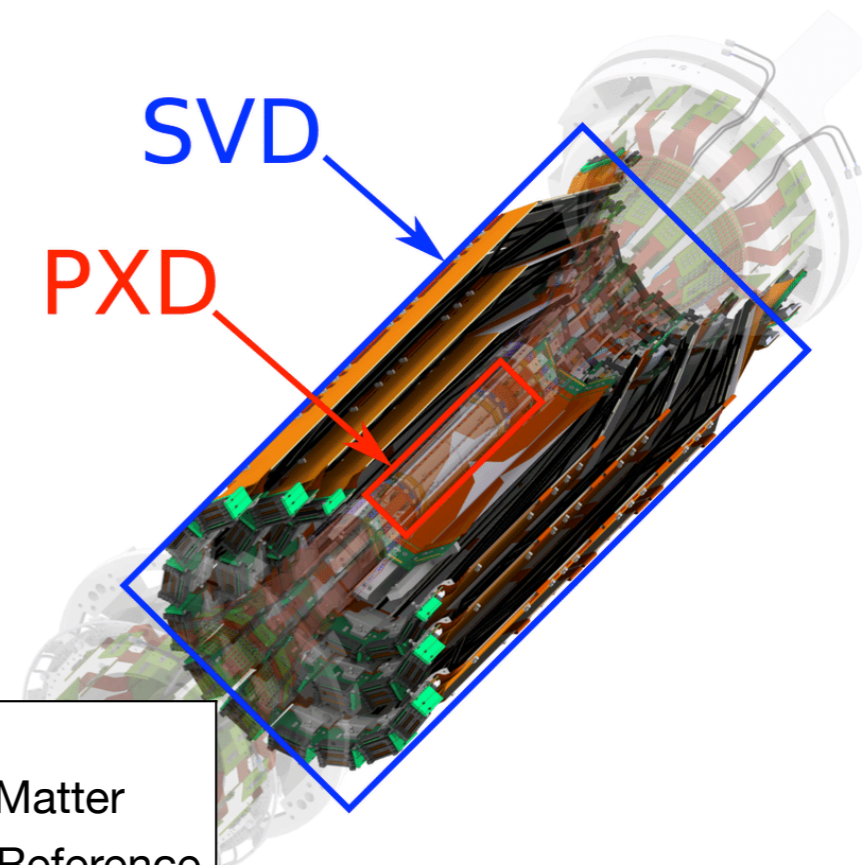
$$\begin{aligned} \alpha_S(t) &= e^{\frac{1}{2}(\chi+\bar{\chi})t} e^{-i\lambda_S t} (\alpha_S^0 + \zeta \alpha_L^0 e^{-i\Delta\lambda t}) \\ \alpha_L(t) &= e^{\frac{1}{2}(\chi+\bar{\chi})t} e^{-i\lambda_L t} (\alpha_L^0 + \zeta \alpha_S^0). \end{aligned}, \text{ where } \zeta = r \left(1 - e^{i\Delta\lambda \frac{Lm}{p}} \right)$$

K^0 REGENERATION

Estimations for Belle II experiment.

Be — 1 mm, Si — $L_{1,2} = 50 \mu\text{m}$, $L_{3-6} = 300 \mu\text{m}$

Материал	σ_{tot} , (mb)	$\text{Re}\Delta f$, fm	$\text{Im}\Delta f$, fm
Si	553.0	-7.5	-12.9
Be	219.1	-3.9	-6.2



UNCERTAINTY DUE TO D -MIXING

In the decay channel $D^0 \rightarrow K_S^0 \pi^0$ there is potential bias due to charm mixing. Parameters a, b become dependent on D^0 decay time. Both evolution of K_S^0 and D^0 should be taken into consideration.

$$a^+(t') \equiv \langle \bar{K}^0 \pi^0 | H | D_{phys}^0(t') \rangle = A_{D^0} [f_+(t') - \sqrt{r_f} e^{i(\delta+\phi)} f_-(t')]$$

$$b^+(t') \equiv \langle K^0 \pi^0 | H | D_{phys}^0(t') \rangle = A_{D^0} [\sqrt{r_f} e^{i(\delta-\phi)} f_+(t') - f_-(t')]$$

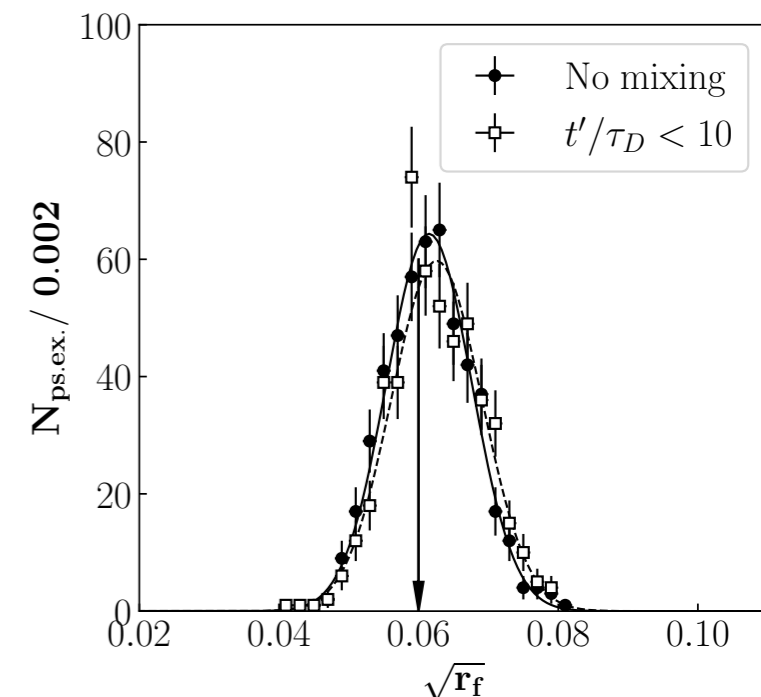
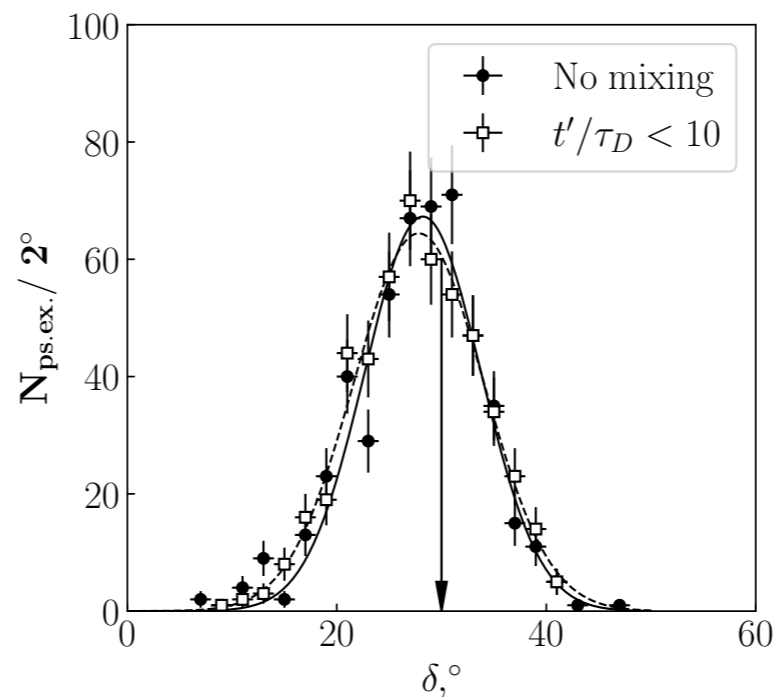
World average:

$$x = (0.43_{-0.11}^{+0.10}) \%$$

$$y = (0.60 \pm 0.06) \%$$

$$\phi = (0.08 \pm 0.31)^\circ$$

Smallness of mixing parameters and experimental factor (obtaining proper D^0 decay time resolution) brought us to consider integration over D^0 decay time. We found the bias from mixing to be negligible (compare to stat. uncertainty).



MEASUREMENTS IN LHCb

LHCb experiment meet many requirements for the experiment to perform proposed measurement. However for typical momenta of kaons $\sim 5 \text{ GeV}/c$ LHCb tracker is very “short”, only few lifetimes could be analysed.

